Notes on “A Fast Machine Learning Algorithm for the MaxCut Problem”

I Introduction:

The Max-Cut problem is a graph partitioning problem that involves finding the best way to divide a graph's vertices into two sets so that the number of edges between the two groups is maximized.

It is an NP hard problem (nondeterministic-polynomial time). Basically means it is a complex problem to solve/come up with the most efficient solution for. A ML algorithm can be useful for this purpose.

If we assign each node to a binary value (+1) or (-1), the cut-size of the graph can be written as:

$$Cut(G) = \frac{1}{2} \sum_{(i,j)\in E} (1 - \sigma(i)\sigma(j))$$

Although there are no known polynomial-time methods for the Max-Cut problem, there are heuristic algorithms that provide more practical approaches, such as the Greedy or simulated annealing algorithms. However, they do not provide the most efficient solution(s) that exist. The implementation of a ML algorithm in a similar problem, as well as an explanation of its Max-Cut application is in the rest of the paper.

II Ising-Spin Glass Model

The Ising-Spin Glass Model is a statistical model that describes the behavior of spin glasses, which are systems where atoms have randomly oriented spins.

The Hamiltonian of the spin system (aids understanding the correspondence of ISG Model and Max-Cut):

$$H(\sigma) = -\sum_{i<j} w(ij)\sigma(i)\sigma(j)$$

Since i and j are both connected in the graph for this problem, w can be assigned a value of -1, so the Hamiltonian can be reduced to this:

$$H(\sigma) = \sum_{(i,j)\in E} \sigma(i)\sigma(j)$$

And using this equivalency:

$$\begin{aligned} \text{Cut}(G) &= \frac{1}{2} \sum_{(i,j)\in E} (1 - \sigma_i\sigma_j) \\ &= \frac{1}{2} \sum_{(i,j)\in E} 1 - \frac{1}{2} \sum_{(i,j)\in E} \sigma_i\sigma_j \\ &= \frac{1}{2}|E| - \frac{1}{2}H(\boldsymbol{\sigma}) \end{aligned}$$

We get that maximizing the cut size of graph G is equivalent to minimizing the Hamilton of the spin system. For the spin system, there are already established insights from statistical mechanics (Boltzmann Distribution) to help solve it, unlike the Max-Cut problem. So, using the Ising-Spin Glass Model as a well established reference, we can refer to it for the Max-Cut ML algorithm.