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# Quantum Anisotropic Synaptic Annealing - Stochastic Anisotropic Quantum Annealing of Synaptic Weights

#### THESIS

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# Preface

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# Table of Contents

	Page
Preface	iii
List of Figures	vi
List of Tables	vii
List of Symbols	viii
List of Abbreviations	ix
Abstract	X
I. Introduction	1-1
II. Background	2-1
2.1 Related Works in Simulated Annealing	2-1
2.1.1 Reheating	2-2
2.2 Related Works in Quantum Mechanics	2-2
2.2.1 Quantum Tunneling	2-2
2.2.2 Quantum Annealing	2-2
2.3 Related Works in Artificial Neural Networks	2-3
2.4 Notation and Terminology Conventions	2-3
III. Methodology	3-1
3.1 Traversing the Error Manifold	3-1
3.1.1 Unidimensional Weight Perturbation	3-1
3.1.2 Omnidimensional Weight Perturbation	3-1
3.1.3 Constrained Step-Size Omnidimensional Weight Perturbation	3-1

			Page
	3.1.4	Quantum Weight Perturbation	3-1
	3.1.5	Stochastic-Anisotropic Quantum Weight Perturbation	3-1
	3.1.6	Quantum Annealing	3-1
Appendix A.	First ap	opendix title	A-1
A.1	In an a	ppendix	A-1
Vita		Ţ	/ITA-1

# List of Figures

Figure

# $List\ of\ Tables$

Table Page

# $List\ of\ Symbols$

Symbol

# $List\ of\ Abbreviations$

Abbreviation

AFIT	/GE	/ENG	1/16
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### Abstract

Abstract goes here.

# Quantum Anisotropic Synaptic Annealing - Stochastic Anisotropic Quantum Annealing of Synaptic Weights

### I. Introduction

In chapter three the traversal a error manifold in the problem configuration space is discussed at length. Several traversal methodologies are proposed and evaluated.

### II. Background

This chapter serves as an comprehensive review of the physical and computational concepts relevant to the topic of this thesis. Relevant work in simulated annealing will be summarized. Next, artificial neural networks and their application will be discussed. The chapter concludes with a very brief overview of the quantum mechanical concepts employed throughout the document. Finally, the notation and terminology conventions to be used throughout the rest of the document are presented.

### 2.1 Related Works in Simulated Annealing

Simulated annealing (SA) is a stochastic optimization algorithm which can be used to find the global minimum of a cost function mapped from the configurations of a combinatorial optimization problem. The concept of simulated annealing was introduced in (Kirkpatrick) as an application of the methods of statistical mechanics to the problem of discrete combinatorial optimization. Specifically, SA is an extension of the Metropolis-Hastings (Metropolis) algorithm which is used to estimate the ground energy state of a many-body systems at thermal equilibrium. Kirkpatrick et al. applied the Metropolis-Hastings algorithm sequentially, with decreasing temperature values in order to approximate a solid slowly cooling to low temperatures. Later work by Geoff (Geoff) and Cortana et al. (Cortana) extended simulated annealing to the continuous domain. The algorithm, as it will be used in this paper, is presented in [algo1].

### [algo1] j

In the parlance of simulated annealing (Kirkpatric) a system at its maximum temperature is said to be "melted." In the melted state, most perturbations of the configuration of the system are accepted by the algorithm. Analogously, a system that has a temperature of zero, which indicates that the algorithm cannot move to any higher-error state, is said to be "frozen." Note that a frozen system may still be perturbed into a lower-energy state.

When considering only the influence of classical thermal fluctuations in particle energy levels, the probability of a particle overcoming a barrier of height  $\Delta V$  at a temperature

T is on the order of:

$$\mathcal{P}_t = e^{-\frac{\Delta V}{T}} \tag{2.1}$$

2.1.1 Reheating.

#### 2.2 Related Works in Quantum Mechanics

The discipline of quantum mechanics is the study of the physical laws of nature at very small scales, at which quantum effects are not negligible. Several techniques described in this document are either inspired by, or are simple models of quantum mechanical processes. These concepts are very briefly reviewed in this section.

