Homework #1

Due: **Sunday, February 11, 2024 at 11:59 pm**

Turn in, on Canvas, a well organized and clear-to-read pdf file with your calculations, explanations, plots, as well as a ready-to-run .m files containing the MATLAB codes (when applicable).

Problem 1

Two vectors $\mathbf{v}$ and $\mathbf{w}$, are represented in two coordinate reference frames $\mathcal{A}$ anb $\mathcal{B}$ as shown below:

$$\mathbf{v}_{\mathcal{A}} = [1, 2, 3]^T \,,\, \mathbf{w}_{\mathcal{A}} = [-1, 2, 1]^T$$

$$\mathbf{v}_{\mathcal{B}} = [3.56186, 1.13448, 0.16150]^T \,,\, \mathbf{w}_{\mathcal{B}} = [1.33712, 1.31501, -1.57572]^T$$

(a) Find the inner product $\mathbf{v} \cdot \mathbf{w}$ in each reference frame.

(b) Find the cross product $\mathbf{v} \times \mathbf{w}$ in each frame using only matrix multiplication.

(c) Find the coordinate transformation matrix $R_{\mathcal{B}\mathcal{A}}$.

Problem 2

Given the following coordinate transformation matrix between reference frames $\mathcal{I} = \{O, \hat{\mathbf{i}}_\mathbf{1}, \hat{\mathbf{i}}_\mathbf{2}, \hat{\mathbf{i}}_\mathbf{3}\}$ and $\mathcal{B} = \{O, \hat{\mathbf{b}}_\mathbf{1}, \hat{\mathbf{b}}_\mathbf{2}, \hat{\mathbf{b}}_\mathbf{3}\}$

$$R_{\mathcal{B}\mathcal{I}} = \begin{bmatrix} -0.17101007 & 0.46984631 & -0.86602540 \\ 0.98432795 & 0.04305861 & -0.17101007 \\ -0.04305861 & -0.88169745 & -0.46984631 \end{bmatrix}$$

(a) Find the **3-2-1** rotation-sequence Euler angles that parametrize $R_{\mathcal{B}\mathcal{I}}$.

(b) Find the **2-3-2** rotation-sequence Euler angles that parametrize $R_{\mathcal{B}\mathcal{I}}$.

Problem 3

(a) Take the MATLAB Onramp online course https://matlabacademy.mathworks.com/details/matlab-onramp/gettingstarted (~ 2hrs) and upload the certificate (pdf file) for proof of completion.

(b) Write a MATLAB function with the following inputs/outputs:

Input: Euler Angles (γ, β, α) in degrees and any feasible rotation sequence (*e.g. 3-2-1*)
Output: Coordinate Transformation Matrix $R_{\mathcal{B}\mathcal{I}}$.

(c) Write a MATLAB function with the following inputs/outputs:

Input: Coordinate Transformation Matrix $R_{\mathcal{B}\mathcal{I}}$ and any feasible rotation sequence (*e.g. 3-2-1*)
Output: Euler Angles (γ, β, α) in degrees.

Use the part (c) script to assess the correctness of Problem 3.

Problem 1

Two vectors $\mathbf{v}$ and $\mathbf{w}$, are represented in two coordinate reference frames $\mathcal{A}$ anb $\mathcal{B}$ as shown below:

$$\mathbf{v}_{\mathcal{A}} = [1, 2, 3]^T, \mathbf{w}_{\mathcal{A}} = [-1, 2, 1]^T$$

$$\mathbf{v}_{\mathcal{B}} = [3.56186, 1.13448, 0.16150]^T, \mathbf{w}_{\mathcal{B}} = [1.33712, 1.31501, -1.57572]^T$$

(a) Find the inner product $\mathbf{v} \cdot \mathbf{w}$ in each reference frame.

(b) Find the cross product $\mathbf{v} \times \mathbf{w}$ in each frame using only matrix multiplication.

