Lecture 17

ARO 4090 - WEEK 11

- M. MAGGIA

Lecture 17

LAST TIME WE SAW THAT, THE TORQUE-FREE MOTION OF A RIGID BODY-LIKE S/C IS:

i) NEUTRALLY STABLE IF THE S/C SPINS ABOUT ITS AXIS OF MINOR OR MAJOR INERTIA.

ii) UNSTABLE IF THE S/C SPINS ABOUT ITS AXIS OF INTERMEDIATE INERTIA.

COMMENTS:

1) IN REALITY, THE S/C IS NOT PERFECTLY RIGID AND THE MOTION IS NOT TORQUE-FREE (DUE TO DISTURBANCES THAT WE'LL DISCUSS LATER). S/C HAVE FLEXIBLE ELEMENTS WHICH DEFORM DUE TO EXTERNAL TORQUES AND DUE TO THE MOTION ITSELF. THIS CAUSES A KINETIC ENERGY DISSIPATION OVER TIME.

$$\dot{T} < 0$$

Example: LET'S ASSUME $I_{xx} = I_{yy} = I_t$, $I_{zz} = I_a$, $\underbrace{\vec{M}^c = \vec{0} \Rightarrow \vec{H}^c = \text{const.}}$

THE SOURCE OF KIN. ENERGY DISSIPATION IS NOT EXT. TORQUE, RATHER INTERNAL FLEXIBILITY / SLOSHING.

PREVIOUSLY WE FOUND:

- $$T = \frac{1}{2} I_t \omega_t^2 + \frac{1}{2} I_a \omega_a^2$$

MULTIPLYING BY "$2 \cdot I_t$"

$$2 I_t T = I_t^2 \omega_t^2 + I_a I_t \omega_a^2$$

$$I_t^2 \omega_t^2 = 2 I_t T - I_a I_t \omega_a^2$$

- • $$H^2 = I_t^2 \omega_t^2 + I_a^2 \omega_a^2$$

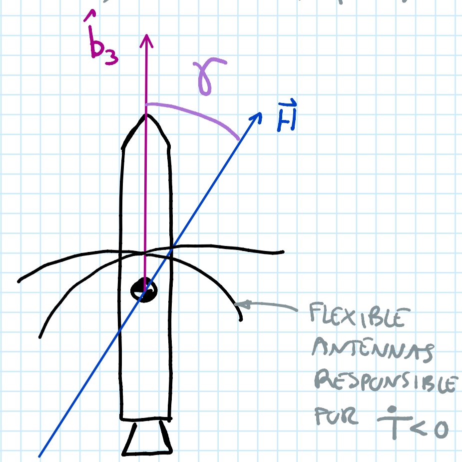
$$= 2 I_t T - I_a I_t \omega_a^2 + I_a^2 \omega_a^2$$

$$H^2 - 2 I_t T = \omega_a^2 I_a^2 \left(\frac{I_a - I_t}{I_a} \right) \qquad (1)$$

$$H^2 - 2I_tT = \omega_a I_a\left(\frac{I_a - I_t}{I_a}\right) \quad (1)$$

$$\therefore \quad \cos\gamma = \frac{H_a}{H} = \frac{I_a\omega_a}{H} \Rightarrow I_a^2\omega_a^2 = H^2\cos^2\gamma \quad (2)$$

(US) EXPLORER 1 (1958)



COMBINING (1) & (2):

$$H^2 - 2I_tT = H^2\cos^2\gamma\left(\frac{I_a - I_t}{I_a}\right)$$

$$T = \frac{H^2}{2I_t}\left(1 - \cos^2\gamma\left(\frac{I_a - I_t}{I_a}\right)\right)$$

$\frac{d}{dt}$

$$\dot{T} = H^2\cos\gamma\cdot\sin\gamma\;\dot{\gamma}\left(\frac{I_a - I_t}{I_aI_t}\right)$$

RECALL THAT γ IS THE NUTATION ANGLE (ANGLE B/W $\vec{H}$ AND $\hat{b}_3$). FOR STABILITY, WE WANT $\dot{\gamma} \leq 0$ (DECREASING NUTATION ANGLE FOR ASYMPT. STABILITY & CONSTANT FOR N. STABILITY)

$$\text{SINCE} \quad \dot{T} < 0 \quad \Longrightarrow \quad \begin{cases} \dot{\gamma} < 0 & \text{IFF} \quad I_a > I_t \\ \dot{\gamma} > 0 & \text{IFF} \quad I_t > I_a \end{cases}$$

