CMPS 130

HW #8

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Exercises 2.11, 2.12, 2.14, 2.30, 2.31, 2.32, 2.35

2.11 G_4 = (V, Σ, R, <EXPR>) E = <EXPR>

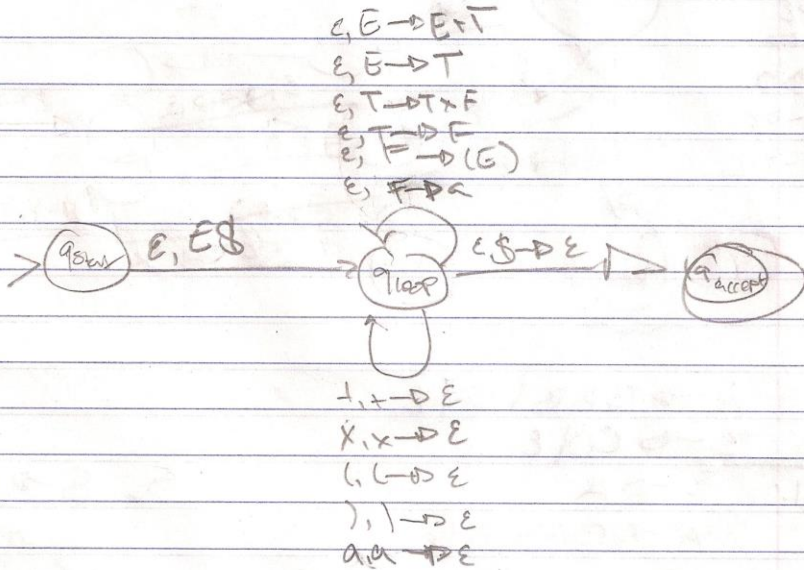
V is {<EXPR>, <TERM>, <FACTOR>} T = <TERM>

Σ is {a, +, x, (,)} F = <FACTOR>

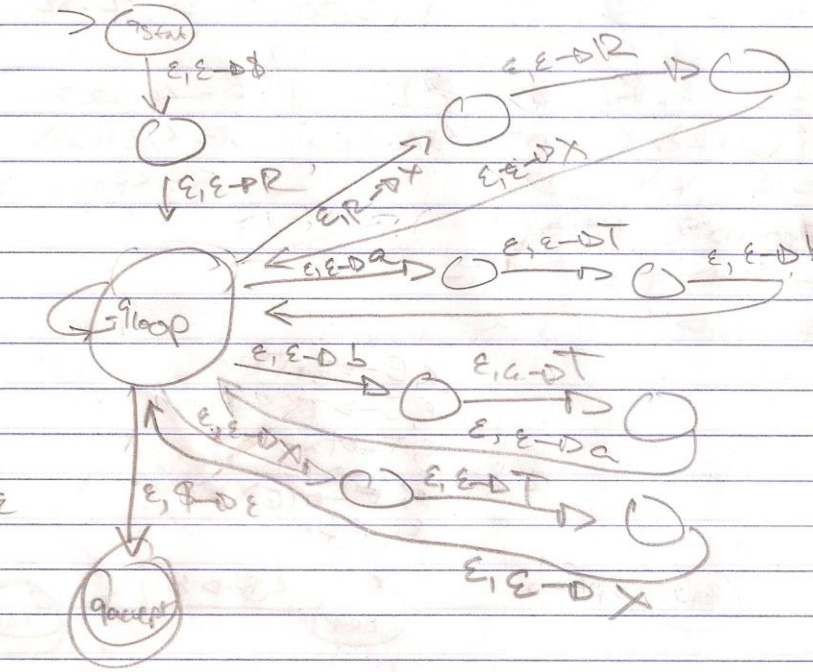
$E \to E + T \mid T$

$T \to T \times F \mid F$

$F \to (E) \mid a$



2.12



$R \to XRX \mid S$
$S \to aTb \mid b \mid bTa$
$T \to XTX \mid X \mid \varepsilon$
$X \to a \mid b$

$\varepsilon, R \to S$
$\varepsilon, T \to X$
$\varepsilon, T \to \varepsilon$
$\varepsilon, X \to a$
$\varepsilon, X \to b$
$a, a \to \varepsilon$
$b, b \to \varepsilon$

