Informatics 1 – Introduction to Computation

Computation and Logic

Julian Bradfield

based on materials by

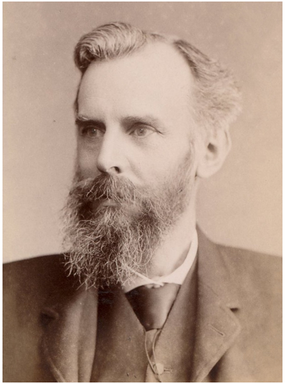
Michael P. Fourman

From Aristotle to Venn:

Aristotelian Syllogisms

and

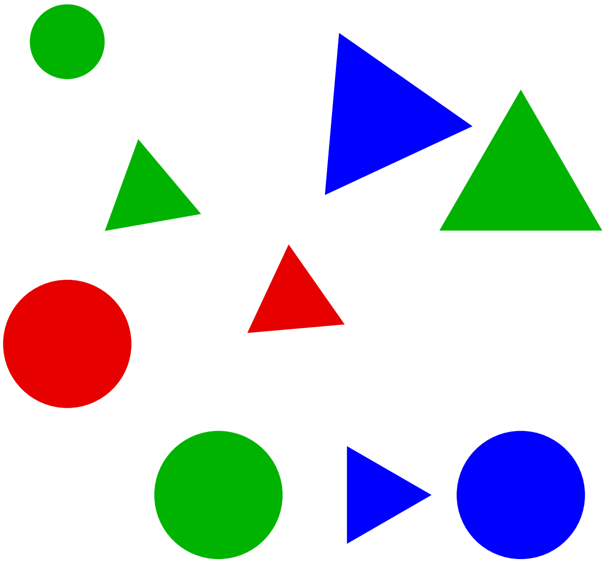
Venn Diagrams



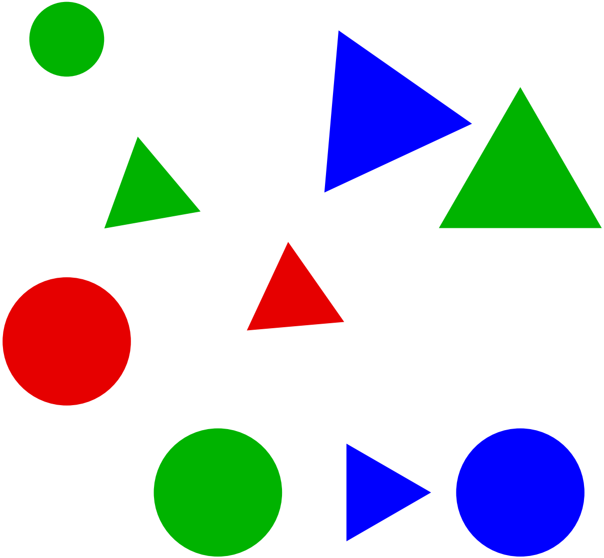
John Venn
1834–1923

A Small Universe

A universe of coloured shapes



Some statements about the universe



Every red triangle is small

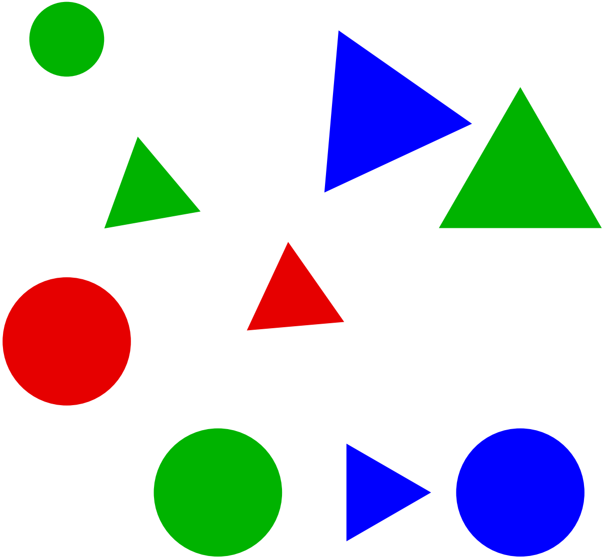
Every small triangle is red

Some big triangle is green

Some small disc is red

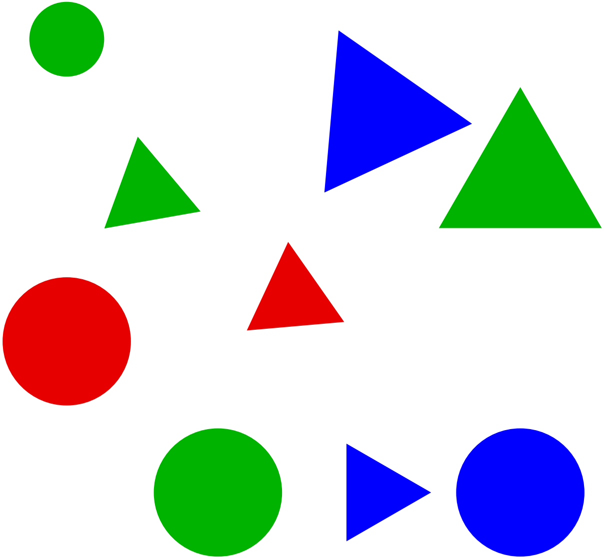
No red thing is blue

Some statements about the universe



- Every red triangle is small ✓
- Every small triangle is red
- Some big triangle is green
- Some small disc is red
- No red thing is blue

Some statements about the universe



Every red triangle is small ✓

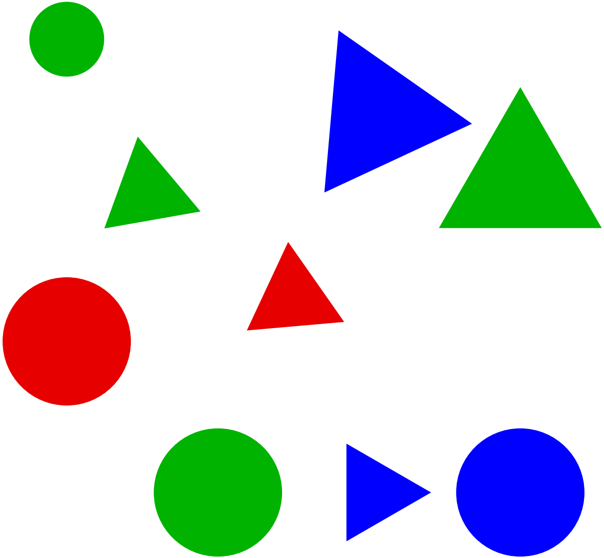
Every small triangle is red ✗

Some big triangle is green

Some small disc is red

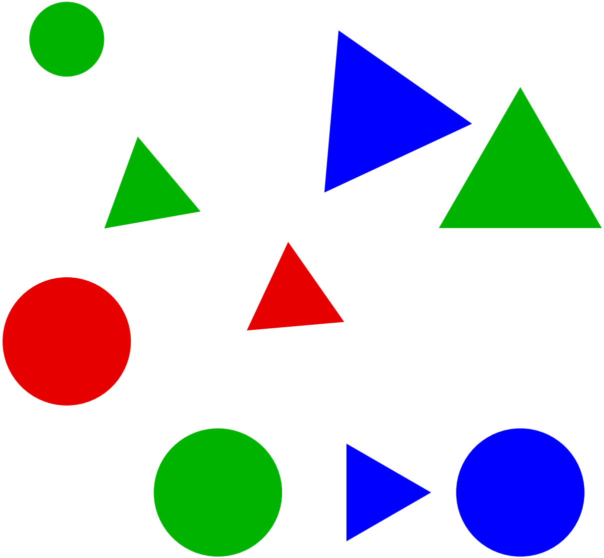
No red thing is blue

Some statements about the universe



Every red triangle is small	✓
Every small triangle is red	✗
Some big triangle is green	?
Some small disc is red	?
No red thing is blue	?

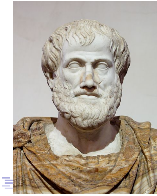
Some statements about the universe



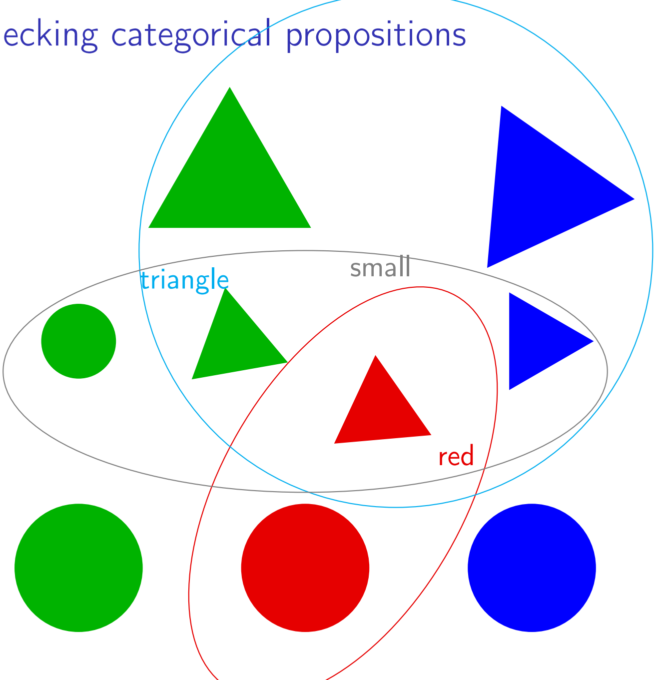
Every red triangle is small	✓
Every small triangle is red	✗
Some big triangle is green	?
Some small disc is red	?
No red thing is blue	?

