EXPECTED COURSE OF THE EXPECTED EQUITY RETURNS

JĀNIS ZVINGELIS

QUANTITATIVE RESEARCH GROUP

ENVESTNET|PMC

1. Introduction

Much has been said regarding the likely future direction of the equity market, which over the last half decade has defied all the nay-sayers by rising ever higher, spurred on by various forms of "unconventional monetary policy". In this note I will stay away from giving advice on whether now is the the time to increase or lower the risk of one's portfolio by boosting or reducing one's allocation to equities.[1] Rather, I present the reader with our estimate on the likely value of a longer term equity return[2] to assist with the decision of whether the additional risk premium expected from equities over the safer investment options is worth the current risk-return tradeoff. As I demonstrate, unless we make some rather extreme assumptions about the future behavior of stock price multiples, the equity returns going forward look decidedly different from those experienced in the past. That is, we can expect that the future returns on equities will offer a premium over less risky assets that is significantly lower than what has been realized in the recent past (e.g., since the financial crisis of 2008) as well as over a longer history.

2. Decomposition of the Rate of Return: Making Forecasting Easier

As the first step in forecasting the average (expected) rate of future return, it is useful to decompose it as shown in the Appendix ("Decomposition of the Total Return"):

$$\begin{aligned} R_{t+1} &= \left(D_{t+1} + P_{t+1}\right)/P_t - 1 \\ &\approx D_{t+1}/P_t + G_{t+1} + \Delta_{t+1}, \end{aligned} \tag{2.1}$$

35 East Wacker Drive, Suite 2400, Chicago, IL, 60601

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[1] If we believe (and we should) that in the long-run more risky assets should command higher risk premia, then the choice of whether and how much of a risky asset to hold at any time comes down to one's risk aversion.

[2] In this paper when referencing future returns, I always have in mind the average of these future returns over an investment horizon. In statistics the word "expected" (as in "expected return") has a different meaning from its everyday meaning of "likely to happen". Here I use the word "expected" with its statistical meaning of "average". Thus, an "expected return" (i.e., average return) might not be expected (i.e., likely) to occur in any given year. Still, estimating an expected (i.e., average) return is a very useful concept for the purposes of forming a forecast of likely average future performance over an investment horizon.

where R_{t+1} is the return during time period $t+1$; P_{t+1} is the price at the end of period $t+1$; D_{t+1} is the dividend paid during $t+1$. Also, G_{t+1} is the net growth rate of the earnings, and Δ_{t+1} is the net growth rate of price-earnings multiple during time period t to $t+1$. We also assume that time period t is the current time period (e.g., we can observe P_t), and that one period in the above equation is a year (i.e., t counts years).

The reason that it is more convenient to try to forecast net growth rates in earnings and pricing multiples is because these statistical series tend to be more stable (and therefore forecastable) than the capital gains series. Also, since the future rates of returns R_{t+1}, R_{t+2}, ... are random, rather than trying to forecast these rates, which might be quite volatile, we would be better served to try to forecast the average rate of return over a longer period of time. This would be helpful from a statistical point of view (Law of Large Numbers kicks in, making the average a lot more stable than the rates of returns of individual years) as well as from the point of view of being able to appeal to economic processes that should hold over a "long horizon" or "full market cycle".

Using the expectations notation,[3] we can re-write the above equation as follows,

$$E_t\left(R_{t+1}\right) \approx E_t\left(D_{t+1}/P_t\right) + E_t\left(G_{t+1}\right) + E_t\left(\Delta_{t+1}\right), \tag{2.2}$$

where the E_t refers to the "expected" or average value taken starting at time t. Thus, to construct a forecast for the average future return ($E_t\left(R_{t+1}\right)$), I will construct forecasts for each of its components in the above equation: $E_t\left(D_{t+1}/P_t\right)$, $E_t\left(G_{t+1}\right)$, and $E_t\left(\Delta_{t+1}\right)$.

3. Are Price Multiples Mean-Reverting?..and Why It Matters

Before I proceed with constructing the forecasts of the return components in equation 2.2, we need to consider a crucial question: "Are price multiples mean-reverting?"

By "mean-reversion" in pricing multiples I do not have in mind a strict mean-reversion in statistical sense (e.g., assuming covariance stationarity would result in mean-reversion), which implies that the process reverts to a constant mean, but rather(and admittedly with much less statistical precision) something much less stringent. That is, I will assume that the mean of the pricing multiples is time-varying, with the current mean potentially being much higher than fifty or even twenty years ago. Thus, under this scenario, the mean of pricing multiples could be moving gradually higher through time, and the realized price multiples form an undulating series around this gradually increasing mean series.

What I do want to rule out, however, is a case, where pricing multiples are free to float ever higher without any reference to their historical norms, as a notion that investors would be willing to pay an ever-higher price for a dollar of revenues (in real terms) seems very nonintuitive and indefensible. Admittedly, a prolonged upward trend in pricing multiples could be caused by, for example, decreasing discount rates. As the market experience over the last almost ten years attests, decreasing discount rates (in this case driven by the coordinated

[3]Again, "expectation" or "expected return" here refers to "average" or "average return", rather than something that is "expected to happen". Also, there is technical distinction between the average of future returns (which is a random variable) and the expected value of future returns (which is a constant). In fact, what we are trying to estimate is the expected value of the average of future returns (i.e., "average of average"). But these technical distinctions are not of main importance here, so for the remainder of the note I will equate "average future return" with "expected future return".

actions of Central Banks the world over) have sent the pricing multiples on an upward trend, which seems to defy the mean-reversion. Still, unless we are willing to entertain a scenario where the discount rates wander deep into negative territory, the effect of discount rates on the pricing multiples has a finite effect. There are other causes for persistently high pricing multiples (e.g., investors' attitudes towards risk, expectations for future earnings growth rates), however none of them provide an intuitive and defensible explanation for a pricing multiple that grows without bounds. Maybe unsurprisingly this conclusion is also overwhelmingly supported by the literature (see, for example, Campbell & Shiller (2001)).

