CL exercise for Tutorial 8

Introduction

Objectives

In this tutorial, you will:

- learn to apply the Tseytin transformation
- use the arrow rule to count satisfying valuations

Tasks

Exercises 1 and 2 are mandatory. Exercise 3 is optional.

Submit

a file called `cl-tutorial-8` with your answers (image or pdf).

Deadline

16:00 Tuesday 16 November

Reminder

Good Scholarly Practice

Please remember the good scholarly practice requirements of the University regarding work for credit.

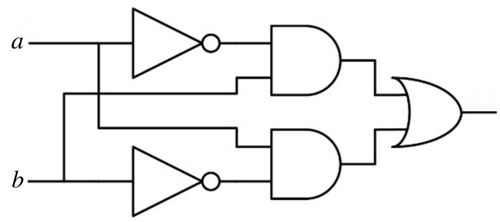
You can find guidance at the School page

https://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct.

This also has links to the relevant University pages. Please do not publish solutions to these exercises on the internet or elsewhere, to avoid others copying your solutions.

Exercise 1 –mandatory—marked–

Consider the following circuit:



Give an equivalent logical expression.

Apply the Tseytin transformation to give an equisatisfiable CNF expression.

Exercise 2 –mandatory—marked–

Read Chapter 23 (*Counting Satisfying Valuations*) of the textbook.

Use the arrow rule to count the number of satisfying assignments for the CNF expression

$$(E \vee F) \wedge (\neg A \vee B) \wedge C$$

Exercise 3 –optional—marked–

A *boolean algebra* is a set B containing elements $\mathbf{0}$ and $\mathbf{1}$, together with operations $\wedge$, $\vee$ and $\neg$ that satisfy the boolean algebra axioms on slide 4 of the week 8 lectures. The set $\mathbb{B} = \{0, 1\}$ with the usual operators is the simplest (non-trivial) boolean algebra.

For example, $\mathbb{B} \times \mathbb{B}$ with $\mathbf{0} = (0, 0)$, $\mathbf{1} = (1, 1)$ and pointwise operators (that is, $(a, b) \wedge (c, d) = (a \wedge c, b \wedge c)$ etc.) is a boolean algebra.

On slide 2, we saw several (though not all) of the 16 possible binary boolean operators: in other words, functions $\mathbb{B} \times \mathbb{B} \to \mathbb{B}$.

Show how to view the set $\mathbb{B} \times \mathbb{B} \to \mathbb{B}$ as a boolean algebra: that is, identify the $\mathbf{0}$ and $\mathbf{1}$ elements, and define the $\wedge$, $\vee$ and $\neg$ operations on elements.