Categorical propositions say:
(Every/some/no) A is (not) B.

Aristotle
384–322 B.C.



Checking categorical propositions



Every red triangle is small ✓

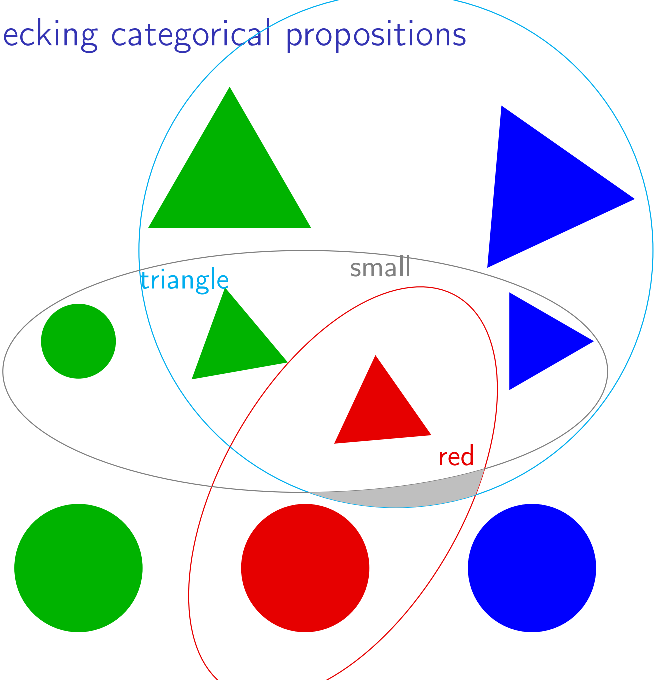
Every small triangle is red ✗

Some big triangle is green ?

Some small disc is red ?

No red thing is blue ?

Checking categorical propositions



Every red triangle is small ✓

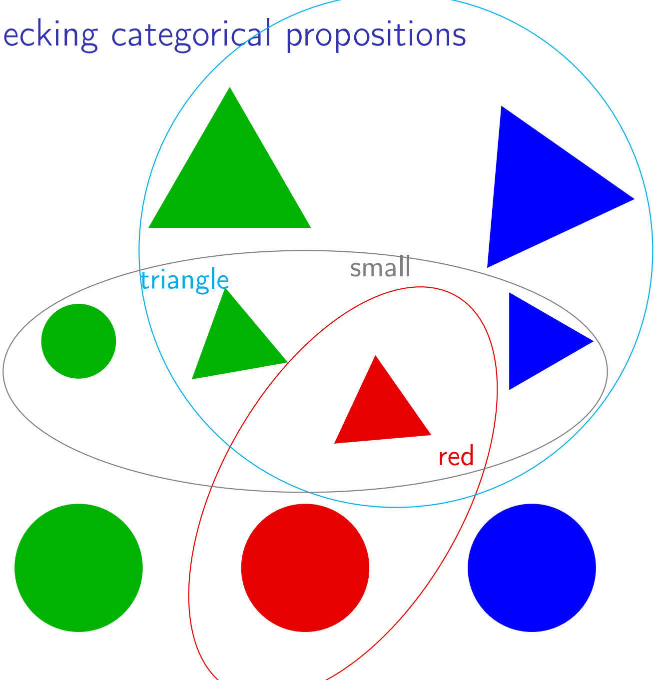
Every small triangle is red ✗

Some big triangle is green ?

Some small disc is red ?

No red thing is blue ?

Checking categorical propositions



Every red triangle is small ✓

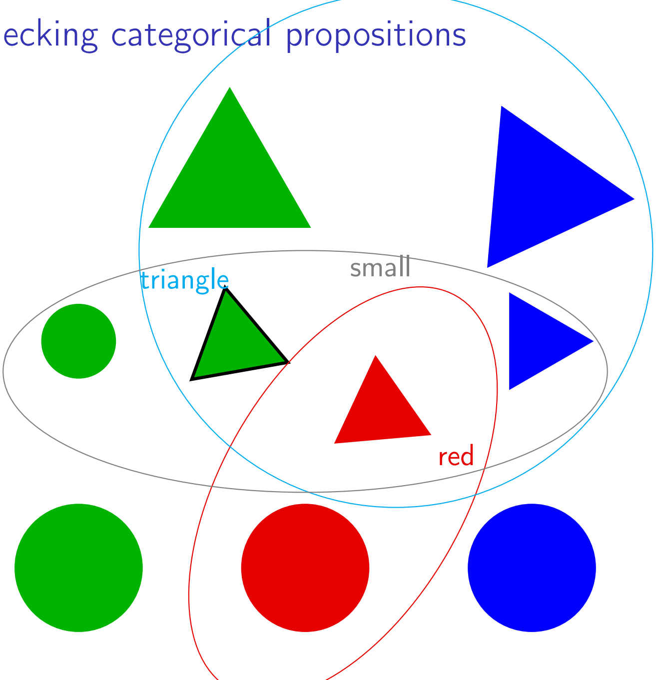
Every small triangle is red ✗

Some big triangle is green ?

Some small disc is red ?

No red thing is blue ?

Checking categorical propositions



Every red triangle is small ✓

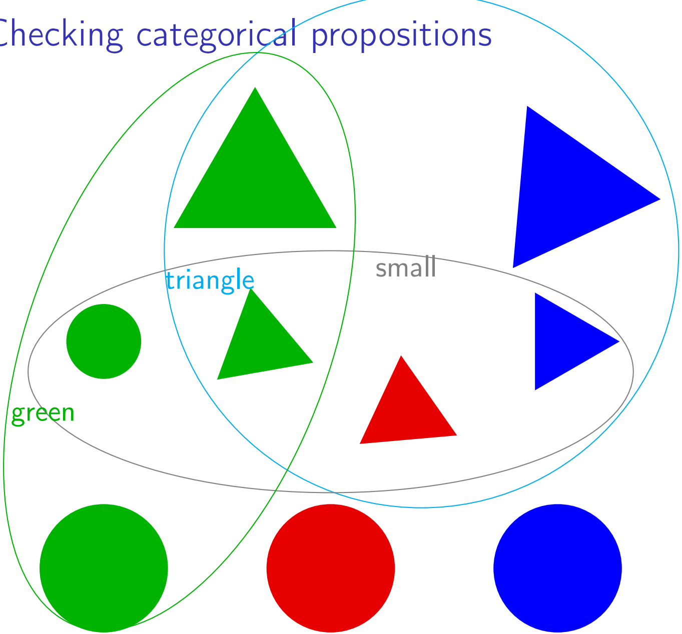
Every small triangle is red ✗

Some big triangle is green ?

Some small disc is red ?

No red thing is blue ?

Checking categorical propositions



Every red triangle is small ✓

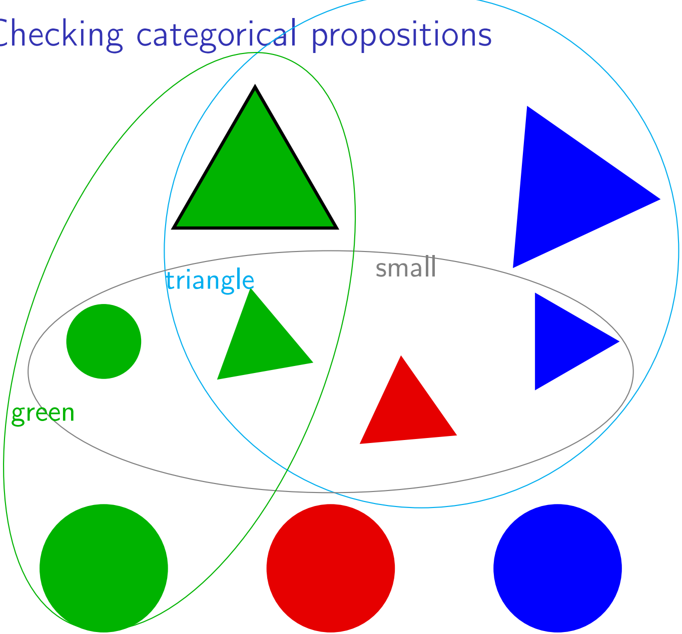
Every small triangle is red ✗

Some big triangle is green ?

Some small disc is red ?

