HW7 - S15
Warmuth

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1 Consider the longest common sub-sequence problem between two strings x_1,...,x_n and y_1,...,y_m. Define a graph over the n x m grid (plus possibly some vertices around the edges) s.t. the longest common sub-sequence corresponds to the longest path in this graph.

The nodes in the graph would be the index of the of the grid, so node (i,j) is the vertex at row i and column j.

(a) Clearly describe the condition for the presence of an edge between two vertices on the grid.

In the grid starting from the top left at the first letters of the two strings, we will compare them. If there is a match then it will increase the count and have a diagonal edge. If there is no match then it will be either a horizontal or a vertical edge indicating that we are checking the next letter index.

(b) How should the edges be labeled?

In the graph if it is possible to reach the node then there is an edge connecting the node with the previously reached node. If it is no possible to reach it then there will be no edge between the two.

(c) How do you find the longest path?

Using a Breadth-First Traversal it is possible to see if there exists a path from node (m,n) to node (0,0). If there exists a path then there is a match between the two common sub-sequences, otherwise there is not.

(d) Is this algorithm more efficient than the dynamic programming algorithm?

This algorithm is not more efficient because we still will have to check build the grid of possible values that will be used for mapping out the graph.

2 KT 7.3

(a) What is the value of the flow? Is this a maximum flow in the graph?

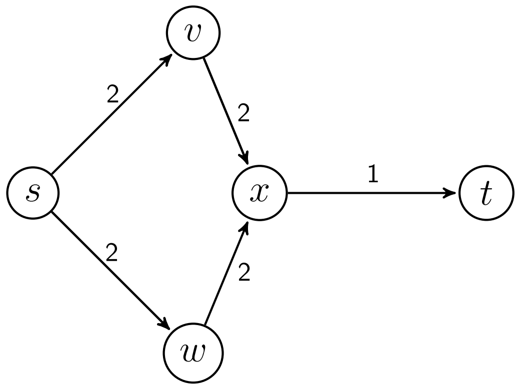
The flow shown is $5 + 5 = 10$. This is not a maximum flow in the graph, it is possible to get a flow of $5 + 5 + 1 = 11$, by rerouting flow from s$-$ >a and a$-$ >c, and then using s$-$ >d, d$- > t$ for 1 more unit of flow.

(b) Find a minimum s-t cut in the flow network and say what its capacity is.

The minimum cut is s,a,b,c and d,t with capacity of 11.

3 KT 7.4

False, as in the example the max flow is 1 and does not saturate the edges coming from the source.



4 KT 7.5

False, as seen in Figure 1, the minimum cut is 3 choosing the s and the rest of Figure 1. But in Figure 2 the minimum cut is now 6 with t and the rest of the network.

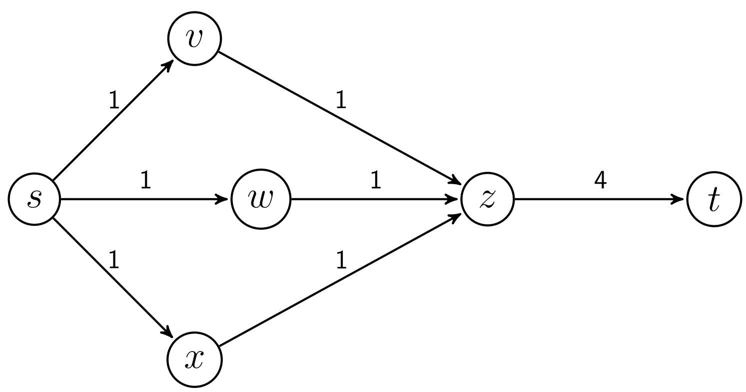


Fig. 1

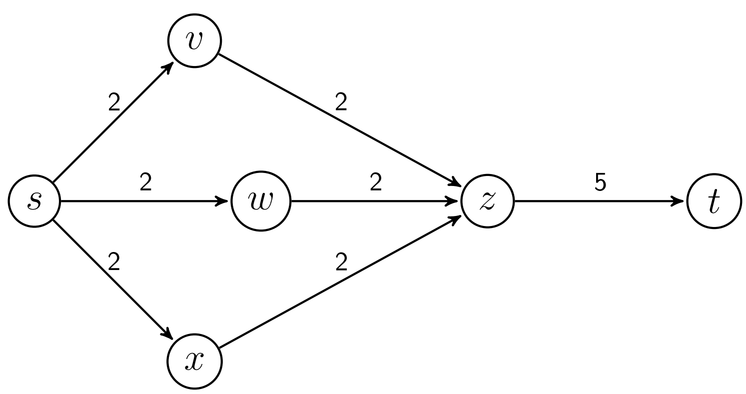


Fig. 2

5 KT 7.10 Reducing an edge capacity by 1.

If the edge e^* is not already saturated with flow then reducing it will not change the maximum flow, but if it is then we will have to check different cases.

After reducing the capacity we have to adjust the rest of the graph. All the edges that enter the node that e^* begins from will be reduced by 1 as well as all the edges that exit from the node that e^* ends at. The flow is already acyclic so any edge that is traversed will have a reduced flow by 1 unit. Using the new flow f' since f was already a maximum flow we can look to see if there is a single augmenting path from source to sink. In the case there is not one, then f' is the maximum flow. Otherwise the flow must have been augmented to be $\leq$ f because the new flow network can't be larger than the old one and is a new maximum flow. We will have to traverse each node and edge so the running time is O(n+m).

6 Problem 26.1-6, CRLS p 714 The professor and his sons

The source of the network flow would be the professor's house and the sink would be the location of the school. Each street corner is a vertex and the streets are edges each with capacity of 1, so that only one of them would be able to walk through each street. Since the children are fine with meeting at street corners there could be directed edges leaving a vertex. If there is a flow of 2 and all vertices have an integer value on them, then it means that there are two edge-disjoint paths from the source to sink.