Chapter 6 Introduction to Time Series

The preceding three chapters of this book dealt with regression methods applied to cross sectional data. When we have cross sectional data our approach to forecasting is to use the explanatory variables to forecast the value of the dependent variable. This approach to forecasting is often termed *interpolation*.

The other broad method of quantitative business forecasting is to use the history of the data we wish to forecast. We examine the historical pattern of the data we wish to forecast and try to find a discernible pattern. Using this pattern, we forecast forward the future pattern of the data. This is method of forecasting is often termed *extrapolation.* The use of the historical pattern of the data to be forecast is known as *time series methods*.

Chapter 6

The future lies ahead.
– Mort Sahl

Stage 1

Collection and Analysis of Times Series Data

A time series of data is data collected sequentially over equal periods of time. There are many forms of time series: time series can be collected as daily data (such as daily stock market data), weekly (such as weekly receipts data), monthly (such as monthly sales data), quarterly (such as quarterly revenue data), or yearly (such as annual profits data).

Remember though, the time period of data collection must be consistent. If our time series data are both in monthly and quarterly form then forecasting can be exceedingly difficult. When you collect time series data be certain that the time period of collection will remain consistent.

To create a time series of data from our example of Sales Volume data we now collect a series of Sales Volume data from ***just one location over consecutive monthly periods.*** We collected a data set of four years, 48 consecutive months (48 periods) of Sales Volume.

Table 6.1

Four Years of Time Series Data

Period	Year 1	Period	Year 2	Period	Year 3	Period	Year 4
1	95.5	13	255.0	25	325.0	37	440.0
2	127.0	14	263.0	26	333.0	38	472.0
3	118.0	15	253.0	27	348.0	39	494.5
4	153.0	16	263.0	28	355.0	40	512.0
5	175.5	17	274.0	29	248.0	41	519.0
6	180.0	18	278.0	30	363.0	42	526.0
7	202.0	19	284.5	31	370.0	43	530.5
8	190.0	20	300.0	32	388.0	44	558.0
9	220.5	21	305.5	33	412.0	45	557.5
10	245.0	22	310.5	34	435.0	46	590.0
11	260.0	23	372.0	35	490.5	47	636.4
12	320.0	24	412.0	36	532.5	48	677.4

Graphically displaying Time Series Data: **Time Series Plot**

The corresponding ***time series plot*** or ***time series graph*** of the Sales Volume is in Figure 6-1 below.

Tips on Graphing Times Series Data

The Practice

Good graphical presentations of time series data and forecasts can be very useful and very powerful in the presentation of ideas. A good layout for a time series graph is always helpful in the data analysis, model fitting, and forecasting.

Notice in our graph, that we allowed for additional space to the right of the last observation of our data series. We have 48 observations but allowed for 54 observations. Notice too that because the time series graph reveals an upward direction of the data series we also allowed for additional space above the last observation. The value of the last observation is 677.4 but allowed for values up to 800. In other words we have provided for plenty of “white space" for our graph.

Along the horizontal or time period axis we created horizontal reference lines at every six periods. It would be too dense to create horizontal reference lines at every of 54 periods. And it would be too sparse to form lines at every 12th period. These are monthly data so every 6 periods denotes a half year. Period 6 is a June, Period 12 is a December, Period 18 is June again, and so on.

Along the vertical or Sales Volume axis we set vertical reference lines at every 100 units. Units of 100 allow for sufficient spacing so the graph does not become crowded, and yet detailed enough so that we can estimate values with reasonable accuracy.

In summary, when producing time series graph, try to make the graphs uncluttered without sacrificing accuracy or information.

Time Series Data: **Time Series Notation**

When discussing observations in our data set we think of the data in the form

$$Y_1, Y_2, Y_3, \ldots, Y_T$$

Y_1 is the first observation, Y_2 is the second observation, and so on. Y_1 is the oldest observation down through Y_T which is the most recent observation.

In our example data series,

$$\begin{aligned} Y_1 &= 95.5, \\ Y_2 &= 127.0 \\ \mathrm{Y}_3 &= 118.0 \\ &\vdots \\ Y_{48} &= 677.4 \end{aligned}$$

The uppercase letter T of Y_T will always mean the most recent observation in our time series. Y_{T+1} means then an observation one period into the future. Thus, Y_{T+1}, Y_{T+2}, $Y_{T+3,}$ $\ldots$ denote future observations one, two, three $\ldots$ periods beyond the present time Y_T. Similarly, Y_{T-1} denotes an observation one period in the past, so that Y_{T-1}, Y_{T-2}, $Y_{T-3,}$ $\ldots$ denote observations one, two, three, $\ldots$ periods in the past.

$\ldots$, $Y_{T-3}, Y_{T-2}, Y_{T-1}$	Y_T	$Y_{T+1}, Y_{T+2}, Y_{T+3}, \ldots$
past or historical observations	most recent observation	future observations

We can generalize this notation to

$Y_{T+\ell}$ a future observation ℓ periods ahead

$Y_{T-\ell}$ a past, historical observation ℓ periods back

As we distinguish between Y_i and $\hat{Y}_i$ in regression models, we shall distinguish between $Y_{T+\ell}$ and $\hat{Y}_T(\ell)$ in time series models.

$Y_{T+\ell}$ is the future observation of Y, ℓ periods ahead, and unknown at time T.

$\hat{Y}_T(\ell)$ is the forecast of the future observation, ℓ periods ahead, made at time T.

Stage 1 Continued. **Collecting and Analyzing Data**
The Theory

We devote most of this chapter to Stage 1 of the Forecasting Process. With time series we recommend considerable investigation and analysis, to really understand the time series data, to really understand what the data are "telling us" before attempting to develop forecasting models. Consequently we shall begin this time series analysis with some easy, straightforward methods of analysis.

Smoothing Out Time Series Data

Almost all time series data are rough, having a stochastic element to the series. We shall discuss methods of "smoothing out" the data so that we may observe an underlying structure or pattern. The first, and simplest, method of smoothing time series data is through averaging.

Smoothing Out the Data by Averaging

If we believe there exists some underlying trend or pattern in the time series data, then by "smoothing the data" we smooth out (or average out) the random variations to reveal the underlying pattern in the series.

