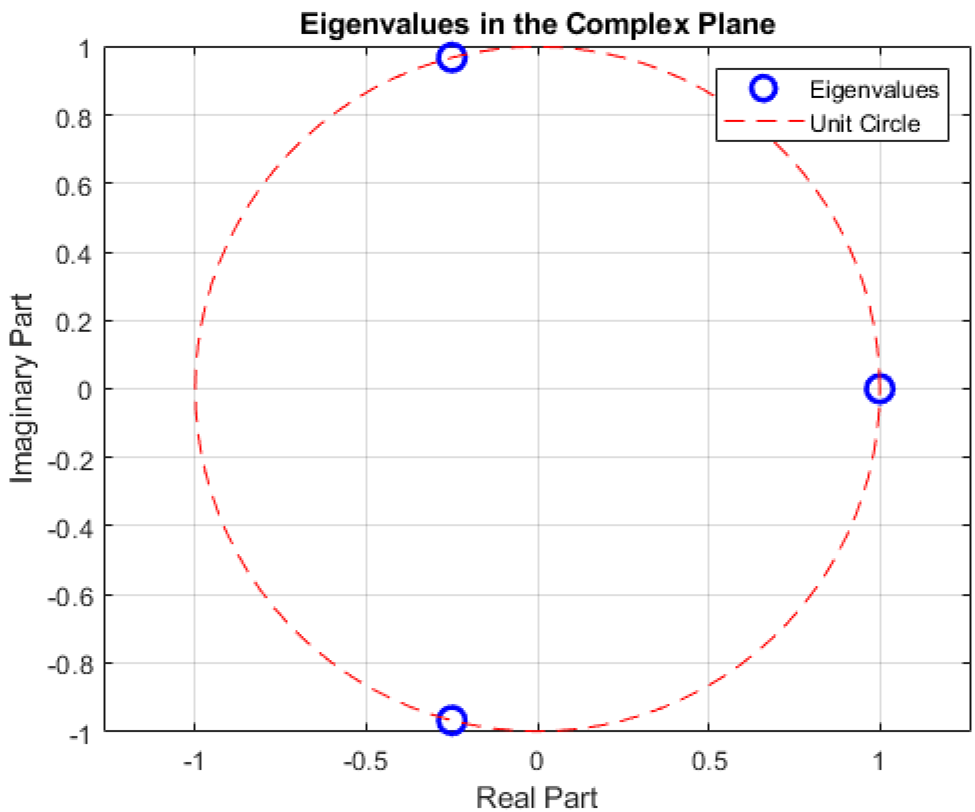
```
% ARO 4090 - Space Vehicle Dyn. & Cntrl. | Dr. Maggia | Justin Millsap |
Homework 5 %
clc; clear; close all;
```

Problem 1

```
clc; clear; close all
R = [0 1/2 sqrt(3)/2 ; -1 0 0 ; 0 -sqrt(3)/2 1/2];

I = [1 0 0 ; 0 1 0 ; 0 0 1];

% ~~~~~~~~~~~~~~ PART A ~~~~~~~~~~~~~~ %

% Show that R is a rotation Matrix
    % Must satisfy det(R) = 1
    % & RR^T = I

condition_1 = det(R);
condition_2 = R*R';

% ~~~~~~~~~~~~~~ PART B ~~~~~~~~~~~~~~ %

% Find eigenvalues
eigenvalues = eig(R)

% Plot the eigenvalues on the complex plane
figure; % Opens a new figure window
plot(real(eigenvalues), imag(eigenvalues), 'bo', 'MarkerSize', 10,
'LineWidth', 2);
xlabel('Real Part');
ylabel('Imaginary Part');
title('Eigenvalues in the Complex Plane');
grid on;
axis equal;
hold on;

% Additionally, plot the unit circle for reference
th = 0:pi/50:2*pi;
xunit = cos(th);
yunit = sin(th);
plot(xunit, yunit, 'r--'); % Unit circle in red dashed line
legend('Eigenvalues', 'Unit Circle');

hold off;

% ~~~~~~~~~~~~~~ PART C ~~~~~~~~~~~~~~ %

% Find the angle of rotation (alpha)

alpha = acosd( (trace(R) - 1) /2 )

% Find axis of rotation (a_hat)
```

```
a = [-1 ; 1 ; -sqrt(3)];

a_hat = -a/norm(a); % Normalize the vector

fprintf('The angle of rotation is alpha = %.2f degrees ',alpha)
disp(' ')
disp('The axis of rotation is a=')
disp(a_hat)




% ~~~~~~~~~~~~~~ PART D ~~~~~~~~~~~~~~ %

% Use alpha and unit vector a to retrive Matrix R

a_x = [ 0 -a_hat(3,1) a_hat(2,1) ; a_hat(3,1) 0 -a_hat(1,1) ; -a_hat(2,1)
a_hat(1,1) 0]

R_prime = cosd(alpha)*I + (1 - cosd(alpha)) * a_hat * a_hat' -
sind(alpha)*a_x

% ~~~~~~~~~~~~~~ PART E ~~~~~~~~~~~~~~ %

% Solve for Eurler Angles by using most opitmal positions to compare between
R_BI & Rotation Sequence Matrix


theta = asind(-R(1,3)) + 360;
phi   = atan2d(R(2,3) , R(3,3));
psi   = atan2d(R(1,2) , R(1,1));

% Check if Eurler angles work
R1 = [ 1 0 0  ; 0 cosd(phi) sind(phi) ; 0 -sind(phi) cosd(phi)];
R2 = [cosd(theta) 0 -sind(theta); 0 1 0; sind(theta) 0 cosd(theta)];
