Homework #2 (PS 2) ARO 3111 year 2024 Spring

PROBLEMS

(*Note:* Use the tables at the end of this book as extensively as you wish to solve the following problems. Also, when the words "pressure" and "temperature" are used without additional modification, they refer to the *static* pressure and temperature.)

3.1 At a given point in the high-speed flow over an airplane wing, the local Mach number, pressure and temperature are 0.7, 0.9 atm, and 250 K, respectively. Calculate the values of p_o, T_o, p^*, T^*, and a^* at this point.

3.2 At a given point in a supersonic wind tunnel, the pressure and temperature are 5×10^4 N/m^2 and 200 K, respectively. The total pressure at this point is 1.5×10^6 N/m^2. Calculate the local Mach number and total temperature.

3.3 At a point in the flow over a high-speed missile, the local velocity and temperature are 3000 ft/s and 500°R, respectively. Calculate the Mach number M and the characteristic Mach number M^* at this point.

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3.1 At a given point in the high-speed flow over an airplane wing, the local Mach number, pressure and temperature are 0.7, 0.9 atm, and 250 K, respectively. Calculate the values of p_o, T_o, p^*, T^*, and a^* at this point.

→ Calculate P_o

using $M_1^2 = \frac{2}{\gamma-1}\left[\left(\frac{P_o}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$

$$\frac{\gamma-1}{2}M_1^2 = \left(\frac{P_o}{P_1}\right)^{\gamma-1/\gamma} - 1$$

$$\Rightarrow \left(\frac{P_o}{P_1}\right)^{\gamma-1/\gamma} = \frac{\gamma-1}{2}M_1^2 + 1$$

$$\Rightarrow P_o = P_1\left\{\frac{\gamma-1}{2}M_1^2 + 1\right\}^{\gamma/\gamma-1}$$

$$= 0.9\,atm\left\{\frac{1.4-1}{2}(0.7)^2 + 1\right\}^{1.4/1.4-1}$$

$P_o = 1.2483\ atm$

→ Calculate T_o

using $\frac{T_o}{T} = 1 + \frac{\gamma-1}{2}M^2$

$$T_o = T\left[1 + \frac{\gamma-1}{2}M^2\right]$$

$$= 250\,K\left[1 + \frac{1.4-1}{2}(0.7)^2\right]$$

$T_o = 274.5\ K$

→ Calculate P^*

using $\frac{P^*}{P_o} = 0.528$

↳ $P^* = 0.528(1.2483\ atm)$

$P^* = 0.659\ atm$

→ Calculate T^*

using $\frac{T^*}{T_o} = 0.833$

↳ $T^* = 0.833(T_o)$

$= 0.833(274.5\ K)$

$T^* = 228.65\ K$

→ Calculate a^*

using $a^{*2} = \gamma R T^* \Rightarrow a^* = \sqrt{\gamma R T^*}$

$$a^* = \sqrt{(1.4)(8.314\ J/kg\ \cancel{K})(51.58\ \cancel{K}}$$

$a^* = 51.58\ m/s$

3.2 At a given point in a supersonic wind tunnel, the pressure and temperature are 5×10^4 N/m^2 and 200 K, respectively. The total pressure at this point is 1.5×10^6 N/m^2. Calculate the local Mach number and total temperature.

$P_1 = 5 \times 10^4 \ N/m^2$ $\qquad P_0 = 1.5 \times 10^6 \ N/m^2$

$T_1 = 200 K$

$$\rightarrow \frac{P_0}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/\gamma-1}$$

$$\left(\frac{P_0}{P_1}\right)^{\gamma-1/\gamma} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$M_1 = \sqrt{\left[\left(\frac{P_0}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1}} = \sqrt{\left[\left(\frac{1.5 \times 10^6 \ N/m^2}{5 \times 10^4 \ N/m^2}\right)^{\frac{1.4-1}{1.4}} - 1\right]\frac{2}{1.4-1}}$$

$$M_1 = 2.865$$

$$\rightarrow \frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$T_0 = T_1\left[1 + \frac{\gamma-1}{2} M_1^2\right]$$

$$= 200 K\left[1 + \frac{1.4-1}{2}(2.865)^2\right]$$

$$T_0 = 528.52 K$$

3.3 At a point in the flow over a high-speed missile, the local velocity and temperature are 3000 ft/s and 500°R, respectively. Calculate the Mach number M and the characteristic Mach number M^* at this point.

$\rightarrow$ Calculate M using $M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T}}$

$T = {}^\circ R \Rightarrow K = {}^\circ R \cdot 5/9 = 500^\circ (5/9) = 277.7 \ K$

$R = 287 \ \frac{J}{kg \cdot K}$

$V = 3000 \ ft/s \cdot \frac{0.3048 \ m}{1 \ ft} = 914.4 \ m/s$

$$M = \frac{914.4 \ m/s}{\sqrt{1.4(287 \ J/kg \cdot K)(277.7K)}} = 2.73$$

$$M = 2.73$$

$\rightarrow$ Calculate M^* using $M^* = \sqrt{\frac{\frac{(\gamma+1)}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}}$

$$M^* = \left[\frac{\frac{(1.4+1)}{2}(2.73)^2}{1 + \frac{1.4-1}{2}(2.73)^2}\right]^{1/2}$$

$$M^* = 1.89$$