Thus, if pricing multiples do not float ever higher like untethered balloons, how does that help with forecasting the average return? First, referring to equation 2.2, if pricing multiples are mean reverting, we can use their current values and compare them to their (recent) historical values to form a forecast of their likely future values. Second, and perhaps more importantly, as first empirically confirmed by Campbell & Shiller (2001) and confirmed later on in this note, *if the pricing multiples are mean reverting, current levels of pricing multiples predict future returns, rather than changes in dividend or earnings levels.* More specifically, low current levels of pricing multiples predict low levels of future returns, rather than high levels of dividends or earnings. As we will show later, the positive relationship between current dividend yields and future realized returns is very strong and stable through time. Applying the above two implications of mean reversion of pricing multiples makes the task of predicting the components of equation 2.2 much more manageable, and the forecasts themselves much more reliable and precise.

4. Estimating The Components of the Equity Return

With the expected return divided into pieces (equation 2.2) whose statistical properties are easier to track, as well as making the highly reasonable assumption of price multiple stationarity, which allows us to tie the levels of these components to their own historical values as well as to each other, we can then proceed with estimating the likely future values of each of these components. These components are Expected Dividend Yield, Expected Earnings Growth Rate, and Expected Pricing Multiple Adjustment.

4.1. **Dividend Yield.** Recent(as of July 26th) dividend yield for S&P 500 is 1.91 percent, which is based on the price of 2,480.2 and (annual) per share dividend of 47.22. The question we would like to answer is whether it is likely for the dividend yield to experience a significant increase from its current level of 1.91 percent. As I will argue below, assuming that current dividend yield of around 2 percent is applicable also for the longer term future investment is already a very generous assumption.

The dividend yield can increase either because the price drops or because the dividend level increases. We will look at these two scenarios next.

4.1.a. *Dividend Yield and Price Movement.* As mentioned above, one of the ways that the dividend yield could increase from its current 1.91 percent is if the price level dropped. If we look at the historical dividend yield plot (see Figure 1), we notice that almost all of the dividend yield increases (the one notable exception is post-World War II increase in dividend yields due to the increase in the level of dividends per share during a sideways market performance environment) have come due to precipitous market level decreases. In fact, the correlation

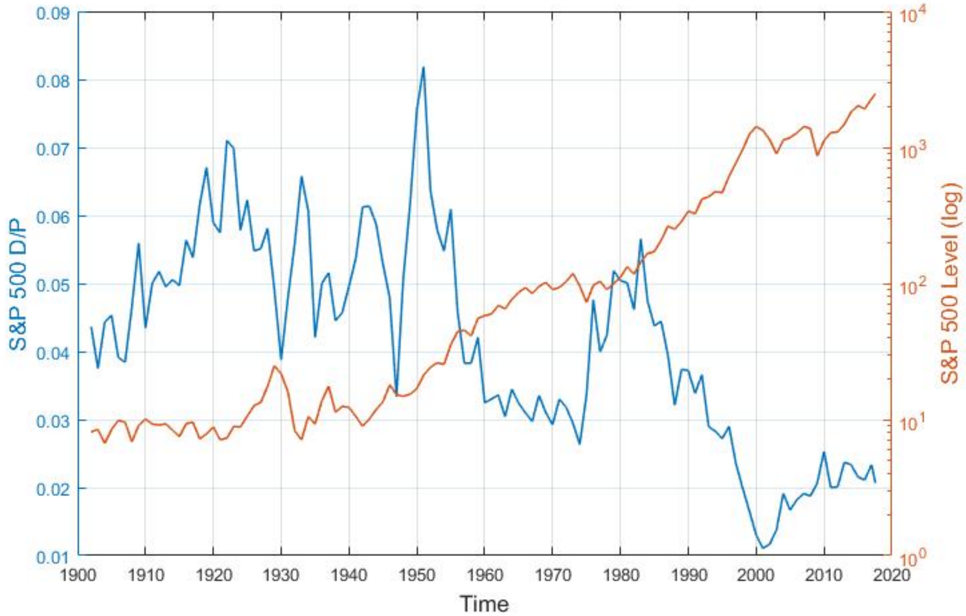

FIGURE 1. Plot of level (log scale) vs its dividend yield (D/P) for S&P 500 (1900/01 - 2017/07, year-end annualized data). *Source: www.multpl.com.*

between the dividend yield and market level since the beginning of the 20th century is -0.7. However, for the dividend yield to increase meaningfully, the price has to drop significantly. Thus, for the dividend yield to increase from the current 1.91 percent to 3 percent (assuming the dividend payment stays the same), say, the price level would have to drop by more than 36 percent. If this type of market correction in fact takes place, then the marginal increase in dividend yield will be an afterthought.

4.1.b. *Dividend Yield and Dividend Size.* The other way that the dividends could increase is if the level of dividend payments spiked relative to the price level. That is, we need an increase in the level of dividend payments without an offsetting increase in prices. This, however, is an unlikely scenario.

Dividends per share (DPS) have been growing at an elevated rate over the last half a decade (real DPS have grown by more than 40 percent over this period) and now stand at an all time high (see Figure 2). The main reason for this almost unprecedented growth in dividends (the only other instance of a comparable rate of DPS increase is late 1920 – right before the Great Depression) has to do with the sluggish capital investment of most firms since the financial crisis of 2008. Capital expenditures as percentage of operating cash-flow for S&P 500 companies have dropped from about 55 percent before the financial crisis to about 40 percent currently. The relatively low levels of capital expenditures (perhaps not surprisingly) coincide with very anemic growth in sales (see Figure 3 for sales per share for S&P 500 companies).