The Practice
Simple Average Smoothing

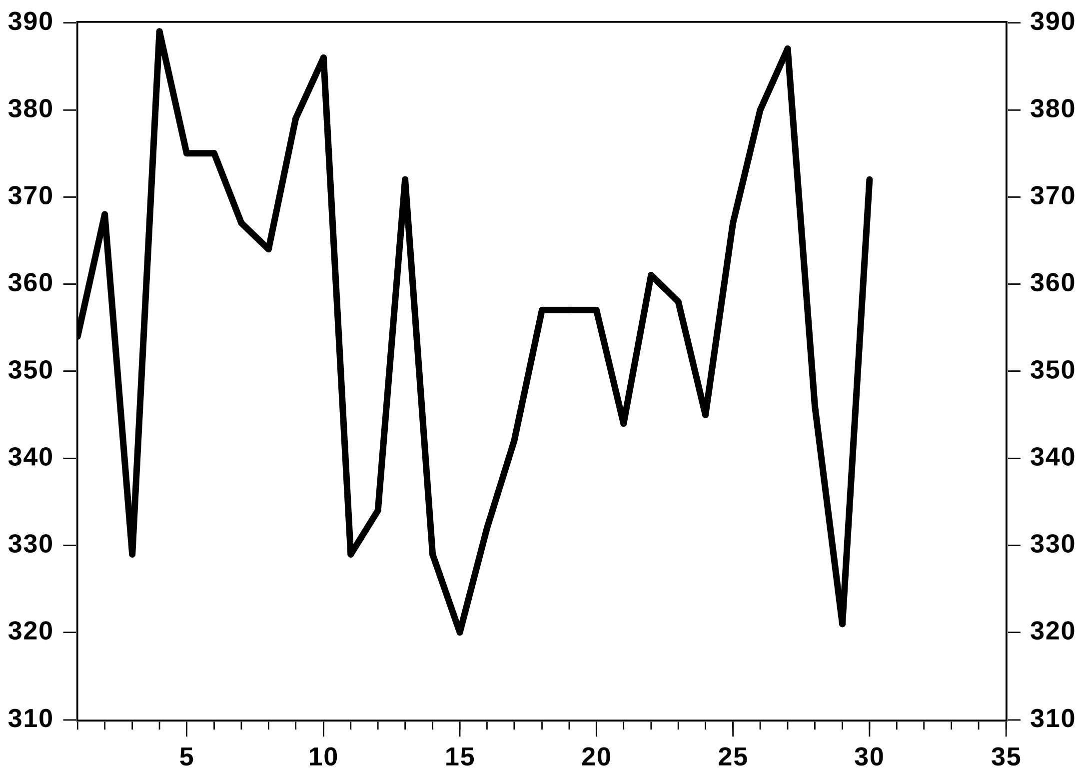
We begin with a time series of 30 observations, $T = 30$, which has a constant mean, or a slowly changing mean, over time, we call this "horizontal data." As in Table 6.2 and Figure 6-3.

Table 6.2

Horizontal Data

Period	Actual	Period	Actual
1	354	16	332
2	368	17	342
3	329	18	357
4	389	19	357
5	375	20	357
6	375	21	344
7	367	22	361
8	364	23	358
9	379	24	345
10	386	25	367
11	329	26	380
12	334	27	387
13	372	28	346
14	329	29	321
15	320	30	372

Table 6.2

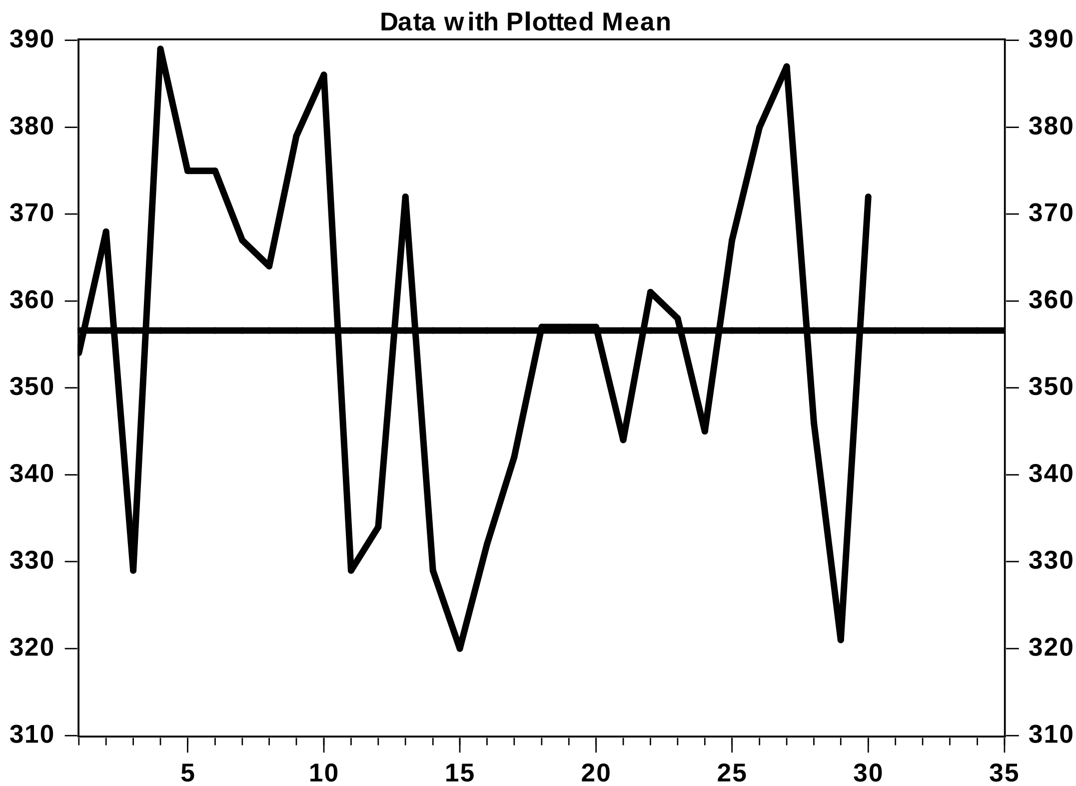


Because the Y_t are varying around a constant mean, a first smoothing of Y_t is just its mean $\bar{Y}$, and so

Smoothing using the sample mean

$$\bar{Y} = \sum_{t=1}^{T} Y_t \qquad \textbf{6.1}$$

In this example, the sample mean is $\bar{Y} = 356.5$.



Moving Average Smoothings

The simple average discussed above is a smoothing method over all observations. A ***moving average*** is the technique of creating successive new averages by dropping the "oldest" observation and adding the most recent observation to calculate the new average.

We shall use the data from Table 6.2 to illustrate this method.