(c) Find the coordinate transformation matrix $R_{\mathcal{BA}}$.

a) Find inner product v·w in each reference frame

a.1) Inner Product in "A" ref. frame

$$\vec{v} \xrightarrow{A} \vec{v}_A = [1, 2, 3]^T = 1\hat{a}_1 + 2\hat{a}_2 + 3\hat{a}_3$$
$$\vec{w} \xrightarrow{A} \vec{w}_A = [-1, 2, 1]^T = -1\hat{a}_1 + 2\hat{a}_2 + 1\hat{a}_3$$

Inner product in A ref frame $\Rightarrow$ $\vec{v}_A \cdot \vec{w}_A = (1\hat{a}_1 + 2\hat{a}_2 + 3\hat{a}_3) \cdot (-1\hat{a}_1 + 2\hat{a}_2 + 1\hat{a}_3)$

$$= -1 + 4 + 3 = 6$$

$$\boxed{\therefore \ \vec{v}_A \cdot \vec{w}_A = 6}$$

a.2) inner product in "B" ref. frame

$$\vec{v} \xrightarrow{B} \vec{v}_B = [3.56186, 1.13448, 0.16150] = 3.56186\,\hat{b}_1 + 1.13448\,\hat{b}_2 + 0.16150\,\hat{b}_3$$
$$\vec{w} \xrightarrow{B} \vec{w}_B = [1.33712, 1.31501, -1.57572] = 1.33712\,\hat{b}_1 + 1.31501\,\hat{b}_2 - 1.57572\,\hat{b}_3$$

Inner Product in B ref frame $\rightarrow$ $\vec{v}_B \cdot \vec{w}_B = (3.56186\,\hat{b}_1 \cdot 1.33712\,\hat{b}_1) + (1.13448\,\hat{b}_2 \cdot 1.31501\,\hat{b}_2) + (0.16150\,\hat{b}_3 \cdot (-1.57572\,\hat{b}_3))$

$$= 4.7626 + 1.4919 - 0.2545 = 6$$

$$\boxed{\therefore \ \vec{v}_B \cdot \vec{w}_B = 6}$$

b) Find outter product v x w in each frame using only Matrix Multiplication

b.1) Outter Product in "A" ref frame

$$\vec{v}_A \wedge \vec{w}_A = (1\hat{a}_1 + 2\hat{a}_2 + 3\hat{a}_3) \wedge (-1\hat{a}_1 + 2\hat{a}_2 + 1\hat{a}_3)$$

$$= \cancel{1 \cdot -1 \cdot \hat{a}_1 \cdot \hat{a}_1} + 1 \cdot 2 \cdot \overbrace{\hat{a}_1 \wedge \hat{a}_2}^{\hat{a}_3} + 1 \cdot 1 \cdot \overbrace{\hat{a}_1 \wedge \hat{a}_3}^{-\hat{a}_2} + \dots$$
$$+ 2 \cdot -1 \cdot \overbrace{\hat{a}_2 \wedge \hat{a}_1}^{-a_3} + \cancel{2 \cdot 2 \cdot \hat{a}_2 \wedge \hat{a}_2} + 2 \cdot 1 \cdot \overbrace{\hat{a}_2 \wedge \hat{a}_3}^{a_1} + \dots$$
$$+ 3 \cdot -1 \cdot \underbrace{\hat{a}_3 \wedge \hat{a}_1}_{a_2} + 3 \cdot 2 \cdot \underbrace{\hat{a}_3 \wedge \hat{a}_2}_{-a_1} + \cancel{3 \cdot 1 \cdot \hat{a}_3 \wedge \hat{a}_3}$$

$$\Rightarrow \vec{v}_A \wedge \vec{w}_A = (2 - 6)\hat{a}_1 + (-3 - 1)\hat{a}_2 + (2 - (-2))\hat{a}_3$$

$$\boxed{\vec{v}_A \wedge \vec{w}_A = [-4, -4, 4]^T = z_A}$$

b.2) Outter Product in "B" ref frame

$$\vec{v}_B \wedge \vec{w}_B = (3.56186\,\hat{b}_1 + 1.13448\,\hat{b}_2 + 0.16150\,\hat{b}_3) \wedge (1.33712\,\hat{b}_1 + 1.31501\,\hat{b}_2 - 1.57572\,\hat{b}_3)$$