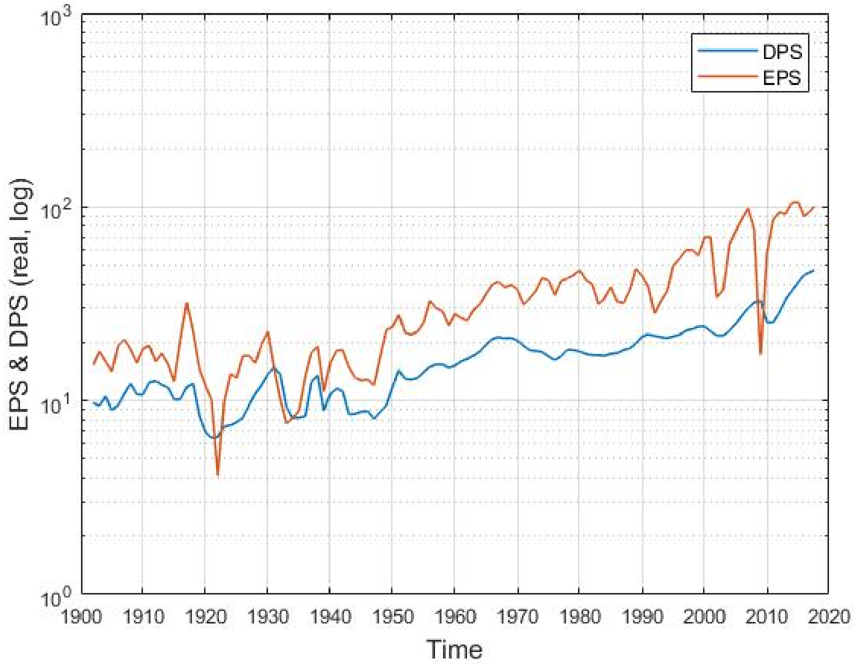

FIGURE 2. Plot of real dividends and earnings per share (log scale) for S&P 500 (1900/01 - 2017/07, year-end annualized data). *Source: www.multpl.com.*

Thus, as companies' profit margins improved after the Great Recession due to aggressive cost cutting measures (see Figure 3), and as companies faced uncertainty due to slow sales growth and therefore slowed down their capital expenditures, they started accumulating cash reserves. Before the recession of 2008, corporate cash made up less then 20 percent of all corporate assets. By 2012, this ratio stood at more than 30 percent, where it has remained ever since. As capital investments remained relatively low and cash reserves hit all time highs, companies started getting rid of accumulated cash by increasing dividends and through stock buy-backs.

However, with companies facing anemic sales growth and earnings growth maxed out (more on this later) as well as new administration's economic plans mired in political maelstrom, it is difficult to see what would prompt firms to greatly increase dividends even further from their current record levels.[4] Note also that for the dividend yield to increase materially over a long-term investment due to increase in the size of dividend payment (i.e., due to increase in the numerator of the dividend yield), what we need is level of dividends, which would also outpace any corresponding likely increase in the price level. This latter requirement perhaps is even more unlikely than any potential further increase in the level of dividends.

[4]Perhaps a change of tax laws that allows the repatriation of the overseas profits amassed by the US multinationals would result in a one-time increase in the DPS, but unlikely to a meaningful change in the rate of growth of DPS.

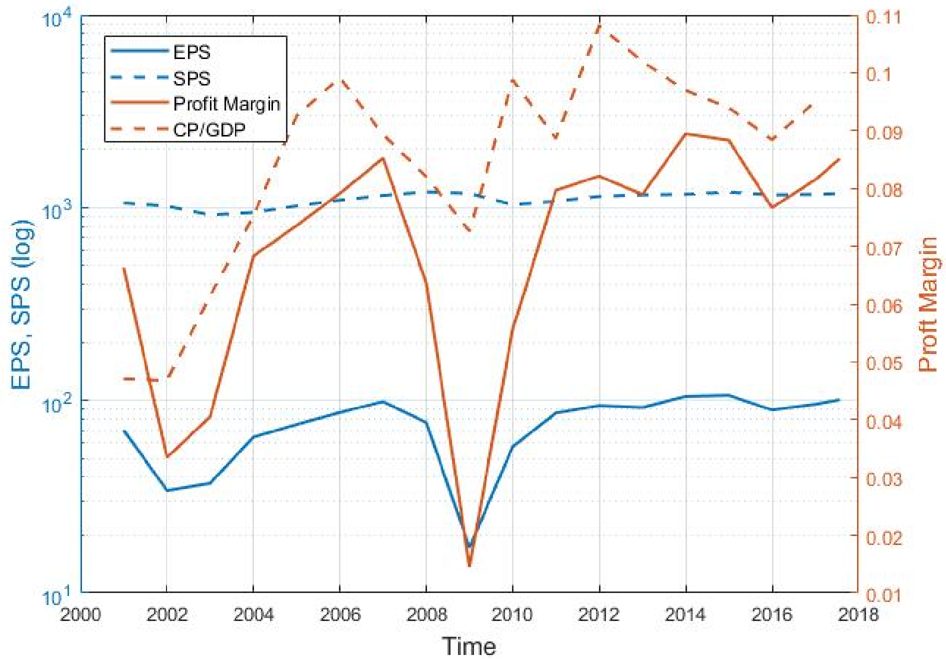

FIGURE 3. Plot of real dividends and earnings per share (log scale) for S&P 500 as well as profit margin and corporate profits as a percent of GDP (1900/01 - 2017/07, year-end annual data). *Source: www.multpl.com, Federal Reserve Bank of St. Louis, & QRG.*

4.2. **Earnings Growth Rate.** The second component in equation 2.2 to estimate is the expected (real) earnings per share (EPS) growth rate. Arguably, it is the anticipated extraordinary growth in this component of the overall return that has taken markets to their present heights in the last half a year. I will argue, using three largely unrelated approaches that the expectations for unprecedented growth in this component are unfounded. The upper range for this component in real terms over a longer investment horizon will be no higher than 1 to 2 percent.

