

CL - Tutoria - 4

Xinyuan Tong S2297280.

Exercise 1.

$$\begin{array}{c} \frac{\frac{\neg C \models A}{\neg A \models C}}{\frac{\frac{\neg C \models B}{\neg B \models C}}{\neg A \vee \neg B \models C}} \vee \text{contraposition} \end{array}$$

$$\begin{array}{c} \frac{\frac{\neg C \models A}{\neg C \models A \wedge B}}{\neg (A \wedge B) \models C} \wedge \text{contraposition} \end{array}$$

$$\Rightarrow \frac{\frac{\neg A \vee \neg B \models C}{\neg (A \wedge B) \models C}}$$

$$\frac{\neg A \vee \neg B \models \neg A \vee \neg B}{\neg A \vee \neg B \models \neg (A \wedge B)} I$$

$$\frac{\neg (A \wedge B) \models \neg (A \wedge B)}{\neg (A \wedge B) \models \neg A \vee \neg B} I$$

$$\Rightarrow \neg (A \wedge B) = \neg A \vee \neg B$$

Exercise 2.

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\overline{x \models x.y}^I \quad \frac{\overline{x.z \models x.y}^I}{\overline{y.z \models x.y}}^I \quad \frac{\overline{y.z \models x.y}}^I}{\overline{x, (x \vee z) \models x.y}}^I}{\overline{y, (x \vee z) \models x.y}}^I}{\overline{(x \vee y), (x \vee z) \models x.y}}^I \quad \vee L \quad \frac{\frac{\frac{\frac{\overline{x \models x.z}^I \quad \frac{\overline{x.y \models x.z}}^I}{\overline{x \vee y \models x.z}}^I \quad \frac{\overline{x \vee y \models x.z}}^I}{\overline{(x \vee y), x \models x.z}}^I}{\overline{(x \vee y), z \models x.z}}^I}{\overline{(x \vee y), (x \vee z) \models x.z}}^I \quad \vee L \\
 \hline \hline
 \frac{\overline{(x \vee y), (x \vee z) \models x.y} \quad \overline{(x \vee y), (x \vee z) \models x.z}}{\overline{(x \vee y), (x \vee z) \models x.y \wedge x.z}}^I \quad \wedge R
 \end{array}$$

$$\begin{array}{c}
 \frac{\overline{(x \vee y), (x \vee z) \models x, y \wedge z}}{\overline{(x \vee y), (x \vee z) \models x \vee (y \wedge z)}}^I \quad \vee R \\
 \hline \hline
 \frac{\overline{(x \vee y), (x \vee z) \models x \vee (y \wedge z)}}{\overline{(x \vee y) \wedge (x \vee z) \models x \vee (y \wedge z)}}^I \quad \wedge L
 \end{array}$$

Exercise 3

$$\begin{array}{l}
 \boxed{x, y, z \models x \wedge z}^I \quad \boxed{x, y, z \models y \wedge z}^I \\
 \hline
 x, y, z \models x \wedge y, z \quad \wedge R \\
 \hline
 \models x \wedge y, \neg x, \neg z, \neg y, z \quad \neg R \\
 \hline
 \models x \wedge y, \neg(x \vee z), \neg y, z \quad \neg \\
 \hline
 \models x \wedge y, \neg(x \vee z), \neg y \vee z \quad \vee R \\
 \hline
 \models x \wedge y, \neg(x \vee z) \vee (\neg y \vee z) \quad \vee R \\
 \hline
 \models (x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z)) \quad \vee R
 \end{array}$$

It's neither a tautology nor a contradiction.