2.2.1 Quantum Tunneling. One of the most interesting phenomena studied in the field of quantum mechanics is quantum tunneling.

A particle with energy E incident upon a potential energy barrier of height  $\Delta V > E$  has a non-zero probability of tunneling through the barrier. Classically, this behavior is forbidden. The probability of tunneling,  $\mathcal{P}_t$ , through a step barrier of height  $\Delta V$  goes as:

$$\mathcal{P}_t = e^{-\frac{w\sqrt{\Delta V}}{\Gamma}} \tag{2.2}$$

where  $\Gamma$  is the tunneling field strength [Ref: Multivariable Opt: QAC - Mukherjee]. Figure [1] depicts a one-dimensional example of quantum tunneling.

[Figure 1]

It is instructive to contrast equation [1] and equation [2]; both describe the same value, but the importance of the width and height of the traversed barrier in the two equations is considerably different. For systems in which quantum tunneling is possible, the probability of traversing a barrier of height  $\Delta V$  is increased by a factor of  $e^{\Delta V}$ , for large values of  $\Delta V$ .

2.2.2 Quantum Annealing. Quantum annealing is the use of quantum, rather than thermal, fluctuations to traverse the free energy landscape of a system. This is accomplished by introducing an additional Hamiltonian term that does not commute with

the classical Hamiltonian. This non-commutation implies that [What does it mean?... The non-commutative causes the quantum effects...but how?]. The term is introduced to account for the presence of a tunneling field which controls the frequency with which quantum fluctuations occur in the system. This term, much like thermal energy in simulated annealing, is gradually reduced over the course of the simulation. [Par Source: Quantum annealing in a kinetically constrained system] The time dependent Schrodinger equation for such a system has the form:

$$[\lambda(t)H' + H_0]\psi = i\hbar \frac{\partial \psi}{\partial t}$$
(2.3)

[Eq from Mult Opt... Murherjee] where  $\lambda(t)$  is the time-variance function of the tunneling field, H' is the Hamiltonian term describing the tunneling field, and  $H_0$  is the classical Hamiltonian.

The fluctuations induced by the tunneling field are tunneling events, which transition the system from one configuration to a different, lower-energy configuration without assuming any of the higher energy configurations between the two. This mechanism enables a system to tunnel through barriers between energy valleys in order to escape local minima.

Strictly speaking, this document does not claim to describe a quantum annealing process as it is presented in the referenced literature.

- 2.3 Related Works in Artificial Neural Networks
- 2.4 Notation and Terminology Conventions

[Physics to Algorithmic translation table]

Free Energy Surface - Error Manifold - Cost Function System Configuration -

### III. Methodology

- 3.1 Traversing the Error Manifold
  - 3.1.1 Unidimensional Weight Perturbation.
  - 3.1.2 Omnidimensional Weight Perturbation.
  - 3.1.3 Constrained Step-Size Omnidimensional Weight Perturbation.
- 3.1.4 Quantum Weight Perturbation. It is shown in Proof [n] that this algorithm is certain to eventually find the minimum possible
  - 3.1.5 Stochastic-Anisotropic Quantum Weight Perturbation.
- 3.1.6 Quantum Annealing. [After discussing the way in which the algorithm is implemented] ...The net effect of this design is to allow the algorithm to move from a local minima configuration, to a different, lower-error configuration, without requiring the evaluation of intervening, higher-error configurations. This means that the probability of "tunneling" to a state

### Appendix A. First appendix title

### A.1 In an appendix

This is appendix section A.1.

Note: I highly recommend you create each chapter in a separate file including the \chapter command and \include the file. Then you can use \includeonly to process selected chapters and you avoid having to latex/preview/print your entire document every time.

### Vita

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