THE STABILITY CAN ONLY BE ACHIEVED IF THE SPINNING AXIS IS OF MAJOR INERTIA

ALSO WE CAN SEE THAT

$\dot{T} < 0$

$$T_{MAX} = \frac{1}{2}\frac{H^2}{I_{MIN}}$$

$$T_{MIN} = \frac{1}{2}\frac{H^2}{I_{MAX}}$$

THE S/C WILL START SPINNING FROM I_{MIN} TO I_{MAX}

RECAP FOR TORQUE-FREE MOTION, S/C SPINNING ABOUT I_{zz}:

	IDEALIZED	REAL
H	CONST.	CONST.
$\dot{T}$	0	< 0
I_{zz} MAJOR	N. STABLE	A. STABLE
I_{zz} INTERM.	UNSTABLE	UNSTABLE
I_{zz} MINOR	N. STABLE	UNSTABLE

2) PERTURBATIONS: TORQUE-FREE MOTION IS JUST AN IDEALIZED SCENARIO. IN FACT, THERE ARE A NUMBER OF EXTERNAL TORQUES ACTING ON A S/C, WHICH DISTURB ITS ATTITUDE MOTION. FOR A S/C IN A LEO THE MAJOR DISTURBANCE TORQUES ARE

(a) MAGNETIC TORQUE (MAG)
(b) SOLAR RADIATION PRESSURE TORQUE (SRP)
(c) AERODYNAMIC TORQUE (AERO)
(d) GRAVITY-GRADIENT TORQUE (GG)

(a) MAGNETIC TORQUE

$$\boxed{M_{MAG} = \underbrace{\vec{m}_{S/C}}_{\text{S/C MAGNETIC DIPOLE}} \wedge \underbrace{\vec{B}}_{\text{PLANET'S MAGNETIC FIELD}}}$$

- $\vec{B} \propto \frac{1}{R^3}$ — $\propto$ PROPORTIONAL TO

- TYPICALLY $\|\vec{m}_{S/C}\| \approx 0.1\ A \cdot m^2$ & $\|\vec{B}\| \simeq \overbrace{3 \cdot 10^{-5}\ \frac{Wb}{m^2}}^{\text{or } 30{,}000\ nT\ (\text{NANOTESLA})}$ (@ 200 Km ALTITUDE)

$$\|\vec{M}_{MAG}\| = \|\vec{m}_{S/C}\| \cdot \|\vec{B}\| = 3 \cdot 10^{-6}\ N \cdot m$$

(b) SOLAR RADIATION PRESSURE

- PHOTONS HAVE MOMENTUM AND WHEN THEY IMPACT WITH THE S/C CREATE A FORCE (MOMENTUM TRANSFER) AND THUS TORQUE.

- THE PRESSURE OF THE PHOTONS @ 1 AU (ASTRONOMICAL UNIT)

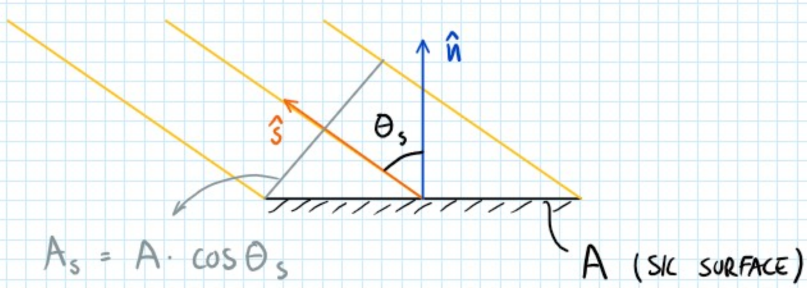
$$P_s = 4.56 \cdot 10^{-6} \ \frac{N}{m^2}$$

- THE FORCE EXERTED ON THE S/C BY THE PHOTONS DEPENDS ON THE INCIDENCE ANGLE SUN RAYS – S/C SURFACE AND ON THE PROPERTIES OF THE S/C SURFACE

$$\underbrace{\rho}_{\text{REFLECTIVITY}} + \underbrace{\tau}_{\text{TRANSMISSIVITY}} + \underbrace{\alpha}_{\text{ABSORPTIVITY}} = 1$$ (VALID $\forall$ SURFACES)

