Homework #9

Due: **Sunday, May 5, 2024 at 11:59 pm**

Turn in, on Canvas, a well organized and clear-to-read pdf file with your calculations, explanations, plots, as well as a ready-to-run .m/.slx files containing the MATLAB/Simulink codes (when applicable).

Problem 1

Design a flywheel (i.e., its size and spin rate) for a dual-spin stabilized spacecraft so that its rotational motion about its body-fixed z-axis is stable.

Spacecraft characteristics without wheel:

Mass:	$m = 1,630\,kg$
Dimensions along body x-y-z axes:	$2.49 \times 2.03 \times 2.24\,m$
Nominal rotational speed about body z-axis:	$n = 5\deg/s$

Show the effectiveness of your design by providing the S/C rotational response when perturbed from the equilibrium state.

Please submit your MATLAB/Simulink code.

Problem 2

Consider a spacecraft (S/C) whose body-fixed reference frame (RF) coincides with its principal RF. If we indicate with $\omega_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}} = [p, q, r]^T$ its angular velocity with respect to an inertial RF, measured in the body-fixed RF coordinates:

(a) Show that $[p, q, r]^T = [n, 0, 0]^T$ represents an equilibrium point for the S/C's torque-free motion. After linearizing the system about such equilibrium point, determine its stability characteristics.

(b) If a momentum wheel (MW) rotating about the S/C $\hat{b}_1$ axis (i.e., the body-fixed x-axis) is added, show that the torque-free S/C rotational dynamics equations of motion become

$$\begin{gathered}
I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + I_w\dot{\Omega} = 0 \\
I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_w r\Omega = 0 \\
I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - I_w q\Omega = 0
\end{gathered}$$

where $I_w > 0$ and Ω are the MW's moment of inertia and angular speed, respectively.

(c) Under which condition(s) is $[p, q, r]^T = [n, 0, 0]^T$ still an equilibrium point for the S/C's torque-free motion? What are its stability characteristics?

If $I_{xx} = 350\ kg \cdot m^2$, $I_{yy} = 280\ kg \cdot m^2$, $I_{zz} = 400\ kg \cdot m^2$, $I_w = 5\ kg \cdot m^2$,$\dot{\Omega} = 0$, and $n = 10\ rpm$:

(d) Find the range of angular speeds Ω (in rotations per minute) so that the S/C is stable. (*Hint: linearization → Laplace transform → poles*).

Problem 1

Design a flywheel (i.e., its size and spin rate) for a dual-spin stabilized spacecraft so that its rotational motion about its body-fixed z-axis is stable.

