Homework #4

Due: **Sunday, March 3, 2024 at 11:59 pm**

Turn in, on Canvas, a well organized and clear-to-read pdf file *with your calculations, explanations, plots, as well as a ready-to-run* .m/.slx files *containing the MATLAB/Simulink codes (when applicable).*

Problem 1
Update the Simulink model from HW3 by adding two more blocks that perform the following tasks:

Block 5. Input: Quaternion vector $\hat{\boldsymbol{\beta}} = [\beta_0,\ \beta_1,\ \beta_2,\ \beta_3]^T$; Output: rotation matrix $R(\hat{\boldsymbol{\beta}}) \in \mathbb{R}^{3\times3}$.
Block 6. Input: rotation matrix $R \in \mathbb{R}^{3\times3}$; Output: Quaternion vector $\hat{\boldsymbol{\beta}} = [\beta_0,\ \beta_1,\ \beta_2,\ \beta_3]^T$.

Problem 2
At a certain time instant t, the angular velocity of the reference frame $\mathcal{B}$ relative to the reference frame $\mathcal{I}$ is:

$$\boldsymbol{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}} = [2, 1, -1]^T\ rad/s$$

If, at t, $\psi = 60\ deg$, $\theta = 30\ deg$, $\phi = 45\ deg$:

(a) Calculate the Euler angle rates $\dot{\psi},\ \dot{\theta},\ \dot{\phi}$ in deg/s.

(b) Compute the direction cosine matrix $R_{\mathcal{BI}}$ and its instantaneous rate of change $\dot{R}_{\mathcal{BI}}$.

(c) Calculate the quaternion vector $\hat{\boldsymbol{\beta}} = [\beta_0,\ \beta_1,\ \beta_2,\ \beta_3]^T$ and its time rate of change $\dot{\hat{\boldsymbol{\beta}}} = [\dot{\beta}_0,\ \dot{\beta}_1,\ \dot{\beta}_2,\ \dot{\beta}_3]^T$.

You can perform the calculations by hand and/or using MATLAB. In the latter case, please include your scripts along with the relevant outputs.

Problem 3
A spacecraft's angular velocity components in the $\mathcal{B}$-RF are recorded to change with time as follows:

$$p(t) = 1 - e^{-t}\ , \qquad q(t) = e^{-t}\ , \qquad r(t) = 2e^{-3t}.$$

Knowing that the attitude of the spacecraft, expressed in Euler angles, is: $\psi(0) = \phi(0) = 0\ deg$, $\theta(0) = 45\ deg$, write a Simulink script to plot $\psi(t),\ \theta(t),\ \phi(t)$ for $t \in (0, 10)\ s$.

Please include all plots in one figure (3-by-1 subplots). Submit your code and a screenshot of the plots.

Problem 1

Update the Simulink model from HW3 by adding two more blocks that perform the following tasks:

Block 5. Input: Quaternion vector $\hat{\boldsymbol{\beta}} = [\beta_0,\ \beta_1,\ \beta_2,\ \beta_3]^T$; Output: rotation matrix $R(\hat{\boldsymbol{\beta}}) \in \mathbb{R}^{3\times3}$.
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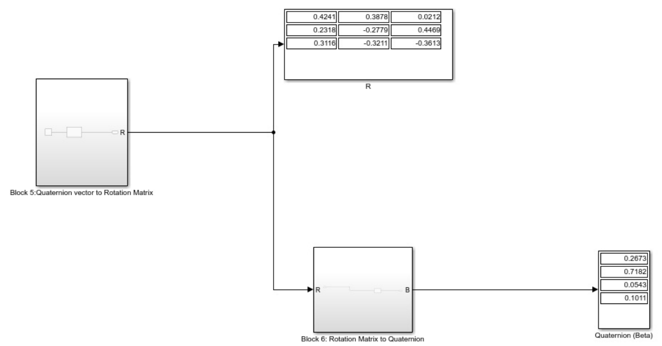

* INPUTS ARE IN ATTACHED .m

Problem 2

At a certain time instant t, the angular velocity of the reference frame $\mathcal{B}$ relative to the reference frame $\mathcal{I}$ is:

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(c) Calculate the quaternion vector $\hat{\boldsymbol{\beta}} = [\beta_0,\ \beta_1,\ \beta_2,\ \beta_3]^T$ and its time rate of change $\dot{\hat{\boldsymbol{\beta}}} = [\dot{\beta}_0,\ \dot{\beta}_1,\ \dot{\beta}_2,\ \dot{\beta}_3]^T$.

$$\vec{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}} = [2,\ 1,\ -1]^T \quad ; \quad \begin{bmatrix} \psi \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 60^\circ \\ 30^\circ \\ 45^\circ \end{bmatrix}$$

(a) Calculate the Euler angle rates $\dot{\psi},\ \dot{\theta},\ \dot{\phi}$ in deg/s.

$$\dot{\vec{\theta}}(t) = B^{-1}(\vec{\theta}(t)) \cdot \vec{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(t)$$

$$= \frac{1}{\cos\theta} \begin{bmatrix} 0 & \sin\phi & \cos\phi \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{\cos 30} \begin{bmatrix} 0 & \sin 45 & \cos 45 \\ 0 & \cos 45\cos 30 & -\sin 45\cos 30 \\ \cos 30 & \sin 45\sin 30 & \cos 45\sin 30 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\dot{\theta}(t) = \begin{bmatrix} 0 \\ 1.4142 \\ 2.000 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4142 \\ 2.000 \end{bmatrix}}$$

(b) Compute the direction cosine matrix $R_{\mathcal{BI}}$ and its instantaneous rate of change $\dot{R}_{\mathcal{BI}}$.

DIRECTION COSINE MATRIX (D.C.M) : $R_{\mathcal{BI}}$

$$R_{\mathcal{BI}} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}\Bigg|_{\substack{\psi=60^\circ \\ \theta=30^\circ \\ \phi=45^\circ}} = \begin{bmatrix} 0.4330 & 0.7550 & -0.500 \\ -0.4356 & 0.6597 & 0.6124 \\ 0.7891 & -0.0474 & 0.6124 \end{bmatrix}$$

$$
R_{BI} = \begin{bmatrix} 0.4330 & 0.7550 & -0.500 \\ -0.4356 & 0.6597 & 0.6124 \\ 0.7891 & -0.0474 & 0.6124 \end{bmatrix}
$$

INSTANTANEOUS RATE OF CHANGE: $\dot{R}_{BI}$

$$
\dot{R}_{BI} = -\left[\vec{\omega}_B^{\,B/I}(t)\right]_\times \cdot R_{BI}
$$

$$
* \quad -\left[\vec{\omega}_B^{\,B/I}\right]_\times = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}
$$

$$
\Rightarrow \quad \dot{R}_{BI} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.4330 & 0.7550 & -0.500 \\ -0.4356 & 0.6597 & 0.6124 \\ 0.7891 & -0.0474 & 0.6124 \end{bmatrix} = \begin{bmatrix} -0.3536 & -0.6124 & -1.2247 \\ 2.0113 & 0.6553 & 0.7247 \\ 1.3042 & -0.5695 & -1.7247 \end{bmatrix}
$$

$$
\dot{R}_{BI} = \begin{bmatrix} -0.3536 & -0.6124 & -1.2247 \\ 2.0113 & 0.6553 & 0.7247 \\ 1.3042 & -0.5695 & -1.7247 \end{bmatrix}
$$

(c) Calculate the quaternion vector $\hat{\beta} = [\beta_0, \beta_1, \beta_2, \beta_3]^T$ and its time rate of change $\dot{\hat{\beta}} = [\dot{\beta}_0, \dot{\beta}_1, \dot{\beta}_2, \dot{\beta}_3]^T$.

$$
R_{BI} = \begin{bmatrix} 0.4330 & 0.7550 & -0.500 \\ -0.4356 & 0.6597 & 0.6124 \\ 0.7891 & -0.0474 & 0.6124 \end{bmatrix}
$$

CALCULATE QUATERNION VECTOR: $\hat{\beta}$

USING SHEPPERD'S METHOD

I)