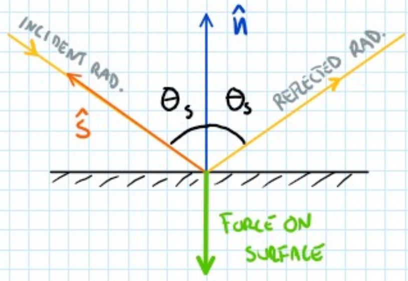
IF:

- OPAQUE SURFACE $\tau = 0$
- GRAY SURFACE $\alpha = \underbrace{\varepsilon}_{\text{EMISSIVITY}}$
- BLACK BODY $\alpha = \varepsilon = 1$ (TOTAL ABSORPTION)

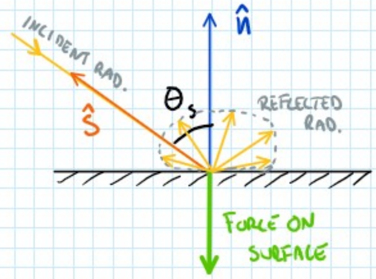


– POSSIBLE MODES OF INTERACTION B/W SOLAR RADIATION AND S/C SURFACE –

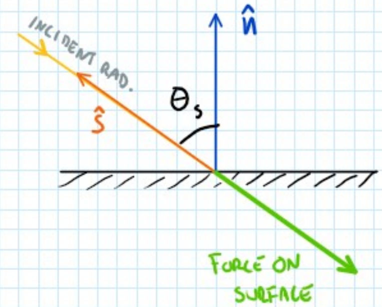
SPECULAR REFLECTION



DIFFUSE REFLECTION



TOTAL ABSORPTION



ASSUMPTION : TOTAL ABSORPTION

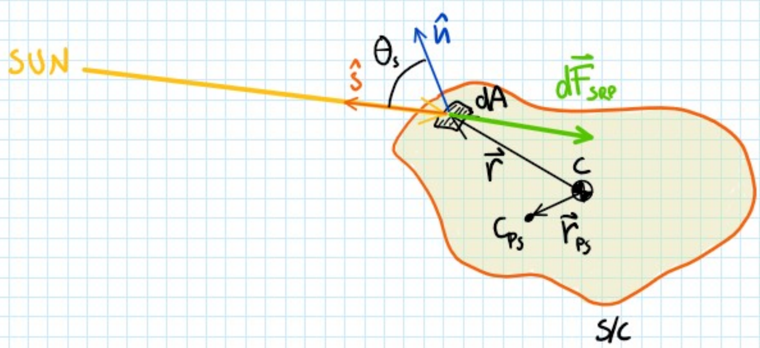
$$d\vec{F}_{SRP} = -P_s \cdot dA \cdot \cos\theta_s \cdot \hat{s}$$

θ_s IS THE ANGLE B/W $\hat{s}$ AND $\hat{n}$ $\Rightarrow$ $\hat{s} \bullet \hat{n} = \hat{s}^T \hat{n} = \cos\theta_s$

INTEGRATING OVER S/C SURFACE

$$d\vec{F}_{SRP} = -P_s \; \hat{s} \bullet \hat{n} \; \hat{s} \; dA$$

$$\vec{F}_{SRP} = -P_s \int_{\underbrace{A_W}_{\text{WETTED (LIT) SURFACE}}} \hat{s} \bullet \hat{n} \, dA \cdot \hat{s}$$



$$\vec{M}_{SRP} = \int_{A_W} \vec{r} \wedge d\vec{F}_{SRP} = -P_s \int_{A_W} \vec{r} \wedge (\hat{s} \cdot \hat{n}) \hat{s} \, dA$$