3-period Centered Moving Average Smoothing for period *t*

The 3-period Centered Moving Average Smoothing for period *t*, denoted $CMA_t(3)$ is

3-period Centered Moving Average Smoothing for period *t*

$$CMA_t(3) = \frac{Y_{t+1} + Y_t + Y_{t-1}}{3} \qquad \textbf{6.2}$$

For example, for period 7, a 3-period Centered (at period 7) Moving Average is

$$CMA_7(3) = \frac{Y_8 + Y_7 + Y_6}{3}$$

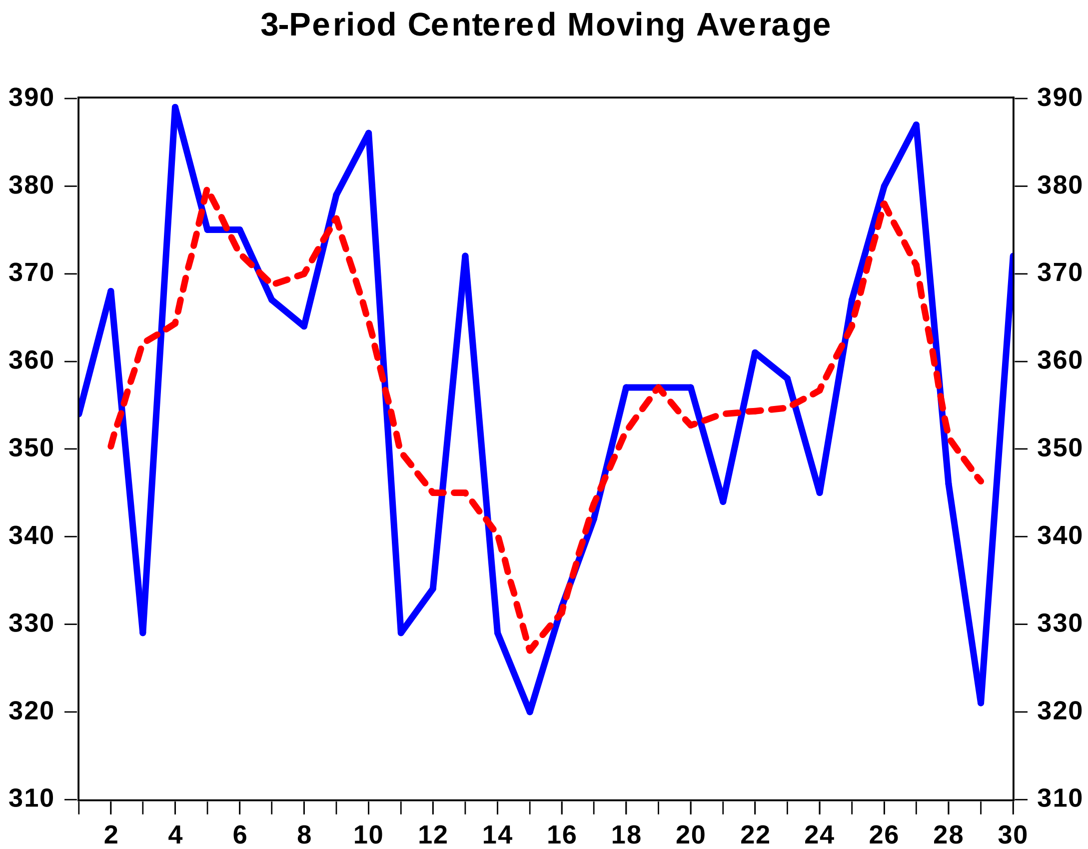
$$CMA_7(3) = \frac{364 + 367 + 375}{3} = 368.67$$

Period	Actual	Smoothing
⋮	⋮	
6	375	
7	367	$= 368.67$
8	364	
⋮	⋮	

This smoothing method is called a 3-period *centered* moving average because the computed value is placed at the center, or middle, of the 3 periods being used in the calculations.

Table 6.3

Period	Actual	$CMA_t(3)$	Period	Actual	$CMA_t(3)$
1	354		16	332	331.3
2	368	350.3	17	342	343.7
3	329	362.0	18	357	352.0
4	389	364.3	19	357	357.0
5	375	379.7	20	357	352.7
6	375	372.3	21	344	354.0
7	367	368.7	22	361	354.3
8	364	370.0	23	358	354.7
9	379	376.3	24	353	356.7
10	386	364.7	25	367	364.0
11	329	349.7	26	380	378.0
12	334	345.0	27	387	371.0
13	372	345.0	28	346	351.3
14	329	340.3	29	321	346.3
15	320	327.0	30	372	



5-period centered moving average smoothing for period *t*

A 5 period moving average smoothing is “smoother" than a 3 period because it uses a larger set of observations.

5-period centered moving average smoothing for period *t*

$$\boldsymbol{CMA_t(5) = \frac{Y_{t+2} + Y_{t+1} + Y_t + Y_{t-1} + Y_{t-2}}{5}} \qquad \textbf{6.3}$$

As an example we smooth period 7 of the data in Table 6.3,

Period	Actual
⋮	⋮
4	389
5	375
6	375
7	367
8	364
9	379
10	386
⋮	⋮

$$CMA_t(5) = \frac{Y_{t+2} + Y_{t+1} + Y_t + Y_{t-1} + Y_{t-2}}{5}$$

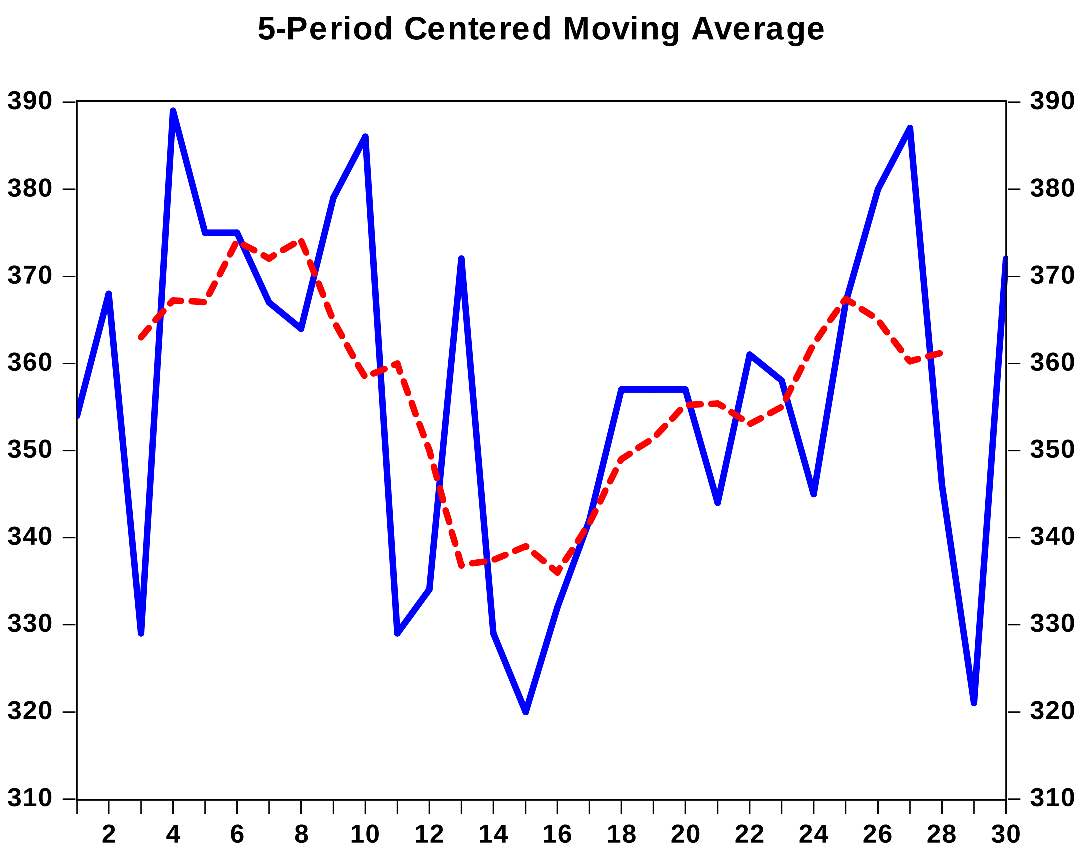
$$CMA_7(5) = \frac{Y_9 + Y_8 + Y_7 + Y_6 + Y_5}{5} = \frac{379 + 364 + 367 + 375 + 375}{5} = 372$$