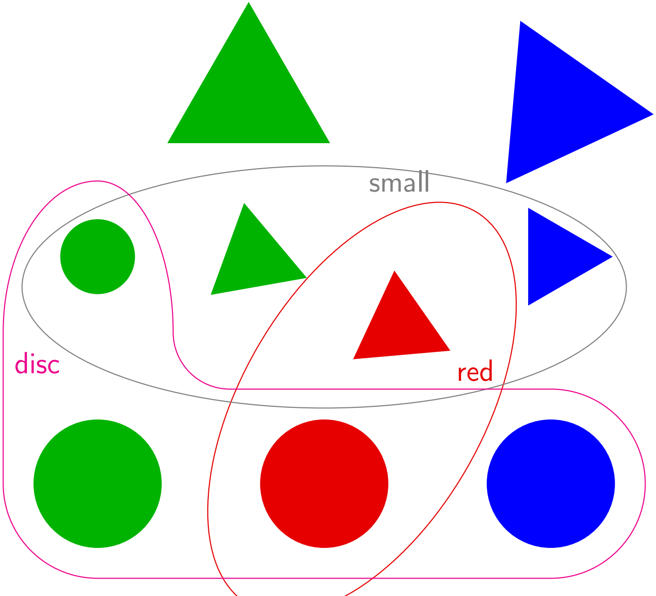
No red thing is blue ?

Checking categorical propositions



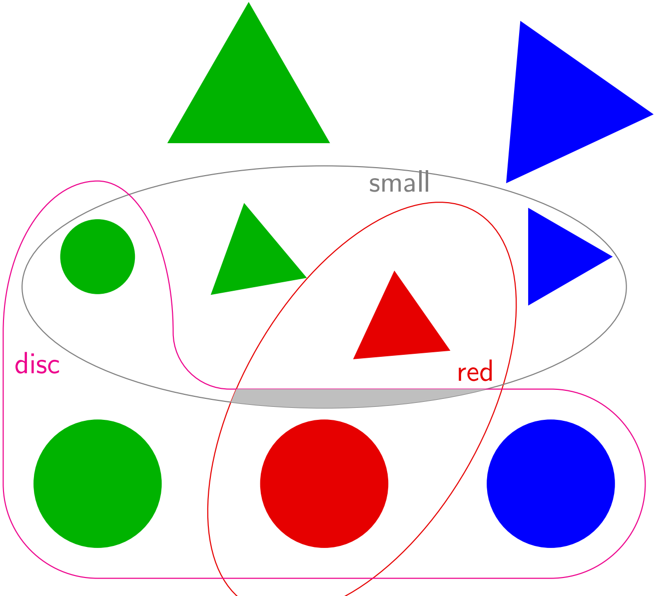
Every red triangle is small	✓
Every small triangle is red	✗
Some big triangle is green	✓
Some small disc is red	?
No red thing is blue	?

Checking categorical propositions



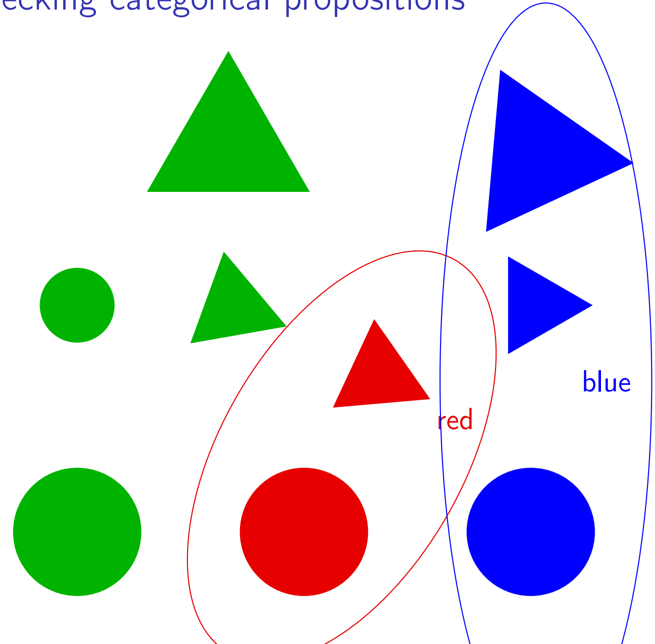
Every red triangle is small	✓
Every small triangle is red	✗
Some big triangle is green	✓
Some small disc is red	?
No red thing is blue	?

Checking categorical propositions



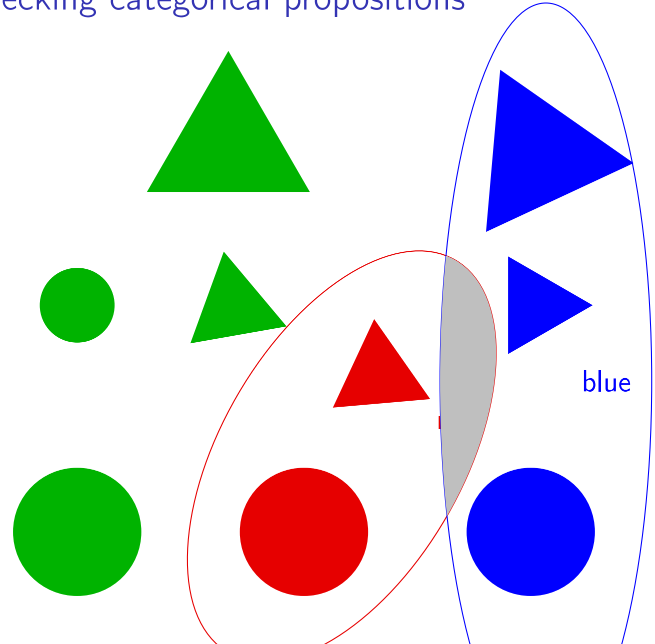
Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ✓
Some small disc is red ✗
No red thing is blue ?

Checking categorical propositions



Every red triangle is small	✓
Every small triangle is red	✗
Some big triangle is green	✓
Some small disc is red	✗
No red thing is blue	?

Checking categorical propositions



Every red triangle is small ✓
Every small triangle is red ✗
Some big triangle is green ✓
Some small disc is red ✗
No red thing is blue ✓

Categorical propositions as predicate logic

Categorical propositions are a very restricted form of predicate logic:

- Every red thing is small
  $\forall x. isRed(x) \rightarrow isSmall(x)$
- Every small triangle is red
  $\forall x. (isSmall(x) \wedge isTriangle(x)) \rightarrow isRed(x)$
- Some small disc is red
  $\exists x. (isSmall(x) \wedge isDisc(x)) \wedge isRed(x)$

Categorical propositions as predicate logic

Categorical propositions are a very restricted form of predicate logic:

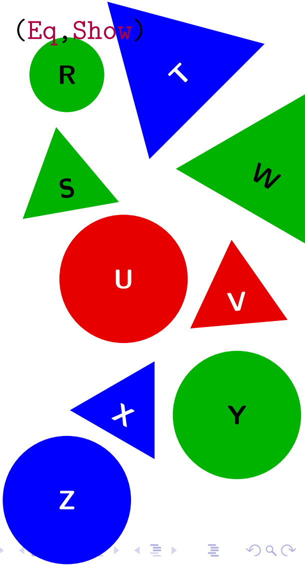
- Every red thing is small
  $\forall x. \mathit{isRed}(x) \rightarrow \mathit{isSmall}(x)$
- Every small triangle is red
  $\forall x. (\mathit{isSmall}(x) \wedge \mathit{isTriangle}(x)) \rightarrow \mathit{isRed}(x)$
- Some small disc is red
  $\exists x. (\mathit{isSmall}(x) \wedge \mathit{isDisc}(x)) \wedge \mathit{isRed}(x)$

Can you write the general form of a categorical proposition?

A universe in Haskell (1)

We need names for the *things* in the universe:

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```



A universe in Haskell (1)

We need names for the *things* in the universe:

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

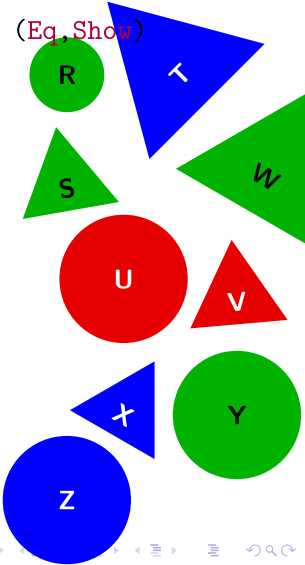
It's tempting to define types for the *features* that things have:

```
data Colour = Red | Blue | Green
data Shape = Disc | Triangle
data Size = Big | Small
```

and then define functions for the features:

```
colour :: Thing -> Colour
shape :: Thing -> Shape
size :: Thing -> Size
colour R = Green
```

etc. etc.



A universe in Haskell (1)

We need names for the *things* in the universe:

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

It's tempting to define types for the *features* that things have:

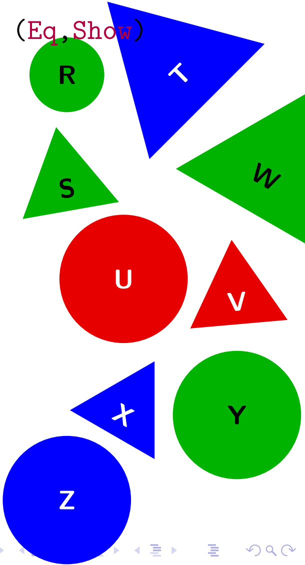
```
data Colour = Red | Blue | Green
data Shape = Disc | Triangle
data Size = Big | Small
```

and then define functions for the features:

```
colour :: Thing -> Colour
shape :: Thing -> Shape
size :: Thing -> Size
colour R = Green
```

etc. etc.