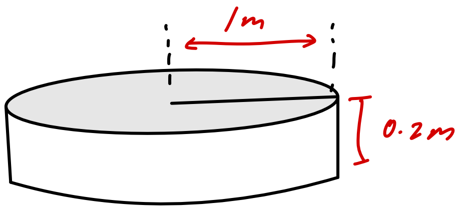
* Mass = 30 kg

* Diameter = 2.0 m

* thickness = 0.2 m

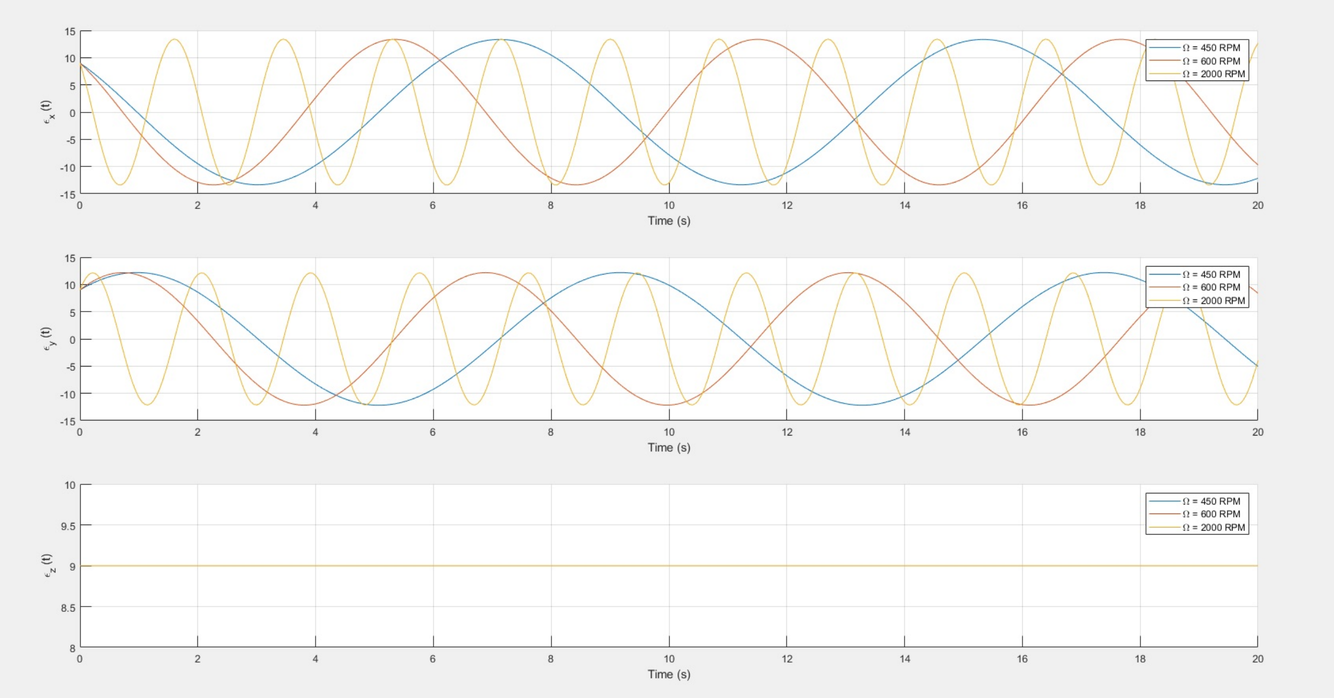
Spacecraft characteristics without wheel:

Mass:	$m = 1,630\ kg$
Dimensions along body x-y-z axes:	$2.49 \times 2.03 \times 2.24\ m$
Nominal rotational speed about body z-axis:	$n = 5 \deg / s$



Show the effectiveness of your design by providing the S/C rotational response when perturbed from the equilibrium state.

Please submit your MATLAB/Simulink code.



* MATLAB CODE ATTACHED

Problem 2
Consider a spacecraft (S/C) whose body-fixed reference frame (RF) coincides with its principal RF. If we indicate with $\omega_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}} = [p, q, r]^T$ its angular velocity with respect to an inertial RF, measured in the body-fixed RF coordinates:

(a) Show that $[p, q, r]^T = [n, 0, 0]^T$ represents an equilibrium point for the S/C's torque-free motion. After linearizing the system about such equilibrium point, determine its stability characteristics.

NONLINEAR SYSTEM

$$\begin{bmatrix} \dot p \\ \dot q \\ \dot r \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}} qr \\ \frac{I_{zz}-I_{xx}}{I_{yy}} pr \\ \frac{I_{xx}-I_{yy}}{I_{zz}} pq \end{bmatrix}\Bigg|_{\substack{p=n \\ q=0 \\ r=0}} \Rightarrow \begin{matrix} \dot p = 0 \\ \dot q = 0 \\ \dot r = 0 \end{matrix} \Rightarrow \boxed{\begin{matrix} p = \text{constant} \\ q = \text{constant} \\ r = \text{constant} \end{matrix}} \rightarrow$$