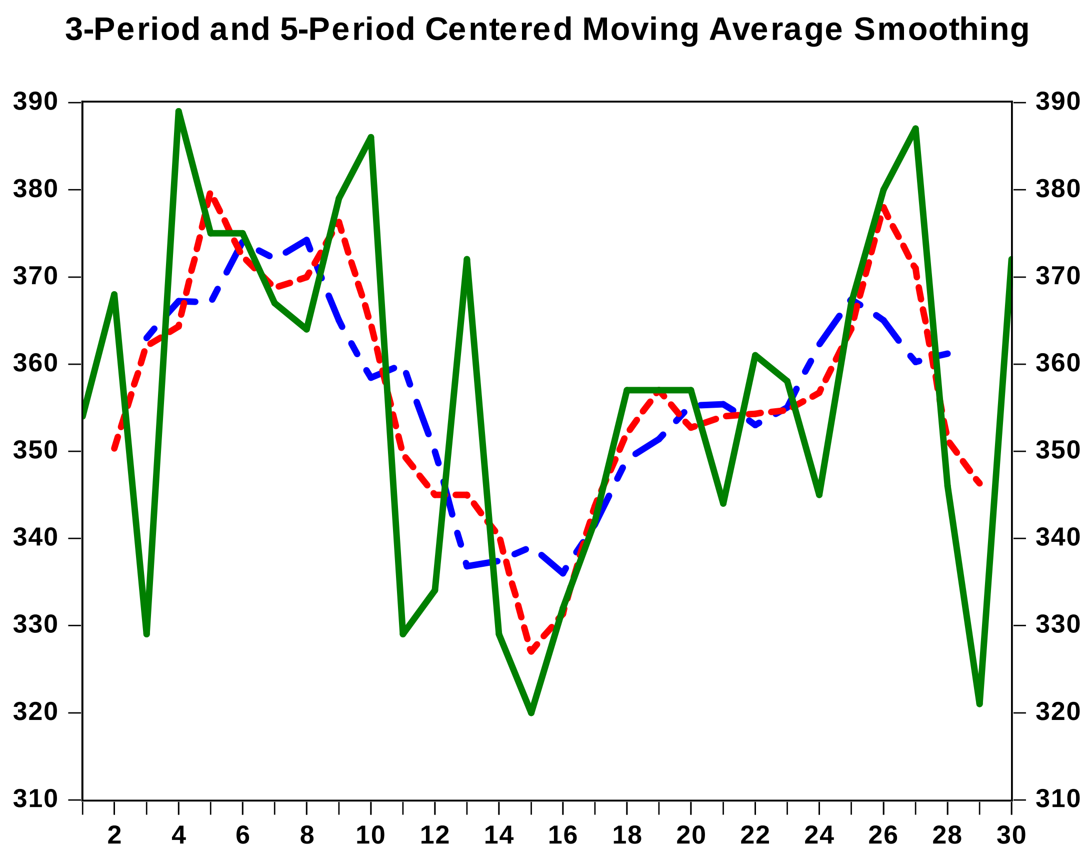
We list below both the 3-period and 5-period moving average smoothing and a graph of the actual values with the two smoothings superimposed.

Table 6.4

5 Period Centered Moving Average Smoothing

Period	Actual	$CMA_t(5)$	Period	Actual	$CMA_t(5)$
1	354		16	332	336.00
2	368		17	342	341.60
3	329	362.00	18	357	349.00
4	389	367.20	19	357	351.40
5	375	367.00	20	357	355.20
6	375	374.00	21	344	355.40
7	367	372.00	22	361	353.00
8	364	374.00	23	358	355.00
9	379	365.00	24	345	362.20
10	386	358.40	25	367	367.40
11	329	360.00	26	380	365.00
12	334	350.00	27	387	360.20
13	372	336.80	28	346	361.20
14	329	337.40	29	321	
15	320	339.00	30	372	





4-Period Centered Moving Average Smoothing

A ***4-period Centered Moving Average Smoothing*** is possible, but the issue of the placing of the results remains. Technically, if (say) the first 4 periods are used

$$CMA_t(4) \;=\; \frac{Y_4 + Y_3 + Y_2 + Y_1}{4}$$

then the placement of $CMA_t(4)$ is between periods 2 and 3 at "period 2.5."

$$CMA_t(4) \;=\; \frac{389 + 329 + 368 + 354}{4} \;=\; 360.00$$

The next 4-period centered smoothing is placed at "period 3.5"

$$CMA_t(4) \;=\; \frac{Y_5 + Y_4 + Y_3 + Y_2}{4}$$

$$CMA_t(4) \;=\; \frac{375 + 389 + 329 + 368}{4} \;=\; 365.25$$

Then the "adjusted centered smoothing" for period 3 is the average of period 2.5 and period 3.5 smoothing.

