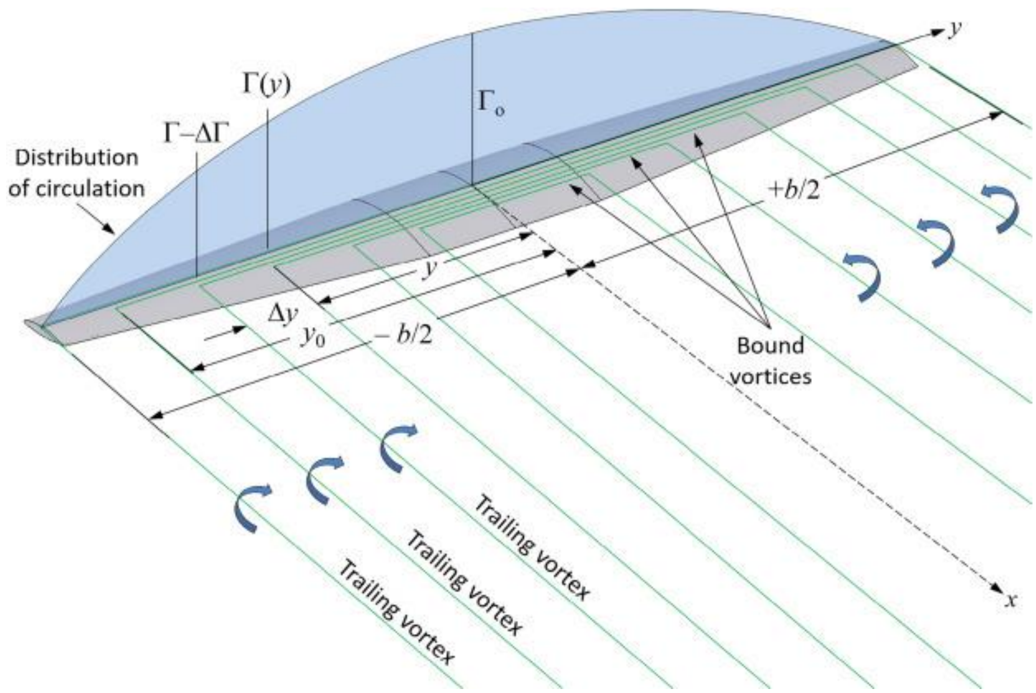
# Numerical Lifting Line theory (NLLT) Computer Program Project

**Class: ARO 3011 – Fluid Mechanics & Low Speed Aerodynamics**

**Section #02**

**Instructor: Dr. Tony Lin**





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## Summary:

The numerical lifting line theory is used to analyze and predict aerodynamic characteristics of a wing. This method represents a wing by a number of horseshoe vortices each with a different bound vortex.

A computer program was created to computer total wing lift coefficient ($C_L$), total wing induced drag coefficient ($C_{Di}$), spanwise lift distribution normalized with total wing lift coefficient $\left({}^{C_l(y)}/_{C_L}\right)$ , spanwise lift distribution of bound circulation $\Gamma(y)$, and $\delta$. This program was created on MATLAB and can calculate these values for 3 different geometries such as a rectangular wing, tapered wing, and an elliptical wing.

The approach to create this code started off by defining known values consisting of twist $(\varepsilon)$, angle of attack $(\alpha)$, aspect ratio, wingspan (b), and section-lift curve slope $(C_{L\alpha})$. The distance between each of the horseshoe vortices ($y_{c,i}$), were distributed into a "cosine space" fashion in order to get more precise data at the wing tips. We then set control points ($y_{c,i}$) along these cosine spaced horseshoe vortices. The angle of attack $\alpha(j)$ will vary at each $y_{c,i}$ value depending on the twist of the wing. After we obtained these values the code then proceeds by iterating values for the $C_{i,j}$ which will be used to iterate values for $B_j$ and $A_{j,i}$ which are nxn matrices in which are vital in order to compute the vortex strengths $(\gamma_i)$ using the governing equation $\gamma_i = \left[A_{j,i}\right]^{-1}[B_j]$. The code then calculates the down-wash velocity & down-wash angle of attack $(w_j$ & $(\alpha_{down-wash})_j)$. With all these values obtained above, the code is finalized with calculating ($C_L$), ($C_{Di}$), $\left({}^{C_l(y)}/_{C_L}\right)$ , $\Gamma(y)$, and $\delta$ using equations in the numerical lifting line theory.

From the values produced by the computer program above, plots were produced that create a story of how different wing geometries result in different aerodynamic behaviors. To do this, the computer program generates a plot of ${}^{C_l(y)}/_{C_L})$ vs. $y_c$ as well as $\Gamma(y)$ vs $y_c$ which are listed below.

Upon examining the generated plots, it was evident that the rectangular wing exhibited the highest spanwise lift distribution with respect to the control points. However, it also converged to zero rapidly, resulting in the lowest spanwise bound circulation. The elliptical wing displayed the second highest spanwise lift distribution relative to the control points but encountered a singularity at $y_c = 1$, accompanied by the second highest spanwise bound circulation. Lastly, the tapered wing demonstrated the lowest spanwise lift distribution while having the highest spanwise bound circulation.

In conclusion, various wing geometries come with their own set of advantages and disadvantages. The selection of a particular wing geometry should be pinpointed depending on specific objectives. The numerical lifting line theory proves to be a precise method for comprehending diverse aerodynamic behaviors, which vary based on the distinct characteristics of a wing. This outlines the importance of employing accurate techniques to analyze and visualize how different wing features impact overall performance.