R3 = [ cosd(psi) sind(psi) 0 ; -sind(psi) cosd(psi) 0 ; 0 0 1];


Rot_Seq = R1*R2*R3;          % [3-2-1] Rotation Sequence
disp(' The Eurler Angles for the given R Matrix for a given Rotation
Sequence are')
fprintf('psi = %.2f\n', psi)
fprintf('theta = %.2f\n' , theta)
fprintf('phi = %.2f\n' , phi)


eigenvalues =

  -0.2500 + 0.9682i
  -0.2500 - 0.9682i
   1.0000 + 0.0000i
```

```
alpha =

  104.4775

The angle of rotation is alpha = 104.48 degrees
The axis of rotation is a=
    0.4472
   -0.4472
    0.7746


a_x =

         0   -0.7746   -0.4472
    0.7746         0   -0.4472
    0.4472    0.4472         0


R_prime =

         0    0.5000    0.8660
   -1.0000         0    0.0000
   -0.0000   -0.8660    0.5000

 The Eurler Angles for the given R Matrix for a given Rotation Sequence are
psi = 90.00
theta = 300.00
phi = 0.00
```



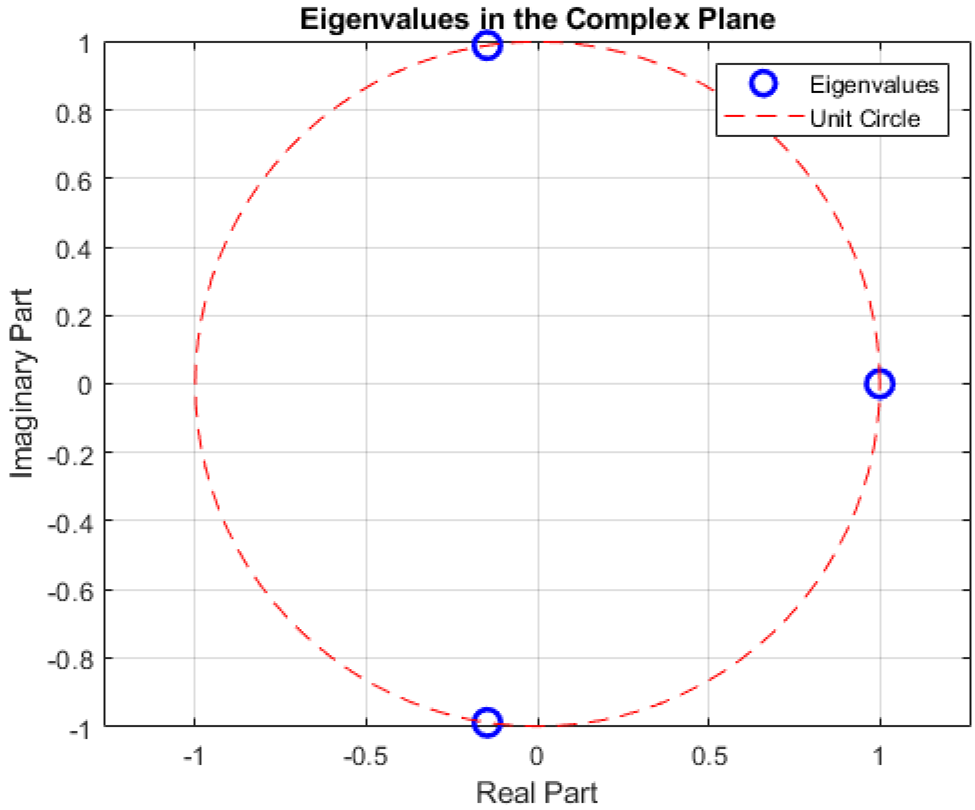
Problem 2

```
clc; clear; close all
R = [0 sqrt(2)/2 -sqrt(2)/2 ; 0 sqrt(2)/2 sqrt(2)/2 ; 1 0 0];

I = [1 0 0 ; 0 1 0 ; 0 0 1];

% ~~~~~~~~~~~~~~ PART A ~~~~~~~~~~~~~~ %

% Show that R is a rotation Matrix
    % Must satisfy det(R)  = 1
    % & RR^T = I

condition_1 = det(R)
condition_2 = R*R'

% ~~~~~~~~~~~~~~ PART B ~~~~~~~~~~~~~~ %

% Find eigenvalues
eigenvalues = eig(R)

% Plot the eigenvalues on the complex plane
figure; % Opens a new figure window
plot(real(eigenvalues), imag(eigenvalues), 'bo', 'MarkerSize', 10,
'LineWidth', 2);
```

```
xlabel('Real Part');
ylabel('Imaginary Part');
title('Eigenvalues in the Complex Plane');
grid on;
axis equal;
hold on;

% Additionally, plot the unit circle for reference
th = 0:pi/50:2*pi;
xunit = cos(th);
yunit = sin(th);
plot(xunit, yunit, 'r--'); % Unit circle in red dashed line
legend('Eigenvalues', 'Unit Circle');

hold off;

% ~~~~~~~~~~~~~~ PART C ~~~~~~~~~~~~~~ %

% Find the angle of rotation (alpha)

alpha = acosd( (trace(R) - 1) /2 )

a = [1 ; 0 ; 0]
% Find axis of rotation (a_hat)
if alpha >= 0
    if alpha <= 180
        a = (1/(2*sind(alpha))) * [R(2,3) - R(3,2) ; R(3,1) - R(1,3) ;