$$CMA_3(4) \;=\; \frac{360.00 + 365.25}{2} \;=\; 362.63$$

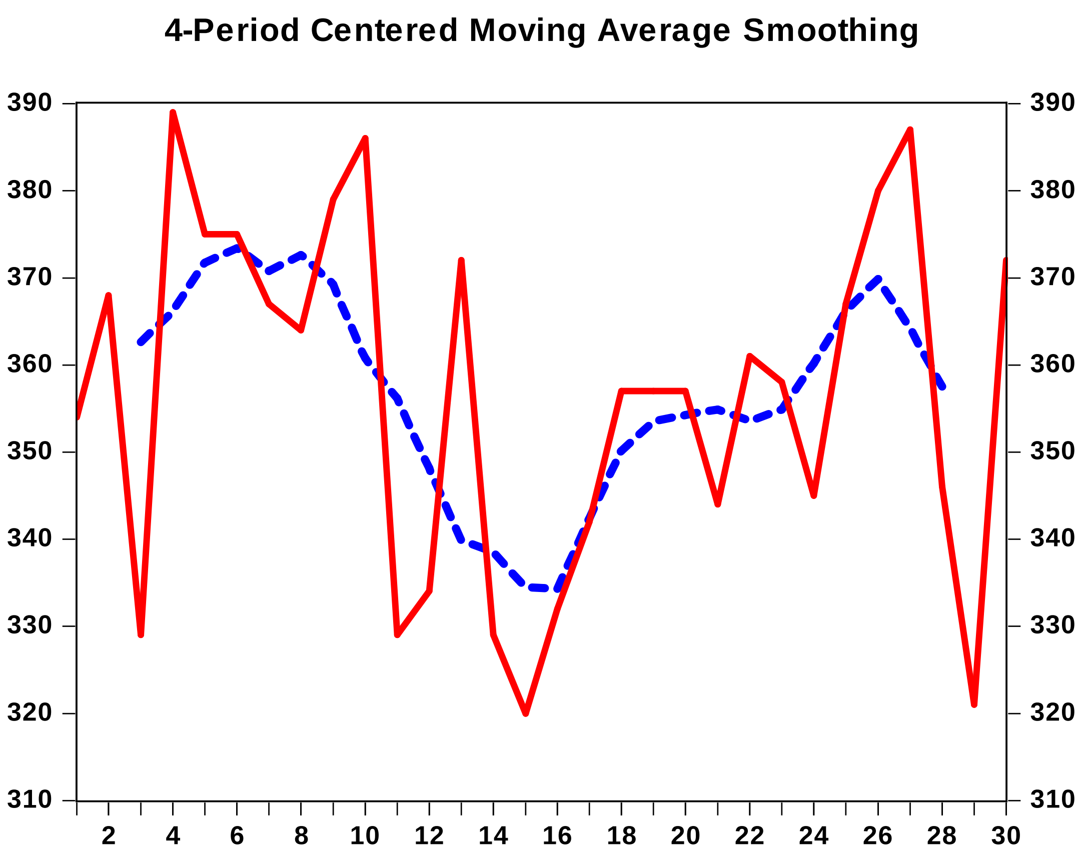
Algebraically, it can be shown that the adjusted moving average of 4 periods, starting with period $t \;=\; 3$ is

4-period Centered Moving Average Smoothing for period t, $t \geq 3$

$$CMA_t(4) = \frac{Y_{t+2} + 2Y_{t+1} + 2Y_t + 2Y_{t-1} + Y_{t-2}}{8} \qquad 6.4$$

Table 6.5
Adjusted 4 Period Centered Moving Average Smoothing

Period	Actual	$CMA_t(4)$	Period	Actual	$CMA_t(4)$
1	354		16	332	334.25
2	368		17	342	342.38
3	329	362.63	18	357	350.13
4	389	366.13	19	357	353.50
5	375	371.75	20	357	354.25
6	375	373.38	21	344	354.88
7	367	370.75	22	361	353.50
8	364	372.63	23	358	354.88
9	379	369.25	24	345	360.13
10	386	360.75	25	367	366.13
11	329	356.13	26	380	369.88
12	334	348.13	27	387	364.25
13	372	339.88	28	346	357.50
14	329	338.50	29	321	
15	320	334.50	30	372	



The 4-period Centered Moving Average Smoothing is especially suited for quarterly time series. However, the Centered Moving Average Smoothing formula may be generalized to any even number of periods. As examples,

6-period Centered Moving Average Smoothing for period *t*, $t \geq 4$

$$\boldsymbol{CMA_t(6) = \frac{Y_{t+3} + 2Y_{t+2} + 2Y_{t+1} + 2Y_t + 2Y_{t-1} + 2Y_{t-2} + Y_{t-3}}{12}} \qquad \textbf{6.5}$$

Or for monthly data, a 12-period Centered Moving Average Smoothing, starting with period 7;

12-period Centered Moving Average Smoothing for period *t*, $t \geq 7$

$$\boldsymbol{CMA_t(12) = \frac{Y_{t+6} + 2Y_{t+5} + 2Y_{t+4} + 2Y_{t+3} + 2Y_{t+2} + 2Y_{t+1} + 2Y_t + 2Y_{t-1} + 2Y_{t-2} + 2Y_{t-3} + 2Y_{t-4} + 2Y_{t-5} + Y_{t-6}}{24}} \qquad \textbf{6.6}$$

Not all smoothings must be placed in the “center" of the data, we have done so because a moving average falls in the “middle" of the data. However, for example, some 4-period weekly smoothings are placed at the last period of the four periods smoothed.

The length of periods of smoothing is usually dependent on the type of time series data being analyzed. For example, some analyses of stock market and bond market time series data use 13-period and 39-period smoothing of weekly data.

Weighted Moving Average Smoothings

Simple moving averages assume equal weight to the observations being used in the calculations. There may be situations, however, in which it is better to assign greater weight to the most recent observation and less to observations in the past.

3-period Centered Weighted Moving Average

For example, a 3-*period Centered Weighted Moving Average*, denoted $CWMA_t(3)$, is

$$CWMA_t(3) = .6Y_{t+1} + .3Y_t + .1Y_{t-1}$$

All that is required is that the sum of the weights equal 1, as in

$$.6 + .3 + .1 = 1$$

In general terms, we denote the weights by ω_1, ω_2, ω_3 (“omega sub 1," “omega sub 2," “omega sub 3")

3-period Centered Weighted Moving Average Smoothing for period *t*

$$\boldsymbol{CWMA_t(3) = \omega_1 Y_{t+1} + \omega_2 Y_t + \omega_3 Y_{t-1}, \text{ with } \sum \omega_i = 1} \qquad \textbf{6.7}$$

