CMPS 130
COMPUTATIONAL MODELS

Homework #5: THE PUMPING LEMMA
1.29, 1.30, 1.42, 1.46, 1.47, 1.55 (e, f, i, j)

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1.29a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Assume A_1 is regular.
Let p be the pumping length given by the pumping lemma
$S = 0^p 1^p 2^p$
Since $S \in A_1$ and $|s| > p$, the pumping lemma states that S can be split into xyz that satisfy all 3 conditions

Case 1: y is only made up of 0's, 1's, or 2's
so when pumping it creats strings that are in A_1
since it would create too many of either 0, 1, 2.

Case 2: y is a pattern of 01, 012, or 12 so it would violate the $0^n 1^n 2^n$ pattern when y is pumped.
Contradiction, A_1 is not regular

1.29b) $A_2 = \{www \mid w \in \{a, b\}^*\}$

Assume A_2 is regular
Let p be the pumping length given by the pumping lemma
$S = a^p b a^p b a^p b$
S can be divided into 3 pieces $S = xyz$ where $|xy| \leq p$
So xy contains only a's
Since $|y| > 0$, let $y = a^k$, $k > 0$
But $xy^2z = a^{p+k} b a^p b a^p b$ so $p+k > p \notin A_2$
Thus it can not be pumped, contradiction
A_2 is not regular

1.29c $A_3 = \{a^{2^n} \mid n \geq 0\}$ a^{2^n} means a string of 2^n a's

Assume A_3 is regular

Let p be the pumping length given by the pumping lemma

$S = a^{2^p}$

Since $S \in A_3$ and $|s| > p$, the pumping lemma states that S can be split into xyz to satisfy all 3 conditions

If $y = a^{p-1}$ so $0 < |y| < p$.

Then $|xyyz| = 2^p + (p-1)$ which is not a power of 2

$|xyyz| \notin A_3$ a contradiction A_3 is not regular

1.30 0^*1^* is not regular error

If p is the pumping length of 0^*1^*

$S = 0^p1^p$ and divided into xyz

pumping is possible since $xy^jz = 0^{p+(j-1)|y|}1^p$

which is still possible in 0^*1^*

1.42 For languages A and B let the shuffle of A and B be the language $\{w \mid w = a_1b_1 \ldots a_kb_k$ where $a_1 \ldots a_k \in A$ and $b_1 \ldots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$ Show that the class of regular languages is closed

Let $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ be DFAs representing A and B.

The NFA for the A B shuffle uses both M_A and M_B as input and will need to keep track of the current states of A and B. If after the whole string is processed, both A and B are in accept states, it will be accepted, Otherwise it is rejected, it will also accept empty string

The NFA N will be defined as:

(1) $Q = (Q_A \times Q_B) \cup \{q_0\}$ All possible states and if nothing is read

(2) $q = q_0$

(3) $F = (F_A \times F_B) \cup \{q_0\}$ Both A and B accepts and empty string

(4) For $a \in \Sigma$, δ is:

$$\delta((p_A, p_B), a) = \{(\delta_A(p_A, a), p_B), (p_A, \delta_B(p_B, A))\}$$

(i) Otherwise δ is $\emptyset$

1.46 a) $A = \{0^n 1^m 0^n \mid m, n \geq 0\}$

Assume A is regular

Let p be the pumping length by the pumping lemma

$S = 0^p 1 0^p = xyz$

$|s| \geq p$ and $s \in L$

xy contains only 0's.

let $y = 0^k$ where $k > 0$

So $xy^0z = 0^{p-k} 1 0^p$

which is not in A.

Contradiction.

b) $B = \{0^m 1^n \mid m \neq n\}$

$\overline{B} = \{0^m 1^n \mid m = n\}$ → as example 1.73 $\{0^n 1^n \mid n \geq 0\}$

↑ this is not regular from the pumping lemma $0^p 1^p$.

$s = xyz$, y can't be all 0's or 1's and can't have 0's and 1's.

So $\overline{B} \cap 0^* 1^* = \{0^n 1^n \mid n \geq 0\}$

If B were regular $\overline{B}$ would be too, but since it is known that $\overline{B}$ is not B is not.

c) $C = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$

$\overline{C} = \{w \mid w \in \{0,1\}^*$ is a palindrome$\}$

Assume $\overline{C}$ is regular

Let p be the pumping length by the pumping lemma.

$S = 0^p 1 0^p = xyz$

xy contains only 0's

let $y = 0^k$ where $k > 0$

So $xy^0z = 0^{p-k} 1 0^p$

Which is not a palindrome

Contradiction.

d) $D = \{wtw \mid w, t \in \{0,1\}^+\}$

Assume D is regular

Let p be the pumping length by the pumping lemma

$S = 0^p 1 1 0^p 1$

$w = 0^p 1$ and $t = 1$

So XY contains only 0's

$y = 0^k$ where $k > 0$

$xy^iz \in D$ when $i \geq 0$

when $i = 2$, $xy^2z = xyyz = 0^{(p+k)} 1 1 0^p 1$

Which can't be in D

Contradiction D is not regular

1.47) Let $\Sigma = \{1, \#\}$ and let

$Y = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k$ for $k \geq 0$, each $x_i \in 1^*$ and $x_i \neq x_j$ for $i \neq j\}$. Prove Y not regular.

Assume Y is regular

Let p be the pumping length by the Pumping Lemma

$S = 1^p \# 1^{p+1} \# \ldots \# 1^{2p}$

$S = xyz$ since $|xy| \leq p$

$x = 1^j$

$y = 1^k$

and $z = 1^l \# 1^{p+1} \# \ldots \# 1^{2p}$ where $j + k + l = p$ and $k \neq 0$

Pump y with $i = 2 \rightarrow S^n = 1^{p+k} \# 1^{p+1} \# \ldots \# 1^{2p}$

Since $j + k + l = p$ and $k \neq 0$ then $0 < k \leq p$

So $p < k \leq 2p$

$S^n \notin Y$ since 1^{p+1} can equal $1^{p+1}, 1^{p+2}, \ldots$ or 1^{2p}

Contradiction Y is not regular

1.55 e) $(01)^*$

minimum pumping length is 1, since according to the Pumping Lemma condition (2) $|y| > 0$. With length 1 or more, it will be at least 01 and then pumped with $x = \varepsilon$ $y = 01$ and z is the rest.

f) $1^*01^*01^*$

With a pumping length of 1 or 2, we could get the strings 0, 00 but there is no cycle in those cases. So the minimum pumping length must be 3, since the string could be 001, 010, 100, and the ones would be able to be pumped.

i) 1011

The minimum pumping length is 5. 1011 cannot be pumped, so there are no strings in this language with length ≥ 5.

j) Σ^* set of all possible strings over Σ

The minimum pumping length is 1, since $|y| > 0$ according to the Pumping Lemma condition (2). With length 1 or more it can cover $x = \varepsilon$ $y =$ the alphabet z is the rest.