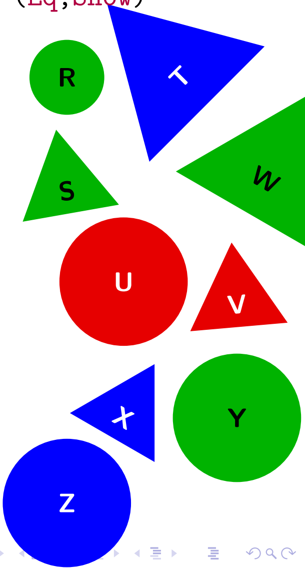
However, because of all the types, this ends up being hard to work with.



A universe in Haskell (2)

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.



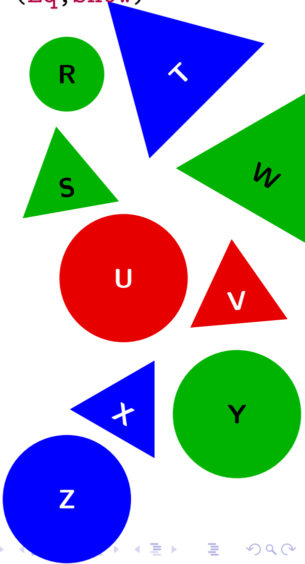
A universe in Haskell (2)


```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.

We could define the type of predicates on things:

```
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```



A universe in Haskell (2)

```
data Thing = R | S | T | U | V | W | X | Y | Z deriving (Eq,Show)
things = [ R, S, T, U, V, W, X, Y, Z ]
```

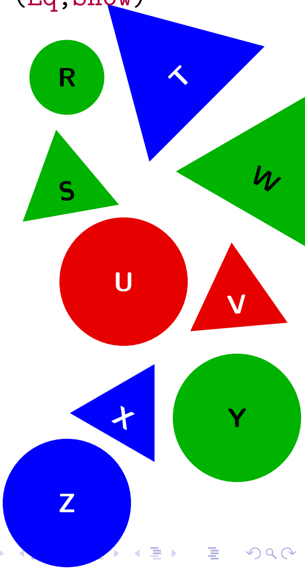
Instead of features, we define predicates, the basic propositions of logic. Every feature has a predicate, e.g. `isGreen`.

We could define the type of predicates on things:

```
type ThingPredicate = Thing -> Bool
isGreen :: ThingPredicate
```

but it's more general and convenient to do:

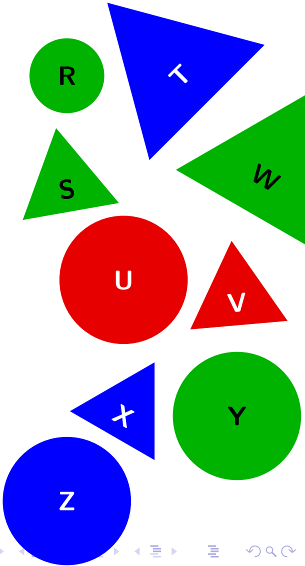
```
type Predicate u = u -> Bool
isGreen :: Predicate Thing
```



Defining the predicates

This is the simplest way to establish the predicates:

```
isGreen R = True
isGreen S = True
isGreen T = False
```



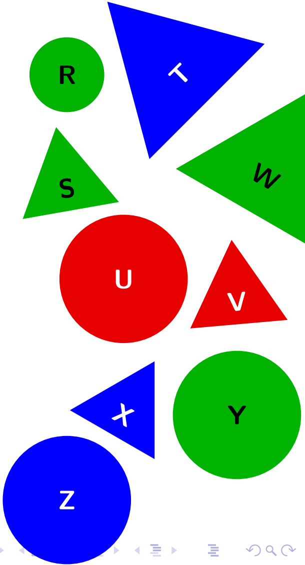
Defining the predicates

This is the simplest way to establish the predicates:

```
isGreen R = True
isGreen S = True
isGreen T = False
```

A lazier[1] way is:

```
isGreen x = x `elem` [ R, S, W, Y ]
isRed x = x `elem` [ U, V ]
```



[1] *The three chief virtues of a programmer are laziness, impatience, and hubris*
– Larry Wall

Defining the predicates

This is the simplest way to establish the predicates:

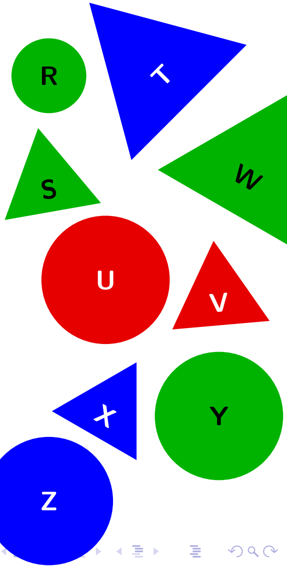
```
isGreen R = True
isGreen S = True
isGreen T = False
```

A lazier[1] way is:

```
isGreen x = x `elem` [ R, S, W, Y ]
isRed x = x `elem` [ U, V ]
```

Is this too lazy? (What happens when we extend the universe?)

```
isBlue x = not (isGreen x || isRed x)
```

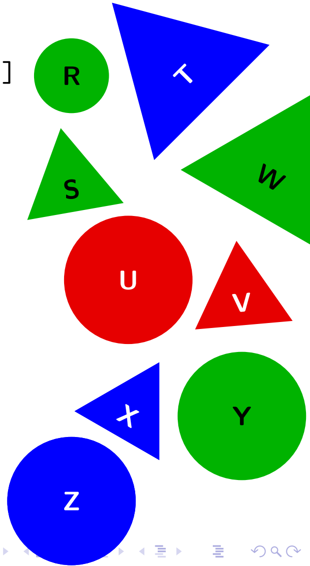


[1] *The three chief virtues of a programmer are laziness, impatience, and hubris*
– Larry Wall

Representing statements with list comprehension

Haskell's *list comprehension* gives a powerful way of representing statements:

```
[ x | x <- things, isBlue x || (isBig x && isDisc x) ]
```

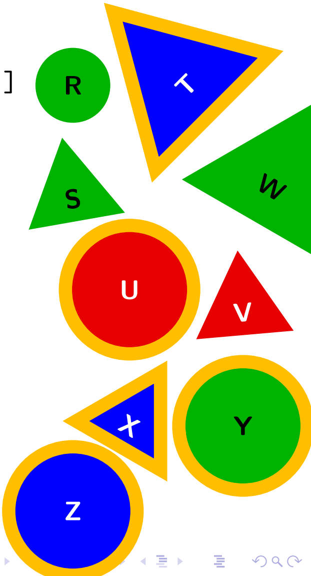


Representing statements with list comprehension

Haskell's *list comprehension* gives a powerful way of representing statements:

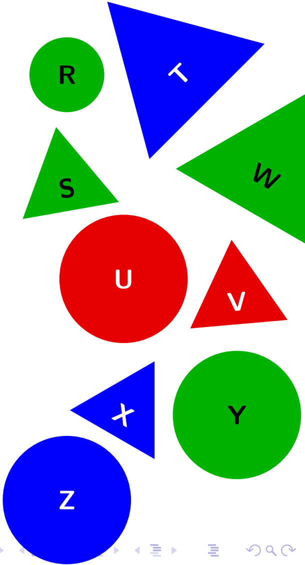
```
[ x | x <- things, isBlue x || (isBig x && isDisc x) ]
```

'the set (list) of things that are either blue or are big discs'



Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. ✗



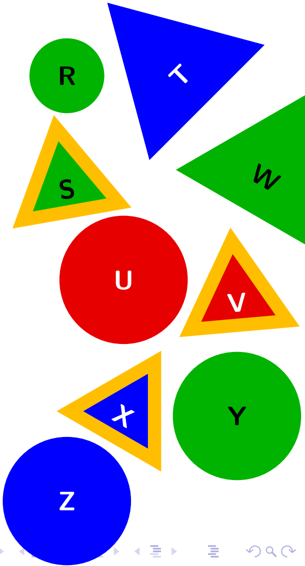
Categorical statements with Haskell

Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. ✗

```
[ x | x <- things, isTriangle(x) && isSmall(x) ]
[S,V,X]
```

'The set of things that are small triangles.'



Categorical statements with Haskell

Combining list comprehension with boolean operators on lists lets us express categorical statements.

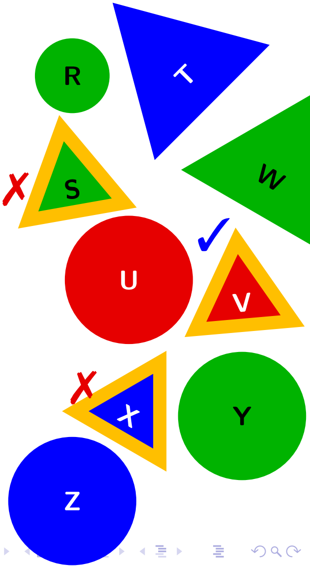
Every small triangle is red. ✗

```
[ x | x <- things, isTriangle(x) && isSmall(x) ]
[S,V,X]
```

‘The set of things that are small triangles.’

```
[ isRed(x) | x <- things, isTriangle(x) && isSmall(x) ]
[False,True,False]
```

‘Whether each small triangle is red.’



