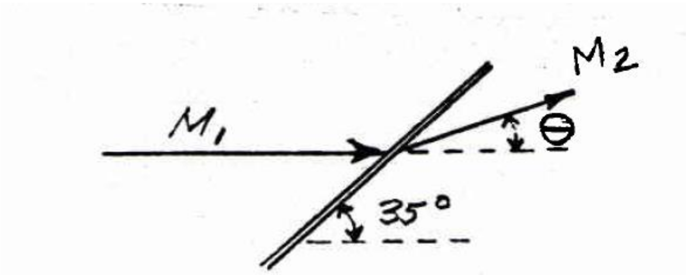
CHAPTER 4

4.1



$a_1 = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(520)}$

$a_1 = 1118$ ft/sec

$M_1 = v_1/a_1 = 3355/1118 = 3.0$

$M_{n_1} = M_1 \sin \beta = 3.0 \sin (35)° = 1.72$

From Table A.l.

$P_2/p_1 = 3.285$

$T_2/T_1 = 1.473$

$M_{n_2} = 0.6355$

Hence:

$p_2 = 3.285\ (2000) = \boxed{6570 \text{ lb/ft}^2}$

$T_2 = 1.473\ (520) = \boxed{766°R}$

$M_{t_1} = M_1 \cos \beta = 3 \cos (35°) = 2.457$

$w_1 = 2.457\ (1118) = 2747$ ft/sec $= w_2$

$a_2 = \sqrt{\gamma RT_2} = \sqrt{(1.4)(1718)(766)} = 1357.3$ ft/sec

$u_2 = M_{n_2}\ a_2 = 0.6355\ (1357.3) = 862.6$ ft/sec

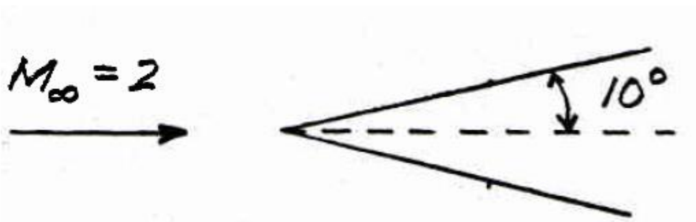
$\tan(\beta - \theta) = u_2/w_2 = 862.6/2747 = 0.314$

$\beta - \theta = 17.4°$

$\theta = \beta - 17.4 = 35 – 17.4 = \boxed{17.6°}$

$V_2 = \sqrt{u_2^{\ 2} + w_2^{\ 2}} = \sqrt{(862.6)^2 + (2747)^2} = \boxed{2879 \text{ ft/sec}}$

4.2



From the θ-β-M diagram: $\beta = 39.3°$

$M_{n_1} = M_1 \sin\beta = 2 \sin 39.3° = 1.2668$

From Table A.2: $p_{o_2} / p_{o_1} = \boxed{0.9842}$

4.3 From the θ-β-M diagram, at $M_1 = 3$, $\theta_{max} = 34.1°$ and $\beta = 66°$. Hence, the wedge can be no larger than a half-angle of 34.1°.

$M_{n_1} = M_1 \sin\beta = (3.0) \sin 66° = 2.74$

From Table A.2: $p_2/p_1 = 8.592$

$p_1 = 1.01 \times 10^5$ N/m_2 (standard sea level pressure)

$$p_2 = \frac{p_2}{p_1} p_1 = 8.592\,(1.01 \times 10^5) = 8.678 \times 10^5 \ \frac{N}{m^2}$$

This is the maximum pressure. Any higher pressure would correspond to a wedge half-angle > θ_{max}, the shock would be detached.

4.4 $\frac{p_2}{p_1} = 5$. From Table A.2, $M_{n_1} = 2.2$, $M_{n_1} = M_1 \sin\beta$. Thus,

$$\sin\beta = \frac{M_{n_1}}{M_1} = \frac{2.2}{3.5}, \quad \beta = 38.94°$$

From the θ-β-M diagram, for $M_1 = 3.5$ and $\beta = 38.94°$,

$$\boxed{\theta = 23.7°}$$

4.5 From the θ-β-M diagram, $\boxed{\beta = 38°}$

$$M_{n_1} = M_1 \sin\beta = (3.0) \sin 38° = 1.85$$

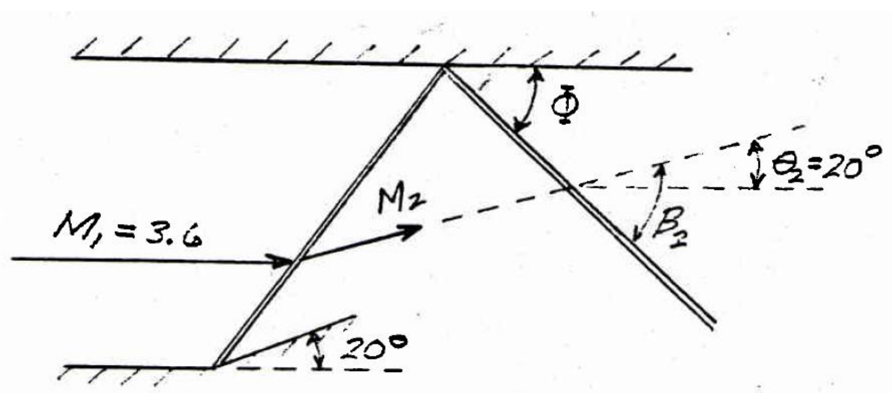
From Table A.2, for $M_{n_1} = 1.85$; $p_2/p_1 = 3.826$, $T_2/T_1 = 1.569$ and $M_{n_2} = 0.6057$

$$P_2 = 3.826\ p_1 = 3.826\ (2116) = \boxed{8096\ \text{lb/ft}^2}$$

$$T_2 = 1.569\ T_1 = 1.569\ (519) = \boxed{814°\text{R}}$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.6057}{\sin(38 - 20)} = \boxed{1.96}$$

