Homework #5

Due: **Monday, March 11, 2024 at 11:59 pm**

Turn in, on Canvas, a well organized and clear-to-read pdf file with your calculations, explanations, plots, as well as a ready-to-run .m/.slx files containing the MATLAB/Simulink codes (when applicable).

Problem 1

Consider a homogeneous, rigid cube of side $l = 30\ cm$ and density $\rho = 1000\ kg/m^3$. Knowing that at $t = 0$ its attitude and angular velocity are:

$$\vec{\boldsymbol{\theta}}(0) = [50, 30, 60]^T\ deg,$$

$$\vec{\boldsymbol{\omega}}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(0) = [200, 30, 10]^T\ deg/s,$$

create a SIMULINK model to find $\hat{\boldsymbol{\beta}}(t)$, $\vec{\boldsymbol{\theta}}(t)$ and $\vec{\boldsymbol{\omega}}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(t)$, for $t \in (0, 60)\ s$, knowing that the external toque acting on the cube is equal to:

a) $\vec{\boldsymbol{M}}_{\mathcal{B}}(t) = 10^{-2} \cdot [-6p(t), 0, -5r(t)]^T\ N \cdot m,$

b) $\vec{\boldsymbol{M}}_{\mathcal{B}}(t) = 10^{-2} \cdot [-5, 0, 0]^T\ N \cdot m$

where $p(t)$, $q(t)$, and $r(t)$ are the components of $\vec{\boldsymbol{\omega}}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(t)$.

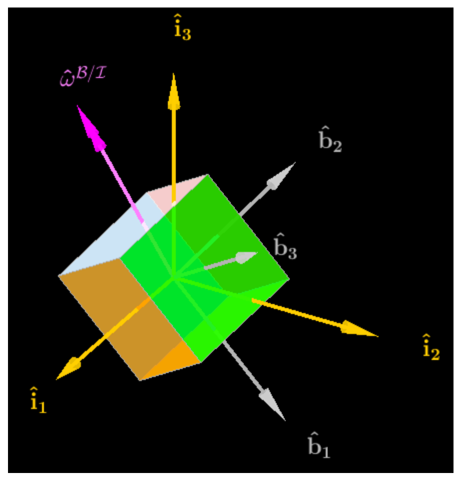


Fig.1 Cube's initial attitude and angular velocity.

- Comment on your results for both case a) and b)
- The kinematics must be written using the quaternions parametrization.

Please include all plots in one figure (4-by-3 subplots). Submit your code and a screenshot of the plots.

Problem 2

Consider a rigid body, whose moments of inertia matrix, calculated with respect to the center of mass and in $\mathcal{B}$-RF coordinates, is:

$$I_{\mathcal{B}}^{C} = \begin{bmatrix} 10 & -2 & -8 \\ -2 & 4 & -1 \\ -8 & -1 & 5 \end{bmatrix}.$$

Find the directions of the principal axes of inertia (i.e., find $R_{\mathcal{BP}}$} and the principal moments of inertia matrix $I_{\mathcal{P}}^{C}$.

Problem 3

(a) Show that the angular velocity of a generic rigid body <u>does not</u> remain constant if no external moments act on it. What is the external moment vector required for the angular velocity to be constant?

(b) (***Optional***) Consider $R_{\mathcal{BI}}$ parametrized using the 3-2-1 Euler angles $\psi,\ \theta,\ \phi$. Starting from $\dot{R}_{\mathcal{BI}} = -[\vec{\boldsymbol{\omega}}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}]_{\times} \cdot R_{\mathcal{BI}}$, retrieve the direct kinematics equation $\dot{\vec{\boldsymbol{\theta}}} = B^{-1}(\vec{\boldsymbol{\theta}}) \cdot \vec{\boldsymbol{\omega}}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}$.

