HW1 - Fall 15
Bailey CMPS 130

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0.1 Informal description of the set

a. odd natural numbers

b. even integers

c. even natural numbers greater than 1

d. natural numbers which are multiples of 6

e. all binary palindromes

f. the empty set

0.2 Formal Description of Sets

a. { 1, 10, 100}

b. $\{n \epsilon Z | n < 5\}$

c. $\{n \epsilon N | n < 5\}$

d. $\{"abc"\}$

e. $\{\epsilon\}$

f. $\emptyset$

0.3 A = {x, y, z} and B = {x, y }

a. A is not a subset of B

b. B is a subset of A

c. $A \cup B = \{x, y, z\}$

d. $A \cap B = \{x, y\}$

e. $AxB = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

f. $P(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

0.4 If A has a elements and B has b elements how many elements in A x B?

There will be a x b number of elements due to the fact that each element in A will be matched with an every element in B.

0.5 If C is a set with c elements, how many elements are in the P(C)?

There are 2^c number of elements in the power set of C. This is seen by comparing the number of subsets, the power set, with the binary strings of c. It is possible to uniquely describe the each set in the P(C) with the binary counterpart with a 1 as a valid bit, and is in the subset, and 0 as not valid, meaning it is not in the subset.

0.6 X = $\{1, 2, 3, 4, 5\}$ and Y = $\{6, 7, 8, 9, 10\}$

a. f(2) = 7

b. D(f) = X and R(f) = Y

c. g(2,10) = 6

d. D(g) = X x Y and R(g) = Y

e. g(4,f(4)) = g(4,7) = 8

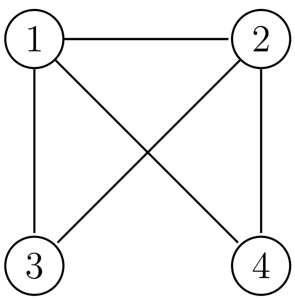
0.7 Relations

a. Reflexive and symmetric but not transitive
R is the relation on the natural numbers from xRy, if and only if $|x - y| \leq 2$

b. Reflexive and transitive but not symmetric
R is the relation on the natural numbers from xRy if and only if x divides y.

c. Symmetric and transitive but not reflexive
R is the relation on real numbers from xRy if and only if $(x - y)^2 < 0$

0.8 Undirected Graph



deg(1) = 3

deg(3) = 2

Path from 3 to 4 is edge $\{2, 3\}$ and then edge $\{2, 4\}$

0.9 Formal description of graph

A bipartite 3-node regular graph

0.10 2 = 1

The error is in the step when you divide each side by (a-b) since a = b that means that a-b = 0. Thus in this step you are effectively dividing by 0 which is an invalid operation.

0.11 All horses are the same color

When looking at the sample that of h when there are only 2 horses the argument that you remove 1 and look at the rest to determine the color doesn't work as when removing 1 when there are only 2, you are only examining one horse so of course that horse is the same color as itself.

0.12 Every graph with 2 or more nodes contains 2 nodes that have equal degrees

We have a graph G with $n \geq 2$ nodes. Each of the n nodes is indexed with $0-(n-1)$. If no 2 nodes have the same index, the labels of $0-(n-1)$ must be the index of each different node. There must be a node withe index 0 meaning it must not be incident to any other node, but another node has index of (n-1) so it is incident to every other node. This is a contradiction, thus at least 2 nodes must have the same degree.

1 DeMorgan's Law Proof

$\neg(A \cap B) = (\neg A \cup \neg B)$

Let x be an arbitrary element of T when T = $\neg(A \cap B)$

1. x $\neg\epsilon(A \cap B)$

2. x $\neg\epsilon A$ or x $\neg\epsilon B$

3. x $\epsilon\neg A$ or x $\epsilon\neg B$

4. x $\epsilon(\neg A \cup \neg B)$

5. x ϵ T

Let y be an arbitrary element of S when S = $\neg A \cup \neg B$

1. y $\epsilon\neg A$ or y $\epsilon\neg B$

2. y $\neg\epsilon A$ or y $\neg\epsilon B$

3. y $\neg\epsilon(A \cap B)$

4. y $\epsilon\neg(A \cap B)$

5. y ϵ S

2 Odd numbers are countable

The integers Z are countable. Every odd number is able to be matched up to a unique member of the integers. In the case of the function odd+1, this will mapp the odds to an even number that exists, thus showing odds are countable.

This relation is reflexive, symmetric and transitive.

3 Proof by induction

1. Base $n=1$

$$1 = \frac{1 \times 2 \times 3}{6}$$

$$1 = 1 \checkmark$$

2. Assume P(n) ~~it~~ is true and we will show P(n+1)

$$\sum_{i=1}^{n+1} i^2 = \left[\frac{n(n+1)(2n+1)}{6}\right] + (n+1)^2$$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)(n(2n+1)+6(n+1))}{6}$$

$$= \frac{n+1(2n^2+n+6n+6)}{6} = \frac{(n+1)(2n^2+7n+6)}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6} = \frac{(n+1)(n+2)(2(n+1)+1)}{6}$$

P(n+1)