4.6



From the θ-β-M diagram, $\beta_1 = 34°$

$$M_{n_1} = M_1 \sin 34° = (3.6)(0.559) = 2.01$$

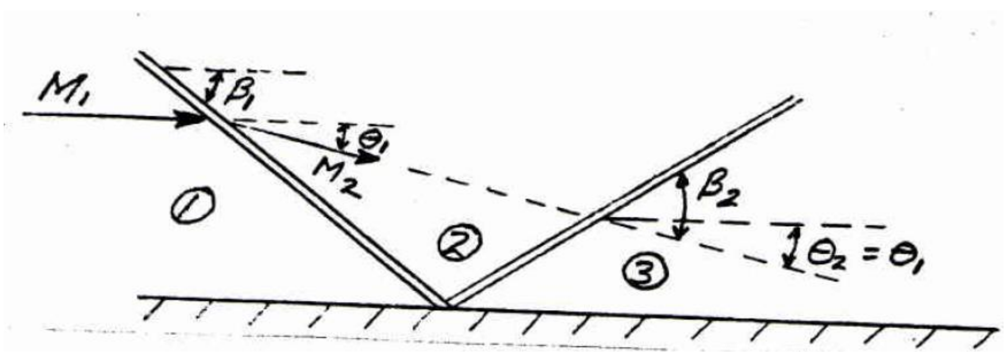
From Table A.2: $M_{n_2} = 0.5752$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.5752}{\sin(34 - 20)} = 2.378 \approx 2.38$$

From θ-β-M diagram, for $M_2 = 2.38$ and $\theta_2 = 20°$, $\beta_2 = 4.45°$. Hence, $\phi = \beta_2 - \theta_2 = 44.5 - 20$

$\boxed{\phi = 24.5°}$

4.7



From the θ-β-M diagram: For $M_1 = 2.8$ and $\beta = 30°$, $\theta_1 = 11°$,

$$M_{n_1} = M_1 \sin 30° = 2.8\ (0.5) – 1.4$$

From Table A.2: $M_{n_2} = 1.4$: $p_2/p_1 = 2.12$, $T_2/T_1 = 1.255$, $p_{o_2} / p_{o_1} = 0.9582$, $M_{n_2} = 0.7297$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.7397}{\sin(30 - 11)} = 2.272$$

From θ-β-M diagram: For $M_2 = 2.272$ and $\theta_2 = 11°$, $\beta_2 = 35.5°$. For the reflected shock,

designate the upstream normal Mach number as $M_{n_2}{}'$.

$$M_{n_2}{}' = M_2 \sin \beta_2 = 2.272 \sin (35.5°) = 1.319$$

From Table A.2 for $M_{n_2}{}' = 1.319$; $p_3/p_2 = 1.866$, $T_3/T_2 = 1.204$, $p_{o_3} / p_{o_2} = 0.9758$, $M_{n_3} = 0.776$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta_2)} = \frac{0.776}{\sin(35.5 - 11)} = \boxed{1.87}$$

$$p_3 = \frac{p_3}{p_2}\frac{p_2}{p_1} p_1 = (1.866)(2.12)(1) = \boxed{3.956 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1} T_1 = (1.204)(1.255)(300) = \boxed{453.3°\text{K}}$$

From Table A.1 for $M_1 = 2.8$, $p_{o_1}/p_1 = 27.14$

$$p_{o_3} = \frac{p_{o_3}}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} = (0.9758)(0.9582)(27.14)(1)$$

$$p_{o_3} = \boxed{25.38 \text{ atm}}$$

As a check, p_{o_3} can be calculated more directly from Table A.1 where for $M_3 = 1.87$,

$$p_{o_3} / p_3 = 6.398$$

$$p_{o_3} = \frac{p_{o_3}}{p_3} p_3 = (6.398)(3.956) = 25.1 \text{ atm}$$

Clean enough within roundoff errors.

4.8 (a) From Table A.2, for $M_1 = 4.0$, $\frac{p_{o_2}}{p_{o_1}} = 0.1388$. From Table A.1, for $M_1 = 4.0$,

$\frac{p_{o_1}}{p_1}$ 151.8. Hence,

$$p_{o_2} = \frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} p_1 = (0.13880(151.8)(1)$$

$$p_{o_2} = 21.07 \text{ atm}$$

(b) $M_{n_1} = M_1 \sin\beta = 4 \text{ Sin } 40° = 2.57$

From Table A.2 for M_{n_1} = 2.57: $\frac{p_{o_2}}{p_{o_1}}$ = 0.4715 and M_{n_2} = 0.5065.

From the θ-β-M diagram, for M_1 = 4 and β = 40: θ = 26.3°

$$M_2 = \frac{M_{n_2}}{\mathrm{Sin}(\beta - \theta)} = \frac{0.5065}{\mathrm{Sin}(40 - 26.3)} = 2.139$$

From Table A.2, for M_2 = 2.139, $\frac{p_{o_3}}{p_{o_2}}$ = 0.6557

$$p_{o_3} = \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.6557)(0.4715)(151.8)(1)$$

$$p_{o_3} = 46.93 \text{ atm}$$

Note that the total pressure in case (b) is higher than case (a), indicating a more efficient shock compression to subsonic flow for case (b). The upstream total pressure is p_{o_1} = (151.8)(1) = 151.8 atm.

For case (a) the loss is total pressure = 151.8 – 21.07 atm = 130.7 atm.

