CL exercise for Tutorial 6

Introduction

Objectives

In this tutorial, you will:

- learn more about Karnaugh maps;
- convert logical expressions to DNF or CNF using Karnaugh maps;
- learn about minimal DNFs and CNFs.

Tasks

Exercises 1 and 2 are mandatory. Exercise 3 is optional. Exercise 4 is optional and will not be marked.

Submit

a file called `cl-tutorial-6` with your answers (image or pdf).

Deadline

16:00 Tuesday 1 November

Reminder

Good Scholarly Practice

Please remember the good scholarly practice requirements of the University regarding work for credit.

You can find guidance at the School page

https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct.

This also has links to the relevant University pages. Please do not publish solutions to these exercises on the internet or elsewhere, to avoid others copying your solutions.

Exercise 1 –mandatory––marked–

Read Chapter 17 (*Karnaugh Maps*) of the textbook.

Consider the following Karnaugh Map of a boolean expression:

ab \ cd	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

(a) Identify (by marking on the map) the blocks of 1s and give the boolean expression in DNF.

(b) Identify (by marking on the map) the blocks of 0s and give the boolean expression in CNF.

Solution to Exercise 1

(a)

ab \ cd	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

$$(\neg a \wedge c) \vee (a \wedge \neg d)$$

This is the minimal answer. There are many other possibilities, for example $(a \wedge \neg c \wedge \neg d) \vee (\neg a \wedge c) \vee (c \wedge \neg d)$. Any right answer is ok, but comment if they give a non-minimal one.

(b)

$ab \backslash cd$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

$$\neg((\neg a \wedge \neg c) \vee (a \wedge d))$$

$$= (a \vee c) \wedge (\neg a \vee \neg d)$$

Again, other answers possible.

Note that thinking directly about intersecting rectangles in order to get blocks does my head in, and I suspect it would do theirs. So I've told them to build a DNF expression for the 0-blocks, and then negate it with De Morgan, instead of (as is done in the book) negating each block and intersecting them.

Exercise 2 –mandatory—marked–

A three-variable Karnaugh map can be drawn with one variable on the left and two on the top, or vice versa. Putting r on the left (as a row variable), and a, b on top (as column variables), give a Karnaugh map for the expression

$$r \leftrightarrow (a \vee b)$$

and produce a DNF equivalent.

Make the DNF *minimal*: with as few occurrences of literals as possible. (Pay attention to the distinction between the number of literals in an expression and the number of occurrences of literals, as the same literal may occur several times.)

Solution to Exercise 2

$r \backslash ab$	00	01	11	10
0	1	0	0	0
1	0	1	1	1

giving minimal DNF

$$(\neg r \wedge \neg a \wedge \neg b) \vee (r \wedge b) \vee (r \wedge a)$$

Getting the minimal DNF is required for a mark of 3 on the mandatory part of this tutorial. (Non-minimal examples will have two singleton blocks.)

Exercise 3 –optional—marked–

Consider four variables a, b, c, d and the following set of two clauses (a CNF clause is a disjunction of literals):

$$a \vee \neg b, \ \ \neg a \vee \neg d$$

(a) Viewing ‘,’ as conjunction, as we do on the left-hand side of a sequent, draw the (single) 4-variable Karnaugh map for these two clauses. Show on your map the blocks of zeros arising from each of the two clauses. (Because the clauses are being and’ed together, the map has zero wherever either of the clauses has zero.)

(b) Use the map to find a new clause δ, different from the given clauses, such that

$$a \vee \neg b, \ \ \neg a \vee \neg d \vDash \delta$$

(Hint: if $\Gamma \vDash \delta$, what do you know about the cells where δ is 0 or 1 in terms of the cells where Γ is 0 or 1?)

How many different δ can you find?

Solution to Exercise 3

a) $a \vee \neg b$, $\neg a \vee \neg d$

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

b) $a \vee \neg b, \ \neg a \vee \neg d \models \delta$
If $\Gamma \vDash \delta$, then δ must be 1 where Γ is 1:
the 0s of δ are a subset of the 0s of Γ.
For example, $\delta = \neg b \vee \neg d$
We can choose δ from:

- 8 1×1 blocks
- 6 1×2 blocks
- 4 2×1 blocks
- 1 2×2 block

Hence, in total, there are 19 possible solutions (clauses) for δ.

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

Exercise 4 –optional—not marked–

Read about race conditions on the Wikipedia Karnaugh maps https://en.wikipedia.org/wiki/Karnaugh_map#Race_hazards and on how race conditions can be avoided by adding clauses.

For the Karnaugh Map you drew in Exercise 3, identify any race conditions and derive both a minimal CNF and a minimal race-free CNF.

Solution to Exercise 4

$a \vee \neg b$, $\neg a \vee \neg d$

There is a race condition when moving from the green block to the red block (which are adjacent but disjoint).

To eliminate the hazard, we add a new clause, $\neg b \vee \neg d$, corresponding to the yellow block.

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

A minimal CNF is $(a \vee \neg b) \wedge (\neg a \vee \neg d)$.

A minimal race-free CNF is $(a \vee \neg b) \wedge (\neg a \vee \neg d) \wedge (\neg b \vee \neg d)$.