ARO 4090 - WEEK 14

- M. MAGGIA

Lecture 22

ACTIVE SPACECRAFT ATTITUDE CONTROL

- ATTITUDE ACCURACY THAT CAN BE OBTAINED WITH PASSIVE MEANS OF STABILIZATION (SINGLE-SPIN, DUAL-SPIN, GRAVITY-GRADIENT TORQUE) IS NOT VERY HIGH. THE ATTITUDE CAN DETERIORATE OVER TIME BECAUSE OF DISTURBANCE TORQUES.

- TO OVERCOME THE LIMITATIONS OF PASSIVE STABILIZATION WE NEED TO RESORT TO ACTIVE ATTITUDE CONTROL

- PASSIVE STABILIZATION IS STILL IMPORTANT
  - ↳ ACTIVE CONTROL DOES NOT NEED TO WORK AS HARD TO MAINTAIN REQUIRED ATTITUDE
  - ↳ IF ACTIVE CONTROL FAILS, THE ATTITUDE REMAINS STABLE.

- ACTIVE CONTROLLED S/C's REQUIRE:

  - SENSORS: RELATIVE VS ABSOLUTE ATTITUDE SENSORS

RELATIVE → GYROSCOPES (IN IMUs)
- PROVIDES ROTATION RATES/ANGULAR POSITION W.R.T AN INITIAL ATTITUDE
- NEED ABSOLUTE SENSORS TO CLEAR ACCUMULATED ERRORS
- USED TO RETRIEVE $\vec{\omega}_B^{B/I}$

ABSOLUTE

↳ HORIZON/EARTH SENSOR
- PROVIDES ORIENTATION W.R.T EARTH ABOUT 2 ORTHOGONAL AXES (PITCH & ROLL)
- DETECTS LIGHT FROM HORIZON (FROM LIMB OF ATMOSPHERE) USING IR CAMERAS.
- CAN BE COUPLED WITH A GYRO FOR YAW MEASUREMENTS.
- $\sim 0.05°$ GEO, $\sim 0.01°$ LEO ACCURACY.

↳ SUN SENSOR
- USE PHOTO CELLS TO DETECT POSITION OF SUN (VOLTAGE PROP. TO ANGLE OF INCIDENCE OF SUN)
- AT LEAST 3 PHOTO CELLS ARE NEEDED
- 0.01° ACCURACY.
- FIELD OF VIEW $\pm 30°$

↳ MAGNETOMETER

- USED TO DETERMINE THE POSITION OF THE EARTH MAGNETIC FIELD
- NOT THAT ACCURATE ($\sim 5°$ AL LEO, $\sim 1°$ AT MEO)

↳ STAR SENSOR (STAR TRACKER)

- DETERMINES POSITION OF KNOWN FIXED STARS USING PHOTO CELLS OR CAMERAS.
- MOST ACCURATE ($<$ 2 arc sec)
- FIELD OF VIEW $\pm 6°$

WHAT TO DO WITH THE SENSOR MEASUREMENTS?

Example: $\hat{s}_{\mathcal{B}}$: DIRECTION OF SUN IN $\mathcal{B}$-RF COORDS (FROM SUN SENSOR)

$\hat{s}_{\mathcal{I}}$: DIRECTION OF SUN IN $\mathcal{I}$-RF COORDS. (FROM ORBIT DETERMINATION)

$\hat{m}_{\mathcal{B}}$: DIRECTION OF MAGN. FIELD IN $\mathcal{B}$-RF COORDS (FROM MAGNETOMETER)

$\hat{m}_{\mathcal{I}}$: DIRECTION OF MAGN. FIELD IN $\mathcal{I}$-RF COORDS (FROM ORBIT DETERMINATION)

$$\left.\begin{aligned} \hat{s}_{\mathcal{B}} &= R_{\mathcal{B}\mathcal{I}}\, \hat{s}_{\mathcal{I}} \\ \hat{m}_{\mathcal{B}} &= R_{\mathcal{B}\mathcal{I}}\, \hat{m}_{\mathcal{I}} \end{aligned}\right\} \Rightarrow \text{ENOUGH TO FIND } R_{\mathcal{B}\mathcal{I}} \text{ (see HW \#1)}$$

$$R_{\mathcal{B}\mathcal{I}} \downarrow \psi, \theta, \phi$$

- ACTUATORS:

↳ THRUSTERS (ATTITUDE CONTROL THRUSTERS, ACT)

- MOST COMMON ARE VERNIER THRUSTERS
- USE FUEL (LIMITED LIFE)
- AT LEAST 4 PER AXIS ARE REQUIRED (TOTAL OF 12)
- THEY ARE FIRED IN ONE DIRECTION TO INITIATE THE ROTATION AND THEN IN THE OPPOSITE WHEN THE SOUGHT ATTITUDE IS ACHIEVED

↳ REACTION / MOMENTUM WHEELS

- ELECTRIC MOTOR - DRIVEN ROTORS
- PRECISE ORIENTATION FOR S/C
- MADE TO SPIN IN THE DIRECTION OPPOSITE TO THAT REQUIRED TO RE-ORIENT THE VEHICLE
- WHEN THEY REACH THEIR MAX ANG. SPEED (SATURATION) THEY NEED TO BE DE-SATURATED (NORMALLY USING ACT)
- YOU NEED A MINIMUM OF 3 RWs (NORMALLY THERE ARE 4 FOR REDUNDANCY, MAY BE MOUNTED IN A PYRAMIDAL CONFIGURATION)

↳ CONTROL MOMENT GYROSCOPE (CMG)

- IT'S A MOMENTUM WHEEL SPINNING AT HIGH ROTATIONAL SPEED MOUNTED ON GIMBALS THAT ALLOW THE CHANGE OF THE DIRECTION OF THE ROTATIONAL AXIS OF THE WHEEL.
- WELL SUITED FOR LARGE S/C
- MORE COMPLEX THAN RWs.

↳ OTHERS

- MAGNETIC TORQUERS
- SOLAR SAILS

⚠ MOMENTUM WHEEL : SPINS AT HIGH ANGULAR VELOCITY TO PROVIDE STABILIZATION (MOMENTUM BIAS, see Lectures ON DUAL-SPIN)

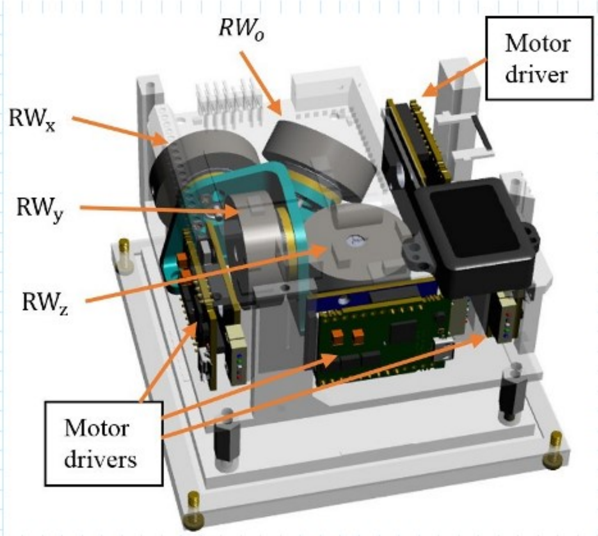
REACTION WHEEL : THEY NEED NOT SPIN AT HIGH RPM. THEIR ANG. SPEED IS CHANGED WHEN A DIFFERENT ATTITUDE NEEDS TO BE ACHIEVED. ROTATION AXIS DIRECTION IS FIXED; MOMENTUM IS CHANGED BY CHANGING THE ROTATION SPEEDS.

CONTROL MOM. GYROS : FIXED ROTATION RATE; MOMENTUM IS CHANGED BY CHANGING THE DIRECTION OF THE ROTATION AXIS.

