

```
library(dplyr)

##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##   filter, lag
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

set.seed(1929)
```

## HW3

### HW3.1

The distribution of photons emitted by  $X$  ( $X \sim \text{Poisson}(100)$ ) and  $Y$  is the number of photons which pass through an object.

If we know the amount of photons emitted by the x-ray beam we can describe the number of photons passing through the object by using the distribution  $Y|X \sim \text{Binomial}(X, 0.4)$

Calculate the probability mass function for  $(X, Y)$  [eg  $p(k, l) = P((X = k, Y = l))$ ]

```
set.seed(1929)
n_reps = 1

#Creating a data frame containing random draws from the Poisson distribution
photons_emitted.df <- data.frame(rpois(n_reps, 100))
names(photons_emitted.df) <- c('Photons')
#Number of photons emitted
photons <- sum(photons_emitted.df$Photons)
#Number of photons from one draw of the Poisson distribution which passes through the material
photons_pass_through_material.str <- rbinom(1, photons, 0.4)
#Probability that a photon is emitted from the x-ray and passes through the object
photons_pass_through_material.str/photons
```

```
## [1] 0.4059406
```

$$p(k, l) = P(X = k, Y = l) = P(X = k) * P(Y = l | X = k)$$

$$P(X = k) * P(Y = l | X = k) = \frac{100^k e^{-100}}{k!} * k! C_l * (0.4)^l (0.6)^{k-l}$$

### HW3.5

Flip a coin 10 times and let  $X$  be the sequence 'HT'. What is  $P(X = 0)$  ?

```
library(stringr)
#Assigning variables
coin_sides <- c('H', 'T')
pat <- setNames(nm = "HT") #Pattern which we are looking for
prob <- c(0.5, 0.5) #The coin is fair
n_reps <- 50000 #Replications of the Experiment
all_flips.df <- 0 #Zeroing out the data frame
```

```

#A simple program which will toss the coin 'flips' times and record the results, creates a
# string from that data, and returns the number of times the pattern 'HT' is seen in the data
coin_pattern_toss <-function(flips){
all_flips.df <- data.frame(sample(coin_sides, flips, replace = TRUE, prob), stringsAsFactors = TRUE)
  names(all_flips.df) <- c('Coin')

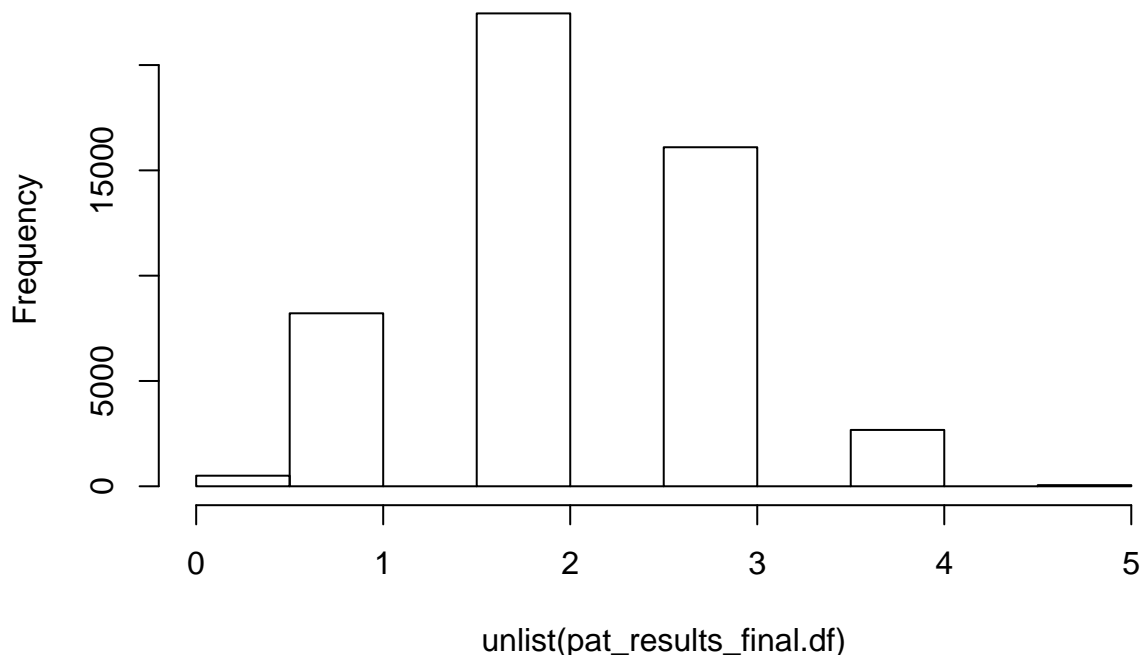
all_flips.str <- paste(unlist(all_flips.df),collapse = '')
  pat_results.df <- data.frame(lapply(pat, str_count, string=all_flips.str))
return(pat_results.df)
}

#A large amount of 10 flip trials
pat_results_final.df <- data.frame(replicate(n_reps, coin_pattern_toss(10)))

#Histogram of the distrubuiton of pattern sucesses
hist(unlist(pat_results_final.df))

```

**Histogram of unlist(pat\_results\_final.df)**



```

#Tabulation of the number of successes in the pattern
pat_results_final.ls <- data.frame(table(unlist(pat_results_final.df)))
#Probability that P(X = 0)
(pat_results_final.ls[1,2]/n_reps)

```

```
## [1] 0.01006
```

What is the expectation  $[E[X]]$ ?

The probability that the pattern 'HT' comes up once in the coin tosses is;

$P(\text{Heads and Tails}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4} = E_i$  Where  $i$  is 1-9 and if one was to calculate the expected value by weighing each coin toss by the probability and iterations we would see;  $E(X) = E_1 + E_2 + \dots +$

$$E_9 = \sum_{i=1}^9 E_i = 9 * E_i = 9 * \frac{1}{4} = 2.25$$

```
#Can be found using the weighted.mean function
x <- c(0, 1, 2, 3, 4, 5) #Number of patterns seen in ten flips
  wt <- (pat_results_final.ls$Freq)/n_reps #Weighted rate of patterns seen

#Expected number of patterns
weighted.mean(pat_results_final.df)
```

```
## [1] 2.24786
```

*If you were to draw one card at a time from a full deck without replacement what is the probability that you will NOT have the pattern 'RB'?*

Use a hypergeometric distribution in order to determine the chance of drawing only black cards first.

Pseudocode; Perform this drawing action 50000 times using a string of R's and B's, do the same pattern searching as above, and then find the probability by density.