Justin Millsap ARO-3111-02 01/29/24 PS-1

1.3 For a calorically perfect gas, derive the relation $c_p - c_v = R$. Repeat the derivation for a thermally perfect gas.

Calorically Perfect gas Derivation:

Enthalpy: $h = e + PV$

Assume: $e = C_v T$; $h = C_p T$ → Lecture 2, Page 27

$\hookrightarrow C_p T = C_v T + PV$ $\quad * \ v = 1/\rho$ $\quad * \ \frac{P}{\rho} = RT$

$$\frac{C_p \cancel{T} = C_v \cancel{T} + R\cancel{T}}{\cancel{T}} \Rightarrow C_p = C_v + R$$

$$C_p - C_v = R$$

Thermally Perfect gas Derivation:

Enthalpy: $h = e + PV$

$\rightarrow dh = de + d(PV)$ $\quad * \ v = 1/\rho$

$\rightarrow dh = de + d\left(\frac{P}{\rho}\right)$ $\quad * \ P = \rho RT \therefore \frac{P}{\rho} = RT$

$\rightarrow dh = de + d(RT) \Rightarrow dh = de + RdT$

$\Rightarrow \frac{dh}{dT} = \frac{de}{dT} + R$ $\quad \left.\begin{array}{l} * \ dh = C_p dT \\ * \ de = C_v dT \end{array}\right\}$ Lecture 2 Page 26

$\therefore C_p = C_v + R$

$$\Rightarrow C_p - C_v = R$$

1.4 The pressure and temperature ratios across a given portion of a shock wave in air are $p_2/p_1 = 4.5$ and $T_2/T_1 = 1.687$, where 1 and 2 denote conditions ahead of and behind the shock wave, respectively. Calculate the change in entropy in units of (*a*) (ft · lb)/(slug · °R) and (*b*) J/(kg · K).

a) $(ft \cdot lb)/(slug \cdot °R)$ [English units $R = 1716 \ ft \cdot lb/slug \cdot R°$]

$$s_2 - s_1 = C_p \ln(T_2/T_1) - R\ln(P_2/P_1)$$

$$* \ C_p = \frac{R\gamma}{\gamma - 1}$$

$$s_2 - s_1 = \frac{R\gamma}{\gamma-1}\ln(T_2/T_1) - R\ln(P_2/P_1)$$

Where: $\gamma = 1.4$; $\frac{T_2}{T_1} = 1.687$

$R = 1716 \ \frac{ft \cdot lb}{slug \cdot R°}$; $\frac{P_2}{P_1} = 4.5$

$$\Rightarrow \Delta s = \frac{\left(1716 \frac{ft \cdot lb}{slug \cdot R°}\right)(1.4)}{1.4 - 1}\ln(1.687) - \left(1716 \frac{ft \cdot lb}{slug \cdot R°}\right)\ln(4.5)$$

$$\Delta s = 559.85 \ \frac{ft \cdot lb}{slug \cdot R°}$$

b) $J/kg \cdot K$ [SI units $R = 287 \ J/kg \cdot K$]

$$s_2 - s_1 = C_p \ln(T_2/T_1) - R\ln(P_2/P_1)$$

$$* \ C_p = \frac{R\gamma}{\gamma - 1}$$

$$s_2 - s_1 = \frac{R\gamma}{\gamma-1}\ln(T_2/T_1) - R\ln(P_2/P_1)$$

Where: $\gamma = 1.4$; $\frac{T_2}{T_1} = 1.687$

$R = 287 \ J/kg \cdot °K$; $\frac{P_2}{P_1} = 4.5$

$$\Rightarrow \Delta s = \frac{(287 \ J/kg \cdot °K)(1.4)}{1.4 - 1}\ln(1.687) - (287 \ J/kg \cdot °K)\ln(4.5)$$

$$\Delta s = 93.6 \ J/kg \cdot °K$$

1.5 Assume that the flow of air through a given duct is isentropic. At one point in the duct, the pressure and temperature are $p_1 = 1800$ lb/ft^2 and $T_1 = 500$°R, respectively. At a second point, the temperature is 400°R. Calculate the pressure and density at this second point.

$P_1 = 1800 \text{ lb/ft}^2$; $P_2 = ?$

$T_1 = 500\,°R$; $T_2 = 400\,°R$

$\rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = (1800 \text{ lb/ft}^2)\left(\frac{400\,°R}{500\,°R}\right)^{\frac{1.4}{1.4-1}}$

$\therefore P_2 = 824.3 \text{ lb/ft}^2$

$\Rightarrow P_2 = \rho_2 R T_2$

$\hookrightarrow \rho_2 = \frac{P_2}{RT_2} = \frac{(824.3 \ \cancel{\text{lb}}/\text{ft}^2)}{\left(1716 \frac{\text{ft}\cdot\cancel{\text{lb}}}{\text{slug}\cdot\cancel{°R}}\right)(400\,\cancel{°R})} = 0.0012 \text{ slug/ft}^3$

$\Rightarrow P_2 = 824.30 \text{ lb/ft}^2$

$\rho_2 = 0.0012 \text{ slug/ft}^3$

Derive the Eq 1.37 :

Eq 1.37 $\rightarrow s_2 - s_1 = C_v \ln(T_2/T_1) + R \ln(v_2/v_1)$

$h = e + Pv$

$dh = de + Pdv + vdP$

$dh - vdP = de + Pdv$ $\quad$ * $Tds = dh - vdP$

$Tds = de + Pdv$ $\quad$ * $de = C_v dt$

$Tds = C_v dt + Pdv$ $\quad$ * $P = \rho RT$; $\frac{1}{T} = \frac{\rho R}{P}$

$ds = C_v \frac{1}{T} dt + P\left(\frac{\rho R}{P}\right) dv$

$ds = C_v\left(\frac{1}{T}\right)dt + \rho R\, dv$ $\quad$ * $v = 1/\rho \Rightarrow \rho = 1/v$

$\int_{s_1}^{s_2} ds = C_v \int_{T_1}^{T_2} \frac{1}{T} dT + R\int_{v_1}^{v_2} \frac{1}{v} dv$

$\therefore s_2 - s_2 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$