ENME303-24S1 Lab Assessment

Name: Justin France

Student Number: 75349339

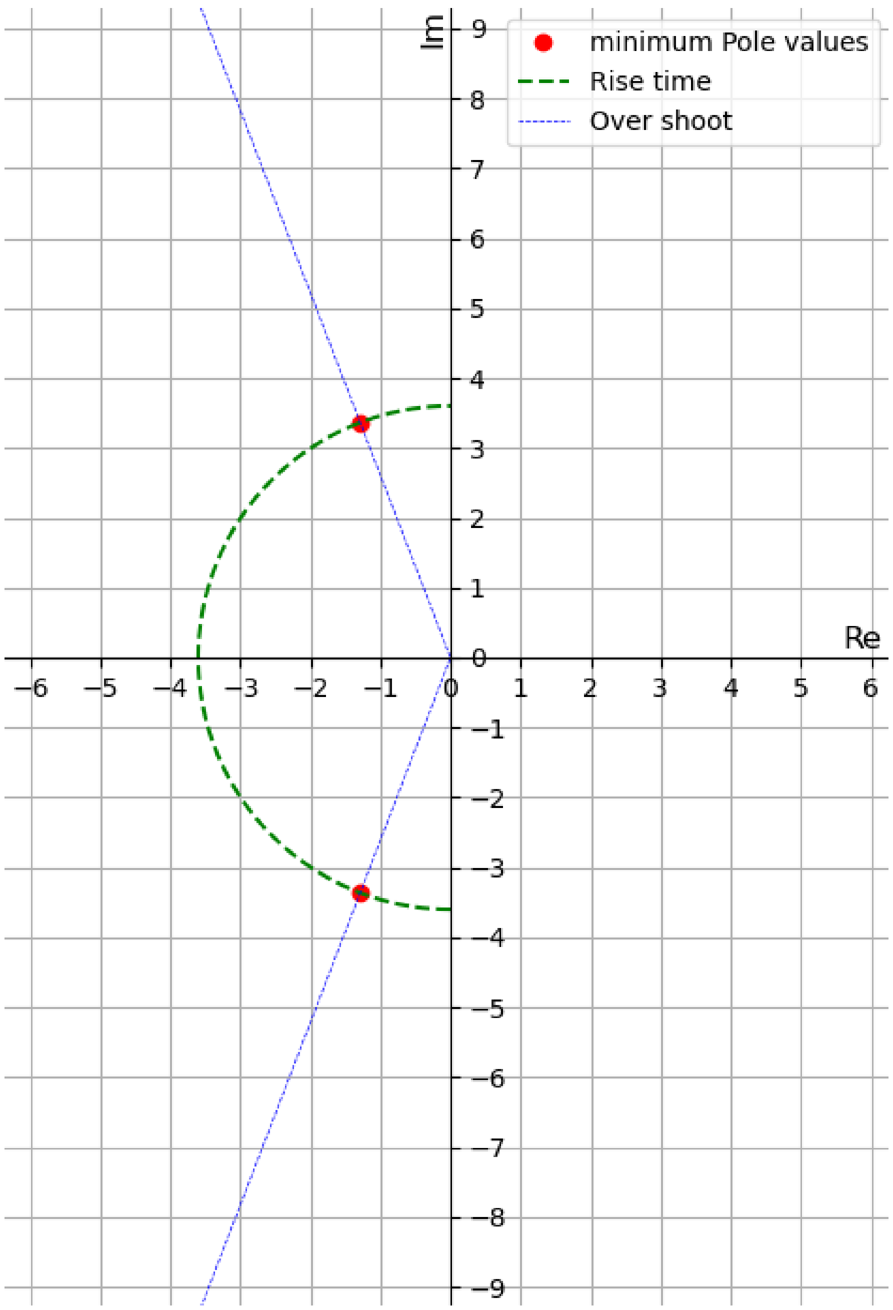
1. PID GAIN DESIGN

Using the specifications outlined in the lab sheet and given below, draw the design region for this system. You can consult Homework 4 if you are unsure on how to find the design region. The specifications you must meet for the system are restated below:

$$\xi > 0.1 \qquad \boldsymbol{M_p} < 30\%$$

$$\boldsymbol{e_{ss}} < 0.1mm \qquad \boldsymbol{t_r} < 0.5s$$

Complete a drawing of the design region in the space below, and ensure you include any calculations you used. **You can choose to plot the design region in Python, draw electronically on the worksheet using a Surface or similar, or draw on paper and scan into this document.**



Rise time

$\omega_n \geq \frac{1.8}{tr}$

$\therefore \omega_n \geq 3.6\ [rads^{-1}]$

Minimum Damping ratio.