$\beta_0^2 = \frac{1}{4}(1 + r_{11} + r_{22} + r_{33})$ → $\beta_0^2 = 0.676$

$\beta_1^2 = \frac{1}{4}(1 + r_{11} - r_{22} - r_{33})$ → $\beta_1^2 = 0.0402$

$\beta_2^2 = \frac{1}{4}(1 + r_{22} - r_{11} - r_{33})$ → $\beta_2^2 = 0.1536$

$\beta_3^2 = \frac{1}{4}(1 + r_{33} - r_{11} - r_{22})$ → $\beta_3^2 = 0.1299$

II)

$$
MAX(\beta_0^2, \beta_1^2, \beta_2^2, \beta_3^2) = \beta_0^2 \rightarrow \beta_0 = \sqrt{0.676} = 0.8224
$$

III)

$$
\beta_1 = \frac{1}{4\beta_0}\left[r_{23} - r_{32}\right] = 0.2006
$$

$$
\beta_2 = \frac{1}{4\beta_0}\left[r_{31} - r_{13}\right] = 0.3919
$$

$$
\beta_3 = \frac{1}{4\beta_0}\left[r_{12} - r_{21}\right] = 0.3604
$$

$$
\Rightarrow \quad \hat{\beta} = \begin{bmatrix} 0.8224 \\ 0.2006 \\ 0.3919 \\ 0.3604 \end{bmatrix}
$$

QUARTERNION RATE OF CHANGE : $\dot{\hat{\beta}}$

$$\dot{\hat{\beta}} = \frac{1}{2} B(\hat{\beta}(t)) \cdot \vec{\omega}_B^{B/I}(t)$$

where

$$B(\hat{\beta}(t)) = \begin{bmatrix} -\beta_1(t) & -\beta_2(t) & -\beta_3(t) \\ \beta_0(t) & -\beta_3(t) & \beta_2(t) \\ \beta_3(t) & \beta_0(t) & -\beta_1(t) \\ -\beta_2(t) & \beta_1(t) & \beta_0(t) \end{bmatrix} = \begin{bmatrix} -0.2006 & -0.3919 & -0.3604 \\ 0.8224 & -0.3604 & 0.3919 \\ 0.3604 & 0.8224 & -0.2006 \\ -0.3919 & 0.2006 & 0.8224 \end{bmatrix} \quad \& \quad \vec{\omega}_B^{B/I}(t) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\hookrightarrow \quad \dot{\hat{\beta}} = \frac{1}{2} \begin{bmatrix} -0.2006 & -0.3919 & -0.3604 \\ 0.8224 & -0.3604 & 0.3919 \\ 0.3604 & 0.8224 & -0.2006 \\ -0.3919 & 0.2006 & 0.8224 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.2163 \\ 0.4462 \\ 0.8719 \\ -0.7028 \end{bmatrix}$$

$$\boxed{\dot{\hat{\beta}} = \begin{bmatrix} -0.2163 \\ 0.4462 \\ 0.8719 \\ -0.7028 \end{bmatrix}}$$

