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EEC 100 Lab 7

11/19/24

(1) Objective: Learn how to use the fourier series to analyze a periodic input signal and calculate the coefficients and phase angles of the different terms. Then see how a filter changes the phase angle of the input signal depending on its frequency.

(2) Prelab:

1. $A = 2.5 \sqrt{2}$, $T = 2 \text{ ms}$ $2.5 / 2 \text{ ms} = 1250$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$
$$a_n = \frac{2}{T} \int_0^T f(t) \cos\left(\frac{2\pi n t}{T}\right) dt$$
$$a_0 = \frac{1}{2 \text{ ms}} \int_0^{2 \text{ ms}} 1250 t dt$$
$$a_n = \frac{2}{2 \text{ ms}} \int_0^{2 \text{ ms}} 1250 t \cos\left(\frac{2\pi n t}{2 \text{ ms}}\right) dt$$

$a_0 = 1.25 \sqrt{2}$

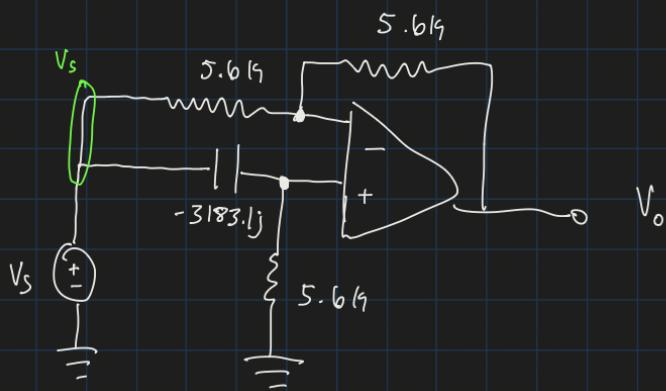
$a_n = 0$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin\left(\frac{2\pi n t}{T}\right) dt$$
$$b_n = \frac{2}{2 \text{ ms}} \int_0^{2 \text{ ms}} 1250 t \sin\left(\frac{2\pi n t}{2 \text{ ms}}\right) dt$$
$$b_n = \frac{5(\sin(2\pi n) - 2\pi n \cos(2\pi n))}{4\pi^2 n^2}$$
$$b_n = \begin{cases} \frac{-5}{2\pi n} & n \text{ is odd} \\ \frac{5}{2\pi n} & n \text{ is even} \end{cases}$$

$V_s(t) = 1.25 - \frac{5}{2\pi} \sin(t) - \frac{5}{4\pi} \sin(2t) - \frac{5}{6\pi} \sin(3t) - \dots$

2.

$$T = 2 \text{ ms}$$



$$f = \frac{1}{2\pi T} = 500 \text{ Hz}$$

$$500 \times 2\pi = 3141.6$$

$$C = \frac{1}{j\omega f} = -3183.1j$$

$$V_p = V_s \left(\frac{5.6k}{-3183.1j + 5.6k} \right) = V_s (0.756 + 0.43j) = V_n$$