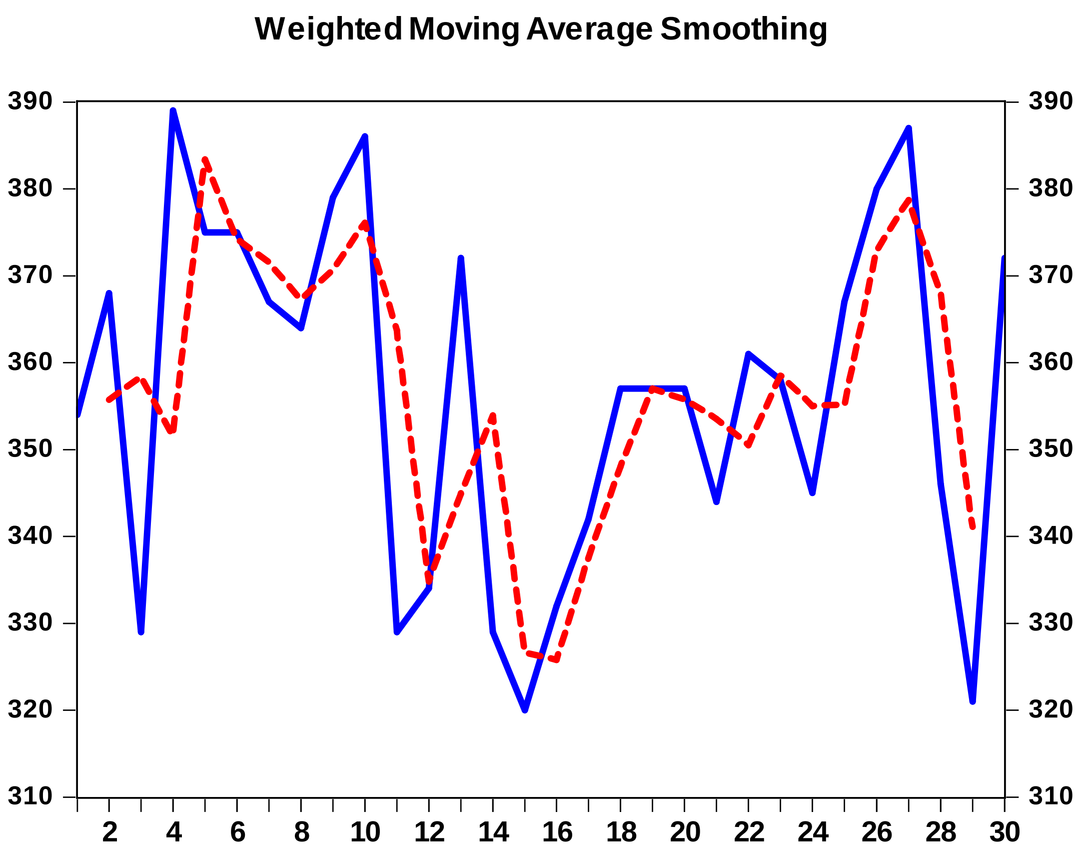
A weighted moving average may be generalized to any number of historical periods and set of weights. Again the choice of the number of historical periods and the weights used are determined, in part, by the Forecast Analyst who has collected the data and has some appreciation of the structure of the data.

Table 6.8 and Figure 6-9 below show the results of the above weighted moving average where $\omega_1 = .6$, $\omega_2 = .3$, and $\omega_3 = .1$.

Table 6.6

3 Period Centered Weighted Moving Average Smoothing
using weights $\omega_1 = .6$, $\omega_2 = .3$, $\omega_3 = .1$

Period	Actual	CWMA(3)	Period	Actual	CWMA(3)
1	354		16	332	336.80
2	368	343.20	17	342	350.00
3	329	368.90	18	357	355.50
4	389	374.60	19	357	357.00
5	375	376.40	20	357	349.20
6	375	370.20	21	344	355.50
7	367	366.00	22	361	357.50
8	364	373.30	23	358	350.50
9	379	381.70	24	345	359.50
10	386	351.10	25	367	372.60
11	329	337.70	26	380	382.90
12	334	356.30	27	387	361.70
13	372	342.40	28	346	335.10
14	329	327.90	29	321	354.10
15	320	328.10	30	372	



Classical Time Series Decomposition

Given an economic time series as shown above in Figure 6-2 one of the most common approaches to understanding such data is through *classical time series decomposition.* The composition of the time series is decomposed into four components: trend, cyclical, seasonal, and irregular components.

The Trend Component

The trend component of a time series at time t, denoted $\boldsymbol{T_t}$, is the upward or downward progression of the data over time. Figure 6-10 below has a trend line superimposed over the actual data. In Chapter 7 we shall discuss how the OLS regression line of Y_t is determined.

The Cyclical Fluctuations Component

The cyclical component of a time series at time t, denoted $\boldsymbol{C_t}$, and illustrated below in Figure 6-11, are the wide, long term up-and-down swings of the series around the trend line. These cycles of high and low usually last more than year, having different lengths and amplitudes. The cyclical component of a time series is usually attributable to some larger aspect of the whole business economy by which this particular data have been affected.

The Seasonal Fluctuations Component

The seasonal component of a time series at time t, denoted $\boldsymbol{S_t}$, illustrated below in Figure 6-12 are the reoccurring fluctuations within a year around the trend/cyclical components. Most business data reveal seasonal fluctuations, or “seasonality," so it is important for business to know and plan for changes in sales or economic activity that will be more or less normal due to the seasonal nature of their product.

Notice in Figure 6-12 that the periods of 11 and 12, 23 and 24, 35 and 36, 47 and 48 are reoccurring local peaks in Sales Volume. This is the seasonality of Sales.

The Irregular Variations Component

The irregular component of a time series at time t, denoted $\boldsymbol{I_t}$, illustrated in Figure 6-13 are the random, unexpected deviations from the trend/cyclical/seasonal components. Irregular changes or “shocks" to a time series are considered nonrecurring chance events, such as strikes, or unusually good (The movie *E.T.* and Reese's Pieces Candy) or unusually bad (Tylenol poisoning) publicity about a product.

In Figure 6-13 below, an irregular variation occurs around period 30 as the series drops abruptly and then returns to its upward trend.

Considering the four components collectively is the method of the ***classical time series decomposition***. In its multiplicative form, at time t, the actual value Y_t is the product of the trend component $\boldsymbol{T}_t$, in physical units, the cyclical component $\boldsymbol{C}_t$, as a percentage of $\boldsymbol{T}_t$, the seasonal component $\boldsymbol{S}_t$, as a percentage of $\boldsymbol{T}_t \times C_t$, and the irregular component, expressed as a percentage of $\boldsymbol{T}_t \times \boldsymbol{C}_t \times \boldsymbol{S}_t$.

Multiplicative Time Series Decomposition

$$Y_t = T_t \times C_t \times S_t \times I_t \qquad \textbf{6.8}$$

Time series decomposition may be additive as in

Additive Time Series Decomposition

$$Y_t = T_t + C_t + S_t + I_t \qquad \textbf{6.9}$$

where all components are expressed in the physical units of Y_t.

Trend, seasonal fluctuations, and irregular variations are usually easily identifiable components of a time series. Calculations of trend and seasonality are quite straightforward, and adjustments for irregular variations can be simply handled. Determining the long term cyclical component of a time series is not so direct. Long term business cycles are beyond the scope of this textbook, so we refer the interested reader to the bibliography.