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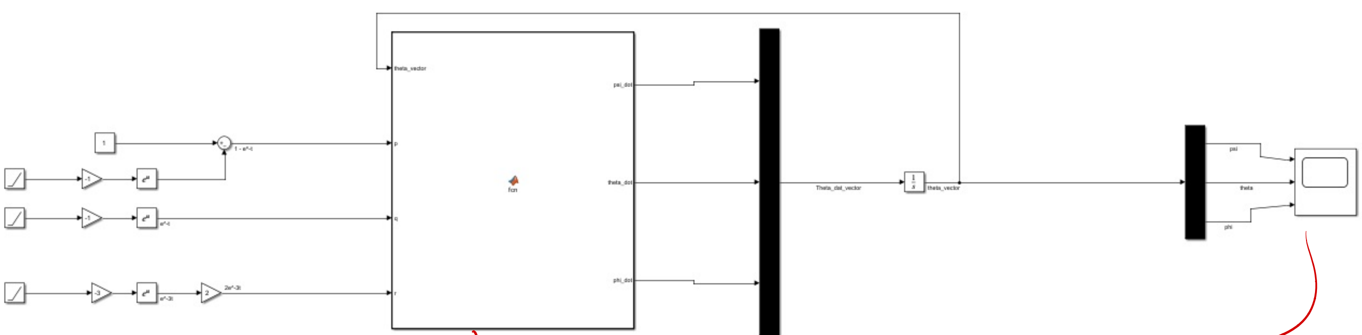
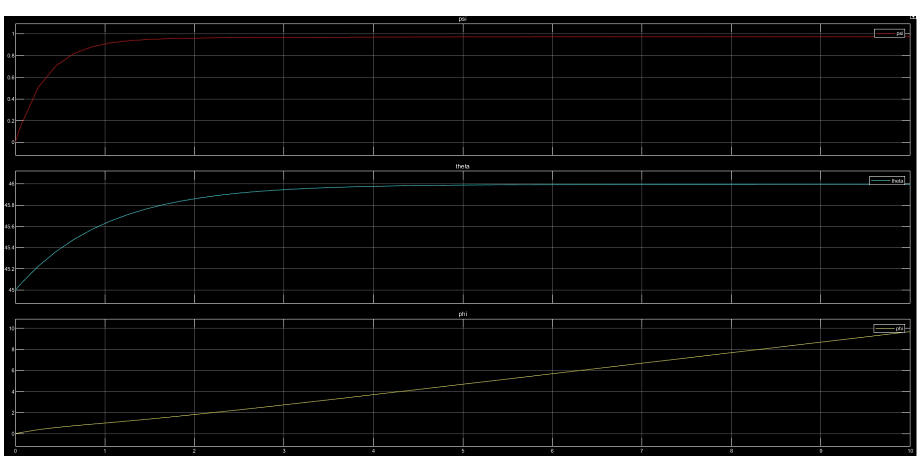



```
function [psi_dot, theta_dot, phi_dot] = fcn(theta_vector, p, q, r)

w_B = [p; q; r];

psi = theta_vector(1);
theta = theta_vector(2);
phi = theta_vector(3);

B_inv = (1/cosd(theta))*[0, sind(phi), cosd(phi);
          0, cosd(phi)*cosd(theta), -sind(phi)*cosd(theta);
          cosd(theta), sind(phi)*sind(theta), cosd(phi)*sind(theta)];

theta_dot_vector = B_inv * w_B;

psi_dot = theta_dot_vector(1);
theta_dot = theta_dot_vector(2);
phi_dot = theta_dot_vector(3);

end
```



```
% ARO 4090 - Space Vehicle Dyn. & Cntrl. | Dr. Maggia | Justin Millsap |
Homework 4 %
```

Problem 1

```
clc; clear;
% Inputs to Block 5 & 6

B_0 = 0.3
B_1 = 0.64
B_2 = 0.242
B_3 = 0.13

Beta = [B_0 ; B_1 ; B_2 ; B_3] ;
```

```
B_0 =

    0.3000


B_1 =

    0.6400


B_2 =

    0.2420


B_3 =

    0.1300
```

