P3

(a) FROM KINETICS:

$$\dot{\vec{\omega}}_B^{B/I} = \left[I_B^c\right]^{-1}\left(\vec{M}_B^c - \left[\vec{\omega}_B^{B/I}\right]_\times I_B^c\, \vec{\omega}_B^{B/I}\right)$$

DROPPING SUB & SUPERSCRIPTS FOR READABILITY:

$$\dot{\vec{\omega}} = I^{-1}\left(\vec{M} - [\vec{\omega}]_\times I\, \vec{\omega}\right) \qquad (1)$$

FOR A GENERIC RIGID BODY $\quad I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$

(1) CAN BE RE-WRITTEN AS:

$$I\dot{\vec{\omega}} = \vec{M} - [\vec{\omega}]_\times I\, \vec{\omega}$$

AND IN THE ABSENCE OF EXTERNAL MOMENTS $(\vec{M} = 0)$ IT BECOMES

$$I\dot{\vec{\omega}} = -[\vec{\omega}]_\times I\, \vec{\omega}$$

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = -\begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

IF WE WANT $\vec{\omega} = \text{CONST.} \implies \dot{\vec{\omega}} = \vec{0}$, THUS:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

EXPANDING:

$$\begin{cases} (I_{yy} - I_{zz})\, qr + (I_{xz}\, q - I_{xy}\, r)\, p + I_{yz}\,(q^2 - r^2) = 0 \\ (I_{zz} - I_{xx})\, rp + (I_{xy}\, r - I_{yz}\, p)\, q + I_{xz}\,(r^2 - p^2) = 0 \\ (I_{xx} - I_{yy})\, pq + (I_{yz}\, p - I_{xz}\, q)\, r + I_{xy}\,(p^2 - q^2) = 0 \end{cases}$$

IF $I_{xx} \neq I_{yy} \neq I_{zz}$ & $I_{xy} \neq 0$, $I_{xz} \neq 0$, $I_{yz} \neq 0$, THE SYSTEM IS SATISFIED ONLY IF:

$$p = q = r = 0 \iff \text{NO ROTATION}$$

THIS IMPLIES THAT IF WE WANT $\vec{\omega} = \text{CONST}$ WE NEED TO APPLY AN EXTERNAL TORQUE:

$$\vec{M} = [\vec{\omega}]_\times \cdot I \cdot \vec{\omega} \qquad \forall\, \vec{\omega}, I$$

$$M_x = -(I_{yy} - I_{zz})\, qr - (I_{xz}\, q - I_{xy}\, r)\, p - I_{yz}\,(q^2 - r^2)$$

$$M_y = -(I_{zz} - I_{xx})\, rp - (I_{xy}\, r - I_{yz}\, p)\, q - I_{xz}\,(r^2 - p^2)$$

$$M_z = -(I_{xx} - I_{yy})\, pq - (I_{yz}\, p - I_{xz}\, q)\, r - I_{xy}\,(p^2 - q^2)$$