Problem 1

Consider a homogeneous, rigid cube of side $l = 30\ cm$ and density $\rho = 1000\ kg/m^3$. Knowing that at $t = 0$ its attitude and angular velocity are:

$$\vec{\theta}(0) = [50, 30, 60]^T\ deg,$$

$$\vec{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(0) = [200, 30, 10]^T\ deg/s,$$

create a SIMULINK model to find $\vec{\beta}(t)$, $\vec{\theta}(t)$ and $\vec{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(t)$, for $t \in (0, 60)\ s$, knowing that the external toque acting on the cube is equal to:

a) $\vec{M}_{\mathcal{B}}(t) = 10^{-2} \cdot [-6p(t), 0, -5r(t)]^T\ N \cdot m,$

b) $\vec{M}_{\mathcal{B}}(t) = 10^{-2} \cdot [-5, 0, 0]^T\ N \cdot m$

where $p(t)$, $q(t)$, and $r(t)$ are the components of $\vec{\omega}_{\mathcal{B}}^{\mathcal{B}/\mathcal{I}}(t)$.

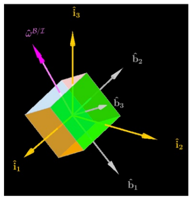
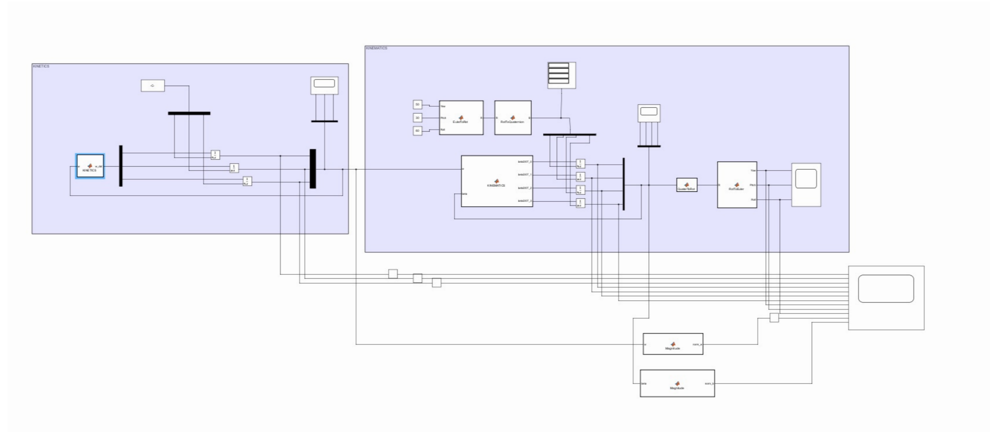


Fig.1 Cube's initial attitude and angular velocity.

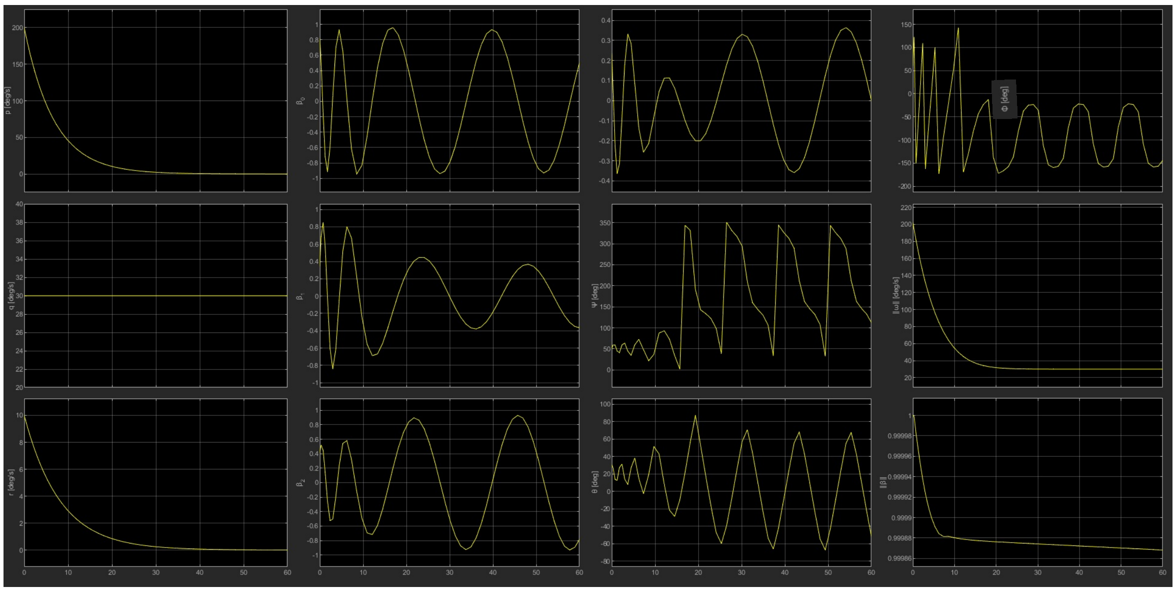
- Comment on your results for both case a) and b)
- The kinematics must be written using the quaternions parametrization.

