Homework #6

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Due: Sunday, March, 11 2024 at 11:59 pm

Problem 1: Define what is meant by the Debye Length of a plasma.

Solution:

The Debye Length (λ_D) is the distance scale factor by which the electric field is reduced to 1/e of the value it would have in the absence of the plasma.

Problem 2: Assume you are in geostationary orbit with a Debye length of 55m. What is the generalized plasma parameter.

Solution:

$$\Lambda = \frac{4 \times \pi \times \eta_e \times \lambda_D^3}{3} \tag{1}$$

Where:

$$\eta_e = 10^7 \text{ m}^3 \; ; \; \lambda_D = 55m$$

Using Equation (1) we get:

$$\Lambda = \frac{4 \times \pi \times 10^7 \times 55^3}{3} = 6.9690 \times 10^{12}$$

Problem 3: Your measured Plasma Frequency during the day is 5 MHz; what region of the Ionosphere are you in and why?

Solution:

$$\eta_e = (\frac{f_{pe}}{K_1})^2 \tag{2}$$

Where:

$$f_{pe} = 5MHz \, ; \, K_1 = 8.979Hzm^{\frac{3}{2}}$$

Using Equation (2) we get:

$$\eta_e = (\frac{5 \times 10^6}{8.979})^2 = 3.1008 \times 10^{11} m^{-3}$$

At a Plasma Frequency during the day of 5 MHz, this results in the F1 region because η_e is in the range of $2 - 5 \times 10^{11} m^{-3}$

Problem 4: What is the range error for a 100MHz transmission to a satellite @ 350 km altitude? Assume a TEC of 1E^{15}

Solution:

$$\Delta r = K_2 \frac{\text{TEC}}{f^2} \tag{3}$$

Where:

$$\text{TEC} = 1 \times 10^{15}\ ;\ f = 100\ \text{MHz}\ ;\ K_2 = 40.31 m^3 Hz^2$$

Using Equation (3) we get:

$$\Delta r = \left(40.31\,\text{m}^3\text{Hz}^2\right) \tfrac{1\times 10^{15}}{(100\times 10^6\,\text{Hz})^2} = 4.031 m$$

Problem 5: If your electron number density is $2E^{11}/\text{m}^3$, what is your Critical Frequency, f$_{pe}$.

Solution:

$$\eta_e = \frac{f_{pe}^2}{k_1^2} \tag{4}$$

Rearranging Equation (4) for f_{pe} gives:

$$f_{pe} = \sqrt{\eta_e \times k_1^2} \tag{5}$$

Using Equation (5) we get:

$$f_{pe} = \sqrt{(2 \times 10^{11} m^{-3})(80.62 m^3 Hz^2)} = 4.015 MHz$$