Stage 1 Analysis Seasonal Indices

The Ratio-to-Moving Average Method

There are many occasions in the analysis of time series when we wish to isolate the seasonality in the data. This is known as determining the *seasonal indices* and then *deseasonalizing the data.* An established, widely used method is the ***Ratio-to-Moving-Average Method***. This was first developed in 1922 by Frederick Macaulay at the National Bureau of Economic Research and then adopted and promoted by Julius Shiskin of the U. S. Bureau of the Census. Often referred to as the Census II decomposition, this method has evolved and now in its most recent version, it is the Census X-11 decomposition. We shall use the time series from Table 6.2 and Figure 6-1 for this example.

Because they are monthly data, we first determine the 12-period Centered Moving Average Smoothing of the data starting with period 7.

Step 1: Determine the 12-period Center Moving Average Smoothing of the Data
We use the formula on page 11. These are the numbers under the label $\mathrm{MA}_t(12)$

Step 2: Calculate the Ratio of the Actual to the Moving Average (hence, the name of the method).

Step 3: We collect the Ratios in a table by Month and determine the Median value of the ratios. These Median values are often termed the Unadjusted Seasonal Indices.

Table 6.7 *Ratio to Moving Average Method*

Obs	Month	Actual	$\mathrm{MA}_t(12)$	Ratio	Seasonal Index	The De-Seasonalized Data
1	1	95.5			0.96	99.07
2	2	127.0			0.96	131.75
3	3	118.0			0.97	121.18
4	4	153.0			0.97	157.12
5	5	175.5			0.94	185.86
6	6	180.0			0.96	186.73
7	7	202.0	197.19	1.02	0.93	216.18

8	8	190.0	209.50	0.91	0.95	199.14
9	9	220.5	220.79	1.00	0.98	224.17
10	10	245.0	231.00	1.06	1.01	241.87
11	11	260.0	239.69	1.08	1.09	238.18
12	12	320.0	247.88	1.29	1.24	258.25
13	1	255.0	255.40	1.00	0.96	264.59
14	2	263.0	263.42	1.00	0.96	272.27
15	3	253.0	271.54	0.93	0.97	257.26
16	4	263.0	277.81	0.95	0.97	270.13
17	5	274.0	285.21	0.96	0.94	290.23
18	6	278.0	293.71	0.95	0.96	294.46
19	7	284.5	300.46	0.95	0.93	304.52
20	8	300.0	306.29	0.98	0.95	311.28
21	9	305.5	313.17	0.98	0.98	310.65
22	10	310.5	320.96	0.97	1.01	306.54
23	11	372.0	323.71	1.15	1.09	340.78
24	12	412.0	326.17	1.26	1.24	332.50
25	1	325.0	333.27	0.98	0.96	337.22
26	2	333.0	340.50	0.98	0.96	344.73
27	3	348.0	348.60	1.00	0.97	353.87
28	4	355.0	358.23	0.99	0.97	364.63
29	5	248.0	368.35	0.67	0.94	262.69
30	6	363.0	378.31	0.96	0.96	384.50
31	7	370.0	388.13	0.95	0.93	396.04
32	8	388.0	398.71	0.97	0.95	402.59
33	9	412.0	410.60	1.00	0.98	418.94
34	10	435.0	423.25	1.03	1.01	429.45
35	11	490.5	441.08	1.11	1.09	449.34
36	12	532.5	459.17	1.16	1.24	429.74
37	1	440.0	472.65	0.93	0.96	456.55
38	2	472.0	486.42	0.97	0.96	488.63
39	3	494.5	499.56	0.99	0.97	502.83
40	4	512.0	512.08	1.00	0.97	525.89
41	5	519.0	524.63	0.99	0.94	549.74
42	6	526.0	536.75	0.98	0.96	557.15
43	7	530.5			0.93	567.83
44	8	558.0			0.95	578.98
45	9	557.5			0.98	566.90
46	10	590.0			1.01	582.47
47	11	636.5			1.09	583.09
48	12	677.5			1.24	546.76

Step 4: We expect the average of the seasonal indices to be 1, so with 12 periods in the season, the sum of the seasonal indices must equal 12. Because their sum is 12.20, we multiply each Unadjusted Seasonal Index by $\frac{12}{12.20}$. For example, for Month 1 (January): $\left(\frac{12}{12.20}\right) \times 0.98 = 0.96$. The sum of the Adjusted Seasonal Indices equals 12, so their average equals 1.

Table 6.8

Ratios of Actual to the 12 Period Centered Weighted Moving Average Smoothing

Month											
1	2	3	4	5	6	7	8	9	10	11	12
						1.02	0.91	1.00	1.06	1.08	1.29
1.00	1.00	0.93	0.95	0.96	0.95	0.95	0.98	0.98	0.97	1.15	1.26

0.98	0.98	1.00	0.99	0.67	0.96	0.95	0.97	1.00	1.03	1.11	1.16		
0.93	0.97	0.99	1.00	0.99	0.98								