$Mp = 30\% \cong 100 * e^{\left(-\frac{\pi*0.358}{\sqrt{1^2-0.358^2}}\right.}$

$\therefore \zeta = 0.358$

Overshoot

$\theta = \sin^{-1}(\zeta)$

$\therefore \theta = 0.366\ [rad]$

Figure 1: Design region for robotic cart from specifications of $t_r < 0.5\ [s]$ and $MP < 30\%$

Using the design-region you have sketched as a starting point, choose an initial location for your poles and plot the response. Iteratively make small adjustments to your gains until you achieve the response you want. Once you have a set of gains you are happy with, record them in the table below. You should prepare at least three sets of gains (P, PD, PID), but if you wish to test more, there is extra space in the table.

Gain Set	P	I	D
1	126		
2	40		16.7
3	60	200	40
4	100	100	8
5	100	50	8
6	100	10	10

Table 1 - Gains chosen for testing.

2. Results

When presenting your results ensure you show both the **simulated and real response to your chosen gains on the same plot.** If you tested more than one of the same type of controller, you should show these on the same plot. You may also choose to plot the controller voltage as a measure of 'effort' if you feel it is relevant.

2.1 Proportional Control

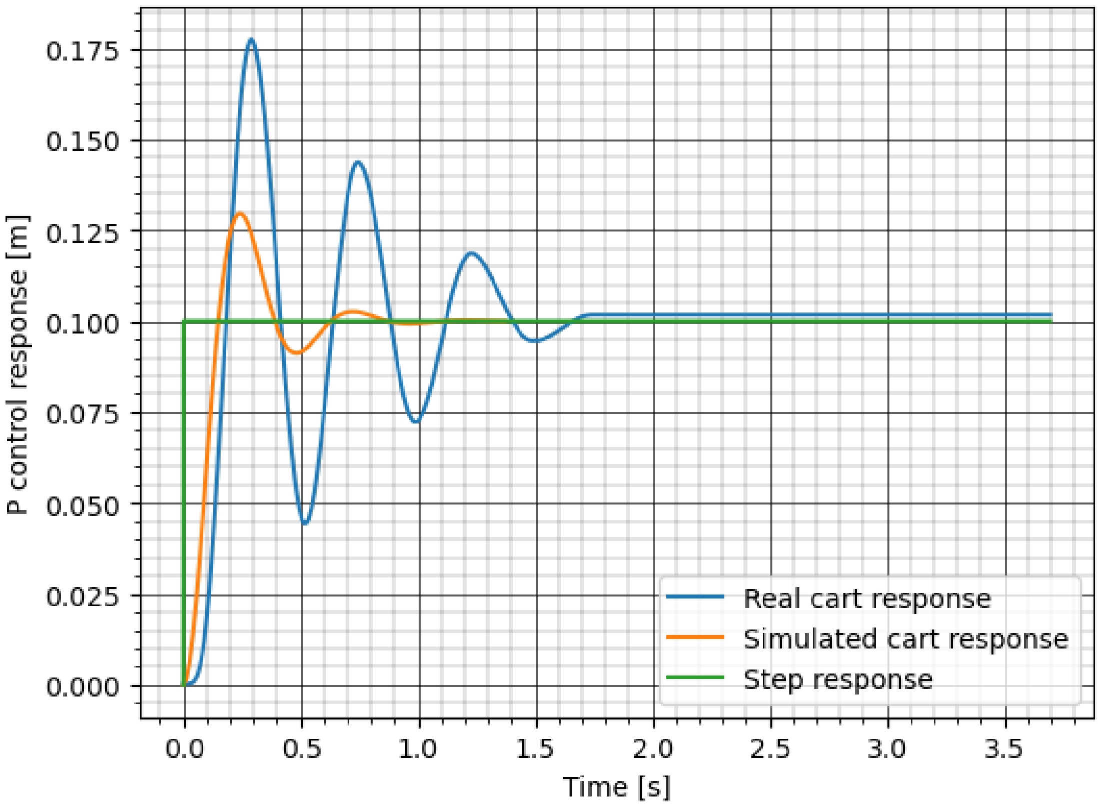


Figure 2: Proportional control response with a kp value of 126 from gain set 1.

