Informatics 1 – Introduction to Computation
Computation and Logic

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based on materials by
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More Syllogisms
and more about syllogisms

Reprise

From the fundamental rule *barbara*

$$\frac{a \models b \quad b \models c}{a \models c}$$

together with *contraposition* and *double negation*, we got five sound syllogisms about *universal categorical statements*.

Contraposition is negating and swapping the two parts of a sequent:

$$a \models b \quad \longrightarrow \quad \neg b \models \neg a$$

Barbara is the feminine form of the Greek βάρβαρος (barbaros) 'foreign'. Taken into Latin, it was used as the name of a mythical early Christian martyr, daughter of a pagan (barbarian).

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In England you can buy alcohol at any time. In some countries you can't buy it (legally) at all.

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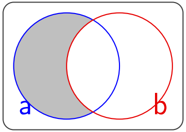
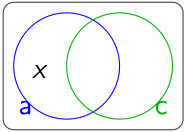


$a \models b$

some a is not c



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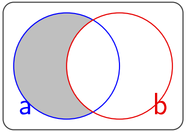

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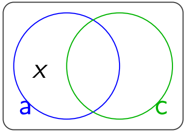


$a \nvDash c$

Why does contraposition work between the conclusion and *one* premise at a time?

What is the relation between the two premises?

Can we combine them into one? What happens with contraposition then?

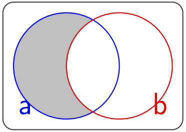
What is the difference between $a \vDash b$ and $a \to b$?

These two syllogisms are *bocardo* and *baroco*.

(not exactly) Aristotle's categorical propositions

universal affirmative

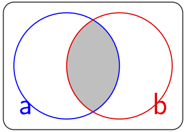
every *a* is *b*



$a \models b$

universal negative

no *a* is *b*



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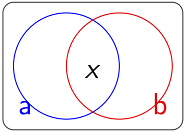
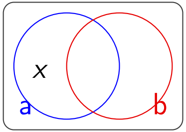


some a is b

particular affirmative

$a \not\models b$



some a is not b

particular negative

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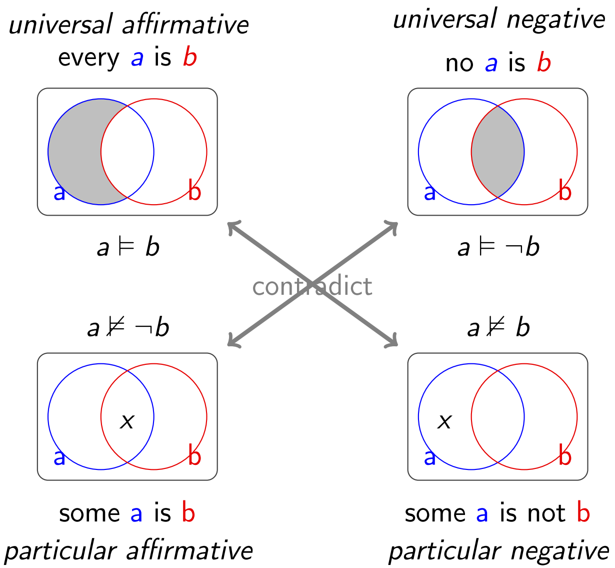

Why not exactly?

Aristotle made the *existential assumption*: if you say 'all a are b', or 'no a is b', that means that some a exists. So for him, universal affirmative implies particular affirmative, and universal negative implies particular negative.

Checking syllogisms (again)

This syllogism comes from *barbara* via contrapositions (do that!), so it should be sound. Check with Venn diagrams:

$$\frac{c \vDash \neg b \qquad a \nvDash \neg c}{a \nvDash b}$$

No mathematician is infallible
Some programmers are mathematicians
$\therefore$ *Some programmers are fallible*

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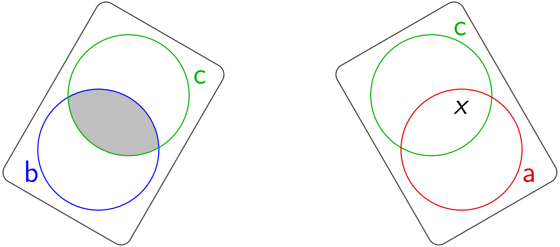

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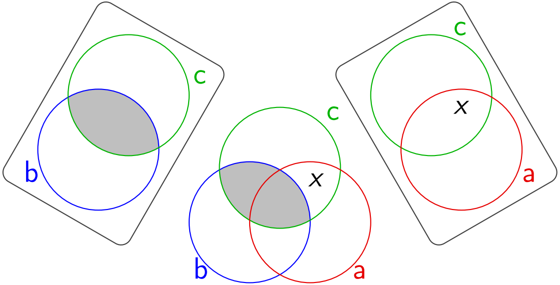

