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## PAGE 0: RECONFIGURABLE REINFORCEMENT LEARNING NETWORKS

In humans, the structure and form of learning is not only driven by the environment and structure of the brain. The growing of the brain-structure itself defines what learning may take place, and thus the conditions and patterns which direct brain formation are primary and total for the success of learning. Thus, as artifical intelligence research continually generates and publishes on novel structures discovered by humans, this work is centered around how the discovery of this structure may occour automatically. This subject is frequently included in the subject of general intelligence, and is famous for both its philosopical and computational complexity, as well as its difficulty in finding funding. There have been previous works on this subject, such as [Consciousness as a State of Matter] [] [].

Specifically, This work presents a unification method for online learning (Reinforcement Learning) and offline learning (Backpropagation). In addition, this work demonstrates an apporach to the self-structuring of parametric models. First, it is demonstrated that Concurrent Markov Decision Processes (CMDPs) can discover parametric structure and optimal bhaviour with even when subject to large state spaces and generous state uncertainty. Second, it is shown that a variation of CMDPs called Reconfigurable Learning Networks (RLNs) can learn parametric decision networks. RLNs in structure and behaviour turn out to be equilivent to the structure and behaviour of feed-forward neural networks. Lastly, a few empirical examples are demonstrated, beginning with the MINST dataset. Two main contributions are made: First, RLNs can be trained offline and online, using Reinforcement Learning and then Backpropagation; online learning stimulates network growth and adaptation immediately, whereas backpropagation seems to be an ideal phase for network pruning. Second, an empty RLN can enjoy empirical success even when the reward function for the system is changed. Thus both a degree of empirical success and general learning have been achieved.

In order for a generally intelligent system to operate, solutions to several open problems need to be solved analytically and/or heuristically. In this work, we present the related problem categories in the Introduction (Section 1), and include background on each area. Second, most of this work is focused around the reconfiguration of existing CMDP problems, so Section 2 includes work on transfer learning and analytical analysis. Third, we express how convergence of behaviour policies can be preserved despite online RLN restructuring (Section 3). The tradeoff between network structure and computation time in learning is expressed analytically (Section 5). Lastly, it is shown that RLNs are actually just feed-forward Neural Networks, which adds the ability to use back propagation and other techniques on discovered models (Section 6).

In this work due to the difficate of the subject matter initally, models are assumed noiseless and stochastically stable. It is expected that later work will broaden this work by considering state uncertainty, and non-stationary problems.

#### **NOTATION**

In general, most online optimization problems can be expressed as fully observable Markov Decision Processes (MDPs) as  $\langle S, A, T, R, \pi \rangle$  tuples:

•  $S \subseteq \mathbb{R}^n$ : A discrete collection of states

- $A \subseteq \mathbb{R}^n$ : A discrete collection of actions
- $T(s|a,s') \in R$ : A stably stochastic transition function, where  $\sum_{s \in S} T(s|a,s') = 1$
- $R(s|a, s') \in R$ : A stable stochastic reward fuction
- $\pi: S \times A \to R$ : A non-negative behaviour policy with the general property,  $\sum_{a \in A} \pi(s,a) = 1$

#### Reward R

In general, we can express behaviour in this domain as a policy  $\pi: S \to R$  [? looked like this, but would  $\pi: S \to A$  make more sense?]. Particular attention is given to the optimal strategy.

In prior work the issue of tractability and subsequent decomposition have been articulated. In this work the subject of learning and generalizing this decomposition work into a General framework is discussed.

## Special topics:

Temporal Difference (A1): how to discover & change time basis/scale

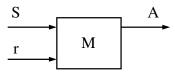
Transitional Learning (A2): how to re-use and generalize transitional models

Financial Systems (A3): how to use with financial systems

Origins (E1-E4): original examples and sketches

Transitional Encoding (E5-E6): Continuous bnns [?] & applications

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assume a Markov decision process M which can be completely represented as a tuple  $M = \langle S, A, T, R, \pi, \tilde{T}, \tilde{R} \rangle$ 

S – a set of states  $s \in S$  which may be experienced by M

A – a set of actions  $a \in A$  that may be executed

T – a true transitional probability, T(s'|a,s) expressing the probability of executing an action a in state s before ending up in later state s'.

R – is a reward function which quantifies how desirable a transition R(s'|a,s) is.  $R:S\times S\to \mathbb{R}_{\geq 0}$ 

[I changed  $\mathbb{R}^+$  to  $\mathbb{R}_{\geq 0}$  because the former is ambiguous with respect to whether or not 0 is included (online research suggests there is no accepted convention) while the latter is unambiguous.]

 $\pi$  – is an action selection policy, ideally chosen to maximize expected reward, an optimal policy is denoted  $\pi^*$ . Typically

$$\pi^*(s) = \arg\max_{a} \sum_{s'} \underbrace{R(s'|a, s)T(s'|a, s) + \gamma V(s')}_{\text{expected reward}}$$

## **ENCODING**

To encode the expected reward over all states, typically Q-values are kept:  $Q(s,a) \sim \sum R(s'|a,s)T(s'|a,s) + \gamma V(s')$  and  $Q_{t+1}(s,a) \leftarrow Q_t(s,a) + \alpha \left(R(s'|a,s) - Q_t(s,a) + \gamma \arg\max_{a'} Q(s',a')\right)$ .