Please include all plots in one figure (4-by-3 subplots). Submit your code and a screenshot of the plots.

SIMULINK SCHEMATIC



PART A) $\vec{M}_{B}(t) = 10^{-2} \cdot [-6\,p(t), 0, -5\,r(t)]^T$ N.m

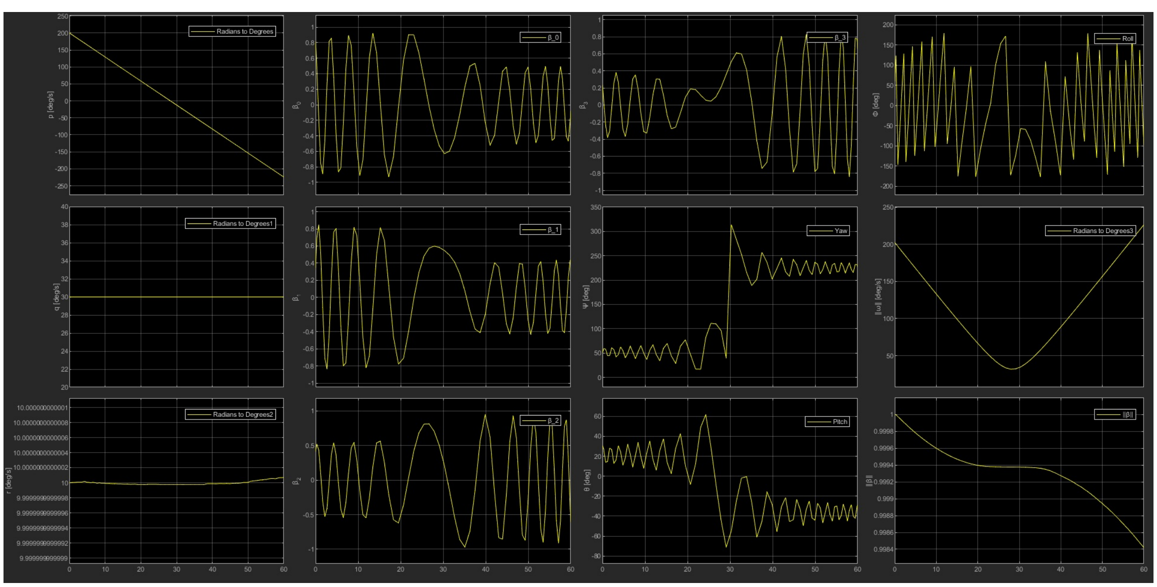


COMMENTS

* p & r change b/c $\vec{M}$ depends on w

* q is 0, M=0.

PART B) $\vec{M}_{B}(t) = 10^{-2} \cdot [-5, 0, 0]^T$ N.m



COMMENTS

* p is linear b/c $\vec{M}$ has a constant decrease

* q & r are constant b/c $\vec{M}$ = 0 at (2) & (3)

M[-5, 0, 0]

Problem 2

Consider a rigid body, whose moments of inertia matrix, calculated with respect to the center of mass and in $\mathcal{B}$-RF coordinates, is:

$$I_{\mathcal{B}}^{C} = \begin{bmatrix} 10 & -2 & -8 \\ -2 & 4 & -1 \\ -8 & -1 & 5 \end{bmatrix}.$$

Find the directions of the principal axes of inertia (i.e., find $R_{\mathcal{BP}}\}$ and the principal moments of inertia matrix $I_{\mathcal{P}}^{C}$.