4.2.a. *Increasing Profit Margins, Not Sales.* There are two sources of growth for the (real) EPS: increase in sales per share (SPS) and increase in profit margins. As Figure 3 demonstrates, since the market crash of 2008, almost all of the growth in EPS has come from the increase in the profit margin. Increasing earnings growth rate through expansion of profit margins, of course, is not sustainable, since most of it comes about due to reduction of real unit costs (mostly labor), which we have arguably hit, as the increasing, albeit slowly, median household income levels have suggested.[5] Another way to increase profit margins is through increases in productivity, which, as we will see shortly, is on a decades long downward trajectory.

[5]Incidentally, the record profit margins also correspond to near record of the ratio of corporate profits to the GDP (Figure 3), which reached its all-time-high in 2012, declined slightly since then, and is again on a trajectory for record highs.

To summarize, sustainable growth in EPS comes from increases in sales per share, which have been moribund at best since the Great Recession, which suggests that it is unlikely that EPS can exhibit further sustained growth rates.

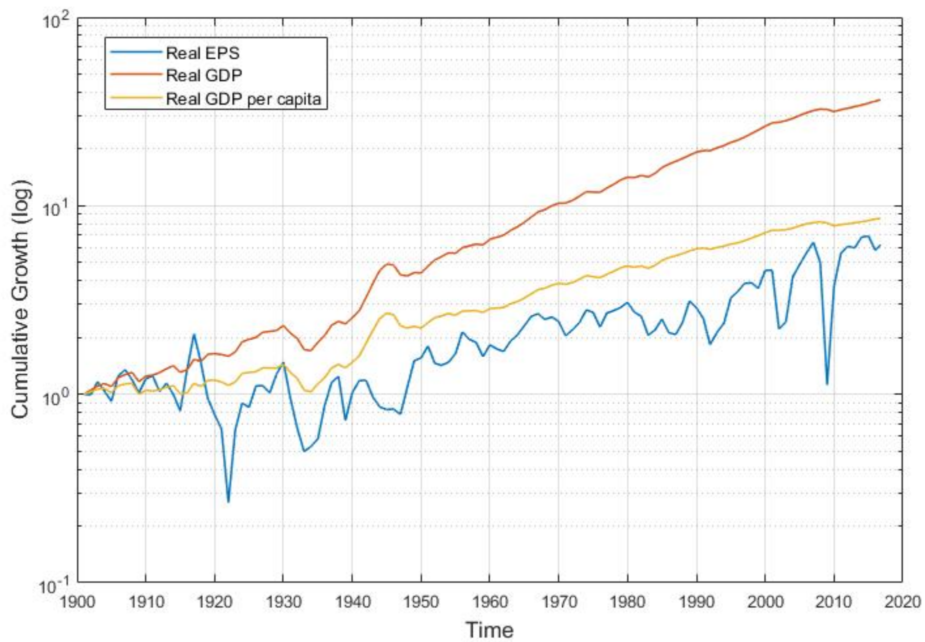

FIGURE 4. Plot of cumulative real growth rates for EPS for S&P 500, GDP, and GDP per capita (log scale) (1900/01 - 2017/07, year-end annual data). *Source: www.multpl.com & www.measuringworth.com.*

4.2.b. *Earnings Growth vs GDP Growth.* Another way to look at the likely EPS growth rates is from a macro point of view. Figure 4 plots cumulative growth rates of real GDP, real GDP per capita, and EPS since the beginning of the 20th century. The salient point here is that real GDP growth rate has far exceeded the real growth rate of GDP per capita as well as real growth rate of EPS. In fact, the cumulative real growth of EPS is much closer to that of GDP per capital rather than the GDP. The growth rate of real GDP since the beginning of the 20th century is slightly higher than 3 percent, the per capital growth rate of real GDP is around 2 percent, while the growth rate of the real EPS is slightly higher than 1.5 percent. As noted by Arnott (2011) and Ilmanen (2011), the main reason for this has to do with the fact that a lot of value generation happens in firms that are not publicly traded, which oftentimes constitute the most dynamic part of the economic growth. This growth, of course, does not get transferred to increases in EPS of publicly traded companies.

While the average growth rate of earnings has been higher than that of the GDP per capita (and the GDP) for the last 25 years, this comes solely due to the performance of EPS growth during the 1990's – a decade that saw unprecedented growth and innovation due to the Internet technology. Over the last 15 years the growth rate of EPS has become extremely

