CMPS 130

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Hw#6

1.51, 1.29 and 1.46 (myhill-Nerode), Minimize DFA, Prove of chaze with myhill-Nerode

1.51 | Showing being indistinguishable by some language L is an equivalence relation is done by showing that the relation between strings is reflexive, symmetric and transitive. So for x and y to be indistinguishable by L as denoted by $x \equiv_L y$ for every string z, $xz \in L$ if and only if $yz \in L$

(1) Reflexive: No string can distinguish x from itself so $x \equiv_L x$ for all x

(2) Symmetric: x is distinguishable from y if y is distinguishable from x

(3) Transitive: if $w \equiv_L x$ and $x \equiv_L y$ then for each z, $wz \in$ L if and only if $xz \in L$ if and only if $yz \in L$ therefore $wz \in L$ if and only if $yz \in L$ so $w \equiv_L y$.

1.29 | a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Consider the set $X = \{0^n \mid n \geq 1\}$, it is clearly infinite

Any two members of X have the form $0^i, 0^j$ where $i \neq j$

The string $z = 1^i 2^i$ distinguishes them since $0^i 1^i 2^i \in A_1$ and $0^j 1^i 2^i \notin A_1$

b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Consider the set $X = \{a^n \mid n \geq 1\}$, it is clearly infinite

Any two arbitrary members of X a^i and a^j where $i \neq j$

The string $z = ba^iba^ib$ will be appended to a^i and a^j

$a^iba^iba^ib \in A_2$ while $a^jba^iba^ib \notin A_2$, they are distinguishable and thus nonregular

1.29c $A_3 = \{a^{2^n} \mid n \geq 0\}$ a^{2^n} string of 2^n a's
Consider the set $X = \{a^n \mid n \geq 1\}$ which is infinite
Any two arbitrary values of X a^i and a^j where $i \neq j$
The string $z = a^{2^i}$ $a^i a^{2^i} \in L$ $a^j a^{2^i} \notin L$

1.46a $\{0^n 1^m 0^n \mid m, n \geq 0\}$
Consider the set $X = \{0^n \mid n \geq 1\}$
Any two arbitrary strings of X 0^i and 0^j where $i \neq j$
The string $z = 1^i 0^i$ so $0^i 1^i 0^i \in L$ but $0^j 1^i 0^i \notin L$

b $\{0^m 1^n \mid m \neq n\} = L_1$
Consider the set $X = \{0^n \mid n \geq 1\}$ which is infinite
Any two arbitrary strings of $X = 0^i$ and $y = 0^j$ where $i \neq j$
The string $z = 1^j$.
xy $0^i 1^j \in L_1$ since $i \neq j$ $(m \neq n)$ but yz $0^j 1^j \notin L_1$.
Therefore x and y are not in the same equivalence class

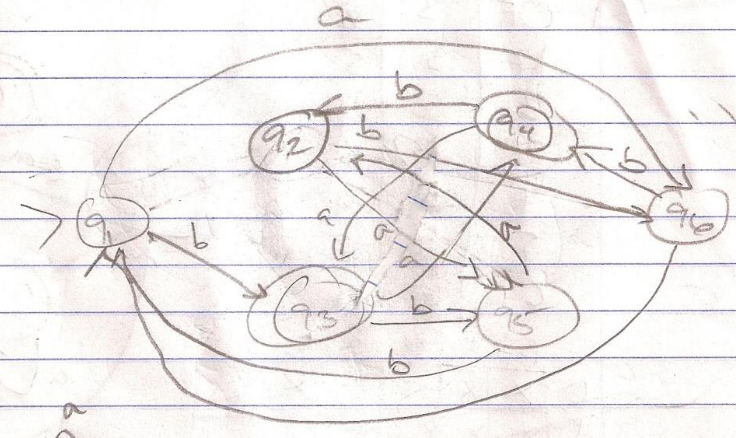
c $\{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$
Consider the set $X = \{a^i \mid i \in \mathbb{N}\}$, which is infinite
Any 2 arbitrary numbers of X are indistinguishable

