

AFIT/GE/ENG/16-..

SIMULATED QUANTUM ANISOTROPIC ANNEALING APPLIED TO ARTIFICIAL  
NEURAL NETWORK WEIGHT SELECTION

THESIS

Justin Fletcher

First Lieutenant, USAF

AFIT/GE/ENG/16-..

Approved for public release; distribution unlimited

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the United States Government.

AFIT/GE/ENG/16-..

SIMULATED QUANTUM ANISOTROPIC ANNEALING APPLIED TO  
ARTIFICIAL NEURAL NETWORK WEIGHT SELECTION

THESIS

Presented to the Faculty of the Electrical and Computer Engineering  
of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science in Computer Science

Justin Fletcher, B.S. CEC

First Lieutenant, USAF

June, 2016

Approved for public release; distribution unlimited

SIMULATED QUANTUM ANISOTROPIC ANNEALING APPLIED TO  
ARTIFICIAL NEURAL NETWORK WEIGHT SELECTION

Justin Fletcher, B.S. CEC

First Lieutenant, USAF

Approved:

<hr/> Dr. Michael J. Mendenhall Thesis Advisor	<hr/> Date
<hr/> Dr. Gilbert L. Peterson Committee Member	<hr/> Date
<hr/> Capt. Charlton D. Lewis Committee Member	<hr/> Date

*Preface*

Justin Fletcher

## *Table of Contents*

	Page
Preface . . . . .	iii
List of Figures . . . . .	vi
List of Tables . . . . .	vii
List of Symbols . . . . .	viii
List of Abbreviations . . . . .	ix
Abstract . . . . .	x
 I. Introduction . . . . .	 1-1
 II. Background . . . . .	 2-1
2.1 Artificial Neural Networks . . . . .	2-1
2.1.1 Back Propagation Training . . . . .	2-1
2.2 Related Works in Simulated Annealing . . . . .	2-1
2.2.1 Reheating . . . . .	2-2
2.2.2 Application of Simulated Annealing to ANN Synaptic Weight Selection . . . . .	2-2
2.3 Related Works in Quantum Mechanics . . . . .	2-2
2.3.1 Quantum Tunneling . . . . .	2-2
2.3.2 Quantum Annealing . . . . .	2-3
2.4 Notation and Terminology Conventions . . . . .	2-4
 III. Methodology . . . . .	 3-1
3.1 Traversing the Error Manifold . . . . .	3-1
3.1.1 Unidimensional Weight Perturbation . . . . .	3-1

	Page
3.1.2 Omni-dimensional Weight Perturbation . . . . .	3-1
3.1.3 Constrained Step-Size Omni-dimensional Weight Per- turbation . . . . .	3-1
3.1.4 Quantum Weight Perturbation . . . . .	3-1
3.1.5 Stochastic-Anisotropic Quantum Weight Perturbation	3-1
3.1.6 Quantum Annealing . . . . .	3-1
Appendix A. First appendix title . . . . .	A-1
A.1 In an appendix . . . . .	A-1
Bibliography . . . . .	BIB-1
Vita . . . . .	VITA-1

*List of Figures*

Figure

Page



*List of Tables*

Table

Page

*List of Symbols*

Symbol

Page

*List of Abbreviations*

Abbreviation	Page
ANN Artificial Neural Network . . . . .	ix
ANN	

AFIT/GE/ENG/16-..

*Abstract*

# SIMULATED QUANTUM ANISOTROPIC ANNEALING APPLIED TO ARTIFICIAL NEURAL NETWORK WEIGHT SELECTION

## *I. Introduction*

In chapter three the traversal a error manifold in the problem configuration space is discussed at length. Several traversal methodologies are proposed and evaluated.

## *II. Background*

This chapter serves as an comprehensive review of the physical and computational concept material to the topic of this thesis. Relevant work in simulated annealing will be summarized. Next, artificial neural networks and their application will be discussed. The chapter concludes with a very brief overview of the quantum mechanical concepts employed throughout the document. Finally, the notation and terminology conventions to be used throughout the rest of the document are presented.

### *2.1 Artificial Neural Networks*

Artificial Neural Networks (ANN)

#### *2.1.1 Back Propagation Training.*

### *2.2 Related Works in Simulated Annealing*

Simulated annealing (SA) is a stochastic optimization algorithm which can be used to find the global minimum of a cost function mapped from the configurations of a combinatorial optimization problem. The concept of simulated annealing was introduced in (Kirkpatrick) as an application of the methods of statistical mechanics to the problem of discrete combinatorial optimization. Specifically, simulated annealing is an extension of the Metropolis-Hastings (Metropolis) algorithm which is used to estimate the ground energy state of a many-body systems at thermal equilibrium. Kirkpatrick et al. applied the Metropolis-Hastings algorithm sequentially, with decreasing temperature values in order to approximate a solid slowly cooling to low temperatures. Later work by Geoff (Geoff) and Cortana et al. (Cortana) extended simulated annealing to the continuous domain. The basic simulated annealing algorithm, as it will be used in this paper, is presented in [algo1].

[algo1]

The physical inspiration for simulated annealing.

Simulated annealing has been successfully applied to numerous domains.

In the parlance of simulated annealing (Kirkpatrick) a system at its maximum temperature is said to be *melted*. In the melted state, most perturbations of the configuration of the system are accepted by the algorithm. Analogously, a system that has a temperature of zero, which indicates that the algorithm cannot move to any higher-error state, is said to be *frozen*. Note that a frozen system may still be perturbed into a lower-energy state.