	M_p	t_r	e_{ss}
Simulated	29.59%	0.13 [s]	0.00002 [m]
Real	75.68%	0.17 [s]	0.00177 [m]

Table 2 - Overshoot, rise time and steady state error comparison for P control

2.2 Proportional Derivative Control

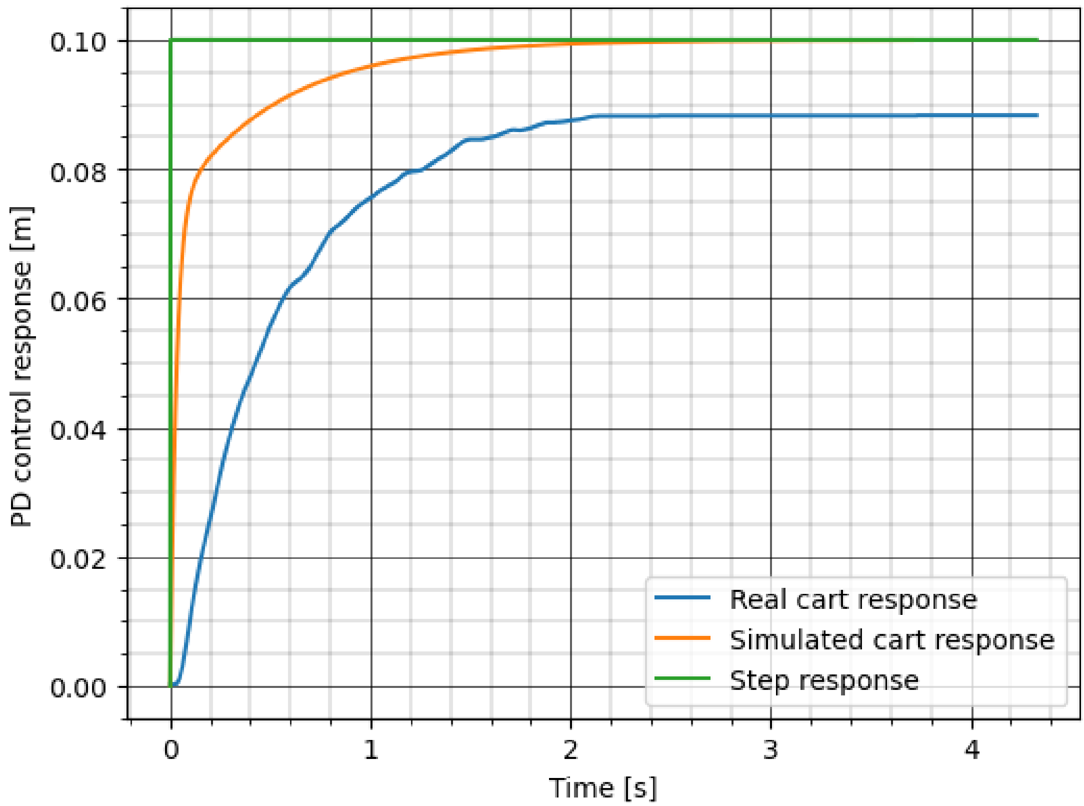


Figure 3: Proportional and derivative control with kp = 40, kd = 16.17 from gain set 2.

	M_p	t_r	e_{ss}
Simulated	0.00%	0.52 [s]	0.00003 [m[
Real	0.00%	Spec not meet	0.01167 [m]

Table 3 - Overshoot, rise time and steady state error comparison for PD control.