d $\{wtw \mid w, t \in \{0,1\}^+\}$

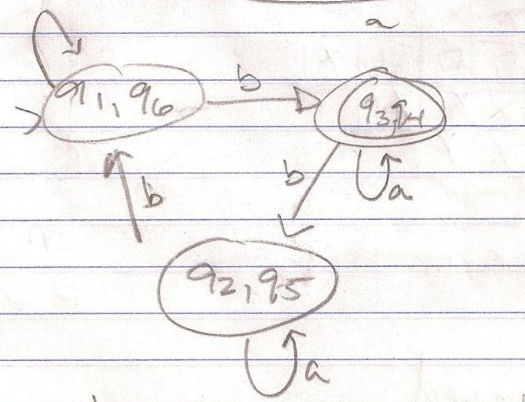
Minimize DFA's

A)

	a	b
1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

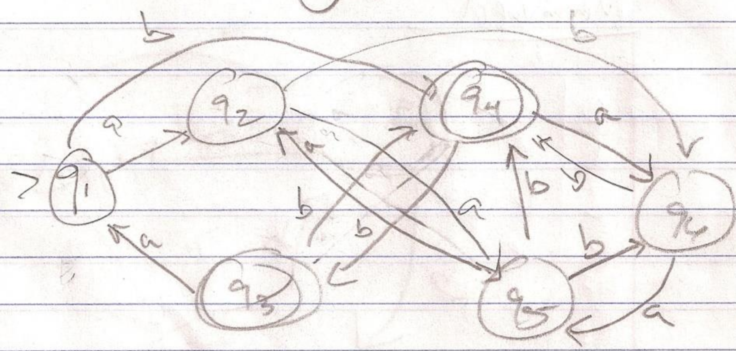


	6	5	4	3	2
1		X	X	X	X
2	X		X	X	
3	X	X			
4	X	X			
5	X				

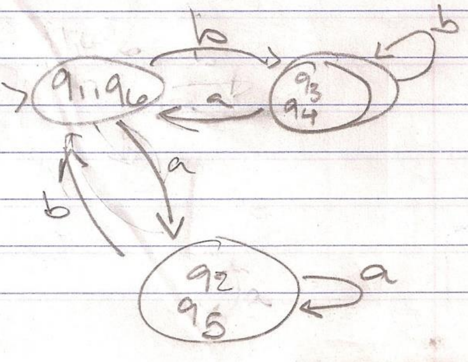


B)

