Butcher Table Solver

February 2, 2020

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

[2]:
```
def Butcher_solver(f,t,x,h,A,b,c):
    Num_stages=len(c)
    m=len(x)
    k=np.zeros([Num_stages,m])
    k[0]=f(t,x)
    for i in range(1,Num_stages):
        Sum=np.zeros([m])
        for j in range(i):
            Sum=Sum+A[i][j]*k[j,:]
        k[i,:]=f(t+c[i]*h,x+h*Sum)
    F=x+h*np.dot(b,k)
    return F
```

[3]:
```
#Butcher Tables
A_Euler=np.array([0])
b_Euler=np.array([1])
c_Euler=np.array([0])

A_Heun=np.array([[0,0],[1,0]])
b_Heun=np.array([0.5,0.5])
c_Heun=np.array([0,1])

A_RK3=np.array([[0,0],[2.0/3,0]])
b_RK3=np.array([1.0/4,3.0/4])
c_RK3=np.array([0,2/3.0])

A_RK4=np.array([[0,0,0],[0.5,0,0],[0,0.5,0],[0,0,1]])
b_RK4=np.array([1/6.,1/3.,1/3.,1/6.])
c_RK4=np.array([0,1/2.,1/2.,1])
```

[4]:
```
def two_body_problem(t,x):
    y1,y2,y3,y4=x
    dxy1=y3
    dxy2=y4
```

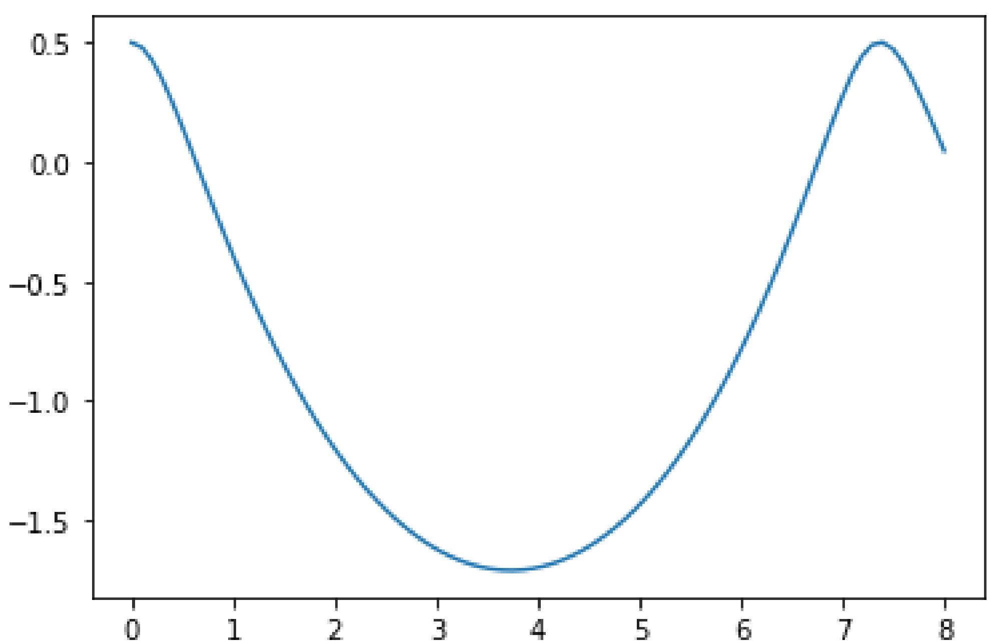
```
    dxy3=-y1/((y1**2+y2**2)**(3/2))
    dxy4=-y2/((y1**2+y2**2)**(3/2))
    return [dxy1,dxy2,dxy3,dxy4]
```