Categorical statements with Haskell

Combining list comprehension with boolean operators on lists lets us express categorical statements.

Every small triangle is red. ✗

```
[ x | x <- things, isTriangle(x) && isSmall(x) ]
[S,V,X]
```

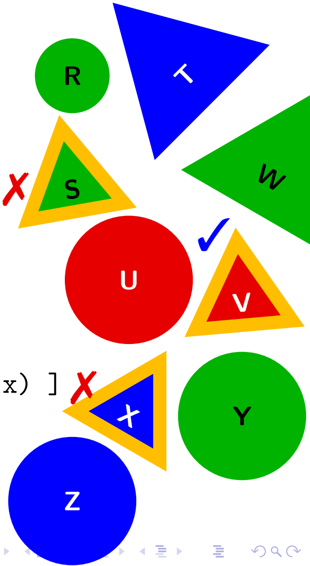
‘The set of things that are small triangles.’

```
[ isRed(x) | x <- things, isTriangle(x) && isSmall(x) ]
[False,True,False]
```

‘Whether each small triangle is red.’

```
and [ isRed(x) | x <- things, isTriangle(x) && isSmall(x) ]
False
```

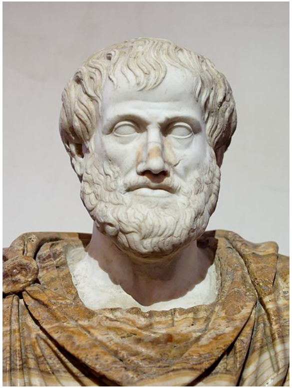
‘Every small triangle is red.’



Aristotle's Syllogistic Reasoning

A syllogism is discourse (logos) in which, certain things being stated, something other than what is stated follows of necessity from those things.

- All Greeks are human
- All humans are mortal
- $\therefore$ All Greeks are mortal



Syllogisms

Can be expressed in many forms:

- $\{ x \mid \mathrm{isGreek}(x) \} \subseteq \{ x \mid \mathrm{isHuman}(x) \}$
- $\{ x \mid \mathrm{isHuman}(x) \} \subseteq \{ x \mid \mathrm{isMortal}(x) \}$
- $\therefore \{ x \mid \mathrm{isGreek}(x) \} \subseteq \{ x \mid \mathrm{isMortal}(x) \}$

All Greeks are human
All humans are mortal
$\therefore$ all humans are mortal

Syllogisms

Can be expressed in many forms:

- $\{\, x \mid \mathrm{isGreek}(x) \,\} \subseteq \{\, x \mid \mathrm{isHuman}(x) \,\}$
- $\{\, x \mid \mathrm{isHuman}(x) \,\} \subseteq \{\, x \mid \mathrm{isMortal}(x) \,\}$
- $\therefore \{\, x \mid \mathrm{isGreek}(x) \,\} \subseteq \{\, x \mid \mathrm{isMortal}(x) \,\}$

In modern logic, we write it as:

$$\frac{\mathrm{isGreek} \vDash \mathrm{isHuman} \qquad \mathrm{isHuman} \vDash \mathrm{isMortal}}{\mathrm{isGreek} \vDash \mathrm{isMortal}}$$

All Greeks are human
All humans are mortal
∴ all humans are mortal

Syllogisms

Can be expressed in many forms:

- $\{\, x \mid \text{isGreek}(x) \,\} \subseteq \{\, x \mid \text{isHuman}(x) \,\}$
- $\{\, x \mid \text{isHuman}(x) \,\} \subseteq \{\, x \mid \text{isMortal}(x) \,\}$
- $\therefore \{\, x \mid \text{isGreek}(x) \,\} \subseteq \{\, x \mid \text{isMortal}(x) \,\}$

In modern logic, we write it as:

$$\frac{\text{isGreek} \vDash \text{isHuman} \qquad \text{isHuman} \vDash \text{isMortal}}{\text{isGreek} \vDash \text{isMortal}}$$

The general form of this syllogism is

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c}$$

All Greeks are human
All humans are mortal
∴ all humans are mortal

Syllogisms

Can be expressed in many forms:

- $\{\, x \mid \text{isGreek}(x) \,\} \subseteq \{\, x \mid \text{isHuman}(x) \,\}$
- $\{\, x \mid \text{isHuman}(x) \,\} \subseteq \{\, x \mid \text{isMortal}(x) \,\}$
- $\therefore \{\, x \mid \text{isGreek}(x) \,\} \subseteq \{\, x \mid \text{isMortal}(x) \,\}$

All Greeks are human
All humans are mortal
∴ all humans are mortal

In modern logic, we write it as:

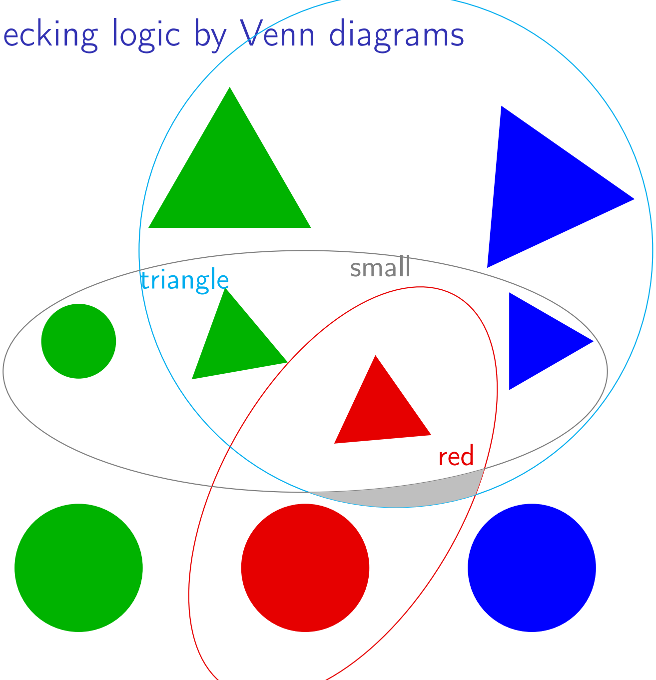
$$\frac{\text{isGreek} \vDash \text{isHuman} \qquad \text{isHuman} \vDash \text{isMortal}}{\text{isGreek} \vDash \text{isMortal}}$$

The general form of this syllogism is

$$\frac{a \vDash b \qquad b \vDash c}{a \vDash c}$$

Is this syllogism sound? I.e. valid in *every* universe?

Checking logic by Venn diagrams



Every red triangle is small ✓

Every small triangle is red ✗

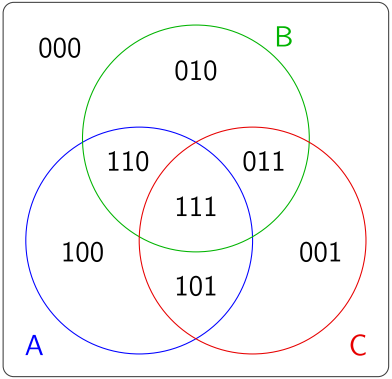
Some big triangle is green ?

Some small disc is red ?

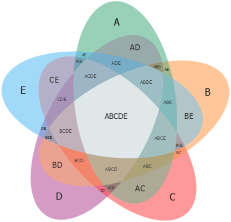
No red thing is blue ?

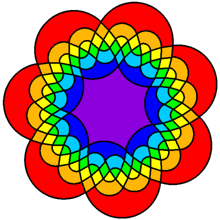
Venn diagrams

Venn diagrams show every possible combination



000	$\bar{A} \cap \bar{B} \cap \bar{C}$
001	$\bar{A} \cap \bar{B} \cap C$
010	$\bar{A} \cap B \cap \bar{C}$
011	$\bar{A} \cap B \cap C$
100	$A \cap \bar{B} \cap \bar{C}$
101	$A \cap \bar{B} \cap C$
110	$A \cap B \cap \bar{C}$
111	$A \cap B \cap C$

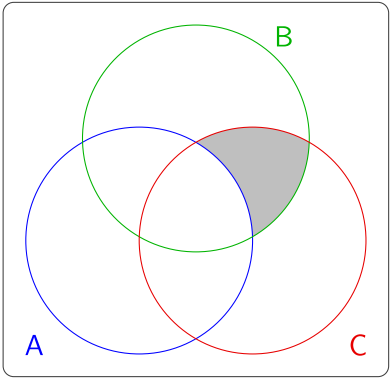




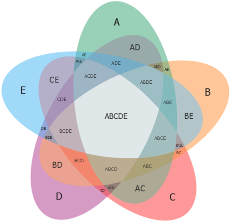
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

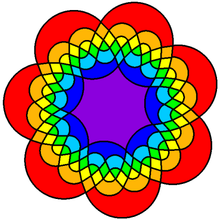
Venn diagrams

We use light shading to show **emptiness** of a region



$$\bar{A} \cap B \cap C = \varnothing$$

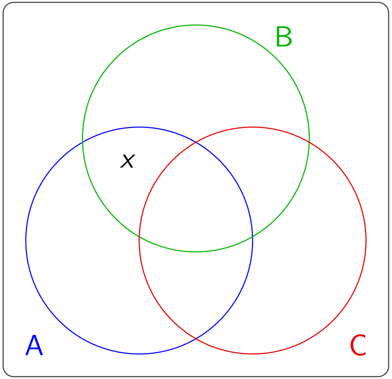




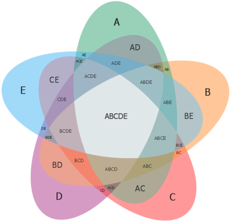
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

