

## Set Seed

In order to obtain the same results

```
set.seed(1964)
```

## HW1.2

*Roll Blue and Red Die, what is the probability that Red Dice < Blue Dice*

## Creation of two independant die – a red and blue die.

```
red_dice_roll <- function(n){
  rolls <- sample(1:6, size = n, replace = TRUE)
  return(rolls)}

blue_dice_roll <- function(n){
  rolls <- sample(1:6, size = n, replace = TRUE)
  return(rolls)}
```

**Determine the probability that the result from the blue die is larger than that of the red die.**

```
roll_dice_B_Greater_R <- function(n){
  blue_is_greater <- 0
  for (i in 1:n){
    if(blue_dice_roll(1) > red_dice_roll(1)){
      blue_is_greater <- blue_is_greater + 1
    }
  }
  return(blue_is_greater/n)}

## The solution will be equivalent to
roll_dice_B_Greater_R(50000)
```

```
## [1] 0.41874
```

## HW 1.3

**One coin is drawn at random (two coins: One is 50% Heads and the second one is 60% Heads)**

Coin A = 50%

Coin C = 60%

*If you flip the coin twice and get heads both times what is the probability that this is the fair coin?*

First we must create the functions for the fair and unfair coin

```

#Function for the fair coin
coin_a <- function(n_flips){
  flips <- sample(c(1,0), size = n_flips, replace = TRUE,
                 prob = c(0.5, 0.5)) ##Fair chance for heads
  return(flips)
}

#Function for the unfair coin (60% heads bias)
coin_c <- function(n_flips){
  flips <- sample(c(1,0), size = n_flips, replace = TRUE,
                 prob = c(0.6, 0.4)) ##Unfair chance for Heads
  return(flips)
}

```

Second we must construct another function to count the number of heads obtained in a coin flips alongside a probability for the chance that the coin is the fair coin if Heads is seen twice (HH).

```

#Function to determine of achieving Heads Heads
fair_coin_HH <- function(n_trials){
  HH_on_flip <- 0
  for(i in 1:n_trials){
    if(coin_a(1) + coin_a(1) == 2){
      HH_on_flip <- HH_on_flip +1
    }
  }
  return(HH_on_flip/n_trials)}

prob_of_fair_coin = fair_coin_HH(100000) * 0.5
#getting two heads with a fair coin multiplied by the chance that the fair coin is selected
prob_of_fair_coin

```

```
## [1] 0.125675
```

*After flipping the coin twice and seeing two heads, what is the probability that the next toss is also heads?*

```

#Addition of the probabilities of the next flip being heads
prob_next_flip_is_heads <-
  prob_of_fair_coin*0.5 + (1-prob_of_fair_coin)*0.6

prob_next_flip_is_heads

```

```
## [1] 0.5874325
```

## HW1.4

*Let A and C be events where . . .*

$$P(A) = 0.4$$

$$P(A|C) = 0.1$$

Explain why it is not possible to have  $P(C) = 0.8$  – more specifically can you prove an upper or lower bound on  $P(C)$ ?

$$P(A|C) = P(A,C) / P(C) = P(C,A) / P(C)$$

$$\Rightarrow P(C,A) = P(A|C) * P(C)$$

$$P(C|A) = P(C,A) / P(A) \Rightarrow P(C,A) = P(C|A) * P(A)$$

$\therefore$

## HW1.5

A diagnostic test for a disease return one of the following answers; Positive (Has the Disease) and Negative (does not have the disease)

*The probabilities of the test outcomes are as follows*

**For a patient with the disease. 0.95 Positive / 0.05 Negative**

**For a patient that does not have the disease. 0.10 Positive / 0.90 Negative**

*Assume 10% of the population has the disease. If everyone is tested, what proportion of the test results are positive?*

```
#Probabilities of achieving a positive result whether or not you have the disease
with_disease_positive_result = 0.1 * 0.95
without_disease_positive_result = 0.9 * 0.1

prob_of_positive_result = with_disease_positive_result + without_disease_positive_result
prob_of_positive_result

## [1] 0.185
```

*For a patient who gets a positive result, what is the probability of having the disease*

Definition of conditional probability:

$$P(A|C) = P(A,C) / P(C) = [P(A)*P(C|A)] / P(C)$$

$P(A)$ : Having the Disease  $P(C)$ : Getting a positive result

```
#Known Terms
with_disease_positive_result = 0.1 * 0.95
without_disease_positive_result = 0.9 * 0.1

#P(A)
prob_having_disease = 0.1
```

```

#P(A~C)
prob_not_having_disease = 0.9
#####

#P(C)
prob_getting_positive_result =
    without_disease_positive_result +
        with_disease_positive_result
#P(A,C)
with_disease_positive_result

## [1] 0.095

#P(A|C) Gets a positive result - prob of having the disease

prob_pos_res_and_having_dis = with_disease_positive_result / prob_getting_positive_result
prob_pos_res_and_having_dis

## [1] 0.5135135

```

Now let  $d$  be the prevalence of the disease in the population

Everyone in the population is screened for the disease. For each person who got a positive result they are brought back for an additional screening. 25% of the returning population test positive. What is  $d$ ?

Q : Second Test is Positive A : First Test is Positive

Probability that you get both tests positive  $P(Q|A) = 0.25$

$$P(Q|A) = P(Q,A) / P(A)$$

$$P(Q|A) = [d(0.95)^2 + (1-d)(0.10)^2] / [d(0.95) + (1-d)(0.1)]$$

$$P(Q|A) = [d(0.9025) + (1-d)(0.01)] / [d(0.95) + (1-d)(0.1)]$$

$$P(Q|A) = [d(0.9025) + (1-d)(0.01)] / [d(0.95) + (1-d)(0.1)]$$

$$P(Q|A) = [0.9025d + 0.01 - 0.01d] / [0.95d + 0.1 - 0.1d]$$

$$P(Q|A) = [0.925d + 0.01] / [0.85d + 0.1]$$

$$d = 0.02105$$