$$= \cancel{(3.56186 \cdot 1.33712)\hat{b}_1 \cdot \hat{b}_1} + (3.56186 \cdot 1.31501)\overbrace{\hat{b}_1 \cdot \hat{b}_2}^{\hat{b}_3} + (3.56186 \cdot (-1.57572))\overbrace{\hat{b}_1 \cdot \hat{b}_3}^{-\hat{b}_2} + \dots$$
$$+ (1.13448 \cdot 1.33712)\overbrace{\hat{b}_2 \cdot \hat{b}_1}^{-b_3} + \cancel{(1.13448 \cdot 1.31501)\hat{b}_2 \cdot \hat{b}_2} + (1.3448 \cdot (-1.57572))\overbrace{\hat{b}_2 \cdot \hat{b}_3}^{b_2} + \dots$$
$$+ (0.16150 \cdot 1.33712)\underbrace{\hat{b}_3 \cdot \hat{b}_1}_{b_2} + (0.16150 \cdot 1.31501)\underbrace{\hat{b}_3 \cdot \hat{b}_2}_{-b_1} + \cancel{(0.16150 \cdot (-1.57572))\hat{b}_3 \cdot \hat{b}_3}$$

$$\Rightarrow \vec{v}_B \wedge \vec{w}_B = [-2.119 + 0.2123]\hat{b}_1 + [0.2159 + 5.612]\hat{b}_2 + [4.6838 + (-1.5169)]\hat{b}_3$$

$$\boxed{\therefore \ \vec{v}_B \wedge \vec{w}_B = [-2, 5.82, 3.16]^T = z_B}$$

c) Find the coordinate transformation matrix R_{BA}

$v_A = [1, 2, 3]^T$ $\quad$ $v_B = [3.56186, 1.13448, 0.16150]^T$

$w_A = [-1, 2, 1]^T$ $\quad$ $w_B = [1.33712, 1.31501, -1.57572]^T$

$\rightarrow v_B = R_{BA} v_A$

$$\underset{v_B}{\begin{bmatrix} 3.56186 \\ 1.13448 \\ 0.16150 \end{bmatrix}} = \underset{R_{BA}}{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}} \underset{v_A}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

$\rightarrow w_B = R_{BA} w_A$

$$\underset{w_B}{\begin{bmatrix} 1.33712 \\ 1.31501 \\ -1.57572 \end{bmatrix}} = \underset{R_{BA}}{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}} \underset{w_A}{\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}}$$

$\rightarrow z_B = R_{BA} z_A$

$$\underset{z_B}{\begin{bmatrix} -2 \\ 5.82 \\ 3.16 \end{bmatrix}} = \underset{R_{BA}}{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}} \underset{z_A}{\begin{bmatrix} -4 \\ -4 \\ 4 \end{bmatrix}}$$

$$\Rightarrow \begin{aligned} r_{11} + 2r_{12} + 3r_{13} &= 3.56186 \\ r_{21} + 2r_{22} + 3r_{23} &= 1.13448 \\ r_{31} + 2r_{32} + 3r_{33} &= 0.16150 \end{aligned}$$

$$\Rightarrow \begin{aligned} -r_{11} + 2r_{12} + r_{13} &= 1.33712 \\ -r_{21} + 2r_{22} + r_{23} &= 1.31501 \\ -r_{31} + 2r_{32} + r_{33} &= -1.57572 \end{aligned}$$

$$\Rightarrow \begin{aligned} -4r_{11} - 4r_{12} + 4r_{13} &= -2 \\ -4r_{21} - 4r_{22} + 4r_{23} &= 5.82 \\ -4r_{31} - 4r_{32} + 4r_{33} &= 3.16 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 3.56 \\ 1.33712 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ -4 & -4 & 4 \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{12} \\ r_{13} \end{bmatrix} \qquad \begin{bmatrix} 1.13448 \\ 1.31501 \\ 5.82 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ -4 & -4 & 4 \end{bmatrix} \begin{bmatrix} r_{21} \\ r_{22} \\ r_{23} \end{bmatrix} \qquad \begin{bmatrix} 0.16150 \\ -1.57572 \\ 3.16 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ -4 & -4 & 4 \end{bmatrix} \begin{bmatrix} r_{31} \\ r_{32} \\ r_{33} \end{bmatrix}$$

$$\boxed{R_{BA} = \begin{bmatrix} 0.4995 & 0.6124 & 0.6119 \\ -0.7493 & -0.0467 & 0.6590 \\ 0.4336 & -0.7886 & 0.4350 \end{bmatrix}}$$

Problem 2

Given the following coordinate transformation matrix between reference frames $\mathcal{I} = \{O, \hat{\imath}_1, \hat{\imath}_2, \hat{\imath}_3\}$ and $\mathcal{B} = \{O, \hat{b}_1, \hat{b}_2, \hat{b}_3\}$

$$R_{\mathcal{B}\mathcal{I}} = \begin{bmatrix} -0.17101007 & 0.46984631 & -0.86602540 \\ 0.98432795 & 0.04305861 & -0.17101007 \\ -0.04305861 & -0.88169745 & -0.46984631 \end{bmatrix}$$

(a) Find the **3-2-1** rotation-sequence Euler angles that parametrize $R_{\mathcal{B}\mathcal{I}}$.