2.3 Proportional Integral Derivative Control

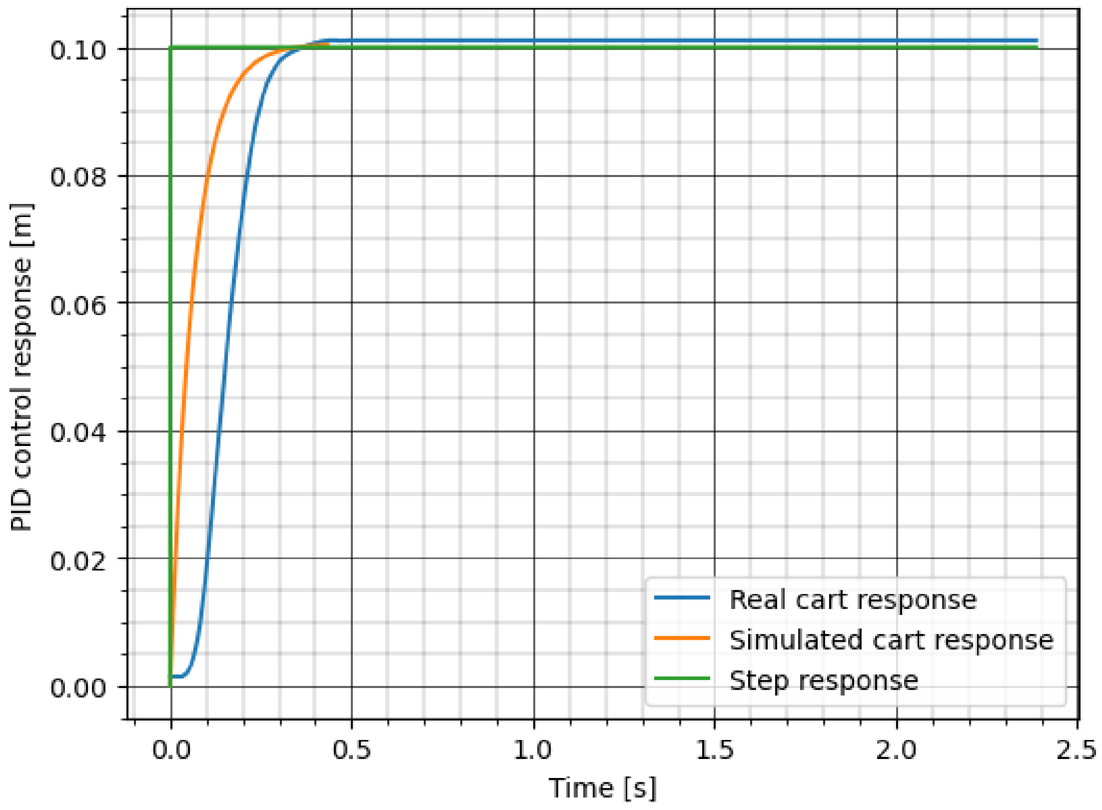


Figure 4: Proportional integral and derivative control with: kp = 100, ki = 10, kd = 10 from gain set 6.

	M_p	t_r	e_{ss}
Simulated	0.00%	0.15 [s]	0.00043 [m]
Real	0.00%	0.25 [s]	0.00109 [m]

Table 4 - Overshoot, rise time and steady state error comparison for PID control.

2.4 Pole locations for Simulated P, PD and PID controllers

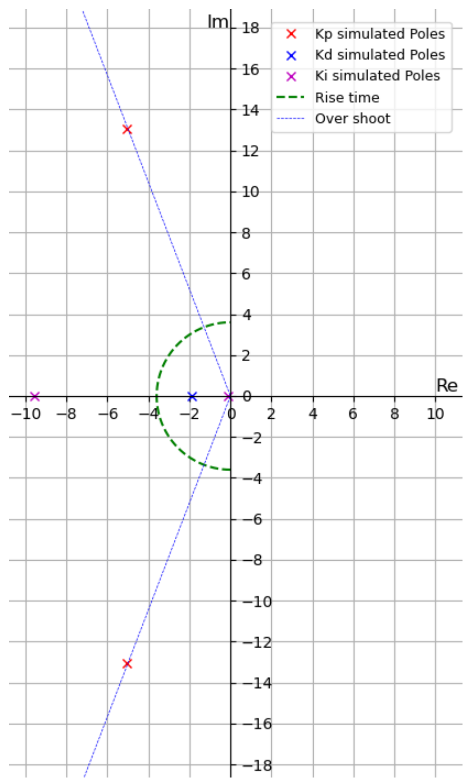


Figure 5: The design region and pole locations for the simulated P, PD and PID control.

3. Discussion

1. Discuss the relation between gains, poles and the real system response.

A system with closed loop feedback including a control block allows, for the gains: kp, kd and Ki to have the ability to shift the pole locations or add poles to achieve the desired system response. Therefore, gain values alter the system response dynamics like natural frequency and dampening, this enables the system behaviour to be changed to be either overdamped, critically damped, underdamped, undamped or unstable depending on the desired response, required from the system.

2. Discuss the effect of P, I and D gains on the system response.

Proportional control P with a large value of kp increases the natural frequency of the system and therefore reduces the steady state error and see appendix for a plot of the simulated range of kp values and their responses. But a to large kp with proportional control may result in a large overshoot but will have a fast response time.

Derivative control d with a value for kd in between 5 to 15 resulted in the response becoming damped with a reduction in overshoot along with a reduced settling time. Values of kd that were large and above 15 started to increase the rise time to becoming out of spec. To little dampening the system acts similarly to proportional control. Note that kd dose not change the steady state error of the system. See appendix for a plot of the simulated range of kd values and their responses.

Integral control I with a value of 140 resulted in system obtaining zero steady state error in a short time frame, but care was required to ensure that a too large value of ki is not used. As this leads to integral wind up and overshoot and a slow settling time. A too smaller of Ki means the system will take a long time to reach a steady state error of zero. See appendix for a plot of the simulated range of ki values and their responses.

3. Did you manage to achieve the system specifications given at the start of the assignment? What trade-offs had to be made? Make reference to your plots if applicable.

From the specifications the proportional with a value of kp = 126, resulted in obtaining adequate rise time with a steady state error being just out of spec at 1.77 [mm]. Not much could be done about the overshoot as lowering the value for kp would result in a larger steady state error. To fix the overshoot a small amount of damping would be required in the control block as the proportional plot the system requires dampening to reduce the system response oscillating.