state variables are constant ∴ $[p, q, r]^T$ represents an equilibrium point

LINEARIZED SYSTEM: $\Delta\dot{\vec{x}} = J(\vec{x}_0) + \Delta\vec{x}$

$$\underbrace{\begin{bmatrix} \Delta\dot p \\ \Delta\dot q \\ \Delta\dot r \end{bmatrix}}_{\Delta\dot{\vec{x}}} = \underbrace{\begin{bmatrix} 0 & \frac{I_{yy}-I_{zz}}{I_{xx}} r_0 & \frac{I_{yy}-I_{zz}}{I_{xx}} q_0 \\ \frac{I_{zz}-I_{xx}}{I_{yy}} r_0 & 0 & \frac{I_{zz}-I_{xx}}{I_{yy}} p_0 \\ \frac{I_{xx}-I_{yy}}{I_{zz}} q_0 & \frac{I_{xx}-I_{yy}}{I_{zz}} p_0 & 0 \end{bmatrix}}_{J(\vec{x}_0)} \underbrace{\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}}_{\Delta\vec{x}}$$

WHERE
$\vec{x}_0 = [p_0, q_0, r_0]^T = [n, 0, 0]^T$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{I_{zz}-I_{xx}}{I_{yy}} n \\ 0 & \frac{I_{xx}-I_{yy}}{I_{zz}} n & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}$$

RECALL:
$$\sigma_x = \frac{I_{zz}-I_{yy}}{I_{xx}}, \quad \sigma_y = \frac{I_{xx}-I_{zz}}{I_{yy}}, \quad \sigma_z = \frac{I_{yy}-I_{xx}}{I_{yy}}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sigma_y n \\ 0 & -\sigma_z n & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}$$

⇒ WE CAN STUDY THE EIGENVALUES OF THE JACOBIAN MATRIX $[J(\vec{x}_0)]$ TO DETERMINE STABILITY.

$$J(\vec{x}_0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sigma_y n \\ 0 & -\sigma_z n & 0 \end{bmatrix}$$

RECALL: $(J(\vec{x}_0) - \lambda I) = \vec{0}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sigma_y n \\ 0 & -\sigma_z n & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \underbrace{\begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & -\sigma_y n \\ 0 & -\sigma_z n & -\lambda \end{bmatrix}}_{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

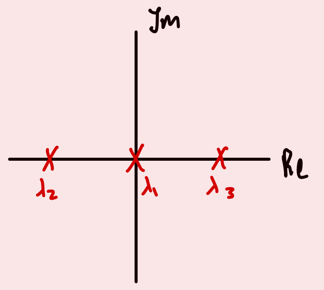
$$\det(B) = -\lambda \begin{vmatrix} -\lambda & -\sigma_y n \\ -\sigma_z n & -\lambda \end{vmatrix} = -\lambda\left((-\lambda)(-\lambda) - (-\sigma_y n)(-\sigma_z n)\right)$$
$$= -\lambda\left[\lambda^2 - n^2\sigma_y\sigma_z\right]$$

$\Rightarrow \lambda_1 = 0$

$$\lambda_{2,3} = \frac{\pm\sqrt{4n^2\sigma_y\sigma_z}}{2} \Rightarrow \lambda_{2,3} = \pm|n|\sqrt{\sigma_y\sigma_z}$$

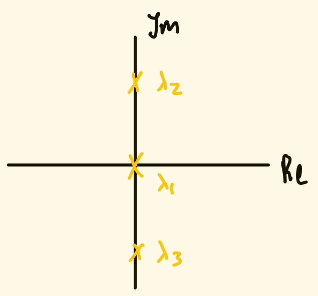
$$\boxed{\begin{cases} \lambda_1 = 0 \\ \lambda_{2,3} = \pm|n|\sqrt{\dfrac{(I_{xx}-I_{zz})(I_{yy}-I_{xx})}{I_{yy} I_{zz}}} \end{cases}}$$

If I_{xx} → INTERMEDIATE AXES
· $I_{yy} < I_{xx} < I_{zz}$ or $I_{zz} < I_{xx} < I_{yy}$