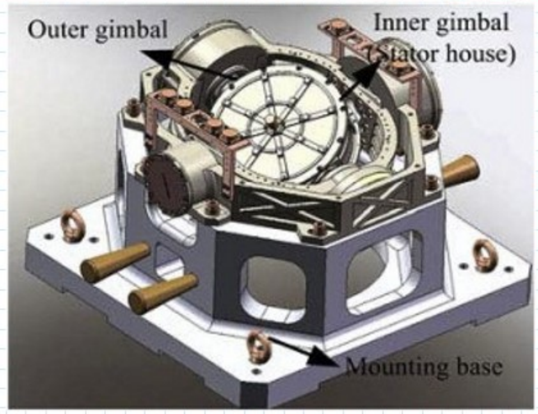
★ USUALLY A S/C USES A COMBINATION OF SENSORS & ACTUATORS FOR REDUNDANCY ★

4 REACTION WHEELS

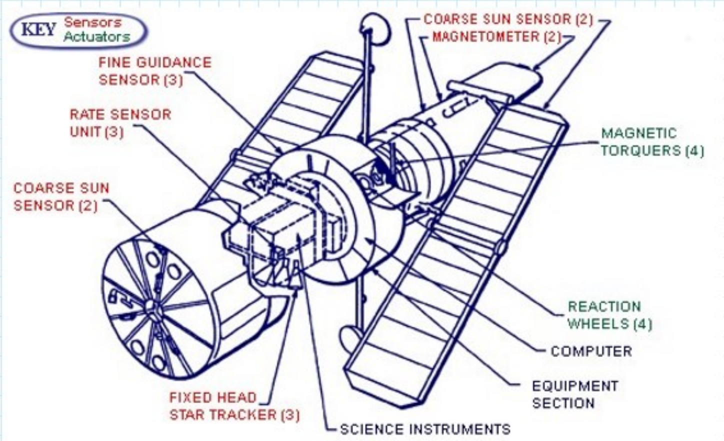
CONTROL MOMENT GYROSCOPE



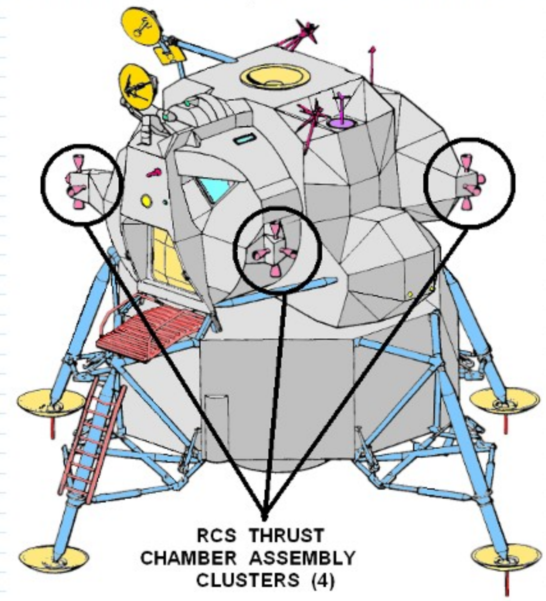
CONTROL MOMENT GYROSCOPE



HUBBLE TELESCOPE EXAMPLE:



REACTION CONTROL SYSTEM (RCS) THRUSTERS



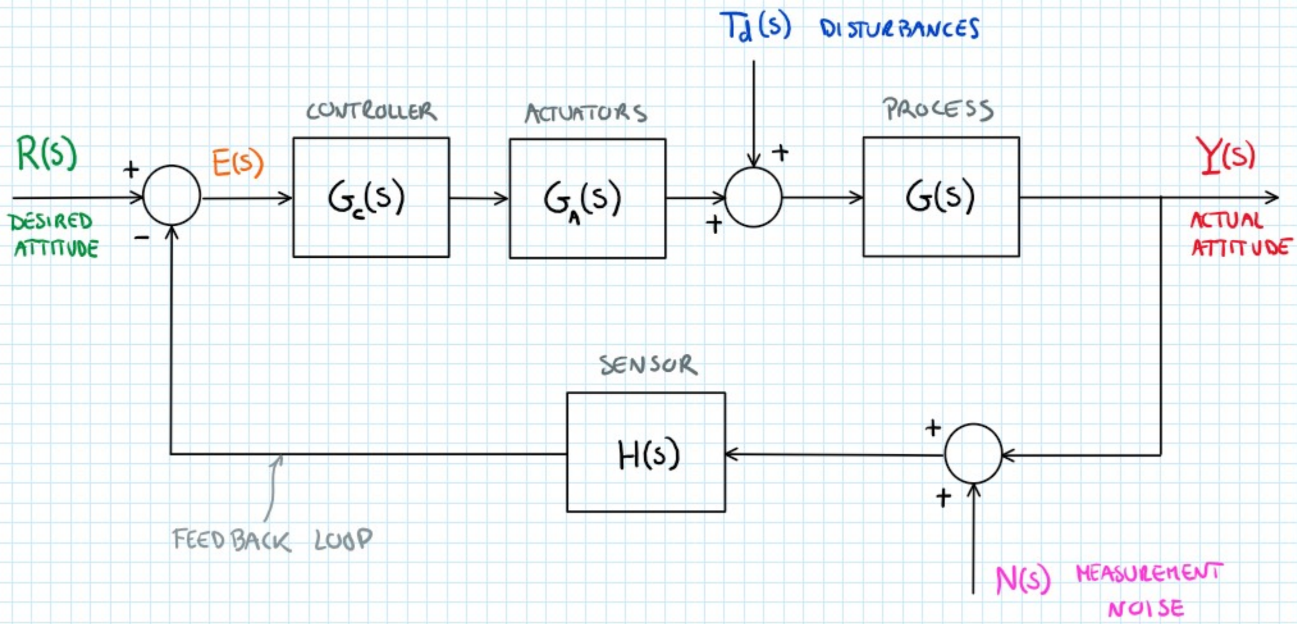
- PROCESSOR/CONTROLLER

  ↳ COMPUTER THAT COMPARES ACTUAL S/C ATTITUDE WITH DESIRED S/C ATTITUDE AND CONSEQUENTLY GENERATES THE CONTROL SIGNAL TO BE SENT TO THE ACTUATORS.

$$I^c \cdot \dot{\vec{\omega}}^{B/N} + [\vec{\omega}^{B/N}]_\times I^c \vec{\omega}^{B/N} = \underbrace{\vec{M}^c_c(t)}_{\text{CONTROL TORQUE}} + \underbrace{\vec{M}^c_d(t)}_{\text{DISTURBANCES TORQUE}}$$

CLOSED-LOOP CONTROL SYSTEMS (FEEDBACK CONTROL)

ASSUMPTION : SYSTEM LINEARIZED ABOUT AN EQUILIBRIUM POINT (e.g. $\theta = \psi = \phi = 0$) SO WE CAN USE THE LAPLACE TRANSFORM AND APPLY THE LINEAR CONTROL ANALYSIS.



$R(s)$: REFERENCE (DESIRED) ATTITUDE

$Y(s)$: ACTUAL ATTITUDE

$G_c(s)$: CONTROLLER TRANSFER FUNCTION

$G_A(s)$: ACTUATOR TRANSFER FUNCTION

$G(s)$: PROCESS TRANSFER FUNCTION (S/C DYNAMICS)

$H(s)$: SENSOR TRANSFER FUNCTION

$T_d(s)$: EXTERNAL DISTURBANCES

$N(s)$: MEASUREMENT NOISE

THE GOAL OF A FEEDBACK CONTROL SYSTEM IS TO MAKE THE ACTUAL OUTPUT EQUAL TO THE DESIRED ONE.

$$\text{GOAL:} \quad R(s) - Y(s) = 0$$

HOWEVER, IN REALITY, WE CANNOT DIRECTLY COMPARE $R(s)$ AND $Y(s)$ DUE TO NOISES INTRODUCED WITH THE MEASUREMENTS AND B/C OF THE DYNAMICS ASSOCIATED WITH