From the specifications of kp = 40, kd = 16.17 the derivative control did not meet 90% of the steady state error and was 11.67[mm] short of the required step response of 100mm as seen from the derivative control plot. This meant the rise time could not be calculated and the rise time for the simulated system was just out of spec with a value of 0.52 seconds. To rectify this problem, it would be recommended to increase kp to 100 and kd to 15 to as this reduces dampening and increases the natural frequency. This would allow for a reduction in steady state error whilst reducing the rise time to be with in the required specifications, with the system still being over dampened.

From the specifications of kp =100 , kd=10, ki =10 the system meets the requirement for rise time of 0.25 seconds with an over damped system. To ensure that the steady state error of 0.1[mm] was achieved it would be recommended to increase the value of ki to a much larger value ie ki = 140 to allow the integral control to build up faster to ensure steady state meets the specification as seen in the integral control plot.

4. Discuss the differences between the data collected from the real system and the simulation. Explain which aspects of the real system may have caused this difference in your results.

The simulated data did not include the pinion backlash, inertia of gearbox and motor, motor slew rates, variance in friction along the track, change in mass due to cable management set up and power limitation of the electric motor. Thus, simulated modelling should be used as a starting point for choosing gains to assist in obtaining the required poles or response from the system. Limitations to the real-world system included surface condition and levelness of the surface in which the wheels run on, the variance in the pinion back lash and the sensor noise from electrical interference. Thus, each step input response may have slightly different response depending each time depending on the location of the track. The system had excessive mechanical dampening, and a design change would be required to fix this if the desired response of the system could not be meet.

5. Comment on how many assembly tasks per hour could be completed using one of your control designs.

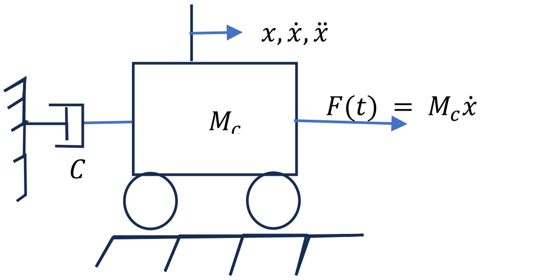
The best control system would PID control as the system position itself within $\approx 0.5\ [s]$ and therefore the system would take $\approx 2.5\ [s]$ per assembly task. Thus, the cart system would complete ≈ 1440 assemblies per hour.

Appendix:

Calculations and method to find simulated system response.

Provided parameters	description	value	Units SI
x $\dot{x}$ $\ddot{x}$	Displacement velocity acceleration		[m] [m/s] [m/s^2]
M_c	Mass or cart	1.5	[kg]
k_m	Back EMF constant for motor	0.017	[V / rad s]
k_g	Gearing ratio	3.7	
V	Applied voltage		[V]
R	Resistance	1.5	[Ω]
r	Radius of pinion	0.018	[m]
D	Dampening		

Design specifications
$\varsigma > 0.1$
$M_p\% < 30$
$e_{ss} < 0.1\ [mm]$
$t_r < 0.5\ [s]$



Equation of motion: $M_c\ddot{x} + D\dot{x} = F(t)$

$\mathcal{L}(M_c\ddot{x} + (C)\dot{x} = f(t)) \quad \rightarrow \quad \mathcal{L}(\ddot{x} + (C)\dot{x} = V(\beta))$

$X(s)[s^2 + (C)(s)] = \beta V(s)$

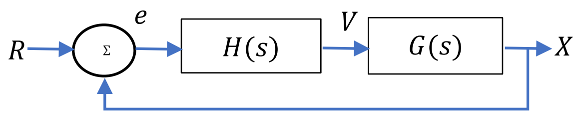


Figure 6: Cart block diagram

$$\beta = \frac{k_m k_g}{M_c R r} = \frac{0.017 * 3.7}{1.5 * 1.5 * 0.018} = 1.5531$$

$$C = \left(\frac{D}{M_c} + \frac{{k_m}^2 {k_g}^2}{M_c R r^2}\right) = \left(\frac{7}{1.5} + \frac{0.017^2 * 0.37^2}{1.5 * 1.5 * 0.018^2}\right) = 10.0938$$

Input output transfer function.		
P control	**PD control**	**PID control**
$H(s) = k_p$ $G(s) = \frac{\beta}{s^2+[C]s}$ $\therefore HG = \frac{\beta k_p}{s^2+(C)s}$ $\frac{X}{R} = \frac{GH}{1+GH}$ $\frac{X}{R} = \frac{\frac{\beta k_p}{s^2+(C)s}}{1+\frac{\beta k_p}{s^2+(C)s}}$ $\frac{X}{R} = \frac{\frac{\beta k_p}{s^2+(C)s}}{\frac{s^2+(C)s}{s^2+[C]s}+\frac{\beta k_p}{s^2+[C]s}}$ $\frac{X}{R} = \frac{\frac{\beta k_p \cancel{(s^2+[C]s)}}{\cancel{s^2+[C]s}}}{s^2+[C]s+\beta k_p}$ $\therefore \frac{X}{R} = \mathbf{0.1}(\frac{\beta k_p}{s^2+[C]s+\beta k_p})$	$H(s) = k_D s + k_p$ $G(s) = \frac{\beta}{s^2+[C+\beta V]s}$ $\therefore HG = \frac{\beta(k_D s+k_p)}{s^2+[C+\beta V)s}$ $\frac{X}{R} = \frac{GH}{1+GH}$ $\frac{X}{R} = \frac{\frac{\beta(k_d s+k_p)}{s^2+(c)s}}{1+\frac{\beta(k_d s+k_p)}{s^2+(C)s}}$ $\frac{X}{R} = \frac{\frac{\beta(k_d s+k_p)}{s^2+(C)s}}{\frac{s^2+[C)s}{s^2+(c)s}+\frac{\beta(k_d s+k_p)}{s^2+(c)s}}$ $\frac{X}{R} = \frac{\frac{\beta(k_d s+k_p)\cancel{(s^2+(c)s)}}{\cancel{s^2+(c)s}}}{s^2+(C)s+\beta k_d s+\beta k_p}$ $\therefore \frac{X}{R} = \mathbf{0.1}(\frac{\beta(k_d s+k_p)}{s^2+(C+\beta k_d)s+\beta k_p})$	$H(s) = \frac{K_i}{s} + k_d s + k_p$ $G(s) = \frac{\beta}{s^2+[C]s}$ $\therefore HG = \frac{\beta(\frac{K_i}{s}+k_d s+k_p)}{s^2+(C)s}$ $\frac{X}{R} = \frac{\frac{\beta(\frac{K_i}{s}+k_d s+k_p)}{s^2+(C)s}}{1+\frac{\beta(\frac{K_i}{s}+k_d s+k_p)}{s^2+(C)s}}$ $\frac{X}{R} = \frac{\frac{\beta(\frac{K_i}{s}+k_d s+k_p)}{s^2+(C)s}}{\frac{s^2+(C)s}{s^2+(C)s}+\frac{\beta(\frac{K_i}{s}+k_d s+k_p)}{s^2+(C)s}}$ $\frac{X}{R} = \frac{\frac{\beta(\frac{K_i}{s}+k_d s+k_p)\cancel{(s^2+(C)s)}}{\cancel{s^2+(C)s}}}{s^2+(C)s+\beta(\frac{K_i}{s}+k_d s+k_p)}$ $\frac{X}{R} = \frac{\beta\left(\frac{K_i}{s}+k_d s+k_p\right)}{s^2+(C+\beta k_d)s+\beta\frac{K_i}{s}+\beta k_p}$ $\frac{X}{R} = \frac{\beta\left(\frac{k_i}{s}+\frac{k_d s^2}{s}+\frac{k_p s}{s}\right)}{\frac{s^3}{s}+\frac{(C+\beta k_d)s^2}{s}+\beta\frac{K_i}{s}+\frac{\beta k_p s}{s}}$ $\frac{X}{R} = \frac{\beta\left(\frac{k_i}{\cancel{s}}+\frac{k_d s^2}{\cancel{s}}+\frac{k_p s}{\cancel{s}}\right)\cancel{s}}{s^3+(C+\beta k_d)s^2+\beta K_i+\beta k_p s}$ $\therefore \frac{X}{R} = \mathbf{0.1}(\frac{\beta(k_d s^2+k_p s+k_i)}{s^3+(C+\beta k_d)s^2+\beta k_p s+\beta K_i})$

Gain Set	P	I	D
1	100	[redacted]	[redacted]
2	100	[redacted]	10
3	100	180	10
4	100	250	5
5	120	250	10
6	80	180	5

Table 3 - Gains which should have been tested as these meet the design specifications for simulated response.

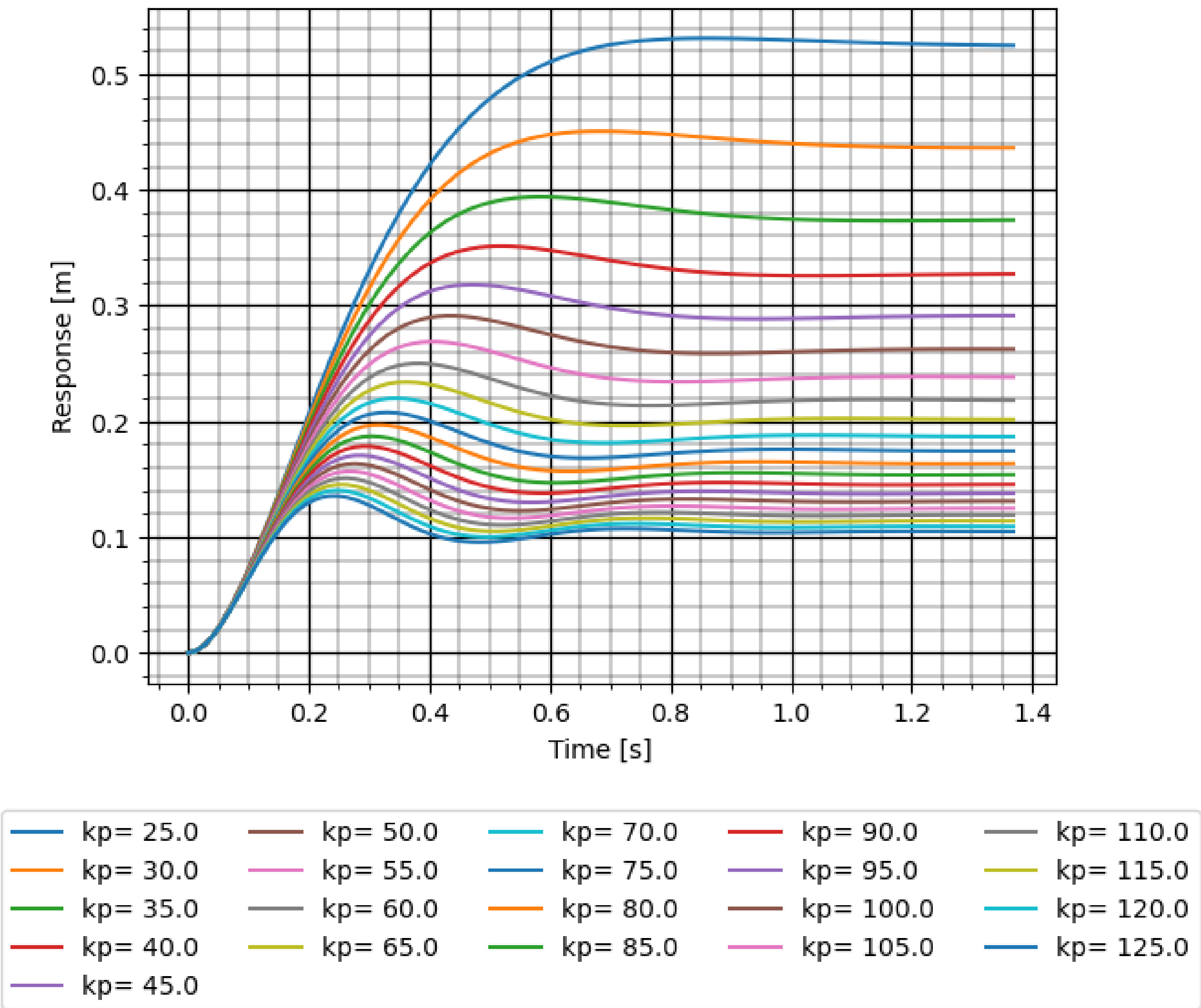


Figure 7: Range of kp values from 5 to 125 for the simulated system response.

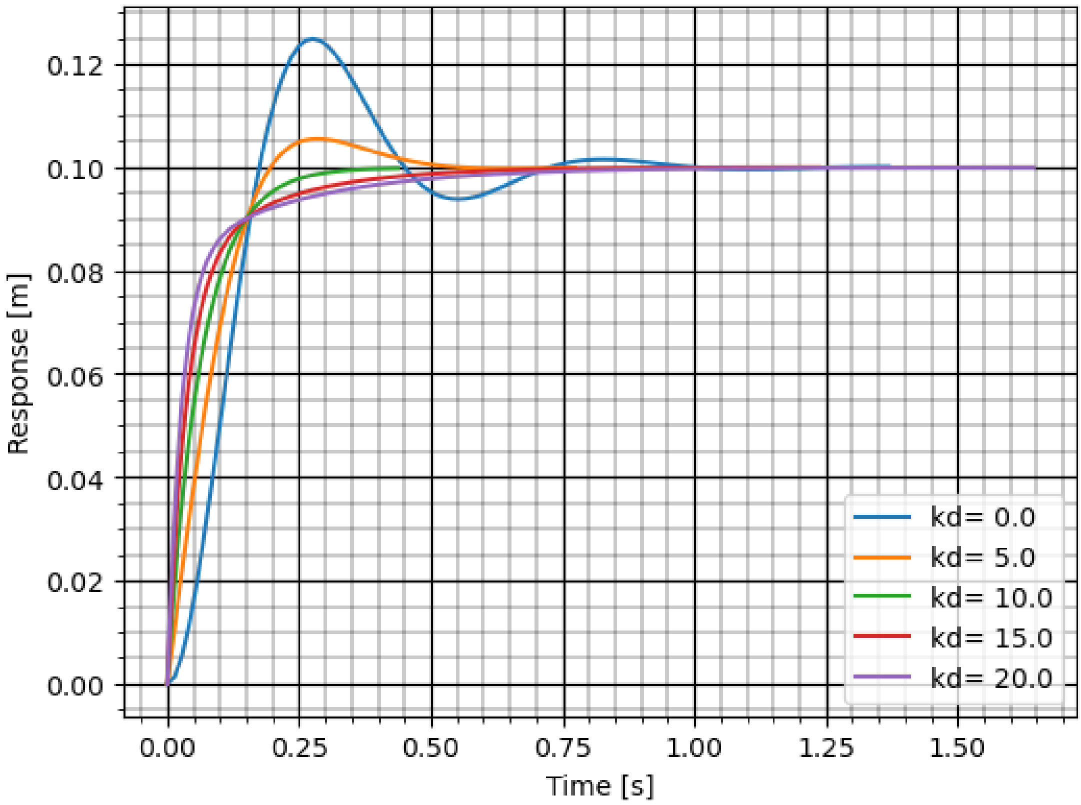


Figure 8; Range of kd values from 0 to 20 for the simulated system response with kp = 100 .

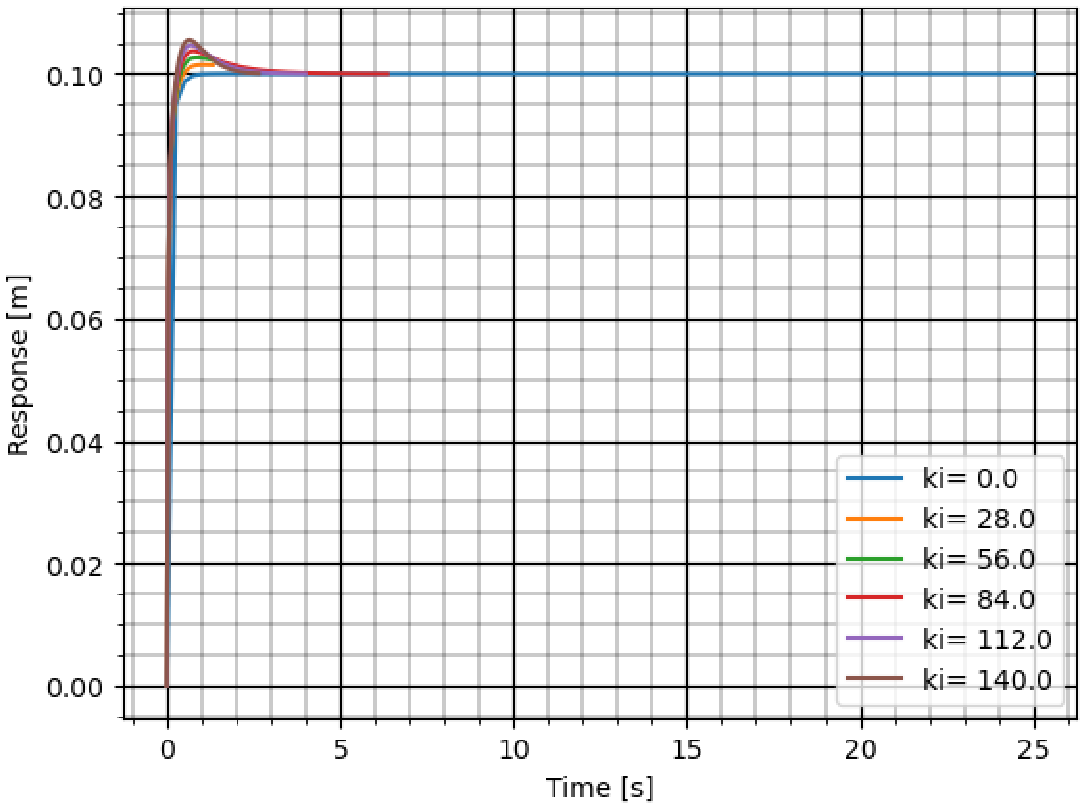


Figure 9: Range of ki values from 0 to 140 for the simulated system responce with kp=100, kd =15.