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Exercise 8

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Due: 21.12.2020

Question 1: Trapezoidal method

Given $I = \int_a^b f(x)dx$ and $|f''(x)| \leq M \quad \forall x \in [a, b]$
Show that the bound on the (absolute) error of the trapezoidal method can be written as

$$|I - T(H)| \leq \frac{M}{8}(b-a)h^2$$

Question 2: Trapezoidal method

Use the trapezoidal method to calculate the approximation of the integral

$$\int_1^2 \frac{1}{x}dx = \ln 2$$

to a tolerance of 10^{-3} exactly.

Question 3: Intergral approximation

a) Compute an approximation of the integral

$$\int_0^{\pi/2} \frac{\sin x}{1+x^2}dx = 0.526978557614\cdots,$$

by performing three steps of the trapezoidal method.

b) Extrapolate according to Romberg-scheme the last two values to an improved value. Compare the result with the exact value.

Question 4: Gauss-formula

Given the Integral

$$I = \int_0^{\pi} \frac{1}{1+\cos^2 x}dx$$

Determine an approximation of I using the 5-point Gauss formula. This formula has the following nodes and weights (rounded to 8 digits):

$\pm x_i$	0	0.53846931	0.90617985
w_i	0.56888889	0.47862867	0.23692689

b) divide the interval $[0, \pi]$ into three equal partial intervals, and apply the 5-point Gauss formula to each partial interval.

Question 5: Programming Task

a) Write a MATLAB program for integral evaluation using Romberg-scheme to some tolerance **tol**

b) Test your program with previous tasks

c) verify your results with the MATLAB command **quad**