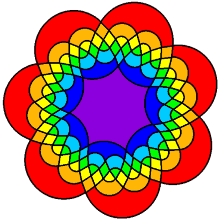
Venn diagrams

We may write a *variable* to show **non-emptiness** of a region



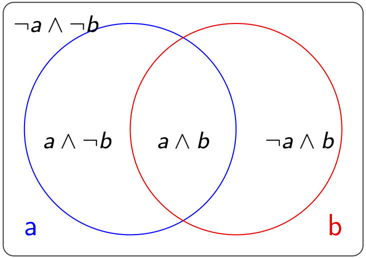
$$x \in A \cap B \cap \bar{C}$$





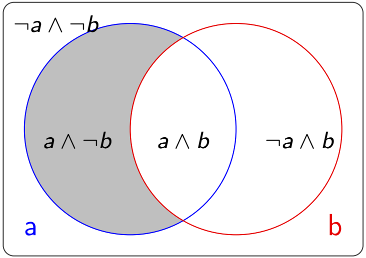
A rotationally symmetric Venn diagram for $n > 1$ sets exists iff n is prime

Venn interpretation of $a \vDash b$



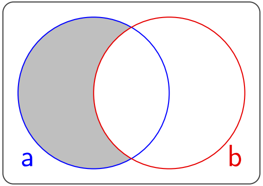
$a \vDash b$	'every a is b'
$a \vDash b$	'no a is not b'
$a \vDash b$	'nothing is a and not b'
$a \vDash b$	$a \cap \bar{b} = \varnothing$
$a \vDash b$	$\neg(a \wedge \neg b)$
$a \vDash b$	$b \vee \neg a$

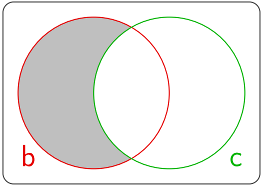
Venn interpretation of $a \vDash b$



$a \vDash b$	'every a is b'
$a \vDash b$	'no a is not b'
$a \vDash b$	'nothing is a and not b'
$a \vDash b$	$a \cap \bar{b} = \varnothing$
$a \vDash b$	$\neg(a \wedge \neg b)$
$a \vDash b$	$b \vee \neg a$

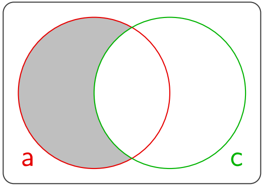
Venn syllogism



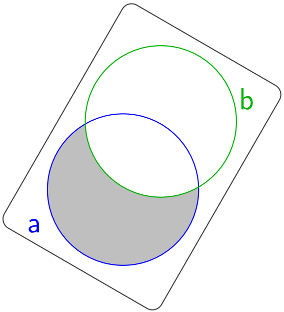


$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

$$\frac{\text{every } a \text{ is } b \qquad \text{every } b \text{ is } c}{\text{every } a \text{ is } c}$$

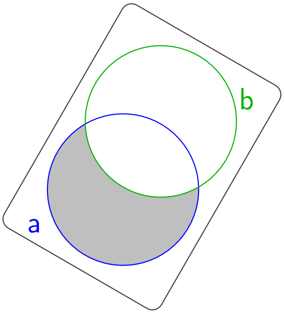


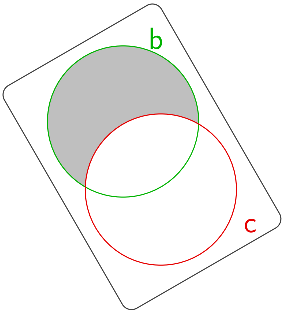
Combining diagrams



$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

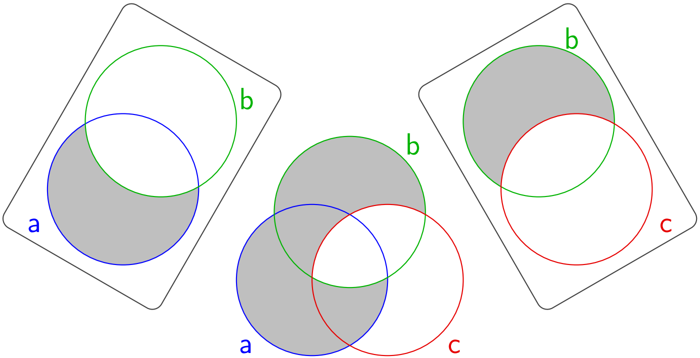
Combining diagrams





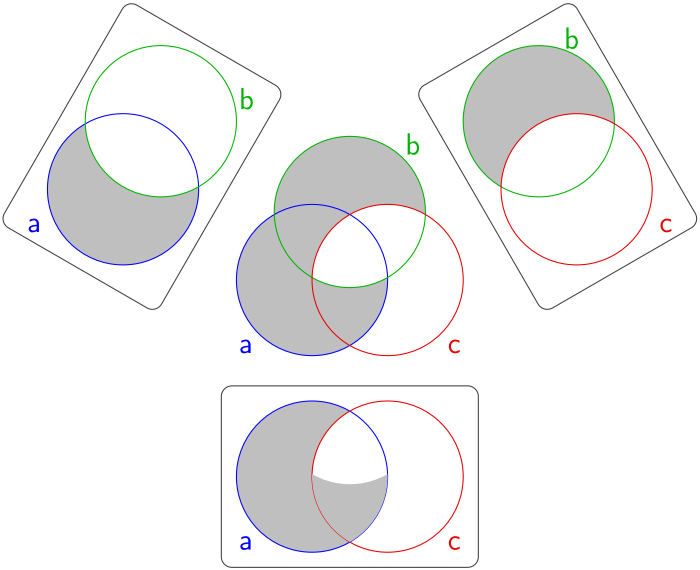
$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Combining diagrams



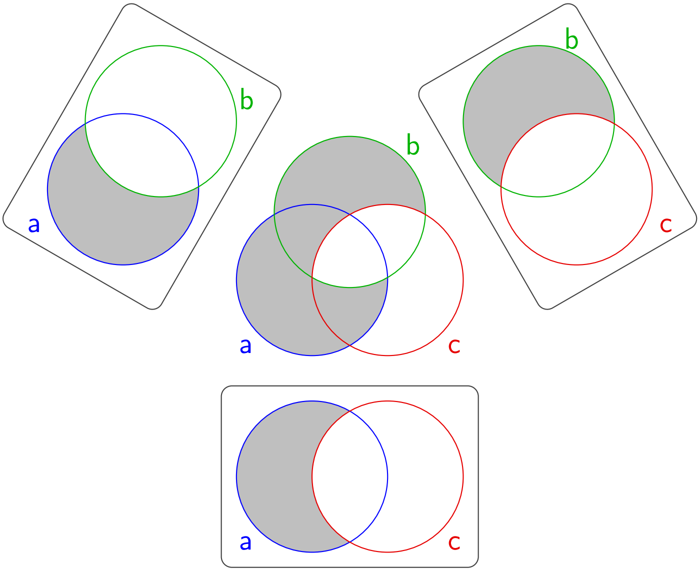
$$\frac{a \models b \quad b \models c}{a \models c}$$

Combining diagrams



$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Combining diagrams



$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Barbara is sound!

$$\text{barbara}\quad \frac{a \vDash b \qquad b \vDash c}{a \vDash c}$$

This rule, as we've seen, is sound:

for *any predicates* a, b, c in *any universe*, we have:

if the premises (above the line) are valid
then the conclusion (below the line) is valid.

Mediaeval logicians gave mnemonic names to syllogisms. This one is *barbara*. Consult Wikipedia to find out what that means – but only if you don't value your sanity!

Aristotle's *universal* propositions

make statements about *all* of something: 'all a are b'.

We can make universal *negative* statements: 'no a is b'.

'no a is b' iff 'every a is $\neg b$' iff $a \vDash \neg b$

Aristotle's *universal* propositions

make statements about *all* of something: 'all a are b'.

We can make universal *negative* statements: 'no a is b'.

$$\text{'no } a \text{ is } b\text{' iff 'every } a \text{ is } \neg b\text{' iff } a \vDash \neg b$$

Here is a syllogism involving universal negatives:

$$\frac{s \vDash r \quad r \vDash \neg f}{s \vDash \neg f}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

Is this an instance of *barbara* (and so valid)?

Aristotle's *universal* propositions

make statements about *all* of something: 'all a are b'.

We can make universal *negative* statements: 'no a is b'.

$$\text{'no } a \text{ is } b\text{' iff 'every } a \text{ is } \neg b\text{' iff } a \vDash \neg b$$

Here is a syllogism involving universal negatives:

$$\frac{s \vDash r \quad r \vDash \neg f}{s \vDash \neg f}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

Is this an instance of *barbara* (and so valid)?

For us modern logicians, it is: $a \equiv s$, $b \equiv r$, $c \equiv \neg f$.

A negated predicate is also a predicate.

