sec 6.1 (10, 14, 24)

(10) $\|\bar{v}\| = \sqrt{36 + 16 + 9} = \sqrt{61}$

$\dfrac{\bar{v}}{\|\bar{v}\|} = \begin{bmatrix} -6/\sqrt{61} \\ 4/\sqrt{61} \\ -3/\sqrt{61} \end{bmatrix}$ unit vector

(14) distance between $\bar{u}$ & $\bar{z}$ $= \|\bar{u} - \bar{z}\| = \left\| \begin{bmatrix} 4 \\ -4 \\ -6 \end{bmatrix} \right\| = \sqrt{}$

$= 2\sqrt{17}$

(24) In $\mathbb{R}^n$, show $\|\bar{u}+\bar{v}\|^2 + \|\bar{u}-\bar{v}\|^2 = 2\|\bar{u}\|^2 + 2\|\bar{v}\|$

$$\begin{aligned}
\|\bar{u}+\bar{v}\| + \|\bar{u}-\bar{v}\| &= (\bar{u}+\bar{v})\cdot(\bar{u}+\bar{v}) + (\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v} \\
&= \bar{u}\cdot\bar{u} + 2\bar{u}\cdot\bar{v} + \bar{v}\cdot\bar{v} + \\
&\quad + \bar{u}\cdot\bar{u} - 2\bar{u}\cdot\bar{v} + \bar{v}\cdot\bar{v} = \\
&= 2\bar{u}\cdot\bar{u} + 2\bar{v}\cdot\bar{v} \\
&= 2\|\bar{u}\| + 2\|\bar{v}\|
\end{aligned}$$

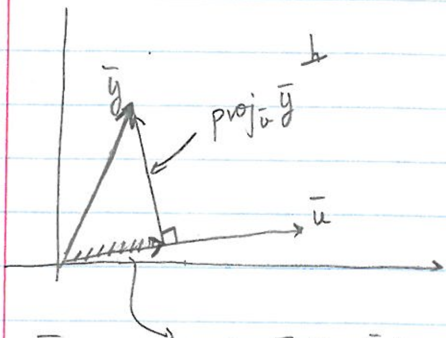
sec 6.2 (2, 14, 27)

(2) $\bar{u}_1 \cdot \bar{u}_2 = 0 - 2 + 2 = 0$ ✓

$\bar{u}_1 \cdot \bar{u}_3 = -5 + 4 + 1 = 0$ ✓

$\bar{u}_2 \cdot \bar{u}_3 = 0 - 2 + 2 = 0$ ✓

(14)



$\bar{a} = \text{proj}_{\bar{u}} \bar{y} = \frac{\bar{y} \cdot \bar{u}}{\bar{u} \cdot \bar{u}} \bar{u} = \frac{20}{50} \begin{bmatrix} 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 14/5 \\ 2/5 \end{bmatrix}$

$\bar{b} = \text{proj}_{\bar{u}} \bar{y}^{\perp} = \bar{y} - \text{proj}_{\bar{u}} \bar{y} = \begin{bmatrix} -4/5 \\ 28/5 \end{bmatrix}$

note $\bar{y} = \bar{a} + \bar{b}$ and $\bar{a} \cdot \bar{b} = 0$

(27) square U has orthonormal columns (nxn m

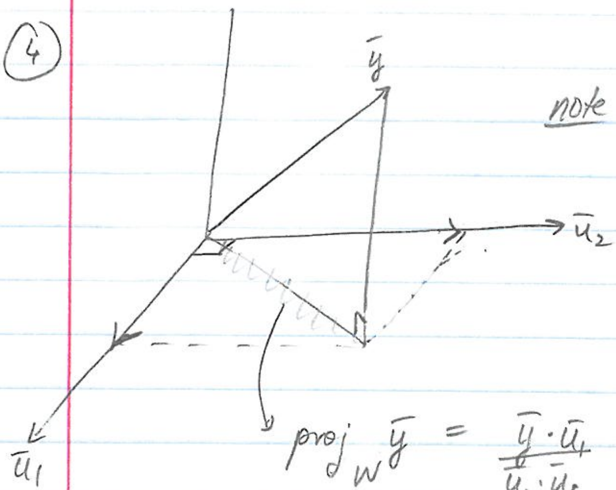
$\Rightarrow$ by theorem 4, columns of U are lin. indep

$\Rightarrow$ columns form a basis for R^n

$\Rightarrow$ by theorem on p. 235, U is invertible.

sec 6.3 (4, 10)

(4)



<u>note</u> $\bar{u}_1 \cdot \bar{u}_2 = 0.$

Let $W = \text{span}\{\bar{u}_1, \bar{u}_2\}$

$$\text{proj}_W \bar{y} = \frac{\bar{y}\cdot\bar{u}_1}{\bar{u}_1\cdot\bar{u}_1}\,\bar{u}_1 + \frac{\bar{y}\cdot\bar{u}_2}{\bar{u}_2\cdot\bar{u}_2}\,\bar{u}_2$$

$$= \frac{30}{25}\begin{bmatrix}3\\4\\0\end{bmatrix} + \frac{-15}{25}\begin{bmatrix}-4\\3\\0\end{bmatrix}$$

$$= \begin{bmatrix}6\\3\\0\end{bmatrix}.$$

(10) Same idea as #4.

$\bar{u}_1 \cdot \bar{u}_2 = 0$

$\bar{u}_1 \cdot \bar{u}_3 = 0$

$\bar{u}_2 \cdot \bar{u}_3 = 0$

Let $W = \text{span}\{\bar{u}_1, \bar{u}_2, \bar{u}_3\}.$

$$\text{proj}_W \bar{y} = \frac{\bar{y}\cdot\bar{u}_1}{\bar{u}_1\cdot\bar{u}_1}\,\bar{u}_1 + \frac{\bar{y}\cdot\bar{u}_2}{\bar{u}_2\cdot\bar{u}_2}\,\bar{u}_2 + \frac{\bar{y}\cdot\bar{u}_3}{\bar{u}_3\cdot\bar{u}_3}\,\bar{u}_3$$

$$= \frac{1}{3}\begin{bmatrix}1\\1\\0\\-1\end{bmatrix} + \frac{14}{3}\begin{bmatrix}1\\0\\1\\1\end{bmatrix} + \frac{-5}{3}\begin{bmatrix}0\\-1\\1\\-1\end{bmatrix}$$

$$= \begin{bmatrix}5\\2\\3\\6\end{bmatrix}.$$

sec 6.4 (6) ; sec 6.5 (4,8)

(6) Let $\bar{v}_1 = \bar{x}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}$ Let $W_1 = \text{span}\{\bar{v}_1\}$

$$\bar{v}_2 = \bar{x}_2 - \text{proj}_{W_1} \bar{x}_2$$

$$= \bar{x}_2 - \frac{\bar{x}_2 \cdot \bar{v}_1}{\bar{v}_1 \cdot \bar{v}_1} v_1$$

$$= \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix} - \frac{-45}{15} \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -3 \\ 0 \end{bmatrix}$$

note $\bar{v}_1 \cdot \bar{v}_2 = 12 - 6 - 6 = 0$

(4) a. normal equation $A^T A \bar{x}^* = A^T \bar{b}$

$$A^T A = \begin{bmatrix} 3 & 3 \\ 3 & 11 \end{bmatrix}$$

$$A^T \bar{b} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

b. Then $\bar{x}^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(8) Least-squares error $= \| \bar{b} - A\bar{x}^* \|$

$$= \left\| \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\|$$