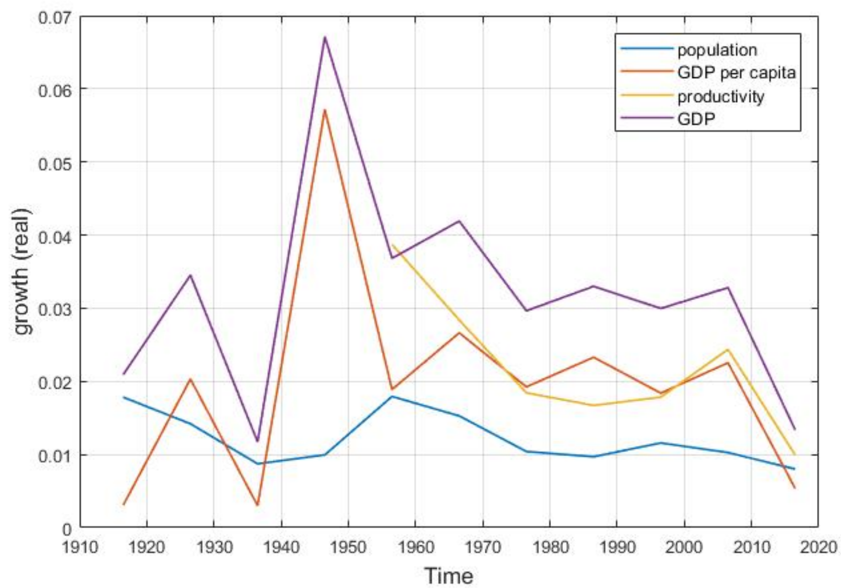

FIGURE 5. Plot of growth of real GDP, population, real GDP per capita and productivity as measured by real output per hour in the nonfinancial corporation sector (1900/01 - 2017/07, except for productivity, which starts in 1947/01; data averaged across non-overlapping and adjacent ten year intervals). *Source: www.multpl.com, www.measuringworth.com, & Federal Reserve Bank of St. Louis.*

volatile, and while the rate of EPS growth compared to its nadir in the financial crash of 2008 is impressive (about 23 percent, compared to GDP growth of 1.47 and GDP per capital growth of 0.71 percent), the growth rate of real EPS over the last ten years has been decidedly sub-par: about negative 0.3, compared to about 1.3 for the real GDP and 0.5 for the real GDP per capita.

If you still believe that the growth rate of EPS will rocket higher any minute now for a sustained long-term trajectory, consider the following historical trends in the growth of real GDP and productivity (see Figure 5), the latter of which, of course, is also a key driver in the growth of EPS, since it directly impacts the profit margin. Growth in real GDP depends on two components: growth in population and growth in productivity, where productivity, usually measured as the real output per hour worked, also closely tracks the GDP per capita. While both population growth as well as productivity have been decreasing since the middle of the 20th century, for our purposes the more important trend is the decrease in productivity.

With the exception of a productivity bump during the 1990s, the productivity growth in the US has been on decades' long downward trend not only in the United States but also in most of developed world. It recorded its first negative growth year during 2016 for the post-financial crises period. Economists and consultants the world over have offered various explanations for this puzzling fact, such as aging of the population, growing inequality of income distribution, and worsening educational performance, among others. In fact, recent study by McKinsey & Company (Manyika, Remes, Mischke, & Krishnan 2017) lists lack of business investment as one of the main reasons for slowdown in productivity. We noted this lack on investment earlier in connection with lack of growth in sales and increasing levels of cash hoarding by businesses.

Still, regardless of the source of this multi-decade long slump in productivity growth, it also, along with the longer term relationships between growth in real EPS and real GDP per capita, signals that it would be overly optimistic to expect the real EPS growth rate to be higher than the growth rate in real GDP. Currently, the most likely path for the foreseeable future for real GDP growth is in the 1 to 2 percent range, which suggests that this is also the ceiling for real EPS growth.

4.2.c. *Current Multiples as Predictors of Earnings Growth.* The third approach to using currently observed values to help us forecast real EPS growth rate relies of building a relationship between the current pricing multiples and future real EPS growth rates and returns. This approach was first used in Campbell & Shiller (2001), and their main analytical tool is reproduced in equation 6.3.

In particular, equation 6.3 shows that the current dividend yield is related (positively) to future returns and (negatively) to future dividend growth. Thus, if the current levels of dividend yields are low, they have to be related to low future returns and/or to high dividend growth levels. Campbell & Shiller (2001) carry out an empirical study of lagged dividend yields on future returns as well as on future dividend growth rates and demonstrate that current dividend yield levels have almost no relationship to future dividend growth levels, i.e., low current dividend yields do not forecast high future dividend growth levels. However, current dividend yields are strongly positively related to future return levels. Thus, current low levels of dividend yields are strongly indicative of low future returns levels rather than high future growth rates in earnings.

I have reproduced the main results of this study (see Figure 6). I measure the relationship between the current period's pricing multiple (here E/P ratio) and the subsequent returns and earnings growth.[6] The time period of study is January of 1971 to present, and the holding period (for the purposes of measuring the subsequent return and earnings growth) is ten years. However, the results are quantitatively and qualitatively similar for other time periods, holding periods, and pricing multiples. In fact, results get stronger with longer holding periods and dividend yields (rather than earnings yields) as the control variables.

To avoid any look-ahead bias, Panel 2 of Figure 6 uses as the control variable the percentile of the current earnings yield in the distribution of earnings yields observed until the particular point in time. Regardless of the approach and other specification details (e.g., time period of analysis, holding period, specific pricing multiple used as control, values of E/P or the percentiles in E/P distribution realized up until that point), the two main results hold. First, panels 1 and 2 of Figure 6 show that we have a very strong (economically and statistically significant) positive relationship between the current level of earnings yield and the future

[6] In their original study Campbell & Shiller (2001) use dividend yield (D/P) as the control variable. This is also reflected in equation 6.3. However, identical logic also applies to earnings yield's relationship to earnings growth rates and future returns. Since we are specifically interested in growth rates in real EPS (equation 2.2), I have chosen to replicate the main result in Campbell & Shiller (2001) using earnings yield rather than dividend yield, without any loss of generality.