For case (b) the loss in total pressure = 151.8 – 46.93 = 104.9 atm.

Hence, case (b) experiences a smaller loss in p_o, and therefore is more efficient. This is generally true. Oblique shock inlets on jet engines are more effective devices than the older normal shock inlets.

4.9 From the θ-β-M diagram, and Figure 4.22: For θ_2 = 20° and M_1 = 3, β = 37.8°.

$$M_{n_1} = M_1 = \sin\beta = (3)\sin(37.8) = 1.839; \quad \frac{p_2}{p_1} = 3.783$$

$$M_{n_2} = 0.6078; \quad M_2 = \frac{M_{n_2}}{\mathrm{Sin}(\beta - \theta)} = \frac{0.6078}{\mathrm{Sin}(37.8 - 20)} = 1.99$$

For $\theta_3 = 15°$ and $M_1 = 3$, $\beta = 32.2$

$$M_{n_1} = M_1 \sin\beta = (3)\sin(32.2) = 1.60; \quad \frac{p_3}{p_1} = 2.82$$

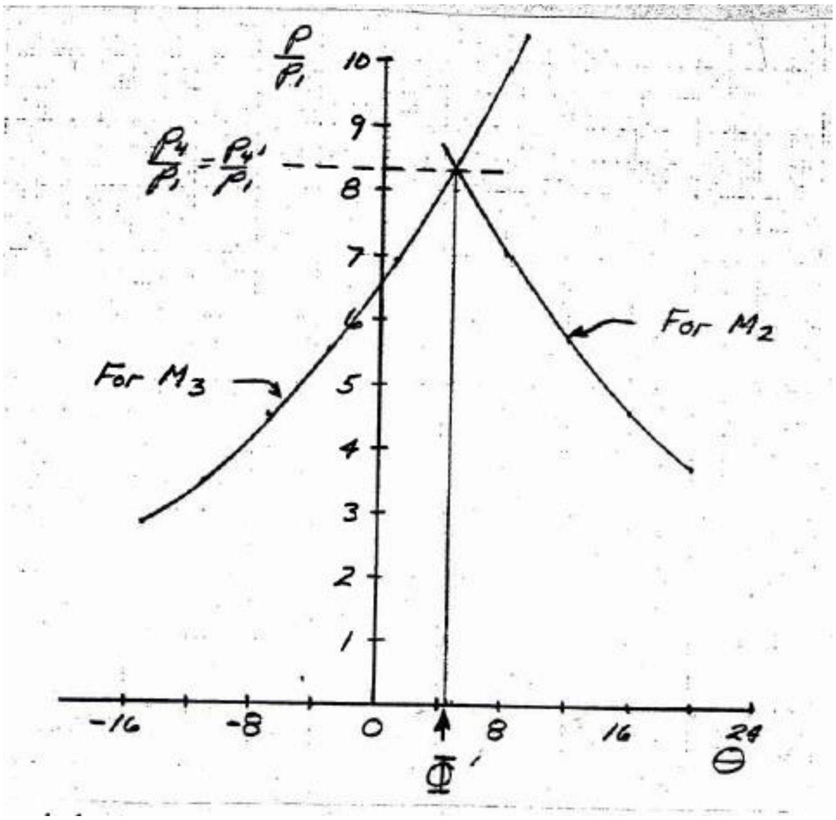
$$M_{n_3} = 0.6684; \; M_3 = \frac{M_{n_2}}{\sin(\beta-\theta)} = \frac{0.6684}{\sin(32.2-15)} = 2.26$$

For the upstream flow represented by region 2, plot a pressure deflection diagram from the following calculations:

β	θ_4'	$M_{n_2} = M_2 \sin\beta$	$\frac{p_4'}{p_2}$	$\frac{p_4'}{p_1} = \frac{p_4'}{p_2}\frac{p_2}{p_1}$	$\theta = \theta_2 - \theta_4'$
30	0	1	1	3.783	20
33.3	4	1.09	1.219	4.61	16
37.2	8	1.20	1.513	5.72	12
41.6	12	1.32	1.866	7.06	8
46.7	16	1.45	2.286	8.65	4
53.5	20	1.60	2.820	10.67	0

For the upstream flow represented by region 3, plot a pressure deflection diagram from the following calculations:

β	θ_4	$M_{n_3} = M_3 \sin\beta$	$\frac{p_4}{p_2}$	$\frac{p_4}{p_1} = \frac{p_4}{p_3}\frac{p_3}{p_1}$	$\theta = \theta_4 - \theta_3$
26	0	1	1	2.82	-15
29	4	1.096	1.23	3.47	-11
33	8	1.23	1.598	4.51	- 7
36.8	12	1.35	1.96	5.53	- 3
41.5	16	1.50	2.458	6.93	1
46.8	20	1.65	3.01	8.49	5
53.7	24	1.82	3.698	10.43	9



From the graph above,

$$p_4 = p_4' = \frac{p_4}{p_1}\, p_1 = (8.3)(1) = \boxed{8.3 \text{ atm}}$$

$$\boxed{\Phi = 4.5°}$$

4.10 From Table A.5, for $M_1 = 2$, $\nu_1 = 26.38°$, $\nu_2 = \nu_1 + \theta = 26.38 + 30 = 56.38°$

From Table A.5: $\boxed{M_2 = 3.37}$

$$T_o = T_{o_1} = T_{o_2} = \text{const; } p_o = p_{o_1} = p_{o_2} = \text{const.}$$

$$p_2 = \left(\frac{p_2}{p_o}\right)_{M_2} \left(\frac{p_o}{p_1}\right)_{M_1} p_1$$

