

Runshi Gu  
HW3 Problem 3

worst cases.

$$\text{Amortized runtime} = \frac{\sum_{k=1}^{\sqrt{n}} \sum_{i=1}^n \sum_{j=0}^i O(1)}{n} + (n - \sqrt{n}) \cdot O(1)$$

$$= \frac{\sum_{k=1}^{\sqrt{n}} \sum_{i=1}^n i \cdot O(1)}{n} + (n - \sqrt{n}) \cdot O(1)$$

$$= \frac{\sum_{k=1}^{\sqrt{n}} \frac{n(n+1)}{2} \cdot O(1)}{n} + (n - \sqrt{n}) \cdot O(1)$$

$$= \frac{\frac{\sqrt{n} \cdot n(n+1)}{2} \cdot O(1)}{n} + (n - \sqrt{n}) \cdot O(1)$$

$$= \frac{\frac{1}{2} n^2 \sqrt{n} + \frac{1}{2} \sqrt{n} n + n - \sqrt{n}}{n} \cdot O(1)$$

$$= \left( \frac{1}{2} n^{\frac{3}{2}} + \frac{1}{2} n^{\frac{1}{2}} + 1 - n^{-\frac{1}{2}} \right) \cdot O(1)$$

$$= \frac{1}{2} n^{\frac{3}{2}} \cdot O(1)$$

$$= \Theta(n^{\frac{3}{2}})$$