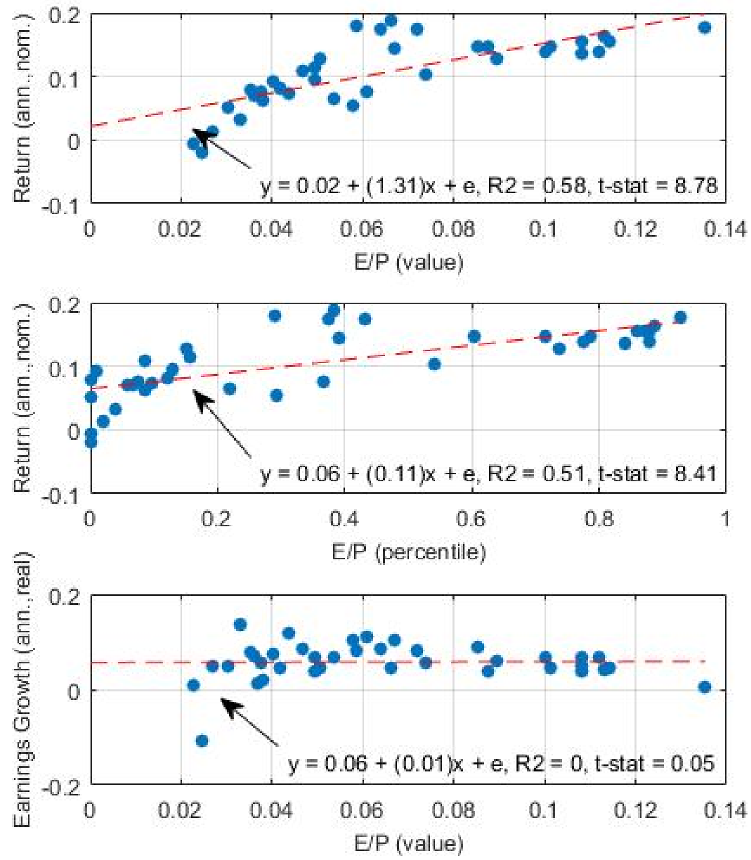

FIGURE 6. Plot of (nominal) return and (nominal) growth of EPS vs E/P (values and percentiles) for S&P 500. Time period: 1971/01 to 2017/07. For all three panels I plot the current value of the control variable (horizontal axis) vs the annualized (nominal) return or (nominal) growth of EPS for the subsequent ten years. In Panels 1 & 3 the control variable is the current E/P value, while in Panel 2 the control variable is the percentile of the current E/P value in the population of E/P values realized up until the current time period. The lines represent OLS regressions, with R2 values and values for t-stats for slope coefficients. *Source: www.multpl.com & QRG.*

rate of return. Second (see panel 3 of Figure 6), there is no detectable relationship between current earnings yields and future earnings growth rates.

To rephrase the above, *the currently low earnings yields are much more likely to signal low future returns than high future earnings growth rates.* In the light of these results, it is perhaps fitting to cite Arnott (2011, p.95), who notes the following regarding the ability of yields below historical levels to predict faster future growth rates: “With this circular logic, we might as well buy at any valuation multiple because our buying creates still higher multiples and the resulting lower yields will imply even faster future growth.” Finally, Asness (2011) comments on the intuitiveness of this result by drawing parallels between it and the better known "value effect", which applies to stocks cross-sectionally.

4.3. **Pricing Multiple Adjustment.** The final component left to be estimated in the equation 2.2 is the rate of mean reversion in pricing multiples. Empirically, this has been the hardest component to pin down, and unsurprisingly it has generated heated debate, which largely centers on the level to which the current multiple should revert.

I will not wade into this debate, but instead make a rather generous assumption that the forecast for the pricing multiple adjustment is zero. This assumption implies that the current levels of pricing multiples constitute this long term average or "center of gravity" for the pricing multiples. As a reminder, if pricing multiples are at the level of their long term averages, it implies that prices are equally likely to either increase or decrease. Most investors (and a sampling of Fed officials should also be included in this list) would probably agree that the market is "somewhat richly priced" at the moment, with the pricing multiples at the comparable levels of where they were at the end of 1997 (see Figure 7) – two years before the Internet Bubble burst.

Thus, when noticing that the current pricing multiple is indeed still not at its historical maximum and thus the long-in-tooth bull market "still has room to run", one should contemplate whether it is worth picking up these last remaining pennies in front of the oncoming steamroller. The answer, as I alluded to at the beginning, really depends on one's risk aversion.

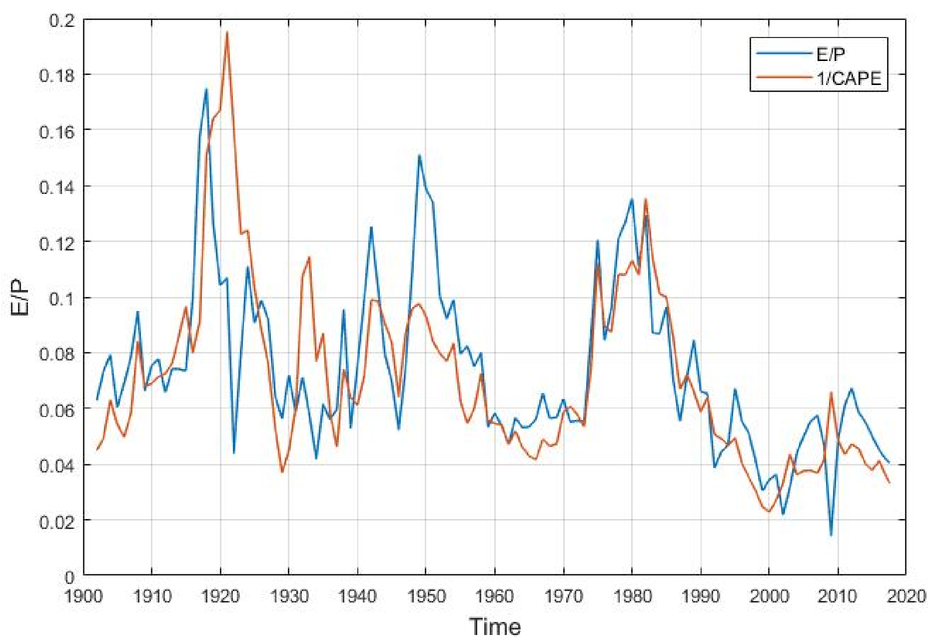

FIGURE 7. Plot of S&P 500 E/P: regular and inverse of Cyclically Adjusted PE (CAPE) (1901/01 - 2017/07, year-end annualized data). *Source: www.multpl.com.*

5. Putting It All Together

To form the overall forecast of long term rate of return for domestic equity, we need to sum up the components given in equation 2.2: dividend yield, real earnings growth, and pricing multiple adjustment. I argued that the upper range for the likely values of these components would be 2 percent, 2 percent, and 0 percent, respectively. This gives us a total of 4 percent as the estimate for real geometric rate of return for the long term domestic equity investment.

