Prof. Dr. I. F. Sbalzarini
TU Dresden, 01187 Dresden, Germany

Exercise 12

Release: 25.01.2021
Due: 01.02.2021

Question 1: Stability

Consider the Linear Differential Equation system

$$\dot{x} = \begin{pmatrix} -1001 & 999 \\ 999 & -1001 \end{pmatrix} x$$

a) We want to apply the explicit Euler and the implicit Trapezoidal procedure to this system. How big must the step size be for the numerical solution for stable qualitative behaviors, when starting with the following initial conditions?
   i) $x(0) = (-1, 1)^T$
   ii) $x(0) = (1, 1)^T$
   iii) $x(0) = (2, 0)^T$

Question 2: Stiff Differential Equations

Consider the IVP,

$$y'(t) = -15y(t)$$

with the initial condition $y(0) = 1$ and $t \geq 0$.

a) Find the exact analytical solution to the problem.

b) Use explicit Euler scheme to solve the problem with step size $h = \frac{1}{4}$ and $h = \frac{1}{8}$

c) Use the implicit Trapezoidal scheme to solve the problem and plot a graph comparing explicit Euler ($h = \frac{1}{4}, \frac{1}{8}$), implicit Trapezoidal ($h = \frac{1}{8}$), and the analytical solution.

Question 3: Classification of PDEs

Classify the following PDEs with respect to their order and type (parabolic, hyperbolic and ellitpic)

$$\frac{\partial u}{\partial t} = D\Delta u$$
$$\Delta u = 0$$
$$u_{tt} - c^2 u_{xx} = 0$$
$$u_{xx} + x u_{yy} = 0$$