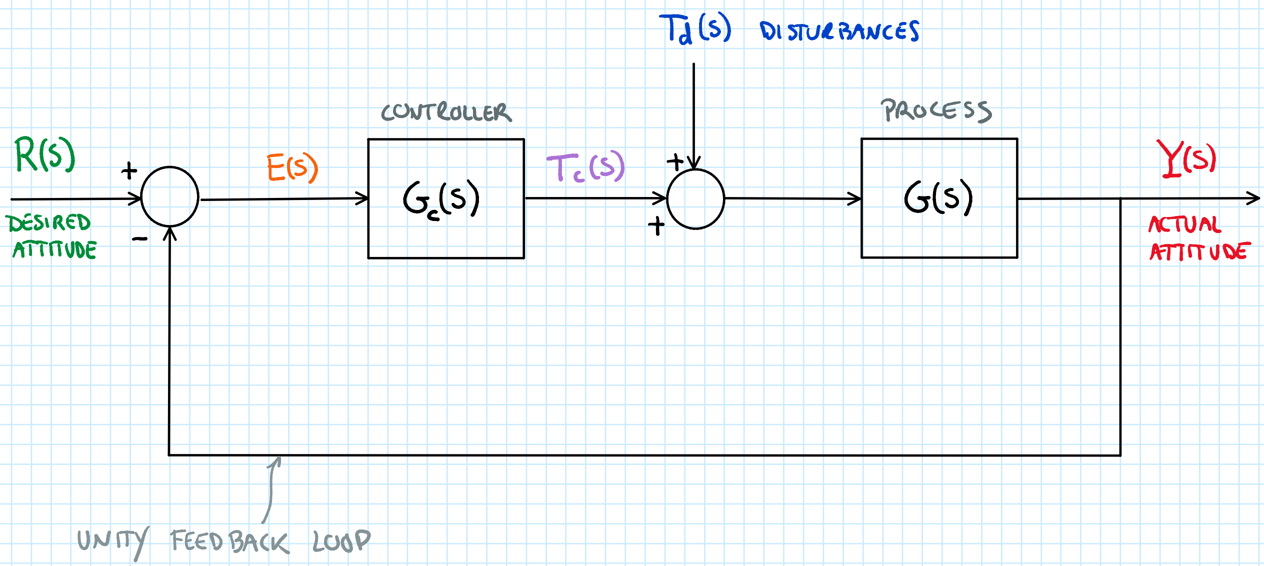
HOWEVER, IN REALITY, WE CANNOT DIRECTLY COMPARE $R(s)$ AND $Y(s)$ DUE TO NOISES INTRODUCED WITH THE MEASUREMENTS AND B/C OF THE DYNAMICS ASSOCIATED WITH THE SENSORS THEMSELVES $(H(s))$

PRACTICALLY WE WILL DRIVE

$$E(s) = R(s) - H(s)\left(N(s) + Y(s)\right) \qquad \text{(ERROR FUNCTION)}$$

TO ZERO

ASSUMPTIONS :
- • NO MEASUREMENT NOISE $(N(s) = 0)$
- •• IDEAL SENSORS $(H(s) = 1 \rightarrow$ UNITY FEEDBACK LOOP$)$
- ••• IDEAL ACTUATORS $(G_A(s) = 1)$



WITH THE ASSUMPTIONS ABOVE :

$$E(s) = R(s) - Y(s)$$

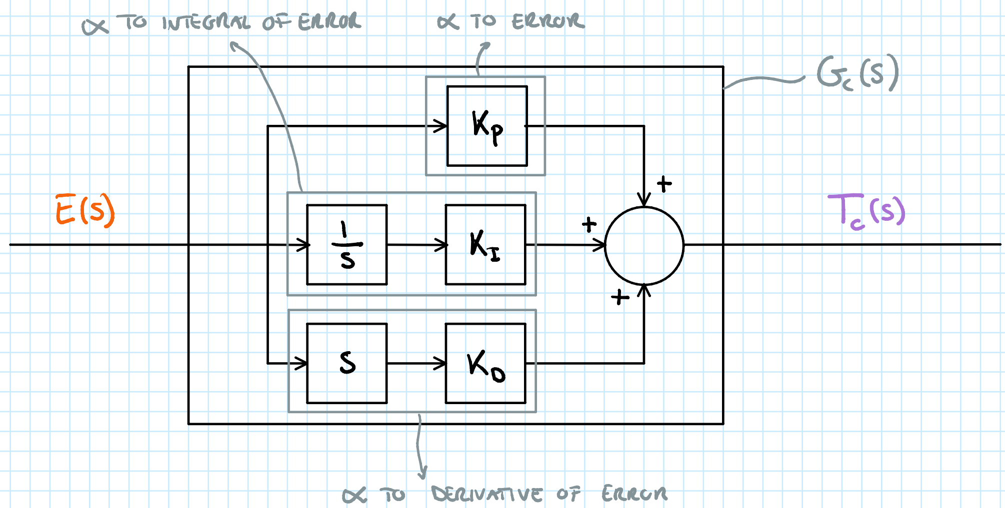
THE CONTROL TORQUE $T_c(s)$ IS GIVEN BY :