To put this rate in perspective, note that the real geometric rate of return from the beginning of the 20th century to present is slightly higher than 6 percent. The real geometric rates of return since the beginning of 1990's and our current decade are approximately 7.5 and 10.5 real geometric rates of return, respectively. Thus, the rate of return of 4 percent, which I believe is an optimistic scenario, might be a rude awakening for a lot of investors, who might have obtained their equity returns forecasts by interpolating the realized rate of return.

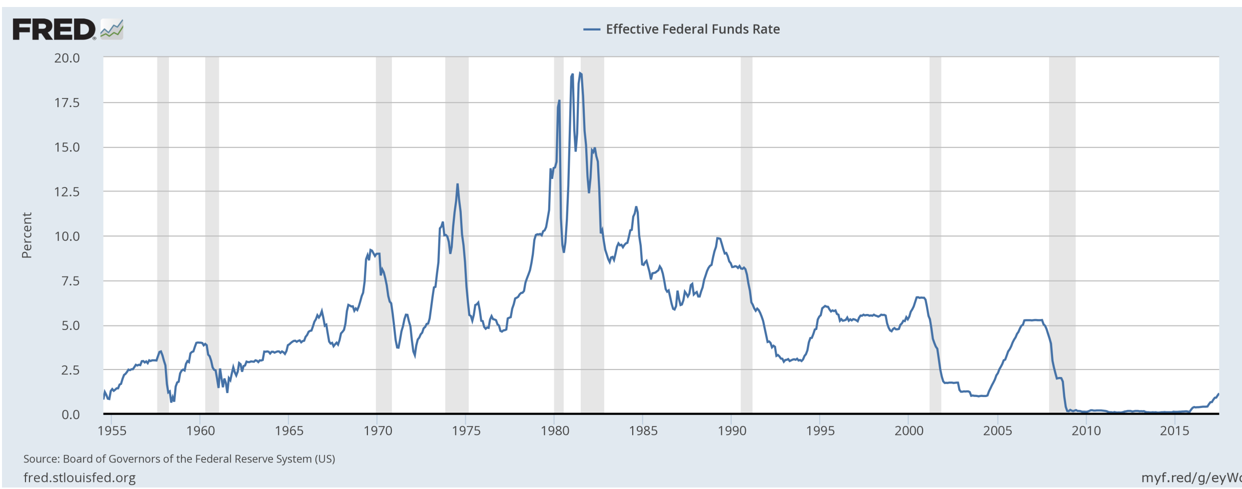

Figure 8. Plot of the effective Federal Reserve funds rate (1954/07 - 2017/07, nominal). *Source: Federal Reserve Bank of St. Louis*

One crucial component that is absent from our discussion so far is the role of the Federal Reserve. It is probably not an exaggeration to note that Fed had pivotal role in shaping the trajectory of the markets over the last two decades. Figures 8 and 9 summarize two main tools used by the Fed to affect its policies: the Federal Funds Rate and the asset purchase programs. Both of these policies result in the reduction of the rate of borrowing for the businesses, which would presumably increase the rate of investment and thus the growth rate of earnings and the economy as a whole. An expected side effect of these policies is that the prices of risky assets rise as yield starved investors abandon relatively safer short-term fixed income investments and in the process drive up the pricing multiples of all risky assets.

Figures 8 and 9 clearly illustrate the "Greenspan put" of 2001-2002, which turned into "Bernanke put" of 2008-2009, as well as the various "Quantitative Easing" rounds of 2008, 2010, and 2012. They also point to the following very important point: because of the current record low levels of the Fed's fund rate as well record high levels of asset holdings from various asset purchase programs, the Fed has a lot less ammunition to ride to the market's rescue

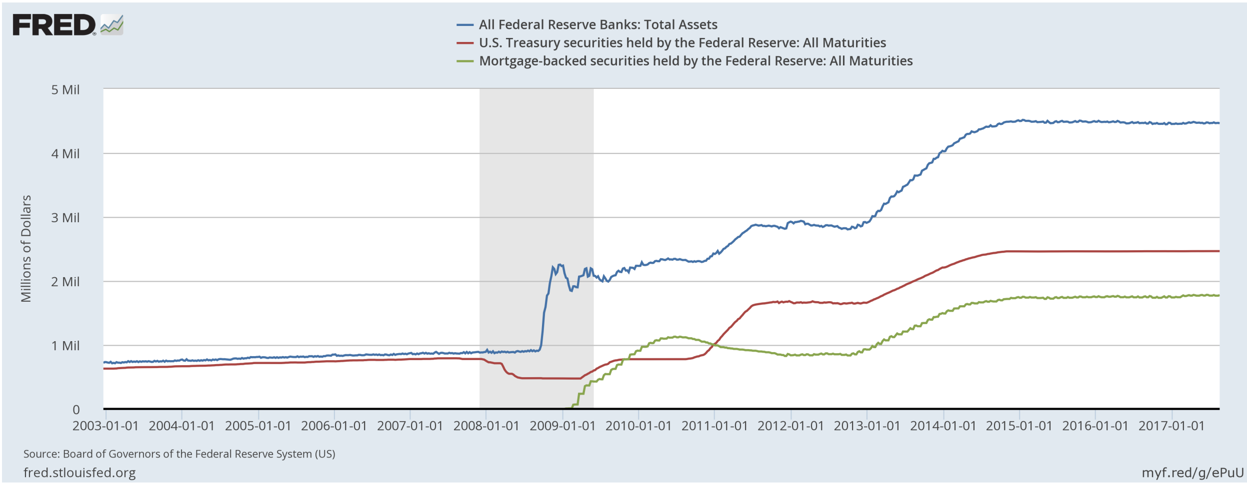

FIGURE 9. Plot of US Treasury holdings, mortgage-backed security holdings and total assets at the Federal Reserve (2003/01 - 2017/07, nominal). *Source: Federal Reserve Bank of St. Louis*