Problem 2

```
clc; clear;

% a) Calculate the Euler Angle Rates (psi_dot , theta_dot , and phi_dot)

w_B_BI = [2; 1; -1]; % angular velocity of the refernce from B realtive to
the reference fram I

psi = 60; % [deg]
theta = 30; % [deg]
phi = 45; % [deg]


B_inv = [0 sind(phi) cosd(phi);
```

```
        0 cosd(phi)*cosd(theta) -sind(phi)*cosd(theta);
        cosd(theta) sind(phi)*sind(theta) cosd(phi)*sind(theta)];

B_inv = (1/cosd(theta))*B_inv

Theta_dot = B_inv*w_B_BI;

% b) Compute the direction cosine matrix (D.C.M) R_BI & its instantaneous
rate of change R_BI_dot

w_B_skew = [0 -1 -1;
            1  0  2;
            1 -2  0]

R_BI = [cosd(theta)*cosd(psi) , cosd(theta)*sind(psi) , -sind(theta);
        sind(phi)*sind(theta)*cosd(psi)-cosd(phi)*sind(psi) ,
sind(phi)*sind(theta)*sind(psi)+cosd(phi)*cosd(psi) , sind(phi)*cosd(theta);
        cosd(phi)*sind(theta)*cosd(psi)+sind(phi)*sind(psi) ,
cosd(phi)*sind(theta)*sind(psi)-sind(phi)*cosd(psi) , cosd(phi)*cosd(theta)]

R_BI_dot = w_B_skew*R_BI

% c) Calculate the quaternion vector Beta and its rate of change Beta_dot

% Sheppards method

    % 1) first calculate B_0^2 ..... B_3^2  will be defined as B_1_check1
    % % ALL VALUES ARE SQUARED

        B_0_check = (1/4)*(1+ R_BI(1,1) + R_BI(2,2) + R_BI(3,3));
        B_1_check = (1/4)*(1+ R_BI(1,1) - R_BI(2,2) - R_BI(3,3));
        B_2_check = (1/4)*(1+ R_BI(2,2) - R_BI(1,1) - R_BI(3,3));
        B_3_check = (1/4)*(1+ R_BI(3,3) - R_BI(1,1) - R_BI(2,2));

        B_check = [B_0_check ; B_1_check ; B_2_check ; B_3_check]

       B_max = max(B_check)


        if B_max == B_0_check % B_0 is max
            B_0 = sqrt(B_0_check);
             B_1 = (1/(4*B_0))*( R_BI(2,3) - R_BI(3,2));
             B_2 = (1/(4*B_0))*( R_BI(3,1) - R_BI(1,3));
             B_3 = (1/(4*B_0))*( R_BI(1,2) - R_BI(2,1));

        elseif B_max == B_1_check % B_1 is max
            B_1 = sqrt(B_1_check)
             B_0 = (1/(4*B_1))*( R_BI(2,3) - R_BI(3,2));
             B_2 = (1/(4*B_1))*( R_BI(2,1) - R_BI(1,2));
             B_3 = (1/(4*B_1))*( R_BI(1,3) - R_BI(3,1));

        elseif B_max == B_2_check % B_2 is max
            B_2 = sqrt(B_2_check);
             B_0 = (1/(4*B_2))*( R_BI(3,1) - R_BI(1,3));
```

```
            B_1 = (1/(4*B_2))*( R_BI(2,1) + R_BI(1,2));
            B_3 = (1/(4*B_2))*( R_BI(3,2) + R_BI(2,3));

        elseif B_max == B_3_check % B_3 is max
            B_3 = sqrt(B_3_check);
            B_0 = (1/(4*B_2))*( R_BI(1,2) - R_BI(2,1));
            B_1 = (1/(4*B_2))*( R_BI(1,3) + R_BI(3,1));
            B_2 = (1/(4*B_2))*( R_BI(3,2) + R_BI(2,3));
        end


B = [B_0 ; B_1 ; B_2 ; B_3];

if B < 0
    B = -1*B
    else B = B
end

% B_dot

B_beta = [ -B(2) , -B(3) , -B(4);
            B(1) , -B(4) , B(3);
            B(4) , B(1) , -B(2);
            -B(3) , B(2) , B(1)]

Beta_dot = (1/2)*B_beta*w_B_BI;


B_inv =

         0    0.8165    0.8165
         0    0.7071   -0.7071
    1.0000    0.4082    0.4082


w_B_skew =

     0    -1    -1
     1     0     2
     1    -2     0


R_BI =

    0.4330    0.7500   -0.5000
   -0.4356    0.6597    0.6124
    0.7891   -0.0474    0.6124


R_BI_dot =

   -0.3536   -0.6124   -1.2247
    2.0113    0.6553    0.7247
    1.3042   -0.5695   -1.7247
```

```
B_check =

    0.6763
    0.0402
    0.1536
    0.1299


B_max =

    0.6763


B =

    0.8224
    0.2006
    0.3919
    0.3604


B_beta =

   -0.2006   -0.3919   -0.3604
    0.8224   -0.3604    0.3919
    0.3604    0.8224   -0.2006
   -0.3919    0.2006    0.8224
```

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