In this paper we rely on a method of extracting dynamic Q-values from an encoded transition and reward function  $(\tilde{T}, \tilde{R})$ . The motivation for this encoding is that it allows mapping the transition function into multiple spaces, and allows the reward function to be altered. The significance of this finding is covered in  $\ref{eq:paper}$  Price wash  $\ref{eq:paper}$ ??

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### 1 RECONFIGURATION

Reconfiguring ??? Process M allows some intractable MDPs to be rendered tractable. As an example, a three dimensional foraging experiment with three thousand positions on the x, y, and z axes respectively will consume over three billion memory locations and may be impossible to explore. If this system is broken into three sub problems, each targets a special axis, the only nine thousand memory locations need be consumed. This decreases memory requirements by an exponential factor.

This paper presents a method of decomposition that, when followed, introduces no degeneration of the found policy  $\pi^*(s, a)$ . The summary of these conditions is presented.

#### **SUMMARY OF REQUIREMENTS**

## INTRODUCTION TO APPROACH

$$d_{M}^{M_{i},M_{k}} = M \longrightarrow \left\{ \begin{aligned} & S_{i} \times (S_{k}/s_{i}) = S, S_{k} \times (S_{i}/s_{k}) = S \\ & A = A_{i} \cup A_{k} \\ & \tilde{T} \sim d^{-1}(d(\tilde{T})), d(\tilde{T}) = \tilde{T}_{i}, \tilde{T}_{k} \\ & \tilde{R} \sim d^{-1}(d(\tilde{R})), d(\tilde{R}) = \tilde{R}_{i}, \tilde{R}_{k} \end{aligned} \right\}$$

where *d*, *d* represent belief mapping functions that decompose and recompose mapping functions. This allows ??? to be mapped as new spaces and observes are encountered. The decomposition process breaks one MDP into a parent and child:

[This diagram hasn't been drawn yet because I'm prioritizing transcribing math and text over doing the diagrams.]

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The system can be broken into the following MDP definitions

$$M_i$$
 – Parent

 $S_i$  – a collection of states,  $s_i \in S_i$ 

 $A_i$  – a collection of actions,  $a_i \in A_i$ 

$$\left. \begin{array}{c} \tilde{T} \\ \tilde{R} \end{array} \right\}$$
 Covered Pages on BII p12-14

 $P(s_i'|s_i,a_i)$  is observed directly

$$R_t \left( \begin{array}{c|c} s_i' & s_i \\ a_k' & a_i & a_k \end{array} \right) = R_t \left( \begin{array}{c|c} s_i' & a_i & s_i \\ s_k' & a_k & s_k \end{array} \right)$$

S,  $T_i$ ,  $S_k$ ,  $S_k'$  are not directly observable

ii) 
$$a_k = \pi_k(s_k)$$

iii) 
$$a_k' = \pi_k(s_k)$$

iiii)  $(s_k, s_k')$  chosen indirectly by  $\pi_k(\cdot)$  in a manner that

$$A^*$$
  $\longrightarrow$   $E[R_{t+1}(\cdot)] \ge E[R_t(\cdot)]$ 

$$M_k$$
 – child

 $S_i'$  – all child states,  $s_k \in S_k$ 

$$a_k \in A_k$$

$$P(s_k'|s_k,a_k)$$

 $R_t(s_k', a_k, s_k) = R_t \begin{pmatrix} s_i, a_i, s_i \\ s_k', a_k, s_k \end{pmatrix}$  s.t.  $s_i, s_i'$  are chosen by another process, and

$$A^*$$
  $\longrightarrow$   $E[R_{t+1}(\cdot)] \ge E[R_t(\cdot)]$ 

## **Definitions**

$$\begin{split} \underline{M} \quad S &= (S_i/S_k) \times (S_k/S_i) \\ A &= A_i \cup A_k \\ T &= P(S \times A \times S) \\ R &= \text{real, positive, convergent stochastic as } t \to \infty \\ R(s',a,s) &= R \left( \begin{array}{ccc} s_i' & a_i & s_i \\ s_k' & a_k & s_k \end{array} \right) \end{split}$$

# Parent MDP

$$\begin{split} \underline{M_i} \quad & S_i, s_i \in S_i \\ & a_i \in A_i \\ & P \left( \begin{array}{c|c} s_i' & a_i & s_i \\ a_k' & a_k & s_k \end{array} \right) \\ & R_t \left( \begin{array}{c|c} s_i' & s_i \\ s_k' & a_i \end{array} \right) \\ & \text{s.t. } s_k, s_k' \text{ are not directly observable} \\ & a_k = \pi_k(s_k) \\ & a_k' = \pi_k(s_k') \end{split}$$

# Child MDP

 $M_k$