when the next market correction comes, which it inevitably will. This point is very important to account for when contemplating the risk-return tradeoff of any future investments in equity, especially in the light of the weak values for the expected dividend yield and earnings growth as well as stretched valuations that are close to their historical peaks.

I will close this note with the same thought that I expressed at the beginning, which is that ultimately the decision to buy or continue to hold equity at the current time depends on the investor's risk aversion. For some, so long as there a potential for upside, that chance is worth taking.

6. Appendix

6.1. **Decomposition of the Total Return.** In this section I give details on how to decompose the expected return into its components.[7] Let's use the following notation: R_{t+1} for the return during time period $t+1$; P_{t+1} for the price at the end of period $t+1$; D_{t+1} for the dividend paid during $t+1$, and EN_{t+1} for earnings per share experienced during the period $t+1$. Then we can decompose the return R_{t+1} in the following way

$$\begin{aligned} 1+R_{t+1} &= \left(D_{t+1}+P_{t+1}\right)/P_t \\ &= D_{t+1}/P_t + \left(P_{t+1}/P_t\right)\cdot\left(EN_{t+1}/EN_{t+1}\right)\cdot\left(EN_t/EN_t\right) \\ &= D_{t+1}/P_t + \underbrace{\left(EN_{t+1}/EN_t\right)}_{1+G_{t+1}}\cdot\underbrace{\left(P_{t+1}/EN_{t+1}\right)\cdot\left(EN_t/P_t\right)}_{1+\Delta_{t+1}} \\ &\equiv D_{t+1}/P_t + \left(1+G_{t+1}\right)\cdot\left(1+\Delta_{t+1}\right) \\ &\approx D_{t+1}/P_t + 1 + G_{t+1} + \Delta_{t+1}, \end{aligned} \tag{6.1}$$

where G_{t+1} is the net rate of change of the earnings, and Δ_{t+1} is the net rate of change of price-earnings multiple during time period t to $t+1$. Note that this decomposition is an accounting identity and not a predictive or modeling identity. This means that for any given period, we will be able to attribute the overall realized return to the above three components: dividend yield, change in earnings, and change in pricing multiples. Then, taking conditional expectations on both sides of equation 6.1 gives us

$$E_t\left(R_{t+1}\right) \approx E_t\left(D_{t+1}/P_t\right) + E_t\left(G_{t+1}\right) + E_t\left(\Delta_{t+1}\right) \tag{6.2}$$

Note that here I have assumed that all the expectations exist.

6.2. **Dynamic Gordon Growth Model.** The following section describes the Dynamic Gordon Growth model obtained in Campbell & Shiller (1988).

Let's use the same notation as above and introduce the following additional notation: $p_t \equiv \log(P_t)$, $r_t \equiv \log(R_t)$, and $d_t \equiv \log(D_t)$. The authors show that if we impose a terminal condition of $\lim_{j\to\infty}\rho^j\left(d_{t+j}-p_{t+j}\right)=0$ (with $0<\rho<1$),[8] then the current log dividend yield can be decomposed as follows:

$$d_t - p_t = -\frac{k}{1-\rho} + E_t\left(\sum_{j=0}^{\infty}\rho^j\left(-\Delta d_{t+1+j}+r_{t+1+j}\right)\right), \tag{6.3}$$

where k is a constant. Thus, equation 6.3 tells us that current dividend yield is positively related to future price returns and negatively related to future dividend growth rates. In other words, low current dividend yields can be expected to lead to either higher future dividend growth and/or lower future price returns.

[7] This is by no means the only way to carry out this decomposition, but this simple approach will be sufficient for illustrating the main points of this note.

[8] This follows from assuming that $d_{t+j}-p_{t+j}$ is stationary, although this assumption is much less restrictive than assuming the stationarity of $d_{t+j}-p_{t+j}$. For example, if we assumed that $d_{t+j}-p_{t+j}$ follows random walk (a non-stationary series), this assumption would be satisfied. It basically prohibits the series $d_{t+j}-p_{t+j}$ from increasing exponentially at a rate that is greater than the inverse of ρ^j, as $j\to\infty$.

Note that if we assume that future expected returns are constant (i.e., $E_t(r_{t+j}) = r, \forall j > 0$) and that the future dividend growth is constant (i.e., $E_t(\Delta d_{t+j}) = g, \forall j > 0$), then equation 6.3 collapses to

$$d_t - p_t \quad = \quad -\frac{k}{1-\rho} + \frac{1}{1-\rho}\left(r - g\right), \tag{6.4}$$

which is the log-linearized (constants k and ρ come from log-linearization of the original Gordon Growth Model) equivalent of its more famous Gordon Growth Model cousin: $D_t/P_t = R - G$, with $E_t(R_{t+j}) = R$, $\forall j > 0$ and $E_t(D_{t+j+1}/D_{t+j}) = 1 + G$, $\forall j > 0$. Thus, equation 6.3 is a significant generalization of the traditional Gordon Growth Model and allows for variable future expected rates of return as well as dividend growth rates.

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