Aristotle differed from us moderns on the relation between 'all' and 'no'. For him, this syllogism contained a universal affirmative and two universal negatives. The mediaeval logicians called it *celarent*.

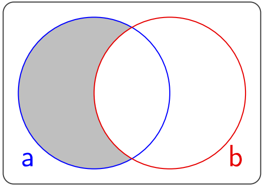
The key difference was the 'existential assumption' – see later.

Universal statements

whether affirmative or negative, say that some region is *empty*:

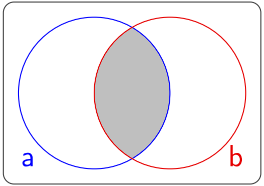
all a are b

$a \vDash b$



no a is b

$a \vDash \neg b$

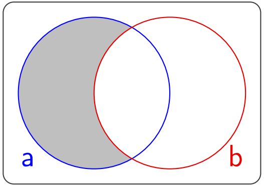


Universal statements

whether affirmative or negative, say that some region is *empty*:

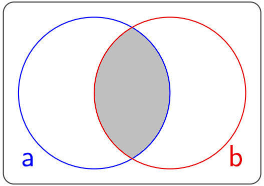
all a are b

$a \models b$



no a is b

$a \models \neg b$



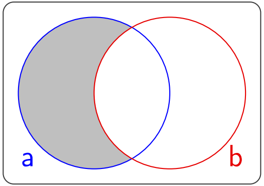
What about $\neg a \models b$ and $\neg a \models \neg b$?

Universal statements

whether affirmative or negative, say that some region is *empty*:

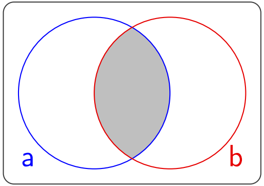
all a are b

$a \vDash b$



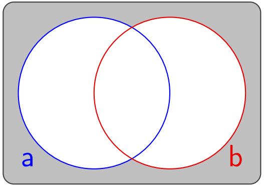
no a is b

$a \vDash \neg b$



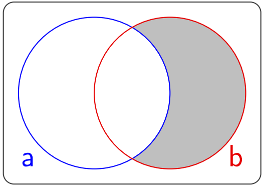
???

$\neg a \vDash b$



???

$\neg a \vDash \neg b$

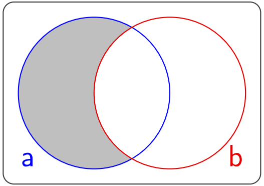


Universal statements

whether affirmative or negative, say that some region is *empty*:

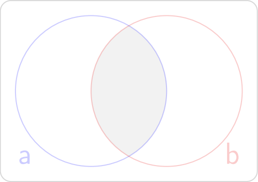
all a are b

$a \vDash b$



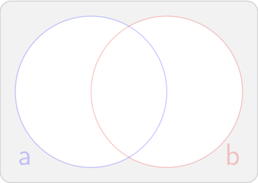
no a is b

$a \vDash \neg b$



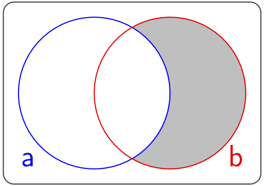
???

$\neg a \vDash b$



???

$\neg a \vDash \neg b$



We can observe:

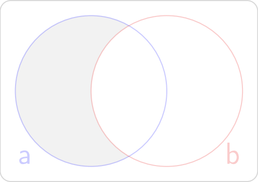
- $a \vDash b$ and $\neg a \vDash \neg b$ are reflections of each other: so $\neg a \vDash \neg b$ is the same as $b \vDash a$. $\neg a \vDash \neg b$ is the contrapositive of $b \vDash a$.

Universal statements

whether affirmative or negative, say that some region is *empty*:

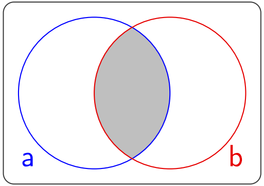
all a are b

$a \vDash b$



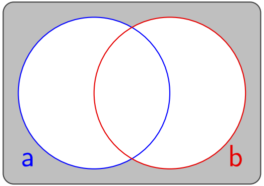
no a is b

$a \vDash \neg b$



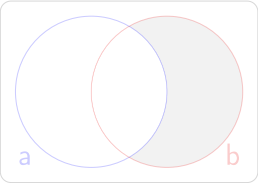
???

$\neg a \vDash b$



???

$\neg a \vDash \neg b$



We can observe:

- $a \vDash b$ and $\neg a \vDash \neg b$ are reflections of each other: so $\neg a \vDash \neg b$ is the same as $b \vDash a$. $\neg a \vDash \neg b$ is the contrapositive of $b \vDash a$.
- $a \vDash \neg b$ is symmetrical, so is the same as $b \vDash \neg a$ – they are contrapositives. Likewise $\neg a \vDash b$ and $\neg b \vDash a$.

Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

Natural languages differ, within and between themselves, on how they treat multiple negatives:

'I didn't never do nothing to nobody!'.

How does your native language/ dialect treat multiple negatives?

Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

Natural languages differ, within and between themselves, on how they treat multiple negatives:

'I didn't never do nothing to nobody!'.

How does your native language/ dialect treat multiple negatives?

Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

The law of contraposition: $a \vDash b$ iff $\neg b \vDash \neg a$.

Natural languages differ, within and between themselves, on how they treat multiple negatives:

'I didn't never do nothing to nobody!'. How does your native language/ dialect treat multiple negatives?

Negation and contraposition

Negation can be tricky – modern classical logic makes it simple.

The law of double negation: $\neg\neg a = a$ (two negatives make a positive).

The law of contraposition: $a \vDash b$ iff $\neg b \vDash \neg a$.

Thus we get $a \vDash b$ iff $\neg b \vDash \neg a$ iff $\neg\neg a \vDash \neg\neg b$ iff $a \vDash b$.

$$\frac{a \vDash b}{\neg b \vDash \neg a}$$

The double line means the rule works both ways.

Natural languages differ, within and between themselves, on how they treat multiple negatives: 'I didn't never do nothing to nobody!'. How does your native language/ dialect treat multiple negatives?

Reprise

So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\vDash$:

- $\neg\neg a = a$ *or* $\dfrac{a}{\neg\neg a}$ (double negation)
- $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$ (*barbara*)
- $\dfrac{a \vDash b}{\neg b \vDash \neg a}$ (contraposition)

Reprise

So far, we have seen (and hopefully agreed on) these **sound** rules about predicates and $\models$:

- $\neg\neg a = a$ *or* $\dfrac{a}{\neg\neg a}$ (double negation) (the rule line is doubled)
- $\dfrac{a \models b \quad b \models c}{a \models c}$ (*barbara*)
- $\dfrac{a \models b}{\neg b \models \neg a}$ (contraposition) (the rule line is doubled)

We also saw a 'different' (for Aristotle) syllogism with negatives got from *barbara* by putting $\neg c$ for c:

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *No snake has fur*

More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c} \qquad \frac{\neg a \vDash b \quad b \vDash c}{\neg a \vDash c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c} \qquad \frac{\neg a \vDash b \quad b \vDash \neg c}{\neg a \vDash \neg c}$$

$$\frac{a \vDash \neg b \quad \neg b \vDash c}{a \vDash c} \qquad \frac{\neg a \vDash \neg b \quad \neg b \vDash c}{\neg a \vDash c}$$

$$\frac{a \vDash \neg b \quad \neg b \vDash \neg c}{a \vDash \neg c} \qquad \frac{\neg a \vDash \neg b \quad \neg b \vDash \neg c}{\neg a \vDash \neg c}$$

More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c} \qquad \frac{\neg a \vDash b \quad b \vDash c}{\neg a \vDash c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c} \qquad \frac{\neg a \vDash b \quad b \vDash \neg c}{\neg a \vDash \neg c}$$

$$\frac{a \vDash \neg b \quad \neg b \vDash c}{a \vDash c} \qquad \frac{\neg a \vDash \neg b \quad \neg b \vDash c}{\neg a \vDash c}$$

$$\frac{a \vDash \neg b \quad \neg b \vDash \neg c}{a \vDash \neg c} \qquad \frac{\neg a \vDash \neg b \quad \neg b \vDash \neg c}{\neg a \vDash \neg c}$$

Aristotle only considered negative predicates on the right of $\vDash$ ($a \vDash \neg b$ means 'no a is b', so he viewed it as a negative statement about positive predicates). This leaves . . .

More syllogisms

By using (un)negated predicates in *barbara*, we get 8 syllogisms:

$$\frac{a \models b \quad b \models c}{a \models c}$$

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

barbara and *celarent*

Aristotle only considered negative predicates on the right of $\models$ ($a \models \neg b$ means 'no a is b', so he viewed it as a negative statement about positive predicates). This leaves ...

