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Exercise 6

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Question 1: Fixed-Point Theorem

For the function

$$f(x) = 2e^{-x/2}$$

a) Graphically show that the function has one fixed point.

b) Show that in the closed interval $I = [0.8, 1.4]$, the requirements for Banach's fixed-point theorem are fulfilled.

c) Find an upper-limit for the number of iterations

$$x_{j+1} = 2e^{-x_j/2}$$

such that, $|x_n - x^*| \leq 10^{-6}$, where $x_0 \in I$ is any starting point in I.

Question 2: Newton method

The Kepler's equation given by,

$$x - e \sin x = t$$

with parameters e and t should be solved for x using Newton method.

a) Formulate the Newton's iteration for the solution of Kepler's equation.

b) Solve the Newton's iteration numerically for $e = 0.4$ and $t = 0.4$ with starting value $x_0 = 0.7$

Question 3: System of non-linear equations

Solve the non-linear system of equations,

$$\begin{aligned} e^{xy} + x^2 + y - 1.4 &= 0 \\ x^2 + y^2 + x - 0.46 &= 0 \end{aligned}$$

with the help of the Newton procedure. Select $(x_0, y_0) = (0.5, 0.4)$. Formulate the procedure and carry out one Newton step by hand.

Question 4: Programming task

a) Implement a robust method for calculating the zeros of a scalar function. The algorithm should work like this :

- Starting from an interval $[a_0, b_0]$, apply bisection-method until the zero is known with an accuracy of $\mathbf{tol}_0$, i.e. $|b_j - a_j| < \mathbf{tol}_0$
- Use Newton-method with starting value $x = \frac{1}{2}(a_j + b_j)$ to determine the zero point with a relative accuracy of **tol**
- STOP, if the Newton procedure exits the interval $[a, b]$

b) Repeat a) with Secant method instead of Newton's procedure

c) Compute the zero(s) of the functions

$$f(x) = x - 2 + \ln x, \quad x > 0,$$
$$g(x) = (x - 1)(x + 1)(x + 2)$$

using the two methods you have implemented in a) and b). First select an appropriate interval $[a_0, b_0]$ and use $\mathbf{tol}_0 = 10, \mathbf{tol} = 10^{-10}$. Now apply the procedures only to $g(x)$ with the parameters $a_0 = -0.9, b_0 = 1.1, \mathbf{tol}_0 = 3, \mathbf{tol} = 10^{-10}$. Explain the result.