	a	b
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

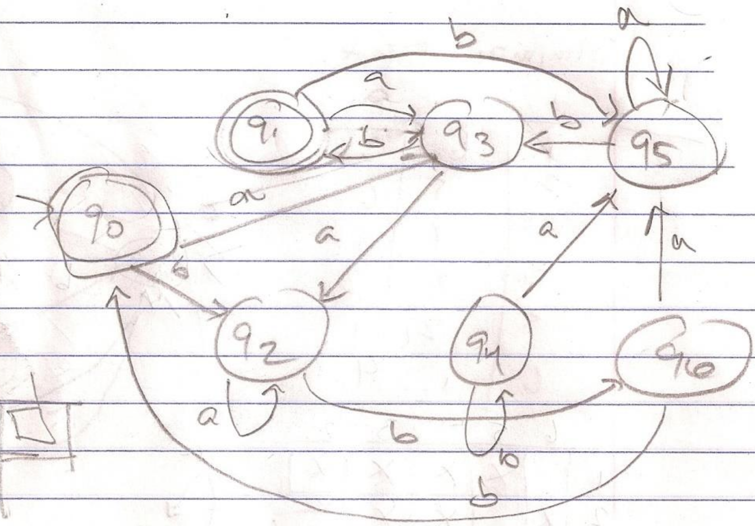


	6	5	4	3	2
1		X	X	X	X
2	X		X	X	
3	X	X			
4	X	X			
5	X				



C

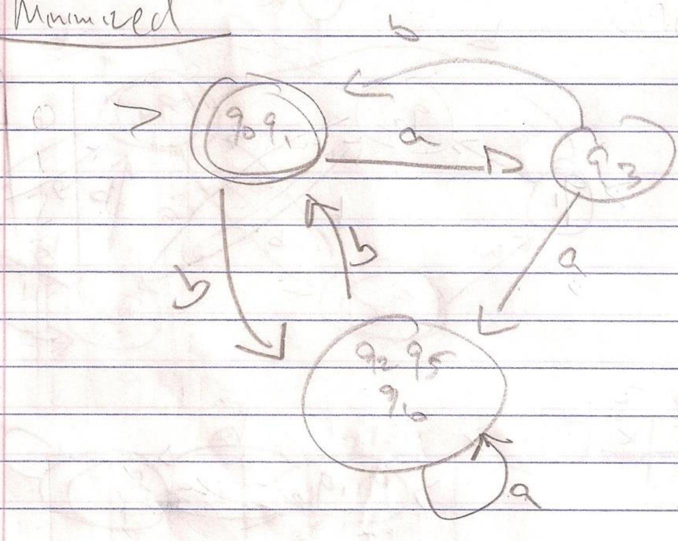
	a	b
0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	[illegible]	3
6	5	[illegible]



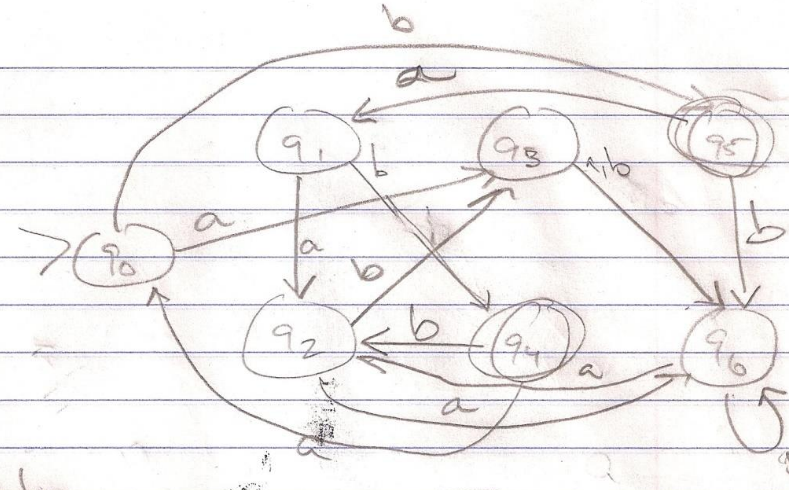
	6	5	4	3	2	1
0	X	X	X	X	X	☐
1	X	X	X	X	X	
2	☐	☐	☐	X		
3	X	X	X			
4	X	X				
5						

q_4 is impossible to reach

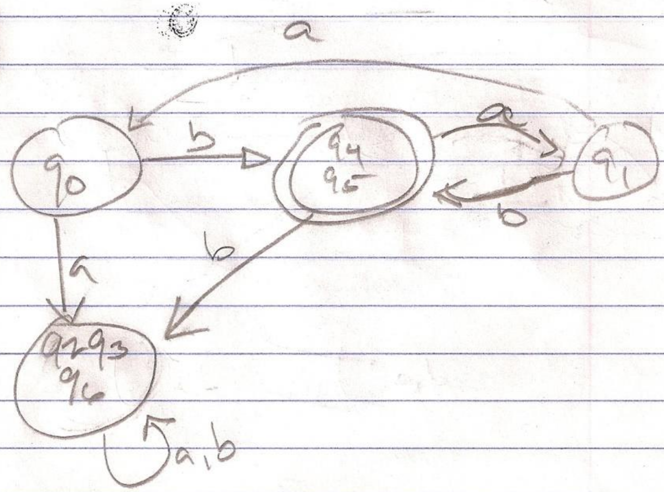
Minimized



D	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6



	6	5	4	3	2	1
0	X	X	X	X	X	X
1	X	X	X	X	X	
2		X	X			
3		X	X			
4	X					
5	X					



Myhill-Nerode Proof

$L = \{0^*10^*\}$ is regular

There are 3 equivalent classes from this language

(1) 0^*

(2) 0^*10^*

(3) $0^*10^*1(0+1)^*$

Since the number of classes that L can create is finite the language L is regular.

No, the pumping lemma can not prove regularity, it can only prove that a language is not regular. This is from the fact that all the conditions of the pumping lemma may be satisfied from a non-regular language.