Using Table A.1:

$$p_2 = \left(\frac{1}{63.33}\right)(7.824)(3) = \boxed{0.37 \text{ atm}}$$

$$T_2 = \left(\frac{T_2}{T_o}\right)_{M_2}\left(\frac{T_o}{T_1}\right)_{M_1} T_1$$

Again using Table A.1:

$$T_2 = \left(\frac{1}{3.27}\right)(1.8)(400) = \boxed{220°K}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o1}}{T_1} T_1 = (1.8)(400) = \boxed{720°K}$$

$$p_{o_2} = p_{o_1} = \frac{p_{o_1}}{p_1} = 7.824\,(3) = \boxed{23.47 \text{ atm}}$$

4.11 From Table A.1, for $M_1 = 3$, $\frac{p_{o_1}}{p_1} = 36.73$. Since $p_{o_2} = p_{o_1}$, then

$$\frac{p_{o_2}}{p_2} = \frac{p_{o_1}}{p_1}\frac{p_1}{p_2} = (36.73)\left(\frac{1}{0.4}\right) = 91.83$$

From Table A.1, for $\frac{p_{o_2}}{p_2} = 91.83$, $M_2 = 3.63$.

From Table A.5; for $M_1 = 3$; $\nu_1 = 49.76°$, $\mu_1 = 19.47°$

for $M_2 = 3.63$, $\nu_2 = 60.55°$, $\mu_2 = 16.00°$

$$\theta = \nu_2 - \nu_1 = 10.79°$$

For the forward Mach line, relative to the freestream, the angle =

$$\boxed{\mu_1 = 19.47.}$$

For the rearward Mach line, relative to the freestream, the

$$\text{Angle} = \mu_2 - \theta = 16.00 - 10.79 = \boxed{5.21°}$$

4.12 From Table A.5, for $M_1 = 4$, $\nu_1 = 65.78$

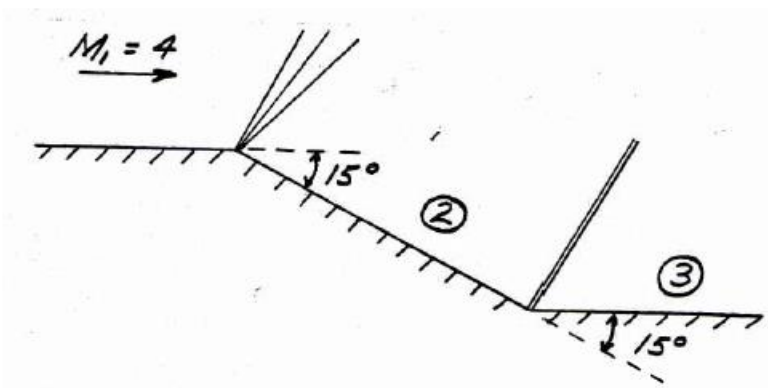
$$\nu_2 = \nu_1 + \theta = 65.78 + 15 = 80.78°$$

From Table A.5, $M_2 = 5.44$.

From Table A.1, for $M_1 = 4$, $\frac{p_{o_1}}{p_1} = 151.8$

$$\text{for } M_2 = 5.44 \ \frac{p_{o_2}}{p_2} = 871.3$$

$$p_2/p_1 = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = \left(\frac{1}{871.3}\right)(1)(151.8) = 0.1742$$



From the θ-β-M diagram, at the compression corner for $M_2 = 5.44$ and $\theta = 15°$, $\beta = 23.6°$

$$M_{n_2} = 5.44 \sin 23.6° = 2.18$$

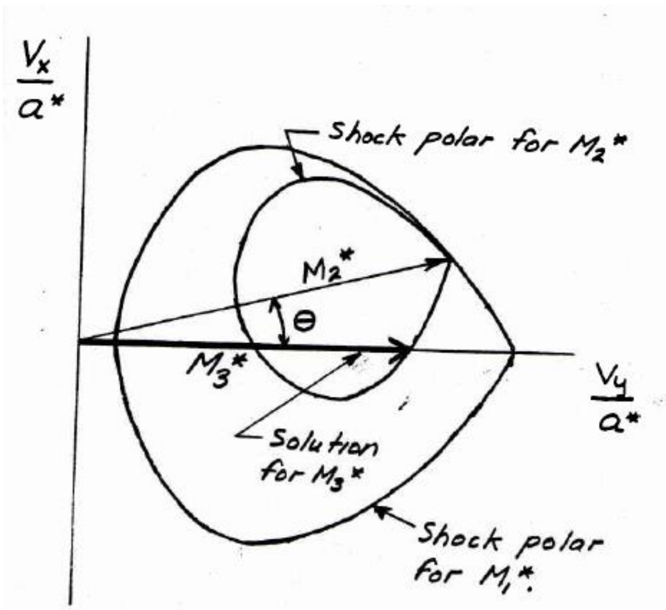
From Table A.2: $M_{n_3} = 0.5498$ and $p_3/p_2 = 5.378$

$$M_3 = \frac{M_{n_3}}{\sin(\beta - \theta)} = \frac{0.5498}{\sin(23.6 - 15)} = \boxed{3.67}$$

$$P_3 = \frac{p_3}{p_2} \frac{p_2}{p_1} p_1 = (5.378)(0.1742)(1) = \boxed{0.937 \text{ atm}}$$

Note that, although the flow directions in regions 1 and 3 are the same, the properties in region 3 are different than in region 1 due to the losses (entropy increase) across the shock wave.