$$R_{BP} I_P^c = I_B^c R_{BP}$$

$$\begin{bmatrix} \hat{p}_{1B} & \hat{p}_{2B} & \hat{p}_{3B} \end{bmatrix} \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} = I_B^c \begin{bmatrix} \hat{p}_{1B} & \hat{p}_{2B} & \hat{p}_{3B} \end{bmatrix}$$

↳ WE CAN FIND $\hat{p}_{1B}$, $\hat{p}_{2B}$, $\hat{p}_{3B}$ BY TAKING EIGENVECTORS → USING $[R_{BP}, I_P^c] = \text{eig}[I_B^c]$

We get the following:

```
R =

    0.5732    0.1322   -0.8087
    0.3382   -0.9371    0.0865
    0.7464    0.3231    0.5819


I_p =

   -1.5969         0         0
         0    4.6269         0
         0         0   15.9700
```

I CHOSE "I_P^c values" based on: For I_B^c, $I_{xx} > I_{zz} > I_{yy}$

$$\Rightarrow \quad \hat{p}_{1B} = \begin{bmatrix} -0.8087 \\ 0.0865 \\ 0.5819 \end{bmatrix}; \quad I_{xx} = 15.97$$

$$\hat{p}_{2B} = \begin{bmatrix} 0.5732 \\ 0.3382 \\ 0.7464 \end{bmatrix}; \quad I_{yy} = -1.590$$

$$\hat{p}_{3B} = \begin{bmatrix} 0.1322 \\ -0.9371 \\ 0.3231 \end{bmatrix}; \quad I_{zz} = 4.6269$$

Problem 3

(a) Show that the angular velocity of a generic rigid body <u>does not</u> remain constant if no external moments act on it. What is the external moment vector required for the angular velocity to be constant?

* ANGULAR VELOCITY DOES NOT REMAIN CONSTANT IF NO EXTERNAL MOMENTS ACT ON IT

$$\dot{\vec{\omega}}_B^{B/I}(t) = [I_B^c]^{-1} \cdot \left(\vec{M}_B^c(t) - [\vec{\omega}_B^{B/I}(t)]_\times I_B^c \omega_B^{B/I}(t) \right)$$

* NO EXTERNAL MOMENTS, HENCE :

$$\vec{M}_B^c(t) = 0$$

$$\rightarrow \dot{\vec{\omega}}_B^{B/I}(t) = [\cancel{I_B^c}]^{-1} \cdot \left(- [\omega_B^{B/I}(t)]_\times \cancel{I_B^c} \vec{\omega}_B^{B/I}(t) \right)$$

$$\boxed{\rightarrow \dot{\vec{\omega}}_B^{B/I}(t) = - [\vec{\omega}_B^{B/I}(t)] \cdot \vec{\omega}_B^{B/I}(t)}$$

$\therefore \dot{\vec{\omega}}_B^{I}(t)$ IS NOT CONSTANT as It depends on $\omega_B^{B/I}$

* AT WHAT EXTERNAL MOMENT MAKES THE ANGULAR VELOCITY TO BE EQUAL

$$\dot{\vec{\omega}}_B^{B/I}(t) = [I_B^c]^{-1} \cdot \left(\vec{M}_B^c(t) - [\vec{\omega}_B^{B/I}(t)]_\times I_B^c \omega_B^{B/I}(t) \right)$$

* CONSTANT ANGULAR VELOCITY $\rightarrow$ $\dot{\vec{\omega}}_B^{B/I}(t) \rightarrow 0$

$$\rightarrow 0 = [I_B^c]^{-1} \left(\vec{M}_B^c(t) - [\vec{\omega}_B^{B/I}(t)]_\times I_B^c \omega_B^{B/I}(t) \right)$$

$$\left(\vec{M}_B^c(t) = [\vec{\omega}_B^{B/I}(t)]_\times I_B^c \omega_B^{B/I}(t) \right)$$

$\therefore \vec{M}_B^c(t) = [\vec{\omega}_B^{B/I}]_\times I_B^c \vec{\omega}_B^{B/I}$ for angular velocity to be constant