$$V_n \left| \frac{V_n - V_s}{5.6k} + \frac{V_n - V_o}{5.6k} = 0 \right.$$

$$-2\tan(\omega_{CR}) + 180^\circ$$

$$V_n - V_s + V_n - V_o = 0$$

$$2V_n = V_s + V_o$$

$$2V_s (0.756 + 0.43j) - V_s = V_o$$

$$\frac{V_o}{V_s} = 0.512 + 0.86j$$

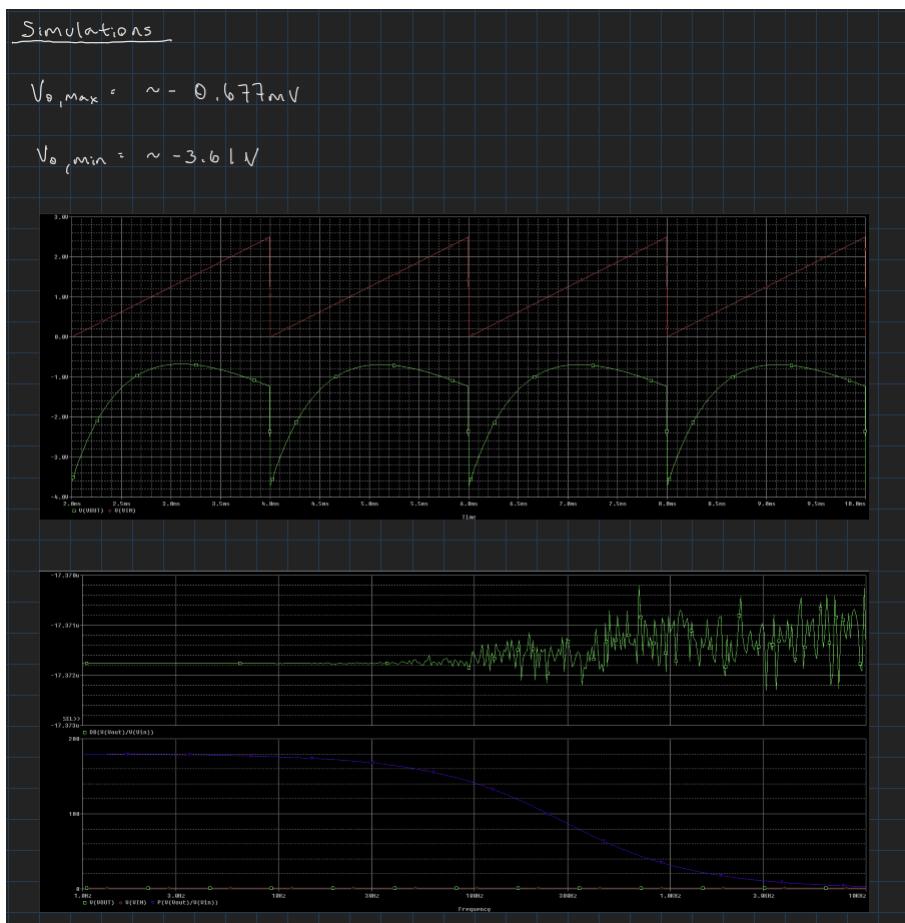
$$\boxed{H(j\omega) = 1 \angle 59.24^\circ}$$

$$\boxed{H(j\omega) = \frac{j\omega - \frac{1}{RC}}{j\omega + \frac{1}{RC}}}$$

This is a all pass filter since $|H(j\omega)| = 1$

$$\begin{aligned}
 S. V_0 &= 1.25 |H(j\omega_0)| \cos\{\angle H(j\omega_0)\} \\
 &= 1.25 (-1) \cos(\pi) \\
 &= 1.25 \\
 V_0(t) &= 1.25 + \sum_{n=1}^{\infty} \frac{-5}{4\pi n^2} |H(j_n\omega_0)| \sin\{\omega_n t + \angle H(j_n\omega_0)\} \\
 &= 1.25 + \left[\frac{-5}{4\pi} (|H(j\omega_0)| \sin\{\omega_0 t + \angle H(j\omega_0)\}) \right] + \left[\frac{-5}{16\pi} (|H(z_j\omega_0)| \sin\{\omega_0 t + \angle H(z_j\omega_0)\}) \right] \\
 &= 1.25 + \left(\frac{-5}{2\pi} \sin(10\pi t + 59.24^\circ) \right) + \left(\frac{-5}{4\pi} \sin(10\pi t + 31.73^\circ) \right) + \\
 &\quad \left(\frac{-5}{6\pi} \sin(10\pi t + 21.46^\circ) \right)
 \end{aligned}$$

(3) Simulation:

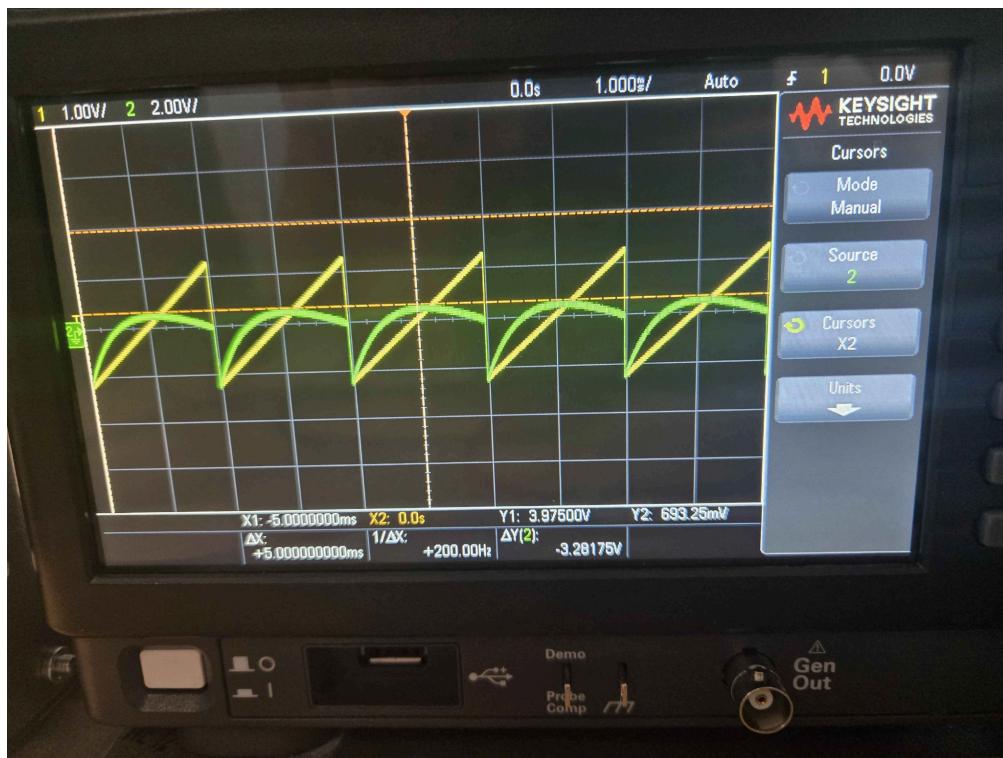


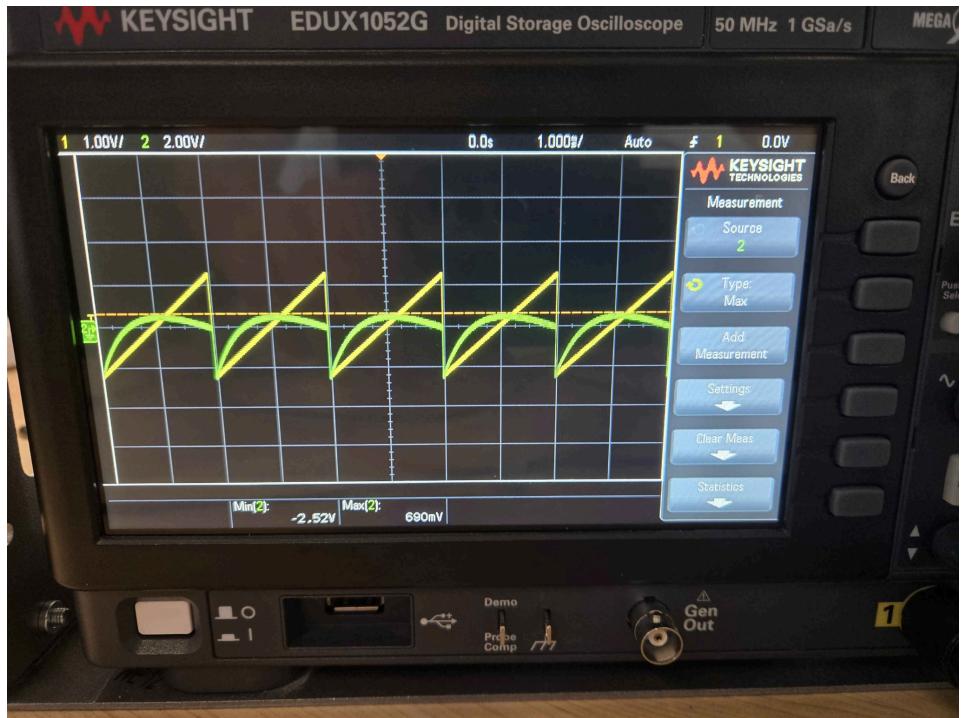
This circuit behaves as expected because we can see from the dB output that it behaves like the sum of a bunch of sinusoids added together, and the bottom graph shows an expected freq cutoff for a filter