4.13



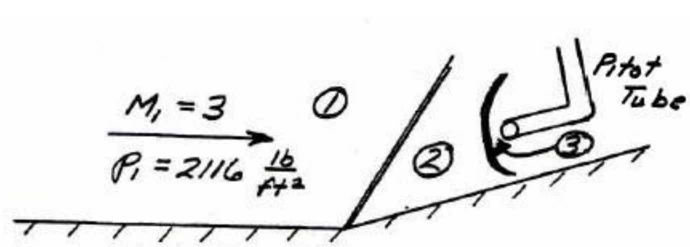
4.14 From the θ-β-M diagram, for $M_1 = 3$ and $\theta = 20°$, $\beta = 37.5°$

$$M_{n1} = M_1 \sin\beta = (3)\sin 37.5 = 1.83$$

From Table A.2: $M_{n_2} = 0.6099$ and $= \frac{p_{o_2}}{p_{o_1}} = 0.7993$

$$M_2 = \frac{M_{n_2}}{\sin(\beta-\theta)} = \frac{0.6099}{\sin(37.5-20)} = \boxed{2.03}$$

M_2 is the Mach number ahead of the normal shock on the Pitot tube.

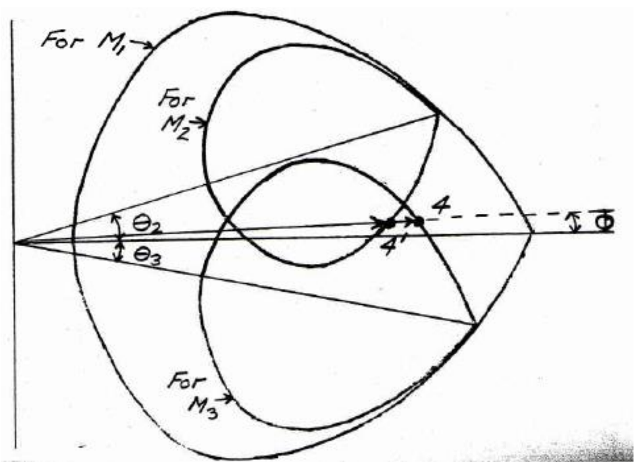


From Table A.2 for M_2 = 2.03: $p_{o_3} / p_{o_2} = 0.7069$

Also, From Table A.1 for M_1 = 3: $\frac{p_{o_1}}{p_1} = 36.73$

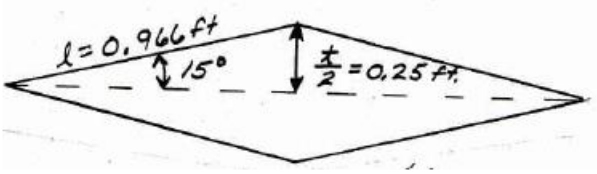
$$p_{o_3} = \left(\frac{p_{o_3}}{p_{o_2}}\right)\left(\frac{p_{o_2}}{p_{o_1}}\right)\left(\frac{p_{o_1}}{p_1}\right)p_1 = (0.7069)(0.7993)(36.73)\ 2166 = 4.39 \times 10^4\ \frac{\text{lb}}{\text{ft}^2}$$

4.15 Shock polars do not give the solution directly.

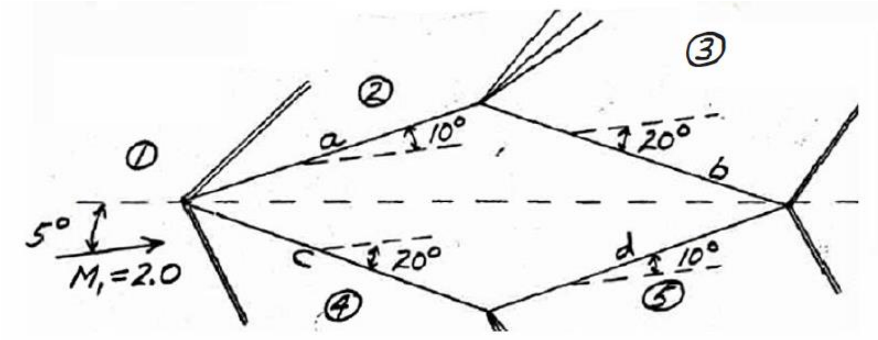


If the solution is <u>known</u>, it can then be sketched on the shock polars as shown above. However, in regions 4 and 4′, even though the flow directions are the same, the flow Mach numbers are different, $M_4^* \neq M_4^{*\prime}$. Therefore, without some additional information, we can not uniquely locate points 4 and 4′ on the shock polars.

4.16 The geometry of the airfoil is given below:



$$\ell = \frac{t/2}{\sin\varepsilon} = \frac{0.25}{\sin(15^\circ)} = 0.966 \text{ ft}$$



For face (a): When $M_1 = 2.0$ and $\theta = 10°$, $\beta = 39.2°$

$$M_{n_1} = M_1 \sin\beta = (2.0) \sin 39.2° = 1.264$$

$$M_{n_2} = 0.8049$$

$$p_2 = \frac{p_2}{p_1} p_1 = (1.698)(2116) = 3593 \text{ lb/ft}^2$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.8049}{\sin(39.12 - 10)} = 1.65$$

$\nu_2 = 16.34°$

For face (b): $\nu_3 = \nu_2 + \theta = 16.34 + 30 = 46.34°$

$\therefore M_3 = 2.83.$ Also $\frac{p_{o_3}}{p_3} = 28.41$

$$p_3 = \frac{p_3}{p_{o_3}} \frac{p_{o_2}}{p_2} p_2 = \left(\frac{1}{28.41}\right)(4.579)(3595) = 579 \text{ lb/ft}^2$$