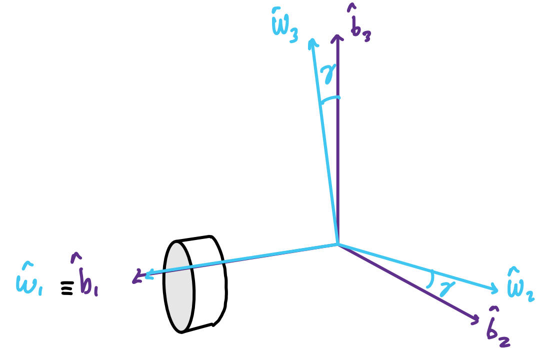
⚠ UNSTABLE SYSTEM

If I_{xx} → MAX/MIN AXES
· $I_{xx} > I_{yy}$ & $I_{xx} > I_{yy}$ or $I_{xx} < I_{yy}$ or $I_{xx} < I_{yy}$



⚠ NEUTRALLY STABLE

(b) If a momentum wheel (MW) rotating about the S/C $\hat{b}_1$ axis (i.e., the body-fixed x-axis) is added, show that the torque-free S/C rotational dynamics equations of motion become



$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + I_w\dot{\Omega} = 0$$
$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_w r\Omega = 0$$
$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - I_w q\Omega = 0$$

$$\Rightarrow \vec{H}^{c}_{S/c_B} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} I_{aw} & 0 & 0 \\ 0 & I_{tw}+M_w\chi_w^2 & 0 \\ 0 & 0 & I_{tw}+M_w\chi_w^2 \end{bmatrix}\begin{bmatrix} p+\Omega \\ q \\ r \end{bmatrix}$$

$$\begin{bmatrix} I_{xx}p \\ I_{yy}q \\ I_{zz}r \end{bmatrix} + \begin{bmatrix} I_{aw}(p+\Omega) \\ I_{tw}\,q \\ I_{tw}\,r \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}\begin{bmatrix} I_{xx}p + I_{aw}(p+\Omega) \\ I_{yy}q + I_{tw}q \\ I_{zz}r + I_{tw}r \end{bmatrix} = \begin{bmatrix} rq(I_{zz}-I_{yy}) \\ rp(I_{xx}-I_{zz}) \\ pq(I_{yy}-I_{xx}) \end{bmatrix} + \begin{bmatrix} 0 \\ rI_{aw}(p+\Omega) - rpI_{tw} \\ pqI_{tw} - qI_{aw}(p+\Omega) \end{bmatrix}$$

$$\frac{{}^{\mathcal{I}}d\vec{H}^{c}_{S/c_B}}{dt} = \dot{\vec{H}}^{c}_{S/c_B} + \vec{\omega}^{B/\mathcal{I}}_B \wedge \vec{H}^{c}_{S/c_B} = \vec{M}^{c}_B$$

$$= \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} I_{aw} & 0 & 0 \\ 0 & I_{tw} & 0 \\ 0 & 0 & I_{tw} \end{bmatrix}\begin{bmatrix} \dot{p}+\dot{\Omega} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}\left(\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} I_{aw} & 0 & 0 \\ 0 & I_{tw} & 0 \\ 0 & 0 & I_{tw} \end{bmatrix}\begin{bmatrix} p+\Omega \\ q \\ r \end{bmatrix}\right) = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$= \begin{bmatrix} I_{xx}\dot{p} \\ I_{yy}\dot{q} \\ I_{zz}\dot{r} \end{bmatrix} + \begin{bmatrix} I_{aw}(\dot{p}+\dot{\Omega}) \\ I_{tw}\dot{q} \\ I_{tw}\dot{r} \end{bmatrix} + \begin{bmatrix} rq(I_{zz}-I_{yy}) \\ rp(I_{xx}-I_{zz}) \\ pq(I_{yy}-I_{xx}) \end{bmatrix} + \begin{bmatrix} 0 \\ rI_{aw}(p+\Omega) - rpI_{at} \\ pqI_{tw} - qI_{aw}(p+\Omega) \end{bmatrix} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$\therefore\ M_x = I_{xx}\dot{p} + I_{aw}(\dot{p}+\dot{\Omega}) + rq(I_{zz}+I_{yy})$$
$$M_y = I_{yy}\dot{q} + I_{tw}\dot{q} + rp(I_{xx}-I_{zz}) + rI_{aw}(p+\Omega) - rpI_{at}$$
$$M_z = I_{zz}\dot{r} + I_{tw}\dot{r} + pq(I_{yy}-I_{xx}) + pqI_{tw} - qI_{aw}(p+\Omega)$$