$$T_c(s) = E(s) \cdot \boxed{G_c(s)}$$

CONTROL LAW

THE CONTROL LAW CAN BE DESIGNED IN MANY WAYS (e.g. COMPENSATORS, PID CONTROLLERS...)

LET'S CONSIDER A PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROLLER



$$T_c(s) = G_c(s) \cdot E(s)$$

$$= \left(K_p + sK_D + \frac{K_I}{s}\right) E(s)$$

$$= K_p E(s) + K_D\, sE(s) + K_I \frac{E(s)}{s}$$

INTEGRAL OF ERROR IN LAPLACE DOMAIN

DERIVATIVE OF ERROR IN LAPLACE DOMAIN

IN TIME DOMAIN:

$$e(t) = r(t) - y(t)$$

$$M_c(t) = K_p e(t) + K_d \frac{de(t)}{dt} + K_I \int_0^t e(t)\, dt$$

$\mathcal{L}$

$$T_c(s) = K_p E(s) + K_d\, sE(s) + K_I \frac{E(s)}{s}$$ (ZERO I.C.'s)

CLOSED-LOOP TRANSFER FUNCTION $T(s)$

$$T(s) := \frac{Y(s)}{R(s)}$$ when other inputs are null (i.e., $T_d(s) = 0$)

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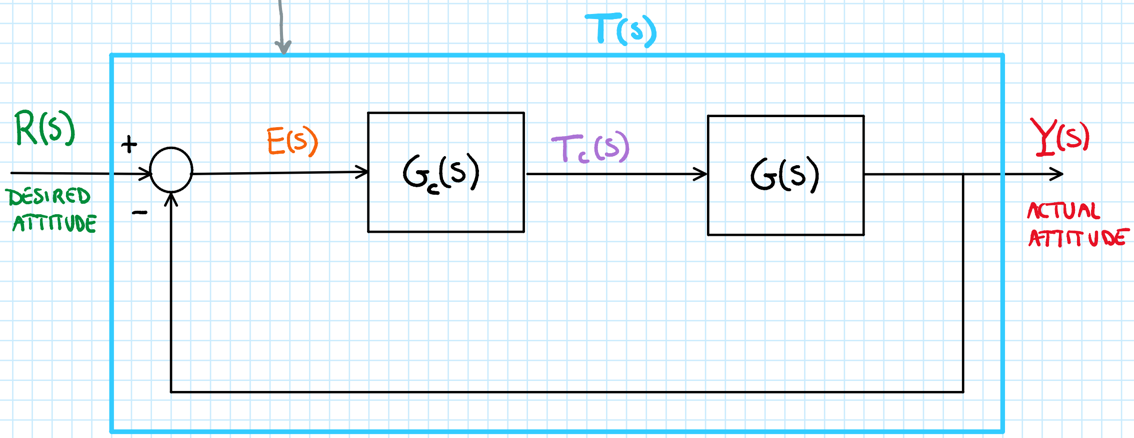


$$
\begin{aligned}
T(s) &= \frac{Y(s)}{R(s)} \\
&= \frac{E(s) \cdot G_c(s) \cdot G(s)}{R(s)} \\
&= \frac{\big(R(s) - Y(s)\big)\, G_c(s)\, G(s)}{R(s)} \\
&= \big[1 - T(s)\big]\, G_c(s)\, G(s)
\end{aligned}
$$

