Stat 244, Aut 2015: HW2

due: Tuesday 10.10.17

1. Suppose that errors in HW solutions obey the following model. There is a 20% chance that there is an error in the posted solutions. If an error is present, then the number of students who send emails about the error, follows a Poisson(3) distribution. If no emails were received, what is the probability that there was no error?

2. Calculate $P(X \text{ is odd})$ in each setting below. Show your calculation or explain your answer for each part.

   (a) $X \sim \text{Geometric}(0.7)$

   (b) $X \sim \text{Binomial}(101, 0.5)$

   (c) First let $Y \sim N(0, 1)$ (a standard normal random variable), and then let X be the answer you get when you round Y to the nearest integer. Your answer does not need to be simplified to a number—you can write it with summations/fractions/integrals etc as needed. However, your final answer cannot have any probability type notation in it, e.g. it cannot include terms like $p_X(3)$ or $f_Y(1)$ or $P(X \leq 1)$ etc.

3. One person draws cards from a full deck (all 52 cards), while a second person draws cards from a red deck (only the 26 red cards). As usual, cards are drawn at random and without replacement. In each problem below, do X and Y have the same distribution?

   (a) X is the indicator variable for the event that the first card drawn from the full deck is a face card i.e. an Ace, King, Queen, or Jack (that is, $X = 1$ if the first card drawn from the full deck is a face card, and $X = 0$ if not). Y is the indicator variable for the event that the first card drawn from the red deck is a face card.

   (b) X is the number of face cards, when you draw 10 cards from the full deck. Y is the number of face cards, when you draw 10 cards from the red deck.

4. Suppose that $X \sim N(0, 1)$. Calculate the following two density functions. For each one, be sure to specify the support i.e. the range of possible values of the random variable.

   (a) Calculate the density of $Y = X^3$.

   (b) Calculate the density of $Z = |X|$.

5. Let the random variable T be the time until some event occurs (e.g. time until an atom decays, time until next rainfall, etc). Suppose it's a continuous random variable supported on $[0, \infty)$. The <u>hazard rate</u> for T is defined as

$$h(t) = \frac{f(t)}{1 - F(t)},$$

   where f and F are the density and CDF for the distribution of T. On an intuitive level, this is the chance that the event will occur in the very near future, given that it has not yet occurred. Hazard rate is a function of time since it can rise or fall as time goes on.

   (a) Calculate the hazard rate $h(t)$ if $T \sim \text{Exponential}(\lambda)$.

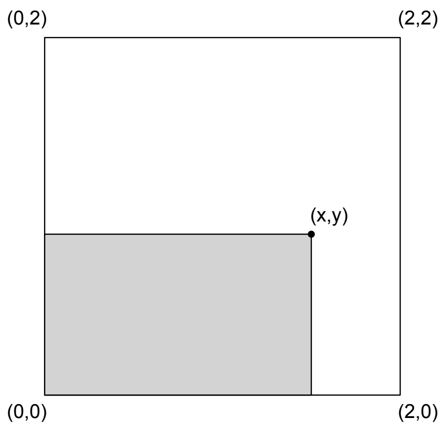
   (b) Now suppose that T follows a Weibull distribution with shape k and scale α, which is defined by the CDF $F(t) = 1 - e^{-(t/\alpha)^k}$. Calculate the density $f(t)$, and the hazard rate function $h(t)$, for this distribution.

   (c) For the Weibull distribution, for which values of k and α is $h(t)$ decreasing over time, increasing over time, or constant over time?

   (d) Next, suppose that you buy a new watch. When you purchase it, you put in a new battery. Let T be defined as the time from putting in the new battery, until the battery runs out. Do you think the hazard rate function for T should be decreasing over time, increasing over time, or constant over time (or some other shape)? Explain. (There may be multiple plausible answers.)

   (e) Finally, a common shape for the hazard function is a "U" shape, where the hazard rate is high initially, goes down to a low rate for a long time, and then rises again later on. Give a real-life example for what T could measure that would likely have this type of hazard rate function, and explain.

6. Suppose that you have a 2-foot-by-2-foot square, and you choose a point on the square, call it (x, y), uniformly at random from the square. Then you draw the rectangle spanning $(0, 0)$ on the bottom left and (x, y) on the top right.



(a) What is $P(A)$, where A is the event that the rectangle is at least twice as tall as it is wide? (Hint: draw a picture of this event.)

(b) What is $P(B)$, where B is the event that the area of this rectangle is ≤ 1? (Hint: draw a picture of this event.)

(c) Are the events A and B independent? No need to do any complicated integrals for this answer—you should be able to explain it without a lot of calculation.