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Exercise 5

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Question 1: Least-Squares formulation, Normal equation

a) Given three points $P_i = (x_i, y_i),\ i = 1, 2, 3$, wherein

x_i	0	1	2
y_i	5.41	5.17	5.93

Determine a linear function $y = f(x) = ax + b$, so that the sum of the error squares in the y-direction

$$\sum_{i=1}^{3} |f(x_i) - y_i|^2 \qquad \text{is minimized}$$

b) Consider the matrix $A\,(3 \times 2)$ and vector $b\,(3 \times 1)$, given by

$$A = \begin{pmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

i) Write the least-square form which leads to the solution of linear system $Ax = b$.

ii) Check if the method of Normal Equations is stable for matrix A and $0 < \epsilon \ll 1$.

Question 2: Least-squares, QR decomposition and SVD

Following are the velocity measurements $f(t)$ in ms^{-1} from the pitot-tube of a descending airplane at time $t_i,\ i = 1, 2, ..., 10$.

t_i	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f_i	100	34	17	12	9	6	5	4	4	2

We express the unknown function $f(t) = \sum_{j=1}^{4} \lambda_j \phi_j(t)$ as the linear combination of known functions $\phi_j(t),\ j = 1, 2, 3, 4$, given by,

$$\phi_1(t) = \frac{1}{t},\ \ \phi_2(t) = \frac{1}{t^2},\ \ \phi_3(t) = e^{-(t-1)},\ \ \phi_4(t) = e^{-2(t-1)}$$

Determine the coefficients λ_j, the linear combination such that

$$\sum_{10}^{i=1} |f(t_i) - f_i|^2 \qquad \text{is minimized}$$

a) using normal equations

b) using the QR decomposition of the matrix A

c) by means of singular value decomposition (SVD)
(MATLAB command `[U, S, V] = svd(A)`)

Question 3: SVD decomposition by hand

Given the matrices A and B,

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

a) Find the rank and singular values of the matrix A and matrix B.

b) Find the SVD decomposition of the matrix A and B. Also comment on the uniqueness of the decomposition.