DEFINING THE CENTER OF SOLAR PRESSURE WE CAN RE-WRITE THE TORQUE AS:

$$\vec{r}_{PS} = \frac{\int_{A_W} \vec{r} (\hat{n} \cdot \hat{s}) \, dA}{\int_{A_W} (\hat{n} \cdot \hat{s}) \, dA} \qquad (C_{PS})$$

$$\boxed{\vec{M}_{SRP} = \vec{r}_{PS} \wedge \vec{F}_{SRP}}$$

(C) AERODYNAMIC TORQUE

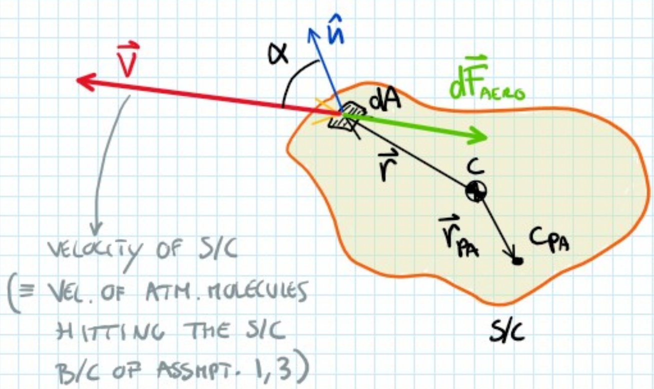
- IN LEO THERE IS RESIDUAL ATMOSPHERE. HOWEVER, THE DENSITY IS SO LOW THAT CONVENTIONAL FLUID DYNAMICS DOES NOT APPLY (CONTINUUM MODEL CAN'T BE APPLIED, INTERACTION FLUID / S/C IS AT MOLECULAR LEVEL)

ASSUMPTIONS :

i) MOMENTUM OF MOLECULES HITTING S/C IS LOST (MOL. STICK TO S/C)

ii) THERMAL MOTION OF ATMOSPHERE ≪ S/C SPEED

iii) THE S/C IS NOMINALLY NON-SPINNING



IT CAN BE SHOWN THAT

$$d\vec{F}_{AERO} = -\underbrace{\rho}_{\text{AIR DENSITY}} V^2 (\hat{n}\cdot\hat{v})\, dA \cdot \hat{v}$$

$$\vec{F}_{AERO} = \int_{A_{Wa}} d\vec{F}_{AERO} = -\rho V^2 \int_{A_{Wa}} \hat{n}\cdot\hat{v}\, dA \cdot \hat{v}$$

A_{Wa}: WETTED AREA FACING THE FLOW

DEFINING THE CENTER OF AERODYNAMIC PRESSURE $\vec{r}_{PA} = \dfrac{\int_{A_{Wa}} \vec{r}\,(\hat{n}\cdot\hat{v})\, dA}{\int_{A_{Wa}} \hat{n}\cdot\hat{v}\, dA}$

$$\boxed{\vec{M}_{AERO} = \vec{r}_{PA} \wedge \vec{F}_{AERO}}$$

$$\boxed{\vec{M}_{AERO} = \vec{r}_{PA} \wedge \vec{F}_{AERO}}$$

(d) GRAVITY-GRADIENT TORQUE

WE'LL SEE THE DETAILS LATER WHEN TALKING ABOUT THE GRAVITY-GRADIENT STABILIZATION. GRAVITY-GRADIENT CAN BE IN FACT TURNED FROM A MERE PERTURBATION, TO A STABILIZING (PASSIVE) TORQUE.

3) THE PROBLEM SEEN IN LECTURES 15 & 16 IS AN EXAMPLE OF PASSIVE CONTROL VIA SINGLE-SPIN STABILIZATION. IT'S NOT THE ONLY OPTION AVAILABLE TO STABILIZE & CONTROL THE S/C ROTATIONAL MOTION.

★ PASSIVE CONTROL (OPEN-LOOP, SENSORS/ACTUATORS/PROCESSOR NOT REQUIRED)

- ↳ SINGLE-SPIN STABILIZATION
- ↳ DUAL-SPIN STABILIZATION

} GYROSCOPIC STABILITY

- ↳ GRAVITY-GRADIENT STABILIZATION

★ ACTIVE CONTROL (CLOSED-LOOP, SENSORS/ACTUATORS/PROCESSOR REQUIRED)

- ↳ REACTION WHEELS
- ↳ CONTROL MOMENT GYROSCOPE
- ↳ MAGNETORQUERS

• AKA STATE FEEDBACK OR STABILITY AUGMENTATION

IN BOTH CASES, THE OBJECTIVE IS TO "MOVE" THE POLES TO THE LEFT-HAND SIDE OF THE PLANE TO ACHIEVE ASYMPTOTIC STABILITY (AT BEST) OR NEUTRAL STABILITY (AT WORST).