R(1,2) - R(2,1)]
    end
end

a_hat = a/norm(a)

 % Normalize the vector

fprintf('The angle of rotation is alpha = %.2f degrees ',alpha)
disp(' ')
disp('The axis of rotation is a=')
disp(a_hat)




% ~~~~~~~~~~~~~~ PART D ~~~~~~~~~~~~~~ %

% Use alpha and unit vector a to retrive Matrix R

a_x = [ 0 -a_hat(3,1) a_hat(2,1) ; a_hat(3,1) 0 -a_hat(1,1) ; -a_hat(2,1)
a_hat(1,1) 0]

R_prime = cosd(alpha)*I + (1 - cosd(alpha)) * a_hat * a_hat' -
sind(alpha)*a_x

% ~~~~~~~~~~~~~~ PART E ~~~~~~~~~~~~~~ %
```

```
% Solve for Eurler Angles by using most opitmal positions to compare between
R_BI & Rotation Sequence Matrix


theta = asind(-R(1,3));
phi   = atan2d(R(2,3) , R(3,3));
psi   = atan2d(R(1,2) , R(1,1));

% Check if Eurler angles work
R1 = [ 1 0 0  ; 0 cosd(phi) sind(phi) ; 0 -sind(phi) cosd(phi)];
R2 = [cosd(theta) 0 -sind(theta); 0 1 0; sind(theta) 0 cosd(theta)];
R3 = [ cosd(psi) sind(psi) 0 ; -sind(psi) cosd(psi) 0 ; 0 0 1];


Rot_Seq = R1*R2*R3;           % [3-2-1] Rotation Sequence
disp(' The Eurler Angles for the given R Matrix for a given Rotation
Sequence are')
fprintf('psi = %.2f\n', psi)
fprintf('theta = %.2f\n' , theta)
fprintf('phi = %.2f\n' , phi)


condition_1 =

    1.0000


condition_2 =

    1.0000         0         0
         0    1.0000         0
         0         0    1.0000


eigenvalues =

  -0.1464 + 0.9892i
  -0.1464 - 0.9892i
   1.0000 + 0.0000i


alpha =

   98.4211


a =

     1
     0
     0
```

```
a =

    0.3574
    0.8629
    0.3574


a_hat =

    0.3574
    0.8629
    0.3574

The angle of rotation is alpha = 98.42 degrees
The axis of rotation is a=
    0.3574
    0.8629
    0.3574


a_x =

         0   -0.3574    0.8629
    0.3574         0   -0.3574
   -0.8629    0.3574         0


R_prime =

    0.0000    0.7071   -0.7071
         0    0.7071    0.7071
    1.0000    0.0000    0.0000

 The Eurler Angles for the given R Matrix for a given Rotation Sequence are
psi = 90.00
theta = 45.00
phi = 90.00
```



Published with MATLAB® R2023b