For face (c): When $M_1 = 2.0$ and $\theta = 20°$, $= 53.5$

$$M_{n_1} = M_1 \sin\beta = (2) \sin 53.5° = 1.61$$

$$M_{n_4} = 0.6655$$

$$p_4 = \frac{p_4}{p_1}\, p_1 = (2.857)(2116) = 6045 \text{ lb/ft}^2$$

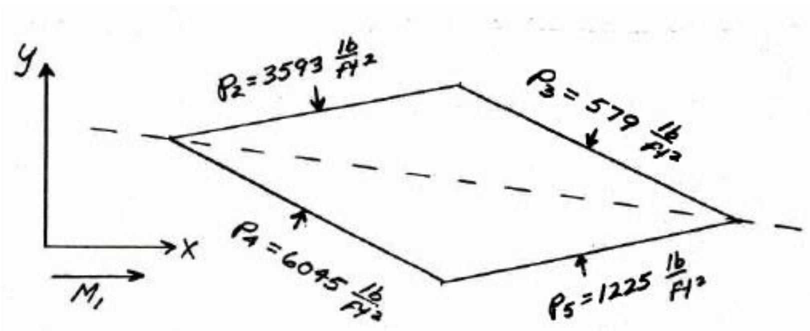
$$M_4 = \frac{M_{n_4}}{\sin(\beta - \theta)} = \frac{0.66559}{\sin(53.5 - 20)} = 1.21$$

$$\nu_4 = 3.806°$$

For face (d): $\nu_5 = \nu_4 + \theta = 3.806 + 30 = 33.81°$

$$M_5 = 2.28. \qquad \text{Also } \frac{p_{o_5}}{p_5} = 12.12$$

$$p_5 = \frac{p_5}{p_{o_5}} \frac{p_{o_4}}{p_4}\, p_2 = \left(\frac{1}{12.12}\right)(2.457)(6045) = 1225 \text{ lb/ft}^2$$



Lift is the component of the total aerodynamic force in the y-direction:

L(per unit span) = ℓ [(p_4-p_3) cos 20° + (p_5-p_2) cos 10°]

= 0.966 [(6045-579) cos 20° + (1225-3593 cos 10°]

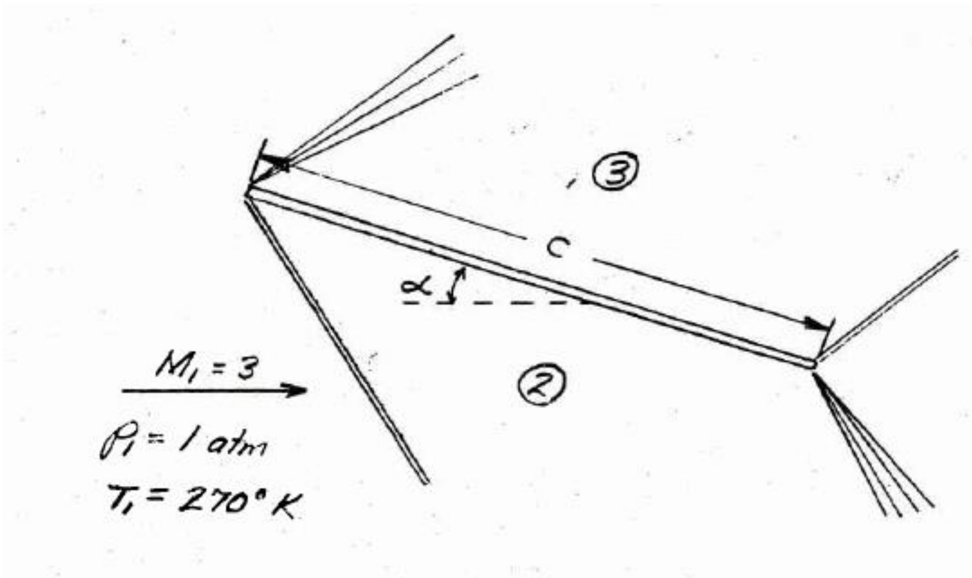
= 0.966 (5136 – 2332) = $\boxed{\text{2708 lb per foot of span}}$

Drag is the component of the total aerodynamic force in the x-direction:

D(per unit span) = ℓ [(p_4-p_3) sin 20° + (p_2-p_5) sin 10°]

= 0.99 (1869 + 411) = $\boxed{\text{2202 lb per foot of span}}$

4.17



α	β	M_{n_1}	$\frac{p_2}{p_1}$	p_2 (atm)	$\frac{T_2}{T_1}$	T_2 (°K)
0	19.6	1	1	1	1	270
5	23.1	1.18	1.458	1.458	1.115	301
10	27.4	1.38	2.055	2.055	1.242	335
15	32.2	1.60	2.820	2.820	1.388	375
20	37.8	1.84	3.783	3.783	1.562	422
25	44.0	2.08	4.881	4.881	1.754	474
30	52.0	2.36	6.331	6.331	2.002	541

