Set Seed

In order to obtain the same results

```
set.seed(1964)
```

HW1.2

Roll Blue and Red Die,what is the probability that Red Dice < Blue Dice

Creation of two independant die – a red and blue die.

```
red_dice_roll <- function(n){
rolls <- sample(1:6, size = n, replace = TRUE) #Drawing a number between 1-6
return(rolls)}

blue_dice_roll <- function(n){
rolls <- sample(1:6, size = n, replace = TRUE) #Drawing a number between 1-6
return(rolls)}
```

Since we are looking for the probability that B > R we can assume that the blue die can be larger except when a six is rolled by the red die (because even if the blue die is 6 it fails).

Determine the probability that the result from the blue die is larger than that of the red die.

```
roll_dice_B_Greater_R <- function(n){
blue_is_greater <- 0
    for (i in 1:n){ #As many throws of both dice
if(blue_dice_roll(1) > red_dice_roll(1)){  #Checking if the blue die is bigger than the red die
blue_is_greater <- blue_is_greater + 1 #Couting occurances where blue d is bigger than red die
}}
  return(blue_is_greater/n)}

## The solution will be equivelent to
roll_dice_B_Greater_R(50000)
```

```
## [1] 0.41874
```

HW 1.3

One coin is drawn at random (two coins: One is 50% Heads and the second one is 60% Heads)

Coin A = 50%

Coin C = 60%

If you flip the coin twice and get heads both times what is the probability that this is the fair coin?

First we must create the functions for the fair and unfair coin

```r
#Function for the fair coin
coin_a <- function(n_flips){
  flips <- sample(c(1,0), size = n_flips, replace = TRUE,
                  prob = c(0.5, 0.5)) ##Fair chance for heads
        return(flips)
}

#Function for the unfair coin (60% heads bias)
coin_c <- function(n_flips){
  flips <- sample(c(1,0), size = n_flips, replace = TRUE,
                  prob = c(0.6, 0.4)) ##Unfair chance for Heads
        return(flips)
}
```

Second we must construct another function to count the number of heads obtained in a coin flips alongside a probability for the chance that the coin is the fair coin if Heads is seen twice (HH).

```r
#Function to detmermine of achieving Heads Heads
fair_coin_HH <- function(n_trials){
HH_on_flip <- 0
  for(i in 1:n_trials){
    if(coin_a(1) + coin_a(1) == 2){
      HH_on_flip <- HH_on_flip +1
    }}
  return(HH_on_flip/n_trials)}

#Getting two heads with a fair coin multiplied by the chance that the fair coin is selected
prob_of_fair_coin = fair_coin_HH(50000) * 0.5
prob_of_fair_coin
```

```
## [1] 0.12542
```

After flipping the coin twice and seeing two heads, what is the probability that the next toss is also heads?

```r
#Addition of the probabilities of the next flip being heads
prob_next_flip_is_heads <-
  prob_of_fair_coin*0.5 + (1-prob_of_fair_coin)*0.6

prob_next_flip_is_heads
```

```
## [1] 0.587458
```

HW1.4

Let A and C be events where . . .

P(A) = 0.4

P(A|C)= 0.1

Explain why it is not possible to have P(C) = 0.8 – more specifically can you prove an upper or lower bound on P(C)?

$P(A) = P(A, C) + P(A, C_c) \leq P(A, C) + P(C_c)$
[Which is equilivent to $P(A, C_c) \leq P(C_c)$]

$P(A) \leq P(A, C) + (1 - P(C))$

$P(A) \leq P(A|C)P(C) + (1 - P(C))$

$P(A) \leq P(A|C)P(C) + 1 - P(C)$

$P(A) \leq (0.1)P(C) - P(C) + 1$

$P(A) \leq (0.9)P(C) + 1$

$(0.4 - 1) \leq (0.9)P(C)$

$-\frac{0.6}{0.9} \leq P(C)$

$P(C) \leq \frac{0.6}{0.9}$

$\therefore P(C) \leq \frac{2}{3}$

Thus $P(C) = / = 0.8$, but instead is bounded by,
$0 < P(C) \leq \frac{2}{3}$

HW1.5

A dianostic test for a disease return one of the following answers; Positive (Has the Disease) and Negative (does not have the disease)

The probabilities of the test outcomes are as follows

For a patient with the disesase. 0.95 Positive / 0.05 Negative For a patient that does not have the disease. 0.10 Positive / 0.90 Negative

Assume 10% of the popualtion has the disease. If everyone is tested, what proportion of the test results are positive?

```
#Probabilities of achieving a positive result whether or not you have the disease
with_disease_positive_result = 0.1 * 0.95
without_disease_positive_result = 0.9 * 0.1

prob_of_positive_result = with_disease_positive_result + without_disease_positive_result
prob_of_positive_result
```

```
## [1] 0.185
```

For a patient who gets a positive result, what is the probability of having the disease

Definition of conditional probability:

P(A|C) = P(A,C) / P(C) = [P(A)*P(C|A)] / P(C)

P(A): Having the Disease P(C): Getting a positive result

```
#Known Terms
with_disease_positive_result = 0.1 * 0.95
  without_disease_positive_result = 0.9 * 0.1

  #P(A)
prob_having_disease = 0.1

#P(A^C)
```

```r
prob_not_having_disease = 0.9
##########################################################################################################

#P(C)
prob_getting_positive_result =
  without_disease_positive_result +
            with_disease_positive_result
#P(A,C)
with_disease_positive_result
```

```
## [1] 0.095
```

```r
#P(A|C) Gets a positive result - prob of having the disease

prob_pos_res_and_having_dis = with_disease_positive_result / prob_getting_positive_result
prob_pos_res_and_having_dis
```

```
## [1] 0.5135135
```

Now let d be the prevelance of the disease in the population

Everyone in the population is screeened for the disease. For each person who got a positive result they are brought back for an additional screening. 25% of the returning population test positive. What is d?

Q : Second Test is Positive A : First Test is Positive

Probability that you get both tests positive

$P(Q|A) = 0.25$

Probability of getting a positive result times the probability of geeting a positive result

$P(Q, A) = d(0.95)^2 + (1 - d)(0.1)^2$

Probability of getting a positive result (1st test)

$P(A) = 0.95d + (1 - d)(0.1)$

$P(Q|A) = P(Q, A)/P(A)$

$P(Q|A) = [d(0.95)^2 + (1 - d)(0.10)^2]/[d(0.95) + (1 - d)(0.1)]$

$P(Q|A) = [d(0.9025) + (1 - d)(0.01)]/[d(0.95) + (1 - d)(0.1)]$

$P(Q|A) = [d(0.9025) + (1 - d)(0.01)]/[d(0.95) + (1 - d)(0.1)]$

$P(Q|A) = [0.9025d + 0.01 - 0.01d]/[0.95d + 0.1 - 0.1d]$

$P(Q|A) = [0.925d + 0.01]/[0.85d + 0.1]$

$d = 0.02105$