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Exercise 1

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Question 1: Floating point representation

Consider a computer with floating point representation $(B = 2, l = 4, k = 2)$, where B : base, and l : number of digits of mantissa and k : length of exponent

a) What is the largest and smallest (normalized) positive number that can be represented?

b) How many numbers can be represented approximately ?

c) Repeat analysis a) and b) for a calculator with floating point representation given by $(B = 10,\ l = 10,\ k = 2)$

Question 2: Machine precision

a) Write a code to determine the machine precision of MATLAB and compare the result with the output of the MATLAB command **eps**.

b) What is the relation between the worst relative rounding error of a floating point representation (B, l, k), and the machine precision ? What floating point precision does MATLAB use, according to the **eps** command ?

Question 3: Condition number

a) Find the condition numbers of the following functions and also comment on the well and ill-conditioned regions

$$f_1(x) = \sin(x)$$
$$f_2(x, y) = x - y$$
$$f_3(x, y) = x \cdot y$$

b) Calculate the condition number of the Matrix C and verify your solution with MATLAB command **cond(C)** / Python's **numpy.linalg.cond(C)**

$$\mathrm{C} = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

c) Given a function $f(x) = \sqrt{1 + x} - 1$

  i) Determine the condition number near $x = 0$ and see if the step-wise evaluation (1) $y = 1 + x$, (2) $z = \sqrt{y}$ and (3) $f(x) = z - 1$ is well conditioned and stable.

  ii) Can you restructure the problem to make it well conditioned in all evaluation steps ?

Question 4: Roundoff and Extinction

Calculate the sum

$$s := \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + ... - \frac{4}{19999} = \sum_{k=1}^{k=10000} \frac{4 \cdot (-1)^{k+1}}{2k-1}$$

in two ways in MATLAB/Python

i) from right to left

ii) from left to right

Explain the difference from the 11th decimal place and compare the exact result π for 20 decimal points i.e.

$$\pi \approx 3.14159265358979323846$$