When considering only the influence of classical thermal fluctuations in particle energy levels, the probability of a particle traversing a barrier of height  $\Delta V$  at a temperature  $T$  is on the order of:

$$\mathcal{P}_t = e^{-\frac{\Delta V}{T}} \quad (2.1)$$

### 2.2.1 Reheating.

### 2.2.2 Application of Simulated Annealing to ANN Synaptic Weight Selection.

## 2.3 Related Works in Quantum Mechanics

Quantum mechanics is the branch of physics concerned with the physical laws of nature at very small scales. There are many aspects of physical reality than are observable only at these scales. Several techniques described in this document are either inspired by, or are simple models of quantum mechanical processes. These concepts are very briefly reviewed in this section.

*2.3.1 Quantum Tunneling.* One of the quantum phenomena for which there is no classical analog is potential barrier penetration, also known as quantum tunneling. This phenomenon arises from the probabilistic and wavelike behavior of particles in quantum physics. Tunneling plays a significant role in the behavior of bound and scattering quantum mechanical systems.

A particle with energy  $E$  incident upon a potential energy barrier of height  $\Delta V > E$  has a non-zero probability of being found in, or past, the barrier. Classically, this behavior is forbidden. The probability of tunneling,  $\mathcal{P}_t$ , through a step barrier of height  $\Delta V$  is described by:

$$\mathcal{P}_t = e^{-\frac{w\sqrt{\Delta V}}{\Gamma}} \quad (2.2)$$

where  $\Gamma$  is the tunneling field strength [Ref: Multivariable Opt: QAC - Mukherjee]. Figure [1] depicts a one-dimensional example of quantum tunneling.

[Figure 1]

*2.3.2 Quantum Annealing.* Quantum annealing is the use of quantum, rather than thermal fluctuations to traverse the free energy landscape of a system. This is accomplished by introducing an additional Hamiltonian term that does not commute with the classical Hamiltonian. This non-commutation implies that [What does it mean?... The non-commutative causes the quantum effects...but how?]. The term is introduced to account for the presence of a tunneling field which controls the frequency with which quantum fluctuations occur in the system. In effect, this term controls the relative importance of quantum effects on the behavior of the modeled system. This term, much like thermal energy in simulated annealing, is gradually reduced over the course of the simulation. [Par Source: Quantum annealing in a kinetically constrained system] The time dependent Schrödinger equation <sup>1</sup> for such a system has the form:

$$[\lambda(t)H' + H_0]\psi = i\hbar \frac{\partial \psi}{\partial t} \quad (2.3)$$

[Eq from Mult Opt... Murherjee] where  $\lambda(t)$  is the time-variance function of the tunneling field,  $H'$  is the Hamiltonian term describing the tunneling field, and  $H_0$  is the classical Hamiltonian.

The fluctuations induced by the tunneling field are tunneling events, which transition the system from one configuration to a different, lower-energy configuration directly, without assuming any of the higher energy configurations between the two. Said differently, the quantum tunneling field enables the penetration of energy barriers. The addition of these quantum fluctuations also ensures that each possible state of the system can be reached (KCS Paper).

---

<sup>1</sup>Note that the presence of the Schrödinger equation in section does not imply that quantum annealing requires the annealed system must be an approximation to a wavefunction. It merely serves as an exposition of the properties of physical system which is modeled.



## *2.4 Notation and Terminology Conventions*

There is a great deal of academic writing describing quantum annealing in the language of physics, but very little writing describing the concept from an algorithmic perspective. For this reason a new, more specific term is introduced in this document. Simulated quantum annealing (SQA) is the quantum mechanical counterpart of simulated thermal annealing.

### *III. Methodology*

#### *3.1 Simulated Quantum Annealing*

It is instructive to contrast equations 2.1 and 2.2. Both describe the same value, but the importance of the width and height of the traversed barrier in the two equations is considerably different. For systems in which quantum tunneling is possible, the probability of penetrating a barrier of height  $\Delta V$  is increased by a factor of approximately  $e^{\Delta V}$ , for large values of  $\Delta V$ . This relationship is depicted graphically in Fig. 2 which shows the probability of barrier traversal for a system which allows quantum fluctuations, divided by the same probability for a system which only considers thermal fluctuations. Therefore, physical models which considers quantum effects are much more likely predict penetration of tall, thin energy barriers than those which only include classical thermal effects.

[Figure 2]

#### *3.2 Traversing the Error Manifold*

##### *3.2.1 Unidimensional Weight Perturbation.*

##### *3.2.2 Omni-dimensional Weight Perturbation.*

##### *3.2.3 Constrained Step-Size Omni-dimensional Weight Perturbation.*

*3.2.4 Quantum Weight Perturbation.* It is shown in Proof [n] that this algorithm is certain to eventually find the minimum possible

##### *3.2.5 Stochastic-Anisotropic Quantum Weight Perturbation.*

*3.2.6 Quantum Annealing.* [After discussing the way in which the algorithm is implemented] ...The net effect of this design is to allow the algorithm to move from a local minima configuration, to a different, lower-error configuration, without requiring the evaluation of intervening, higher-error configurations. This means that the probability of "tunneling" to a state

#### *IV. Results*

## *V. Conclusion*

## *Appendix A. First appendix title*

### *A.1 In an appendix*

This is appendix section A.1.

Note: I highly recommend you create each chapter in a separate file including the `\chapter` command and `\include` the file. Then you can use `\includeonly` to process selected chapters and you avoid having to latex/preview/print your entire document every time.

## *Bibliography*

## *Vita*

Insert your brief biographical sketch here. Your permanent address is generated automatically.

Permanent address: 452 Orchard Drive  
Oakwood, Ohio 45419