(b) Find the **2-3-2** rotation-sequence Euler angles that parametrize $R_{\mathcal{B}\mathcal{I}}$.

a) Find the angles ψ, θ, ϕ so that $R_{BI} = R_1(\phi) \cdot R_2(\theta) \cdot R_3(\psi)$

$$R_{BI} = \begin{bmatrix} -0.17101007 & 0.46984631 & -0.86602540 \\ 0.98432795 & 0.04305861 & -0.17101007 \\ -0.04305861 & -0.88169745 & -0.46984631 \end{bmatrix}$$

$$R_{BI} = \underset{R_1\phi}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}} \cdot \underset{R_2\theta}{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}} \cdot \underset{R_3\psi}{\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$R_{BI} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} = \begin{bmatrix} -0.17101007 & 0.46984631 & -0.86602540 \\ 0.98432795 & 0.04305861 & -0.17101007 \\ -0.04305861 & -0.88169745 & -0.46984631 \end{bmatrix}$$

BY COMPARISON:

- $-\sin\theta = -0.86602540$
  $\theta = \arcsin(0.86602540)$
  $\theta = 60°$

- $\cos\theta\sin\psi = 0.46984631$
  $\cos\theta\cos\psi = -0.17101007$
  $\Rightarrow \dfrac{\cancel{\cos\theta}\sin\psi}{\cancel{\cos\theta}\cos\psi} = \dfrac{0.46984631}{-0.17101007} = \tan(\psi)$
  $\therefore \psi = a\tan\left(\dfrac{0.46984631}{-0.17101007}\right) = 110°$

- $\sin\phi\cos\theta = -0.17101007$
  $\cos\phi\cos\theta = -0.46984631$
  $\Rightarrow \dfrac{\cancel{\cos\theta}\sin\phi}{\cancel{\cos\theta}\cos\phi} = \dfrac{-0.17101007}{-0.46984631} = \tan(\phi)$
  $\therefore \phi = a\tan\left(\dfrac{-0.17101007}{-0.46984631}\right) = 200°$

$\Rightarrow$ The angles for a 3-2-1 sequence are

$$\boxed{\begin{aligned} \theta &= 60° \\ \phi &= 200° \\ \psi &= 110° \end{aligned}}$$

b) Find the angles α, β, γ so that $R_{BX} = R_2(\alpha) R_3(\beta) R_2(\gamma)$

$$R_{BX} = \begin{bmatrix} -0.17101007 & 0.46984631 & -0.86602540 \\ 0.98432795 & 0.04305861 & -0.17101007 \\ -0.04305861 & -0.88169745 & -0.46984631 \end{bmatrix}$$

$$\begin{array}{ccccc} R_2\alpha & & R_3\beta & & R_2\gamma \\ \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} & \cdot & \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} & \cdot & \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} R_2\alpha & & \\ \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} & \cdot & \begin{bmatrix} \cos\beta\cos\gamma + \cancel{\sin\beta(0)} + \cancel{\sin\gamma(0)} & \cancel{\cos\beta(0)} + \sin\beta(1) + \cancel{\cos\gamma(0)} & \cos\beta(-\sin\gamma) + \cancel{\sin\beta(0)} + \cancel{\cos\gamma(0)} \\ -\sin\beta\cos\gamma + \cancel{\cos\beta(0)} + \cancel{0\sin\gamma} & \cancel{-\sin\beta(0)} + \cos\beta(1) + \cancel{0} & -\sin\beta(-\sin\gamma) + \cancel{\cos\beta(0)} + \cancel{0(\cos\gamma)} \\ \cancel{0\cos\gamma} + \cancel{0 \cdot 0} + 1 \cdot \sin\gamma & \cancel{0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0} & \cancel{0 \cdot \sin\gamma} + \cancel{0 \cdot 0} + 1 \cdot \cos\gamma \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} R_2\alpha & & R_3\beta \cdot R_2\gamma \\ \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} & \cdot & \begin{bmatrix} \cos\beta\cos\gamma & \sin\beta & -\cos\beta\sin\gamma \\ -\sin\beta\cos\gamma & \cos\beta & \sin\beta\sin\gamma \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix} \end{array}$$

$$R_2(\alpha) \cdot R_3(\beta) \cdot R_2(\gamma) = \begin{bmatrix} \cos\alpha\cos\beta\cos\gamma + \cancel{0 \cdot (-\sin\beta)\cos\gamma} + -\sin\alpha\sin\gamma & \cos\alpha\sin\beta + \cancel{0\cos\beta} + \cancel{(-\sin\alpha)0} & -(\cos\alpha\cos\beta\sin\gamma) + \cancel{0(\sin\beta\sin\gamma)} + -\sin\alpha\cos\gamma \\ \cancel{0(\cos\beta\cos\gamma)} + 1(-\sin\beta\cos\gamma) + \cancel{0(\sin\gamma)} & \cancel{0(\sin\beta)} + 1(\cos\beta) + \cancel{0(0)} & \cancel{0(-\cos\beta\sin\gamma)} + 1(\sin\beta\sin\gamma) + \cancel{0(\cos\gamma)} \\ \sin\alpha(\cos\beta\cos\gamma) + \cancel{0(-\sin\beta\cos\gamma)} + \cos\alpha(\sin\gamma) & \sin\alpha(\sin\beta) + \cancel{0(\cos\beta)} + \cos\alpha(0) & \sin\alpha(-\cos\beta\sin\gamma) + \cancel{0(\sin\beta\sin\gamma)} + \cos\alpha(\cos\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & \cos\alpha\sin\beta & -\cos\alpha\cos\beta\sin\gamma - \sin\alpha\cos\gamma \\ -\sin\beta\cos\gamma & \cos\beta & \sin\beta\sin\gamma \\ \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\gamma & \sin\alpha\sin\beta & -\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma \end{bmatrix}$$

BY COMPARISON:

- $\cos\beta = 0.04305861$
- $\dfrac{\sin\alpha \cancel{\sin\beta}}{\cos\alpha \cancel{\sin\beta}} = \dfrac{-0.88169745}{0.46984631}$
- $\dfrac{\cancel{\sin\beta}\sin\gamma}{-\cancel{\sin\beta}\cos\gamma} = \dfrac{-0.17101007}{0.98432795}$

$\beta = \operatorname{acos}(0.04305861)$ $\qquad$ $\alpha = \operatorname{atan}\left(\dfrac{-0.88169745}{0.46984631}\right)$ $\qquad$ $\gamma = \operatorname{atan}\left(\dfrac{-0.17101007}{-0.98432795}\right)$

$$\boxed{\beta = 87.53216 \qquad \alpha = 298.05 \qquad \gamma = 189.85}$$

Problem 3

(a) Take the MATLAB Onramp online course https://matlabacademy.mathworks.com/details/matlab-onramp/gettingstarted (~ 2hrs) and upload the certificate (pdf file) for proof of completion.

(b) Write a MATLAB function with the following inputs/outputs:

Input: Euler Angles (γ, β, α) in degrees and any feasible rotation sequence (*e.g. 3-2-1*)
Output: Coordinate Transformation Matrix $R_{\mathcal{BI}}$.

(c) Write a MATLAB function with the following inputs/outputs:

Input: Coordinate Transformation Matrix $R_{\mathcal{BI}}$ and any feasible rotation sequence (*e.g. 3-2-1*)
Output: Euler Angles (γ, β, α) in degrees.

Use the part (c) script to assess the correctness of Problem 3.

a) MATLAB on Ramp Course → completed in ARO-3090 ~ Fall 2023

MathWorks | *Training Services*

Course Completion Certificate

Justin Millsap

has successfully completed **100%** of the self-paced training course

MATLAB Onramp

DIRECTOR, TRAINING SERVICES

31 October 2023

Part a & b ... see attached .m file or attached PDF below.