Even more syllogisms

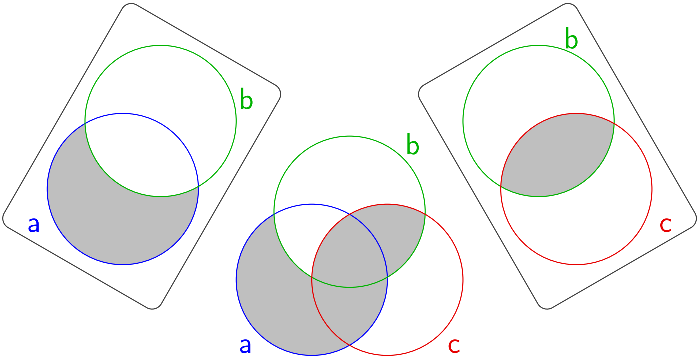
Contraposition lets us generate three more (Aristotelian) syllogisms from *celarent*:

$$\frac{a \models b \quad c \models \neg b}{a \models \neg c} \qquad \frac{a \models b \quad b \models \neg c}{c \models \neg a} \qquad \frac{a \models b \quad c \models \neg b}{c \models \neg a}$$

cesare, *camenes*, *camestres*

That brings us to 5 sound universal syllogisms. That's all!

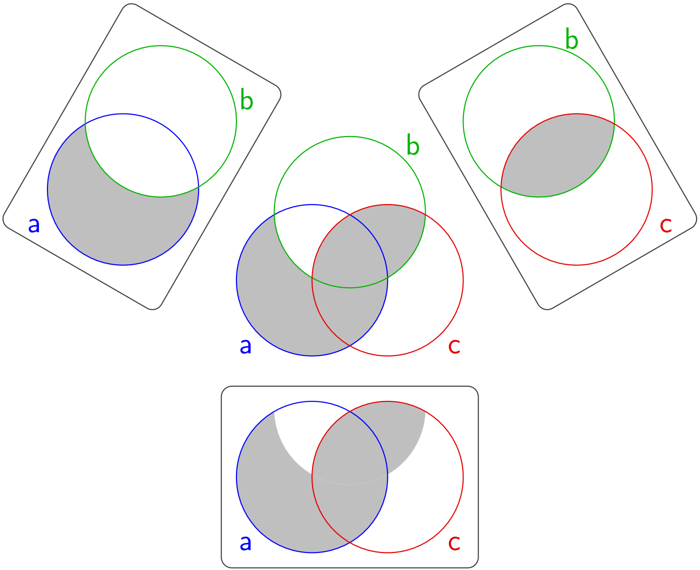
Unsound syllogisms



$$\frac{a \vDash b \qquad b \vDash \neg c}{a \vDash c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

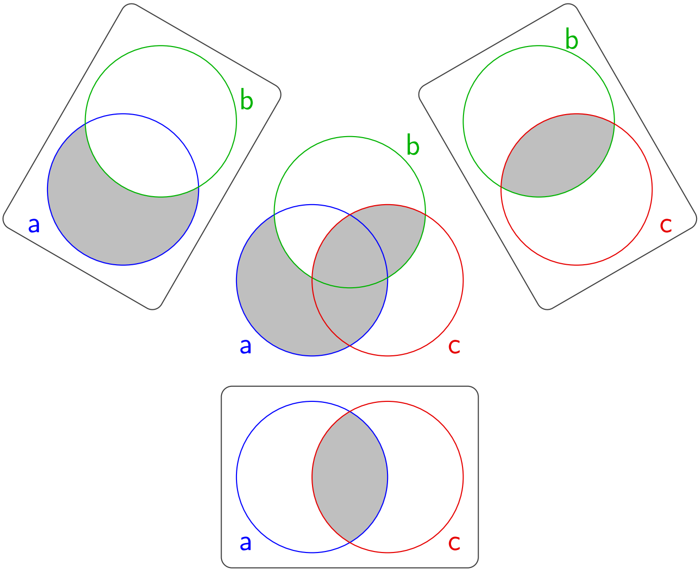
Unsound syllogisms



$$\frac{a \models b \qquad b \models \neg c}{a \models c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

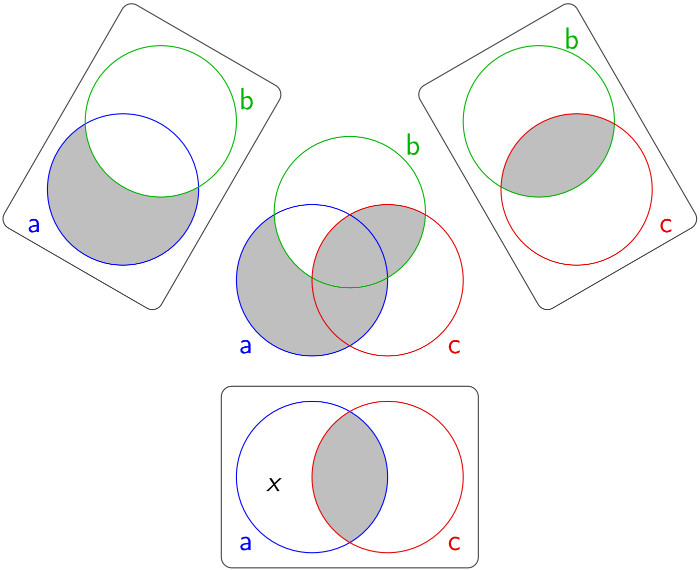
Unsound syllogisms



$$\frac{a \vDash b \qquad b \vDash \neg c}{a \vDash c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

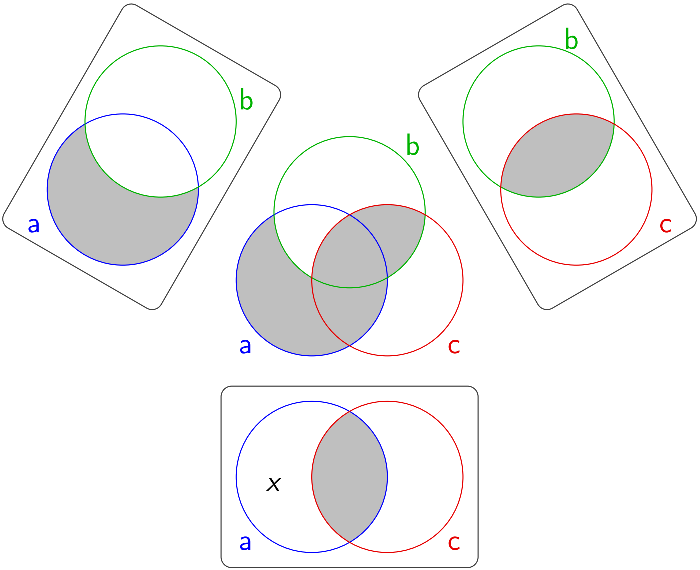
Unsound syllogisms



$$\frac{a \models b \qquad b \models \neg c}{a \models c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

Unsound syllogisms



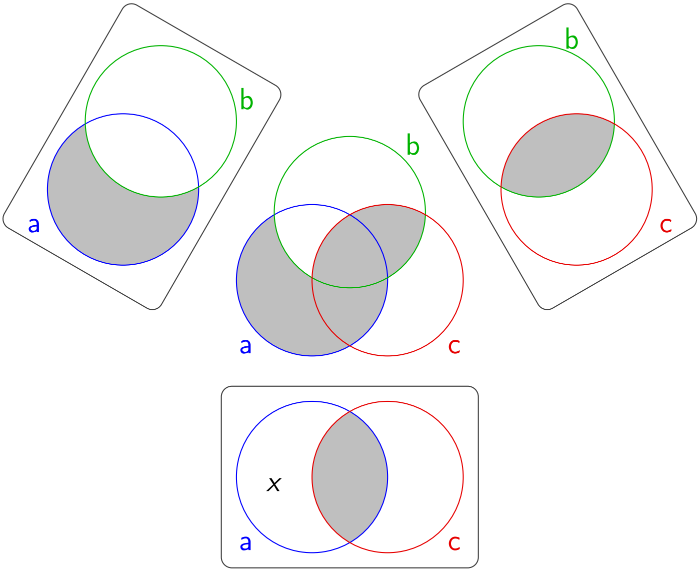
$a \vDash b \qquad b \vDash \neg c$

$a \vDash c$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).

Unsound syllogisms



$$\frac{a \models b \qquad b \models \neg c}{a \models c}$$

All snakes are reptiles
No reptile has fur
$\therefore$ *All snakes have fur*

To disprove a syllogism, we need just one universe where it's invalid (e.g. Edinburgh zoo).

Is there a universe where this syllogism *is* valid?

(Aristotle said 'no'; we moderns differ. Hint: St Patrick.)

Reprise: Sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

$$\frac{a \models b \quad b \models c}{a \models c}$$

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

$$\frac{a \models b \quad c \models \neg b}{a \models \neg c}$$

$$\frac{a \models b \quad b \models \neg c}{c \models \neg a} \quad \text{equivalently} \quad \frac{c \models b \quad b \models \neg a}{a \models \neg c}$$

$$\frac{a \models b \quad c \models \neg b}{c \models \neg a} \quad \text{equivalently} \quad \frac{c \models b \quad a \models \neg b}{a \models \neg c}$$

Reprise: Sound universal syllogisms

From *barbara*, contraposition, and double negation, we have five sound syllogisms about universal statements:

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

Note that the conclusion is negative iff exactly one of the premises is negative – compare the unsound syllogism on the previous slide.

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad b \vDash \neg a}{a \vDash \neg c}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a} \quad \text{equivalently} \quad \frac{c \vDash b \quad a \vDash \neg b}{a \vDash \neg c}$$