$* \; I_{aw} \simeq I_{tw} \simeq I_w$

$$\hookrightarrow M_x = I_{xx}\dot{p} + I_w\dot{\Omega} + \cancel{I_w\dot{p}} + rq(I_{zz}+I_{yy})$$

$$M_x = I_{xx}\dot{p} + rq(I_{zz}-I_{yy}) + I_w\dot{\Omega}$$

$$\hookrightarrow M_y = I_{yy}\dot{q} + I_w\dot{q} + rp(I_{xx}-I_{zz}) + \cancel{rp(I_w)} + r\Omega I_w - \cancel{rpI_{at}}$$

$$M_y = I_{yy}\dot{q} + rp(I_{xx}-I_{zz}) + r\Omega I_w$$

$$\hookrightarrow M_z = I_{zz}\dot{r} + \cancel{I_w\dot{r}} + pq(I_{yy}-I_{xx}) + \cancel{pqI_w} - \cancel{qpI_w} - I_w q\Omega$$

$$M_z = I_{zz}\dot{r} + pq(I_{yy}-I_{xx}) - I_w q\Omega$$

(c) Under which condition(s) is $[p, q, r]^T = [n, 0, 0]^T$ still an equilibrium point for the S/C's torque-free motion? What are its stability characteristics?

If $I_{xx} = 350\ kg \cdot m^2$, $I_{yy} = 280\ kg \cdot m^2$, $I_{zz} = 400\ kg \cdot m^2$, $I_w = 5\ kg \cdot m^2$, $\dot{\Omega} = 0$, and $n = 10\ rpm$:

$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + I_w\dot{\Omega} = 0$$
$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_w r\Omega = 0$$
$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - I_w q\Omega = 0$$

$$\hookrightarrow \quad \dot{p} = \frac{-qr(I_{zz}-I_{yy}) - I_w\dot{\Omega}}{I_{xx}} = \frac{qr(I_{yy}-I_{zz}) - I_w\dot{\Omega}}{I_{xx}}$$

$$\dot{q} = \frac{-rp(I_{xx}-I_{zz}) - I_w r\Omega}{I_{xx}} = \frac{rp(I_{zz}-I_{xx}) - I_w r\Omega}{I_{xx}}$$

$$\dot{r} = \frac{-pq(I_{yy}-I_{xx}) + I_w q\Omega}{I_{zz}} = \frac{pq(I_{xx}-I_{yy}) + I_w q\Omega}{I_{zz}}$$

$*$ use $[p, q, r]^T = [n, 0, 0]$

$$\hookrightarrow \dot{p} = -\frac{I_w\dot{\Omega}}{I_{xx}}$$

$$\hookrightarrow \dot{q} = 0$$

$$\hookrightarrow \dot{r} = 0$$

$\Rightarrow$ If the condition that $[p, q, r]^T = [n, 0, 0]$ is present, this would represent an equilibrium point if $\dot{\Omega} = 0$

$\hookrightarrow$ Neutrally stable due to torque free motion

If $I_{xx} = 350\ kg \cdot m^2$, $I_{yy} = 280\ kg \cdot m^2$, $I_{zz} = 400\ kg \cdot m^2$, $I_w = 5\ kg \cdot m^2$,$\dot{\Omega} = 0$, and $n = 10\ rpm$:

(d) Find the range of angular speeds Ω (in rotations per minute) so that the S/C is stable. (*Hint: linearization → Laplace transform → poles*).

