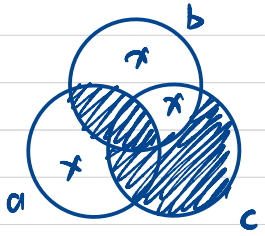
CI - turtorial - 5

Q1 $\dfrac{b \vDash \neg a \quad c \vDash b}{a \nvDash c}$



Existential Assumption ($a \vDash \neg a$) changes the diagram to there is a x in every single area of the diagram.

so $a \nvDash c$ is true and this syllogism is sound.

Q2 $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$

1. Substitute $\neg c$ for c to get: $\dfrac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$

2. Apply contraposition and the double negation law:

$\dfrac{a \vDash b \quad b \vDash \neg c}{\neg\neg c \vDash \neg a}$, which is $\dfrac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}$

3. Denying the conclution with the first premise:

$$\frac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$$

Q3

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{p} \vDash \boxed{p}, q}\,I}{\vDash \boxed{\neg p}, p, q}\,\neg R \qquad \dfrac{\dfrac{}{\boxed{q} \vDash p, \boxed{q}}\,I}{\vDash \boxed{\neg q}, p, q}\,\neg R}{\vDash \neg p \boxed{\wedge} \neg q,\ p,\ q}\,\wedge R}{\vDash \neg p \wedge \neg q,\ p \boxed{\vee} q}\,\vee R}{\vDash (\neg p \wedge \neg q) \boxed{\vee} (p \vee q)}\,\vee R$$

So it's a tautology

$$\frac{}{\Gamma, a \vDash a, \Delta}\,I$$

$$\frac{\Gamma \vDash a, \Delta}{\Gamma, \neg a \vDash \Delta}\,\neg L \qquad \frac{\Gamma, a \vDash \Delta}{\Gamma \vDash \neg a, \Delta}\,\neg R$$

$$\frac{\Gamma, a, b \vDash \Delta}{\Gamma, a \wedge b \vDash \Delta}\,\wedge L \qquad \frac{\Gamma \vDash a, \Delta \qquad \Gamma \vDash b, \Delta}{\Gamma \vDash a \wedge b, \Delta}\,\wedge R$$

$$\frac{\Gamma, a \vDash \Delta \qquad \Gamma, b \vDash \Delta}{\Gamma, a \vee b \vDash \Delta}\,\vee L \qquad \frac{\Gamma \vDash a, b, \Delta}{\Gamma \vDash a \vee b, \Delta}\,\vee R$$

The sequent calculus

Q4

$$\dfrac{\dfrac{\Gamma \vDash p, \Delta}{\Gamma, \neg p \vDash \Delta}\,\neg L \qquad \Gamma, q \vDash \Delta}{\Gamma, q \vee \neg p \vDash \Delta}\,\vee L$$

$$\Gamma, p \to q \vDash \Delta$$

$$\dfrac{\dfrac{\Gamma, p \vDash q, \Delta}{\Gamma \vDash q, \neg p, \Delta}\,\neg R}{\Gamma \vDash q \vee \neg p, \Delta}\,\vee R$$

$$\Gamma \vDash p \to q, \Delta$$

$$\Rightarrow \quad \frac{\Gamma, q \vDash \Delta \qquad \Gamma \vDash p, \Delta}{\Gamma, p \to q \vDash \Delta}\,(\to L) \qquad \frac{\Gamma, p \vDash q, \Delta}{\Gamma \vDash p \to q, \Delta}\,(\to R)$$