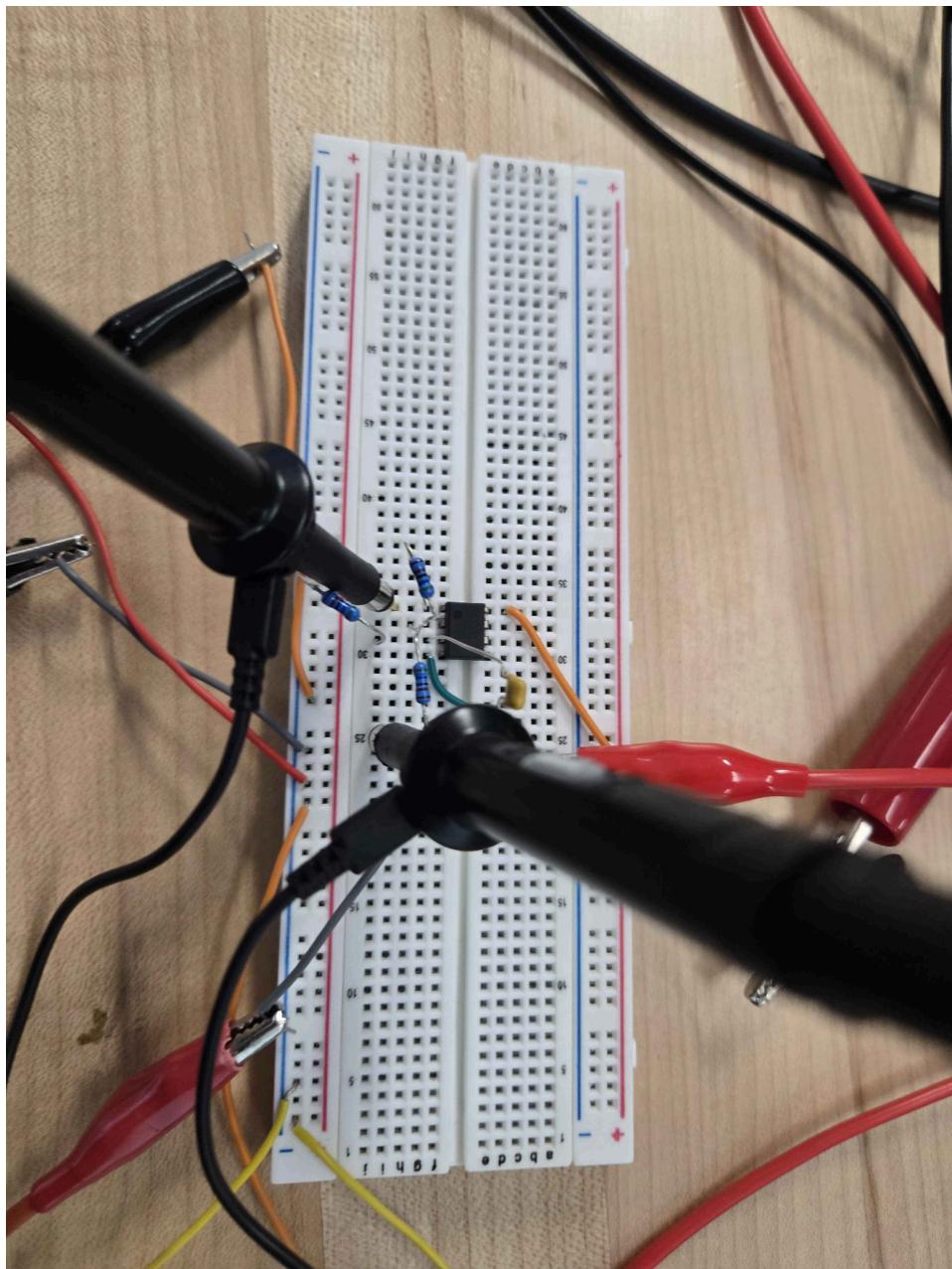
(4) Experiment:

11. On the function generator, change the waveform to be a Sine wave. Select the frequencies in the table below and set the amplitude to be **2 Vpp**. On the oscilloscope, observe how the magnitude and phase of output signal change. Is it as you expected? What is the purpose of this circuit?

Frequency	Does the magnitude $ V_o $ change?	Phase $\angle V_o$ Value (deg)
10 Hz	Yes/No	172
1000 Hz	Yes/No	30
10000 Hz	Yes/No	0
100000 Hz	Yes/No	-7







(5) Conclusion:

As the out input signal frequency changed, the phase change of the output varied, which is to be expected as that is what our calculated fourier series tells us. While our experimental measurements are slightly off (possibly due to non idealities in the circuit such as slightly different resistance and capacitance values, and an non ideal op-amp), they have phase changes as predicted by our calculated fourier analysis.

(6) Checkoff:

