Chapter 8

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We now examine another area of time series forecasting similar to weighted moving averages, called *exponential smoothing methods*. One of the major distinctions between regressions methods on time series data and smoothing methods is that regressions give equal weight to all of the sample data whereas smoothing methods give decreasing weight to older, more distant data.

> "An economist is an expert who will know tomorrow why the things he predicted yesterday didn't happen today."
>
> — Evan Esar

Single Exponential Smoothing

A popular, widely used time series smoothing and forecasting method is *single exponential smoothing* (also called *one-parameter exponential smoothing*). Exponential smoothing is a form of weighted moving average in that recent observations carry more weight than older observations.

Exponential smoothing is based on the concept that a new, one step-ahead forecast can be determined from the *previous forecast* and the *error of the previous forecast*. We illustrate this concept below.

Suppose we have the following series:

Period	Actual(Y_t)	Forecast
1	354	
2	368	
3	329	

And, suppose a one-step ahead forecast for period 4, made at period 3, is 352.5[1].

[1] $\hat{Y}_3(1) = 352.5$

Period	Actual(Y_t)	Forecast
1	354	
2	368	
3	329	
4	NA	352.5

Then suppose period 4's actual value is 389.

Period	Actual(Y_t)	Forecast
1	354	
2	368	
3	329	
4	389	352.5

So, the forecast error is 36.5.

$$\varepsilon_3(1) = Y_4 - \hat{Y}_3(1)$$

$$36.5 = 389 - 352.5$$

In otherwords, we *underforecast* by 36.5.

Period	Actual(Y_t)	Forecast	Error
1	354		
2	368		
3	329		
4	389	352.5	36.5

Since we underforecasted, we will adjust upward our next forecast, based on the error of our previous forecast. We adjust our next forecast upward by 20% of the forecast error. 20% of 36.5 is 7.3.

Hence, the forecast for period 5 is the old forecast of 352.5 adjusted upwarded by 7.3.

$$352.5 + 7.3 = 359.8$$

$$\hat{Y}_4(1) = 359.8$$

Period	Actual(Y_t)	Forecast	Error
1	354		
2	368		
3	329		
4	389	352.5	36.5
5	NA	359.8	NA

The actual value for period 5 is 350, so the one-step ahead period 5 forecast error is

$$\varepsilon_4(1) = Y_5 - \hat{Y}_4(1)$$

$$-9.8 = 350 - 359.8$$

In otherwords, we *overforecast* by 9.8.

Period	Actual(Y_t)	Forecast	Error
1	354		
2	368		
3	329		
4	389	352.5	36.5
5	350	359.8	9.8

The forecast for period 5 then is adjusted by 20% of the forecast error, 20% of -9.8 is -1.96.

The Forecast for period 6 is:

$$359.8 + (-1.96) = 357.8$$

$$\hat{Y}_5(1) = 357.8$$

Each new forecast, $\hat{Y}_t(1)$, is based on the preceding forecast, $\hat{Y}_{t-1}(1)$, plus an adjustment for forecast error.

If we denote the adjustment factor by α, we have:

$$\hat{Y}_t(1) = \hat{Y}_{t-1}(1) + \alpha\varepsilon_{t-1}(1)$$

Algebraically, we re-arrange the last equation as the formal representation of *exponential smoothing.*

Single Exponential Smoothing, one-step exponential smoothing[2]

[2] Formal Notation, equation 8.1

$$\hat{Y}_t(1) = \alpha Y_t + (1-\alpha)\hat{Y}_{t-1}(1)$$

In the case of $\alpha = 0.2$, we have

$$\hat{Y}_t(1) = 0.2Y_t + 0.8\hat{Y}_{t-1}(1)$$

The new forecast is the weighted average of the current observation Y_t and the previous forecast $\hat{Y}_{t-1}$.

Returning to equation 8.1 displayed in the right margin, we illustrate why this approach is called *exponential smoothing.* Since

$$\hat{Y}_t(1) = \alpha Y_t + (1-\alpha)\hat{Y}_{t-1}(1)$$

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we shift back by one period[3]

[3] Equation 8.2

$$\hat{Y}_{t-1}(1) = \alpha Y_{t-1} + (1-\alpha)\hat{Y}_{t-2}(1)$$

by substituting equation 8.2 in for $\hat{Y}_t(1)$ in equation 8.1 we have:

$$\hat{Y}_t(1) = \alpha Y_t + (1-\alpha)\left(\alpha Y_{t-1} + (1-\alpha)\hat{Y}_{t-2}(1)\right)$$

We can than rearrange:

$$\hat{Y}_t(1) = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + (1-\alpha)^2\hat{Y}_{t-2}(1)$$

Using equation 8.1 again as a format, $\hat{Y}_{t-2}(1)$ is written as

$$\hat{Y}_{t-2}(1) = \alpha Y_{t-2} + (1-\alpha)\hat{Y}_{t-3}(1)$$

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