Median values - Unadjusted Seasonal Index

0.98	0.98	0.99	0.99	0.96	0.98	0.95	0.97	1.00	1.03	1.11	1.26	12.20	Sum

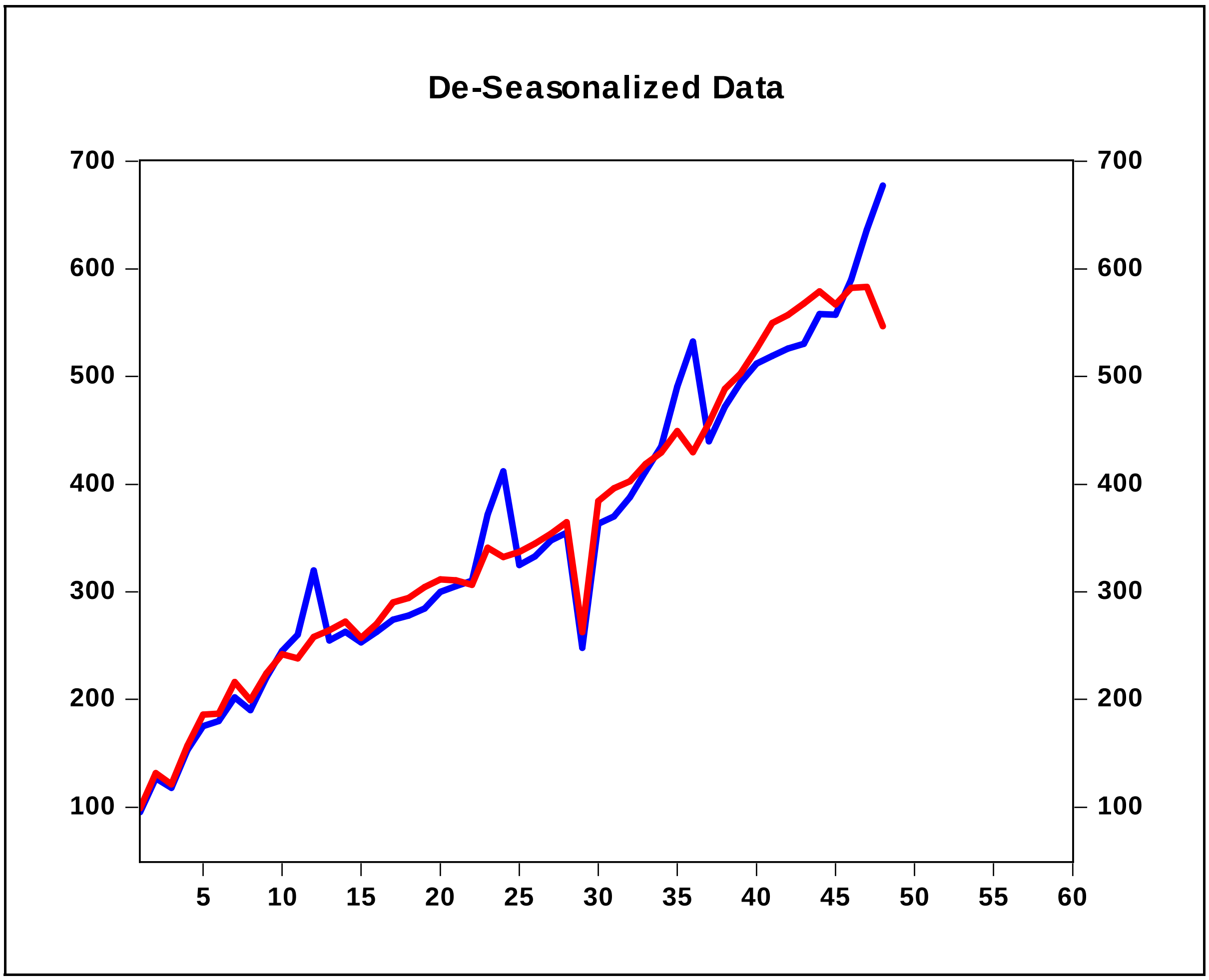
Adjusted Seasonal Index

0.96	0.96	0.97	0.97	0.94	0.96	0.93	0.95	0.98	1.01	1.09	1.24	12.00	Sum

---

As we expected, the 12th month, January, has the largest seasonal index, and the 7th month, July, has the smallest seasonal index.

Step 5: The original, Actual, values are divided by the Seasonal Indices, creating a De-Seasonalized Series.



PROBLEMS AND QUESTIONS

Constant Mean Data

6.1

Listed below is a time series of 24 observations.
Plot the data and determine the mean and variance of the the data set.

Period	Observation	Period	Observation
1	95	13	118
2	100	14	86
3	87	15	86
4	123	16	112
5	90	17	85
6	96	18	101
7	75	19	135
8	78	20	120
9	106	21	76
10	104	22	115
11	89	23	90
12	83	24	92

Smoothing of Data

6.2 Overlay on the time series graph of the data

a. The simple average of the data, as a smoothing.

b. A three period centered moving average.

c. A five period centered moving average.

6.3 Overlay on the time series graph of the data.

a. A four period centered moving average.

b. A three period weighted centered moving average with weights of .6 for the most recent, .3 for one period back, and .1 for two periods back.

6.4 De-seasonalizing Data

The following 24 observations of quarterly data contain trend and seasonality.

Period t	*Quarter* Q_t	*Actual* Y_t	*Period* t	*Quarter* Q_t	*Actual* Y_t
1	1	409	13	1	349
2	2	415	14	2	350
3	3	468	15	3	443
4	4	261	16	4	315
5	1	308	17	1	362
6	2	326	18	2	411
7	3	431	19	3	447
8	4	254	20	4	295
9	1	346	21	1	310
10	2	370	22	2	368
11	3	430	23	3	442
12	4	290	24	4	302

a. Plot the data.

b. Determine a four period centered moving average of the data and overlay it on the original data.

c. Determine the ratio-to moving average seasonal indices for the four quarters.

d. Re-plot the original data and create an overlay plot of the de-seasonalized data.

One-step ahead forecasts

6.4 Determine a set of one-step ahead forecasts of the data, using
- a. Three period moving average forecasts.
- b. Five period moving average forecasts.
- c. Four period moving average forecasts.
- d. Three period weighted moving average forecasts; weights .6, .3 and .1

6.5 Determine a set of one-step ahead forecasts of the data, using
- a. Single exponential smoothing forecasts with $\alpha = .1$.
- b. Single exponential smoothing forecasts with $\alpha = .2$.
- c. Single exponential smoothing forecasts with $\alpha = .7$.

6.6 Using the one-step ahead forecasts of Problem 6.5, determine the forecasts for period 25 in a., b., and c. above.

6.7 Determine the MSE of each of the one-step ahead forecasts in Problem 6.5 Which is the better forecast? Explain.

Data with Trend

6.8

Listed below is a time series of 24 observations.
Plot the data and determine the mean and variance of the the data set.

Period	Observation	Period	Observation
1	60	13	89
2	70	14	120
3	85	15	134
4	60	16	121
5	88	17	93
6	68	18	113
7	106	19	125
8	75	20	136
9	86	21	142
10	124	22	117
11	122	23	132
12	87	24	141

6.9 Determine the one-step ahead forecasts using
- a. Three period moving average
- b. Weighted moving average with weights .5, .3, .2.
  What are the forecasts for period 25 in each case above?
  What is the MSE in each case above?

6.10 Determine the one-step ahead forecasts using
- a. Single exponential smoothing with $\alpha = .3$
- b. What is the forecast for period 25?
- c. What is the MSE of this forecasting model?

6.11 Determine the one-step ahead forecasts using
- a. Double exponential smoothing with $\alpha = .3$.
- b. What is the forecast for period 25?

c. What is the forecast for periods 26 to 30?
d. Determine the 95% confidence intervals of forecast for periods 25 to 30.
e. What is the MSE of the forecasting model?

Linear Regression of a Time Series

6.12 a. Determine a linear regression model for forecasting periods 25 to 30.
b. Determine the 95% confidence intervals of forecast for periods 25 to 30.
c. What is the MSE of this forecasting model?

6.13 How does your “line of sight" forecast compare with the double exponential smoothing model? with the linear regression model?

6.14 Which is the better model, double exponential smoothing or linear regression? Explain.