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# Exercise 3

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## Question 1: Iterative schemes and Convergence

Given the matrices

$$A_1 = \begin{pmatrix} 3 & 0 & 4 \\ 7 & 4 & 2 \\ -1 & 1 & 2 \end{pmatrix}, A_2 = \begin{pmatrix} -3 & 3 & -6 \\ -4 & 7 & -8 \\ 5 & 7 & -9 \end{pmatrix}, A_3 = \begin{pmatrix} 4 & 1 & 1 \\ 2 & -9 & 0 \\ 0 & -8 & -6 \end{pmatrix}, A_4 = \begin{pmatrix} 7 & 6 & 9 \\ 4 & 5 & -4 \\ -7 & -3 & 8 \end{pmatrix}$$

a) For which of these matrices does the Jacobi and Gauss-Seidel methods converge ?

b) In the case when both Jacobi and Gauss-Seidel methods converge, which converges faster ?

## Question 2: Spectral radius and convergence rate

Given the matrix

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

of the linear system of equations $Ax = b$

a) For the iteration matrix T of Successive Over Relaxation(SOR) method, plot the spectral radius as a function of the parameter $\omega, 0 < \omega < 2$. What is the optimum value of $\omega$ for fast convergence ?

b) How many iterations does the SOR method for $\omega = 0.4$ and $1.4$ need to achieve the absolute tolerance of $10^{-10}$.

## Question 3: Programming task

a) Implement Jacobi, Gauss-Seidel and SOR-method in MATLAB/python.
( NOTE : Have a conditional check for spectral radius )

b) Given the linear system

$$A = \begin{pmatrix} 5 & 2 & 1 \\ 4 & 11 & 5 \\ 7 & 8 & 16 \end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix} 8 \\ 20 \\ 31 \end{pmatrix}$$

start with the initial condition of $x^{[0]} = (0.5\ 0.5\ 0.5)^T$ and

i) Check if Jacobi and Gauss-Siedel methods converge ?

ii) Compute the number of steps required to reach a tolerance of $10^{-4}$ and also the CPU time.

iii) Check if SOR method improves the convergence rate.