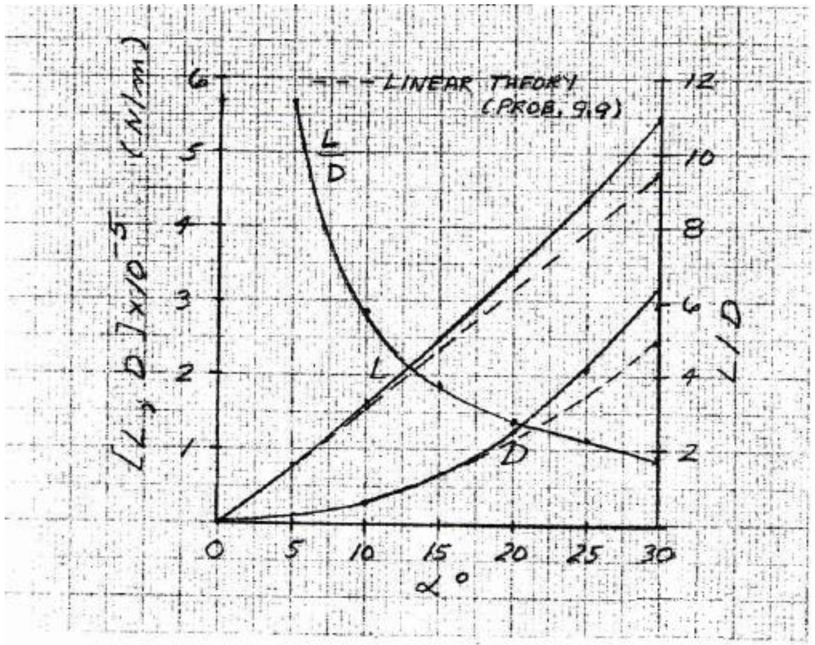
For $M_1 = 3$, $\nu_1 = 49.76$, $\nu_3 = \nu_1 + \alpha$, $p_{o_1}/p_1 = 36.73$, $T_{o_1}/T_1 = 2.8$

α	ν_3	M_3	$\frac{p_{o_3}}{p_3}$	p_3 (atm)	$\frac{T_{o_3}}{T_3}$	T_3 (°K)
0	49.76	3	36.73	1	2.8	270
5	54.76	3.27	54.78	0.670	3.139	241
10	59.76	3.58	85.40	0.430	3.563	212
15	64.76	3.92	136.4.	0.269	4.073	186
20	69.76	4.32	230.6	0.159	4.732	160
25	74.76	4.78	407.8	0.090	5.57	136
30	79.76	5.32	762.8	0.048	6.66	114

Per unit span: $L = c\,(p_2 - p_3)\cos\alpha$, $D = c\,(p_2 - p_3)\cos\alpha$,

$1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2$

α	(p_2-p_3) N/m^2	L N/m	D N/m	L/D
0	0	0	0	-
5	$7.94 \text{ X } 10^4$	7.93×10^4	6.94×10^3	11.4
10	1.64×10^5	1.62×10^5	2.85×10^4	5.68
15	2.58×10^5	2.49×10^5	6.68×10^4	3.73
20	3.66×10^5	3.44×10^5	1.25×10^5	2.75
25	4.84×10^5	4.39×10^5	2.05×10^5	2.14
30	6.35×10^5	5.50×10^5	3.18×10^5	1.73



4.18 From Example 1.8, using the same flat plate with a chord of 3 feet, we have due to the shear stress the force tangential to the flat plate (acting on one side) equal to 39.13 lb. Hence, the skin friction drag due to the shear stress acting on both sides is

$$D_f = 2(39.13)\cos 2° = 78.21 \text{ lb per unit span}$$

For $\alpha = 2°$ and $M_\infty = 2$, from the θ-β-M diagram, we have $\beta = 31.7°$. Hence,

$$M_{n,1} = M_\infty \sin\beta = 2 \sin 31.7° = 1.051$$

From Table A.2, by interpolation,

$$\frac{p_2}{p_1} = 1.12$$

Hence, the pressure distribution over the lower surface is constant, and equal to $p_2 = 1.12\ p_1 = 1.12\ (2116) = 2370$ lb/ft^2. (Note: The standard sea-level pressure is 2116 lb/ft^2.)

The upper surface is an expansion surface. From Table A.5, at Mach 2, $\nu_1 = 26.38°$. Hence,

$$\nu_3 = \nu_1 + \theta = 26.38° + 2 = 28.38°$$

From Table A.5, $M_3 = 2.1$ (rounded to nearest entry). From Table A.1,

$$\text{For } M_1 = 2: \quad \frac{p_o}{p_1} = 7.824$$

$$\text{For } M_3 = 2.1: \quad \frac{p_o}{p_3} = 9.145$$

Hence,

$$p_3 = \frac{p_3}{p_o}\frac{p_o}{p_1}\, p_1 = \left[\frac{1}{9.145}\right](7.824)(2116) = 1810 \text{ lb/ft}^2$$

Thus, the next <u>normal</u> force due to pressure is

$$N = (p_2 - p_3)\,(3) = (2370 - 1810)\,(3) = 1680 \text{ lb per unit span}$$

(a) Lift = y-component of the normal force

$$L = 1680 \cos 2° = \boxed{1679 \text{ lb}} \text{ per unit span}$$

(b) Wave drag = x-component of the normal force

$$D_w = 1680 \sin 2° = \boxed{58.6} \text{ lb per unit span}$$

(c) From the beginning of this problem, the skin-fiction drag is

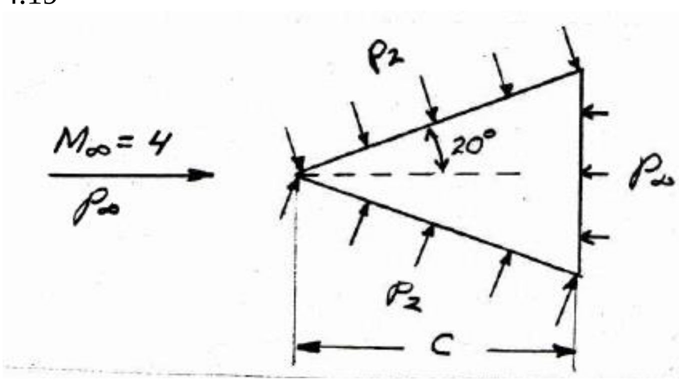
$$D_f = \boxed{78.21} \text{ lb per unit span}$$

In this problem, the total drag is

$$D = D_w + D_f = 58.6 + 78.21 = 136.8 \text{ lb.}$$

Hence, skin-friction is <u>57%</u> of the total drag – a sizeable amount. This is in direct contrast to the case discussed in Example 1.8, where the flat plate is at a larger angle of attack, namely 10°. In Example 1.7, the skin-friction drag is only 5.7% of the total drag, a factor of 10 lower in percentage. Hence, this problem demonstrates the trend mentioned at the end of Example 1.8, namely that at smaller angles of attack, the relative proportion of D_f to D increases. For the case shown here, the angle of attack is small enough that the wave drag is smaller than the skin-friction drag.