NON-LINEAR SYSTEM

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{I_w}{I_{xx}} q\Omega \qquad\qquad \dot{p} = 0$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{I_w}{I_{yy}} p\Omega \quad \xrightarrow{[p,q,r]^T = [n,0,0]} \quad \dot{q} = \frac{I_w}{I_{yy}} (n)\Omega$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} pq \qquad\qquad \dot{r} = 0$$

LINEARIZED SYSTEM

$$\underbrace{\begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix}}_{\Delta\dot{\vec{x}}} = \underbrace{\begin{bmatrix} 0 & \frac{I_{yy} - I_{zz}}{I_{xx}} r_0 + \frac{I_w}{I_{xx}}\Omega & \frac{I_{yy} - I_{zz}}{I_{xx}} q_0 \\ \frac{I_{zz} - I_{xx}}{I_{yy}} r_0 + \frac{I_w}{I_{yy}}\Omega & 0 & \frac{I_{zz} - I_{xx}}{I_{yy}} p_0 \\ \frac{I_{xx} - I_{yy}}{I_{zz}} q_0 & \frac{I_{xx} - I_{yy}}{I_{zz}} p_0 & 0 \end{bmatrix}}_{J(\vec{x}_0)} \underbrace{\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}}_{\Delta\vec{x}}$$

WHERE $\vec{x}_0 = [p_0, q_0, r_0]^T = [n, 0, 0]^T$

$$= \begin{bmatrix} 0 & \frac{I_w}{I_{xx}}\Omega & 0 \\ \frac{I_w}{I_{yy}}\Omega & 0 & \frac{I_{zz} - I_{xx}}{I_{yy}} n \\ 0 & \frac{I_{xx} - I_{yy}}{I_{zz}} n & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{bmatrix}$$

RECALL:

$\Delta p = p - p_0 = \varepsilon_x$

$\Delta q = q - q_0 = \varepsilon_y$

$\Delta r = r - r_0 = \varepsilon_z$

$\sigma_x = \frac{I_{zz} - I_{yy}}{I_{xx}}$

$\sigma_y = \frac{I_{xx} - I_{zz}}{I_{yy}}$

$\sigma_z = \frac{I_{yy} - I_{xx}}{I_{yy}}$

$$\begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\varepsilon}_z \end{bmatrix} = \begin{bmatrix} 0 & \frac{I_w}{I_{xx}}\Omega & 0 \\ \frac{I_w}{I_{yy}}\Omega & 0 & -\sigma_y n \\ 0 & -\sigma_z n & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} \dot{\varepsilon}_x &= \frac{I_w}{I_{xx}} \Omega \varepsilon_y \\ \dot{\varepsilon}_y &= \frac{I_w}{I_{yy}} \Omega \varepsilon_x - \sigma_y n \varepsilon_z \\ \dot{\varepsilon}_z &= -\sigma_z n \varepsilon_y \end{aligned}\right\} \xrightarrow{\mathcal{L}} \begin{aligned} s\varepsilon_x - \varepsilon_x(0) &= \varepsilon_y \left(\frac{I_w}{I_{xx}}\Omega\right) \\ s\varepsilon_y - \varepsilon_y(0) &= \varepsilon_x \left(\frac{I_w}{I_{yy}}\Omega\right) - \varepsilon_z(\sigma_y n) \\ s\varepsilon_z - \varepsilon_z(0) &= -\varepsilon_y(\sigma_z n) \end{aligned}$$