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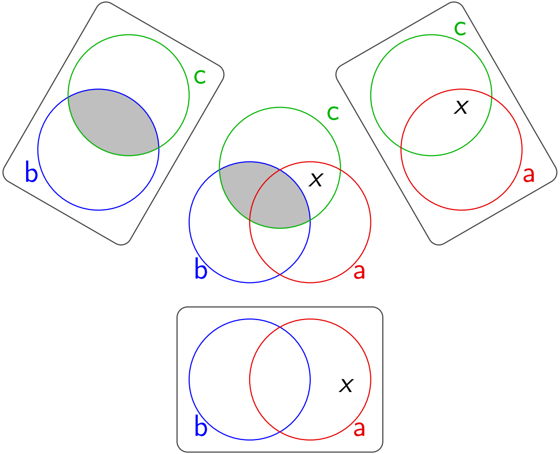

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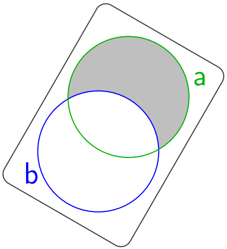

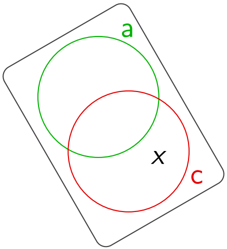

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All plants are fungi
Some flowers are not plants
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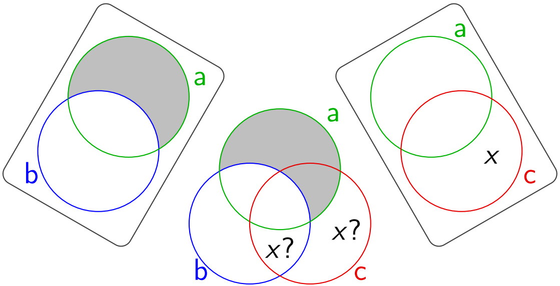

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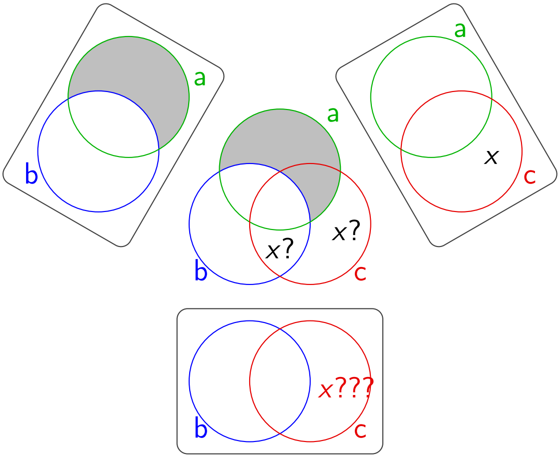

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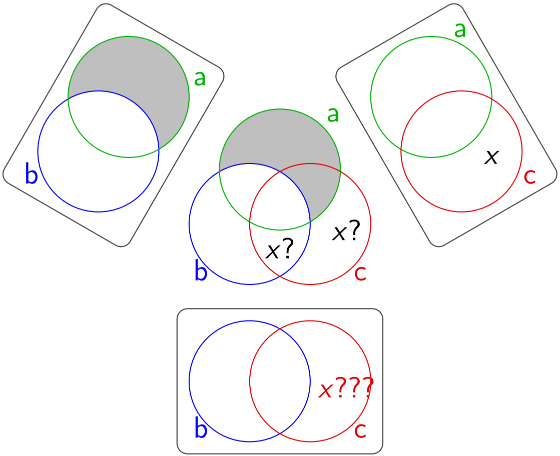

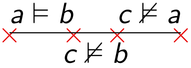

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In the usual meanings, no plant is a fungus, and all flowers are plants. That doesn't matter: the argument doesn't depend on the truth or falsity of the premises in a particular universe.

Suppose a *quonce* is a fungus flower, but not a plant, and nothing else exists. This disproves the syllogism.

Deriving rules

We have used the following to derive sound rules from *barbara*:

- substitution (e.g. q for a, $\neg b$ for b)
- double negation cancellation ($\neg\neg a \longrightarrow a$)
- contraposition within sequents ($a \vDash b \longrightarrow \neg b \vDash \neg a$)
- contraposition between conclusion and a premise

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c} \longrightarrow \frac{a \vDash b \qquad a \nvDash c}{b \nvDash c}$$

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Because these processes are symmetrical, they also derive unsound rules from unsound rules.

All the sound syllogisms

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You can derive all these. Mediaeval students learned them, with the help of this verse:

Barbara celarent darii ferio baralipton
Celantes dabitis fapesmo frisesomorum
Cesare camestres festino baroco
Darapti felapton disamis datisi bocardo ferison

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- All sound syllogisms come from *barbara* via contraposition etc.

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It's all much murkier than this. This existential assumption contradicts other aspects of Aristotle's system. In short, he was most likely confused.

D. W. Mulder, The existential assumptions of traditional logic, *Hist. & Phil. Logic*, 17:1-2, 141–154