4.19



$$D' = 2\left[\frac{c}{\cos\ 20°}\right] p_2 \sin 20° - p_\infty\, 2\, c \text{ Tan } 20° = 2\, c\, (p_2 - p_\infty) \text{ Tan } 20°$$

$$c_d = \frac{D'}{q_\infty S} = \frac{D'}{\frac{\gamma}{2} p_\infty M_\infty{}^2 c}$$

$$c_d = \frac{2c(p_2 - p_\infty)\text{Tan } 20°}{\frac{\gamma}{2} p_\infty M_\infty{}^2 c} = \frac{4}{\gamma M_\infty{}^2}\left(\frac{p_2}{p_\infty}\right) \text{Tan } 20°$$

From the θ-β-M diagram, for θ = 20° and $M_4 = 4$, we have β = 32.5°.

$$M_{n,1} = M_1 \sin\beta = 4 \sin 32.5° = 4\ (0.5373) = 2.149$$

From Table A.2, for $M_{n,1} = 2.149$, $\frac{p_2}{p_\infty} = 5.226$

$$c_d = \frac{4}{(1.4)(4)^2}\ (5.226 - 1)\ (0.364) = \boxed{0.275}$$

4.20 $M_2 - 1 = (3)^2 - 1 = 8$

$$1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{1.2 - 1}{2} (3)^2 = 1.9$$

$$\frac{\gamma - 1}{4} M^4 = \frac{1.2 + 1}{4} (3)^4 = 44.55$$

$$1 + \frac{\gamma + 1}{2} M^2 = 1 + \frac{1.2 + 1}{2} (3)^2 = 10.9$$

$$\lambda = [(M^2 - 1)^2 - 3 (1 + \frac{\gamma - 1}{2} M^2)(1 + \frac{\gamma + 1}{2} M^2) \tan^2\theta]^{1/2}$$

$$= [(8)^2 - 3 (1.9)(10.9) \tan^2 (20°)]^{1/2}$$

$$= [64 - 62.13 (0.1247)]^{1/2} = (55.77)^{1/2} = 7.468$$

$$\chi = \frac{(M^2 - 1)^3 - 9(1 + \frac{\gamma - 1}{2} M^2)(1 + \frac{\gamma - 1}{2} M^2 + \frac{\gamma + 1}{4} M^4) \tan^2 \theta}{\lambda^3}$$

$$= \frac{(8)^3 - 9(1.9)(1.9 + 44.55)(0.13247)}{(7.468)^3} = 0.97667$$

From Eq. (4.19), we have

$$\tan \beta = \frac{M^2 - 1 + 2\lambda \cos[(4\pi\delta + \cos^{-1} \chi) / 3]}{3(1 + \frac{\gamma - 1}{2} M^2) \tan\theta}$$

where $\delta = 1$ for the weak shock solution, and

$$\cos^{-1} \chi = \cos^{-1} (0.97667) = 0.2164 \text{ rad.}$$

and

$$\cos \left[\frac{4\pi(1) + 0.2164}{3} \right] = \cos (4.2609) = -0.4363$$

Hence,

$$\tan \beta = \frac{8 + 2(7.468)(-0.4363)}{3(1.9)\tan 20°}$$

$$\tan \beta = 0.715$$

$\boxed{\beta = 35.57°}$ For $\gamma = 1.2$

As a check on this answer, consider Eq. (4.17)

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

where

$$M_1^2 \sin^2 \beta - 1 = (3)^2 \sin^2 (35.57) - 1 = 2.04534$$

and

$$\gamma + \cos 2\beta = 1.2 + \cos (71.14) = 1.52326$$

$$\tan \theta = 2 \cot (35.57) \left[\frac{2.04534}{(3)^2 (1.52325) + 2} \right]$$

$$= 2(1.39833)(0.13019898) = 0.36412$$

$\theta = 20°$ Check!

For the case of $\gamma = 1.4$, from the θ-β-M diagram for M = 3,

$\boxed{\beta = 37.8°}$ for $\gamma = 1.4$

<u>Conclusion</u>: The effect of chemically reacting flow is to reduce the wave angle for a given deflection angle.

4.21 From Eq. (4.9)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_{n_1}^{\ 2} - 1)$$

For the case of $\gamma = 1.2$, $M_{n_1} = M_1 \sin \beta = (3) \sin 35.57° = 1.745$

$$\frac{p_2}{p_1} = 1 + \frac{2(1.2)}{1.2+1}\,[(1.745)^2 - 1] = 1 + 1.09091\,(2.045)$$

$$\frac{p_2}{p_1} = 3.231 \quad \text{For } \gamma = 1.2$$

For the case of $\gamma = 1.4$, $M_{n_1} = M_1 \sin\beta = (3)\sin 37.8° = 1.8387$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\,(M_{n_1}{}^2 - 1) = 1 + \frac{2(1.4)}{1.4+1}\,[(1.8387)^2 - 1]$$

$$\frac{p_2}{p_1} = 3.778 \quad \text{For } \gamma = 1.4$$

Conclusion: The effect of chemically reacting flow is to reduce the shock strength for a given deflection angle.