$$\Rightarrow \begin{aligned} \varepsilon_x(0) &= s\varepsilon_x - \varepsilon_y \left(\frac{I_w}{I_{xx}}\Omega\right) \\ \varepsilon_y(0) &= s\varepsilon_y + \varepsilon_z(\sigma_y n) - \varepsilon_x \left(\frac{I_w}{I_{yy}}\Omega\right) \\ \varepsilon_z(0) &= s\varepsilon_z + \varepsilon_y(\sigma_z n) \end{aligned} \Rightarrow \begin{bmatrix} \varepsilon_x(0) \\ \varepsilon_y(0) \\ \varepsilon_z(0) \end{bmatrix} = \underbrace{\begin{bmatrix} s & -\frac{I_w}{I_{xx}}\Omega & 0 \\ -\frac{I_w}{I_{yy}}\Omega & s & \sigma_y n \\ 0 & \sigma_z n & s \end{bmatrix}}_{A} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

$$\det(A) = s \begin{vmatrix} s & \sigma_y n \\ \sigma_z n & s \end{vmatrix} - \frac{I_w}{I_{xx}}\Omega \begin{vmatrix} -\frac{I_w}{I_{yy}}\Omega & \sigma_y n \\ 0 & s \end{vmatrix}$$

$$= s(s^2 - n^2\sigma_y\sigma_z) - \frac{I_w}{I_{xx}}\Omega\left(s\left(-\frac{I_w}{I_{yy}}\Omega\right)\right)$$

$$= s(s^2 - n^2\sigma_y\sigma_z) + s\Omega^2\left(\frac{I_w^2}{I_{xx}I_{yy}}\right)$$

$$\hookrightarrow \det(A) = s\left[(s^2 - n^2\sigma_y\sigma_z) + \Omega^2\left(\frac{I_w^2}{I_{xx}I_{yy}}\right)\right]$$

$$\rightarrow s_1 = 0$$

$$s_{2,3} = \pm\sqrt{n^2\sigma_y\sigma_z - \Omega^2 I_w^2\left(\frac{1}{I_{xx}I_{yy}}\right)}$$

· FOR NEUTRAL STABILITY :

$$n^2\sigma_y\sigma_z - \Omega^2 I_w^2\left(\frac{1}{I_{xx}I_{yy}}\right) < 0$$

$$\hookrightarrow \Omega^2 I_w^2\left(\frac{1}{I_{xx}I_{yy}}\right) - n^2\sigma_y\sigma_z > 0$$

$$\hookrightarrow \Omega^2\boxed{\left[I_w^2\cdot\frac{1}{I_{xx}I_{yy}}\right]} + \Omega\boxed{[0]} + \Omega^0\boxed{[-n^2\sigma_y\sigma_z]} > 0$$

$$f(\Omega) = a\Omega^2 + b\Omega + c > 0$$

$I_{xx} = 350\,kg\cdot m^2$, $I_{yy} = 280\,kg\cdot m^2$, $I_{zz} = 400\,kg\cdot m^2$, $I_w = 5\,kg\cdot m^2$,$\dot{\Omega} = 0$, and $n = 10\,rpm$:

$$\rightarrow b^2 - 4ac = -4\left(\frac{I_w^2}{I_{xx}I_{yy}}\right)(-n^2\sigma_y\sigma_z)$$

$$= 4n^2\left(\frac{I_w}{I_{xx}}\right)^2\left(\frac{I_{xx}-I_{zz}}{I_{yy}^2}\right)\left(\frac{I_{yy}-I_{xx}}{I_{zz}}\right)$$

$$= 4(10^2)\left(\frac{5^2}{350}\right)\left(\frac{350-400}{280^2}\right)\left(\frac{280-350}{400}\right) = \frac{5}{1568}\ rpm\cdot kgm^2$$

$$\hookrightarrow a = I_w^2\cdot\frac{1}{I_{xx}I_{yy}} = 2.55\times10^{-4}\,kgm^2$$

$$\hookrightarrow \Omega_{1,2} = \frac{\sqrt{5/1568}\ rpm\cdot kg\cdot m^2}{2(2.55\times10^{-4})\ kgm^2}$$

$$\boxed{\Omega_{1,2} = \pm 110.7}$$