Table of Contents


```
% ARO 4090 - Space Vehicle Dyn. & Cntrl. | Dr. Maggia | Justin Millsap %
clc; clear; close all;
```

Problem 1

```
% Vectors V and W in "A" Reference Frame
v_a = [1 ; 2 ; 3] ;
w_a = [-1 ; 2 ; 1] ;

% Vectors V and W in "B" Reference Frame
v_b = [3.56186 ; 1.13448 ; 0.16150]  ;
w_b = [1.33712 ; 1.31501 ; -1.57572] ;

% Part A) Calculate the inner product in both reference frames

% ~~~~~~~~~~~~~~~~~~~~~~~ A-RF ~~~~~~~~~~~~~~~~~~~~~~~%
A_rf_inner_product = dot(v_a,w_a)

% ~~~~~~~~~~~~~~~~~~~~~~~ B-RF ~~~~~~~~~~~~~~~~~~~~~~~%
B_rf_inner_product = dot(v_b,w_b)


% Part B) Calculate the outter product in both reference frames

% ~~~~~~~~~~~~~~~~~~~~~~~ A-RF ~~~~~~~~~~~~~~~~~~~~~~~%
A_rf_outter_product = cross(v_a,w_a)

% ~~~~~~~~~~~~~~~~~~~~~~~ B-RF ~~~~~~~~~~~~~~~~~~~~~~~%
B_rf_outter_product = cross(v_b,w_b)

% Part c) Find the Coordiante Transformation Matrix R_BA
A = [1 2 3; -1 2 1; -4 -4 4];

% Right-hand side vectors b1, b2, and b3 for each system
b1 = [3.56; 1.33712; -2];
b2 = [1.13448; 1.31501; 5.82];
b3 = [0.16150; -1.57572; 3.16];

% Solving each system
r1 = A\b1; % Solves for r11, r12, r13
r2 = A\b2; % Solves for r21, r22, r23
r3 = A\b3; % Solves for r31, r32, r33
```

```
% Displaying the results
disp('r11, r12, r13:');
disp(r1);
disp('r21, r22, r23:');
disp(r2);
disp('r31, r32, r33:');
disp(r3);
```

```
A_rf_inner_product =

     6


B_rf_inner_product =

    6.0000


A_rf_outter_product =

    -4
    -4
     4


B_rf_outter_product =

   -2.0000
    5.8284
    3.1669

r11, r12, r13:
    0.4995
    0.6124
    0.6119

r21, r22, r23:
   -0.7493
   -0.0467
    0.6590

r31, r32, r33:
    0.4336
   -0.7886
    0.4350
```

Problem 2

```
clc; clear; close all

% INPUTS TO RIB matrix
```

```
r11 = -0.17101007;  r12 = 0.46984631 ;   r13 = -0.86602540;
r21 = 0.98432795 ;  r22 = 0.04305861 ;   r23 = -0.17101007;
r31 = -0.04305861;  r32 = -0.88169745;   r33 = -0.46984631;
R_BI = [r11 r12 r13 ; r21 r22 r23 ;  r31 r32 r33];

% 3-2-1 Rotation Matrix
theta = asind( abs(R_BI(1,3)) );
alpha = atan2d(R_BI(1,2) , R_BI(1,1));
gamma = atan2d(R_BI(2,3) , R_BI(3,3) ) + 360;
R1 = [ 1 0 0  ; 0 cosd(gamma) sind(gamma) ; 0 -sind(gamma) cosd(gamma)];
R2 = [cosd(theta) 0 -sind(theta); 0 1 0; sind(theta) 0 cosd(theta)];
R3 = [ cosd(alpha) sind(alpha) 0 ; -sind(alpha) cosd(alpha) 0 ; 0 0 1];
Rot_321 = R1*R2*R3

% 2-3-2 Rotation Matrx
beta = acosd(R_BI(2,2))
alpha = atan2d(R_BI(3,2) , R_BI(1,2)) + 360
gamma = atan2d(R_BI(2,3) , -R_BI(2,1)) + 360
R1 = [ 1 0 0  ; 0 cosd(alpha) sind(alpha) ; 0 -sind(alpha) cosd(alpha)];
R2 = [cosd(beta) 0 -sind(beta); 0 1 0; sind(beta) 0 cosd(beta)];
R3 = [ cosd(gamma) sind(gamma) 0 ; -sind(gamma) cosd(gamma) 0 ; 0 0 1];
```

```
Rot_321 =

   -0.1710    0.4698   -0.8660
    0.9843    0.0431   -0.1710
   -0.0431   -0.8817   -0.4698


beta =

   87.5322


alpha =

  298.0526


gamma =

  189.8558
```