2.14 $A \to BAB \mid B \mid \varepsilon$
$B \to 00 \mid \varepsilon$

1. $S \to A$
   $A \to BAB \mid B \mid \varepsilon$
   $B \to 00 \mid \varepsilon$
2. $S \to A$
   $A \to BAB \mid B \mid AB \mid BA \mid A \mid \varepsilon$
   $B \to 00$
3. $S \to A \mid \varepsilon$
   $A \to BAB \mid B \mid AB \mid BA \mid A$
   $B \to 00$
4. $S \to BAB \mid AB \mid BA \mid BB \mid B \mid \varepsilon$
   $A \to BAB \mid AB \mid BA \mid BB \mid B$
   $B \to 00$
5. $S \to BAB \mid AB \mid BA \mid BB \mid 00 \mid \varepsilon$
   $A \to BAB \mid AB \mid BA \mid BB \mid 00$
   $B \to 00$

5. $S \to BC \mid AB \mid BB \mid YY \mid \varepsilon$
   $A \to BC \mid AB \mid BA \mid BB \mid YY$
   $C \to AB$
   $B \to YY$
   $Y \to 0$

2.30a | $A = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Let p be the pumping length

$S = 0^p 1^p 0^p 1^p$

S can be split into 5 parts uvxyz

1. for $i \geq 0$, $uv^ixy^iz \in L$
2. $|vy| > 0$
3. $|vxy| \leq p$

vxy can only be like the following

(1) 0^p or 1^p

for each $i=2$ uv^ixy^iz will have more 0's or 1's in the 1st half or 2nd half of s

(2) 0^p1^p in the 1st half of s

for each $i=2$ uv^ixy^iz will have more 0's or 1's in the 1st half of s

(3) 1^p0^p in the middle of s

for each $i=2$, uv^ixy^iz will have more 0's of 1's in the middle of s

(4) 0^p1^p at the 2nd half of s

for each $i=$ uv^ixy^iz will have more 0's or 1's in the 2nd half of s

2.30b | $B = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

Let p be the pumping length

$S = 0^p \# 0^{2p} \# 0^{3p}$

$S = uvxyz$ $|vxy| \leq p$, $|vy| > 0$

(1) V or y can not have # otherwise it will break the pattern of the regex, more than 2#.

(2) Dividing the $0^p, 0^{2p}, 0^{3p}$ into 3 segments it is not possible to keep them all in either V or y it would break the regex ratio.

Thus it is not possible to pump this language

2.30c) $C = \{ w\#t \mid w$ is a substring of t where $w, t \in \{a,b\}^* \}$

Let p be the pumping length

$S = "a^p b^p \# a^p b^p"$, $S = uvxyz$ $|vxy| \le p$, $|vy| > 0$

(1) V and y can not have # in them since there is only #

(2) If V and y are both on the left of the # then it uv^2xy^2z would be longer than the right side of #

(3) If V and y are both on the right of the # then uv^0xy^0z would be shorter than the left side which is a substring

(4) If either V or y is empty then it would be like either (2) or (3) on the same side.

(5) If V and y are both nonempty and the # is inbetween them then the left side of the # will have more b's than the right side making it not a substring

2.30d) $D = \{ t_1\#t_2\# \ldots \#t_k \mid k \ge 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for $i \ne j \}$

Let p be the pumping length

$S = 0^p 1^p \# 0^p 1^p \in L$, $S = uvxyz$, $|vxy| \le p$, $|vy| > 0$

(1) if vxy is only on the left side of the #, then uv^2xy^2z makes $t_1 \ne t_2$

(2) if vxy contains the # then uv^0xy^0z does not contain the #

(3) if x is the # and v is a substring of 1^p while y is a substring of 0^p pumping would make the two sides not equal

2.31 Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0's and 1's. Show B is not context-free

Let p be the pumping lemma

$S = 0^p 1^p 1^p 0^p$, $S = uvxyz$ $|vxy| \leq p$, $|vy| > 1$

1) $0^p 1^p$ or $1^p 0^p$ is $\cancel{v}xy$ and vy must contain both 0's and 1's. But then uv^0xy^0z would provide the reqs since one side of s is changed making it a palindrome no longer

2.32 Let $\Sigma = \{1,2,3,4\}$ and $C = \{w \in \Sigma^* |$ in w, the # of 1's = # of 2's and #3's = #4's$\}$

Let p be the pumping length

$S = 1^p 4^p 2^p 3^p$, $S = uvxyz$ $|vy| > 0$ $|vxy| \leq p$

(1) vxy contains only 1, only 2, only 3, or 4

for each $i > 1$ the # 1's ≠ #'s 2 or # 3's ≠ #4's breaking the reqs

(2) vxy is in $1^p 4^p$ or $4^p 2^p$ or $2^p 3^p$

for each $i > 1$, the #1's ≠ #2's or # 3's ≠ #4's since pumping would increase the numbers unevenly