$$T(s) = G_c(s)\,G(s) - T(s)\,G_c(s)\,G(s)$$

$$T(s)\big[1 + G_c(s)\,G(s)\big] = G_c(s)\,G(s)$$

$$\boxed{T(s) = \frac{G_c(s)\,G(s)}{1 + G(s)\,G_c(s)}}$$

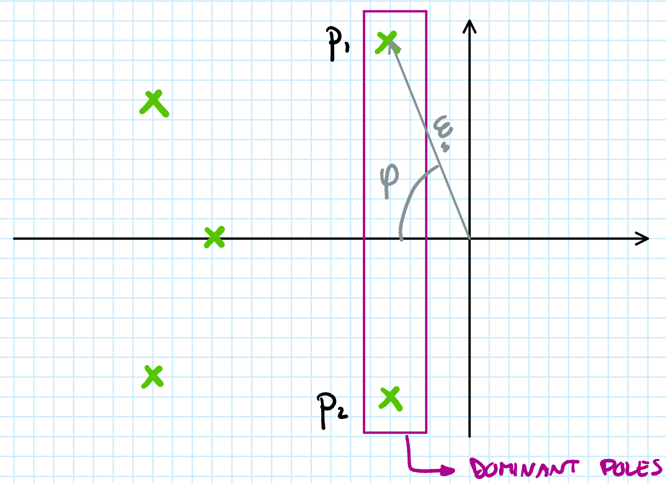
ZEROS OF $T(s)$: s s.t. $G_c(s) \cdot G(s) = 0$

POLES OF $T(s)$: s s.t. $1 + G_c(s)\,G(s) = 0$

NECESSARY & SUFFICIENT CONDITION FOR CLOSED LOOP STABILITY IS THAT THE POLES OF $T(s)$ MUST HAVE NEGATIVE REAL PART (LIE ON LHS OF COMPLEX PLANE)

- $T(s)$ MIGHT HAVE MORE THAN 2 POLES. HOWEVER, THE SYSTEM RESPONSE GENERALLY FOLLOWS THE BEHAVIOR DICTATED BY THE 2 POLES THAT ARE CLOSEST TO THE IMAGINARY AXIS. THESE POLES ARE KNOWN AS **DOMINANT**

e.g.



$\zeta = \cos\varphi$

$$(s-p_1)\cdot(s-p_2) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

where $\omega_n = \sqrt{Re(p_1)^2 + Im(p_1)^2}$ (NAT. FREQ.)

$\zeta = \operatorname{atan}\left(\frac{Im(p_1)}{Re(p_1)}\right)$ (DAMPING RATIO)

IF $\zeta > 1 \rightarrow$ OVERDAMPED SYSTEM

$\zeta = 1 \rightarrow$ CRITICALLY DAMPED SYSTEM

$0 < \zeta < 1 \rightarrow$ UNDERDAMPED SYSTEM

$\zeta = 0 \rightarrow$ UNDAMPED SYSTEM (WE DON'T WANT THIS FOR A. STABILITY)

★ RULE OF THUMB: IF ALL THE REMAINING POLES ARE WELL ENOUGH TO THE LEFT OF THE DOMINANT POLES, THE SYSTEM RESPONSE CAN BE APPROXIMATED WITH THE RESPONSE OF A 2nd-ORDER SYSTEM WITH NATURAL FREQUENCY ω_n AND DAMPING RATIO ζ.

ω_n, ζ ARE ASSOCIATED WITH : PERCENT OVERSHOOT (P.O.)

SETTLING TIME (T_S)

RISE TIME (T_R)

$$P.O. = 100\, e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$T_R = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \qquad (\text{FOR } \zeta < 1)$$

$$T_S \simeq \frac{4}{\zeta\omega_n} \qquad (2\% \text{ CRITERION})$$

$\left\{ \begin{array}{l} t < T_S \ : \text{TRANSIENT} \\ t > T_S : \sim \text{STEADY-STATE} \end{array} \right\}$

ANOTHER IMPORTANT MEASURE OF THE PERFORMANCES IS THE STEADY-STATE ERROR

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) \qquad (\text{FINAL VALUE THEOREM})$$

USUALLY WE HAVE A REQUIREMENT SO THAT $e_{ss} < e_{max}$