Problem 3

```
clc; clear; close all

% Part A) Take MATLAB Onramp online course ~~ FINISHED

% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~Part B~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ %
disp('Part B')
```

```
% Write a MATLAB function with the following inputs/outputs:
    % ~Inputs: Eurler Angles (gamma, beta, alpha) in degrees and any
        %feasible rotation sequence (e.g. 3-2-1)
    % ~Outputs: Coordiate Transformation Matrix R_BI



%%%%%%%%%%%%%%%%%%%%%%%%%%%%      EDIT      %%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Input Eurler Angles [degrees]

gamma = 200 ;
beta = 60   ;
alpha = 110 ;

% Define Rotation Sequence (e.g. [3-2-1] use 321)

Rot_Seq = 321 ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%  DO NOT EDIT    %%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Define the elementary rotation matrices
R1 = [ 1 0 0  ; 0 cosd(gamma) sind(gamma) ; 0 -sind(gamma) cosd(gamma)];
R2 = [cosd(beta) 0 -sind(beta); 0 1 0; sind(beta) 0 cosd(beta)];
R3 = [ cosd(alpha) sind(alpha) 0 ; -sind(alpha) cosd(alpha) 0 ; 0 0 1];



% Determine and compute the rotation matrix based on Rot_Seq
if Rot_Seq == 321
    R_BI = R1*R2*R3; % [3-2-1]
elseif Rot_Seq == 231
    R_BI = R1R3*R2; % [2-3-1]
elseif Rot_Seq == 121
    R_BI = R1*R2*R1; % [1-2-1]
elseif Rot_Seq == 131
    R_BI = R1*R3*R1; % [1-3-1]
elseif Rot_Seq == 132
    R_BI = R2*R3*R1; % [1-3-2]
elseif Rot_Seq == 312
    R_BI = R2*R1*R3; % [3-1-2]
elseif Rot_Seq == 212
    R_BI = R2*R1*R2; % [2-1-2]
elseif Rot_Seq == 232
    R_BI = R2*R3*R2; % [2-3-2]
elseif Rot_Seq == 213
    R_BI = R3*R1*R2; % [2-1-3]
elseif Rot_Seq == 123
    R_BI = R3*R2*R1; % [1-2-3]
elseif Rot_Seq == 313
    R_BI = R3*R1*R3; % [3-1-3]
elseif Rot_Seq == 323
    R_BI = R3*R2*R3; % [3-2-3]
else
```

```
    error('Invalid rotation sequence');
end

disp('The Coodinate Transformation Matrix R_BI = ')
disp(R_BI)




% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~Part C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ %
disp(' Part C')
% Write a MATLAB function with the following inputs/outputs:
    % ~Inputs: Coordiante Tranformation Matrix R_BI and the 3-2-1
    %          Rotation Sequence.
    % ~Outputs: Eurler angles ( psi , theta , and phi ) in degrees.


% INPUTS TO R_BI matrix Inputs
r11 = -0.17101007;  r12 = 0.46984631 ;   r13 = -0.86602540;
r21 = 0.98432795 ;  r22 = 0.04305861 ;   r23 = -0.17101007;
r31 = -0.04305861;  r32 = -0.88169745;   r33 = -0.46984631;

% R_BI Matrix
R_BI = [r11 r12 r13 ; r21 r22 r23 ;  r31 r32 r33];


Rot_Seq = R1*R2*R3;           % [3-2-1] Rotation Sequence

% Solve for Eurler Angles by using most opitmal positions to compare between
R_BI & Rotation Sequence Matrix


theta = asind(-R_BI(1,3));
phi   = atan2d(R_BI(2,3) , R_BI(3,3)) + 360 ;
psi   = atan2d(R_BI(1,2) , R_BI(1,1)) ;

% Check if Eurler angles work

Rot_Seq = R1*R2*R3;
disp(' The Eurler Angles for the given R_BI Matrix for a given Rotation
Sequence are')
disp(theta) ; disp(phi) ; disp(psi)
```

```
Part B
The Coodinate Transformation Matrix R_BI =
   -0.1710    0.4698   -0.8660
    0.9843    0.0431   -0.1710
   -0.0431   -0.8817   -0.4698

 Part C
 The Eurler Angles for the given R_BI Matrix for a given Rotation Sequence
are
   60.0000
```

```
  200.0000

  110.0000
```

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