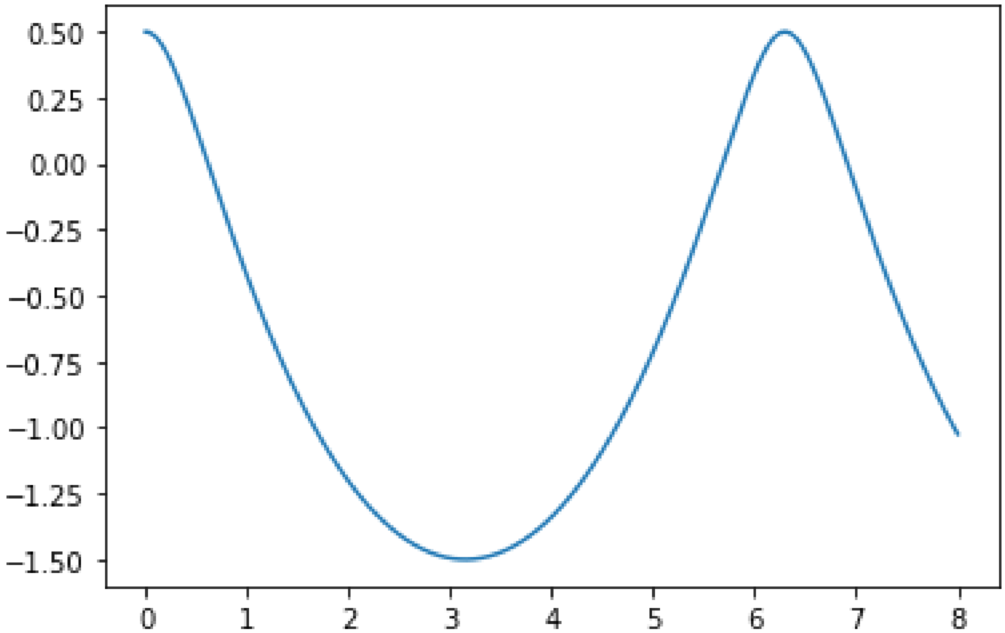
[5]:
```
def Butcher_Stepper(f,t0,tf,y0,h,A,b,c):
    m=len(y0)
    n=int((tf-t0)/h)
    sol=np.zeros([n,m])
    sol[0]=y0
    for i in range(1,n):
        sol[i]=Butcher_solver(f,t0,sol[i-1],h,A,b,c)
    T=np.linspace(t0,tf,n)
    return sol,T
```

[6]:
```
RK4_sol,T=Butcher_Stepper(two_body_problem,0,8,[0.5,0,0,np.sqrt(3)],0.
 ↪1,A_Heun,b_Heun,c_Heun)
```

[7]:
```
RK4_sol1,T1=Butcher_Stepper(two_body_problem,0,8,[0.5,0,0,np.sqrt(3)],0.
 ↪01,A_Heun,b_Heun,c_Heun)
```

[8]:
```
plt.plot(T,RK4_sol[:,0])
plt.show()
plt.plot(T1,RK4_sol1[:,0])
plt.show()
```

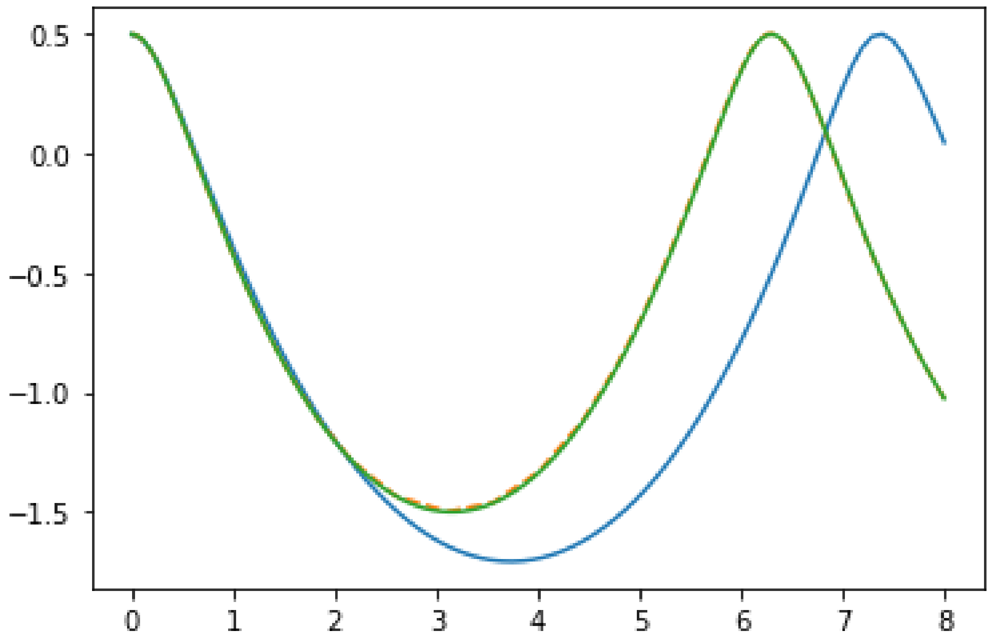




[9]:
```
from scipy.integrate import odeint,RK23,solve_ivp
```

[10]:
```
sol =solve_ivp(two_body_problem, [0, 8], [0.5,0,0,np.sqrt(3)], RK23)
```

[11]:
```
plt.plot(T,RK4_sol[:,0])
plt.plot(sol.t,sol.y[0],'--')
plt.plot(T1,RK4_sol1[:,0])
plt.show()
```



[12]:
```
#RK45 is DOPRI5
#RK45??
#RK23 is not classic RK4
#RK23??
```

[13]:
```
def Adaptive_Butcher(f,t,x,h,A,b,b_hat,c,p,Tol,Fs):
    Num_stages=len(c)
    m=len(x)
    k=np.zeros([Num_stages,m])
    k[0]=f(t,x)
    for i in range(1,Num_stages):
        Sum=np.zeros([m])
        for j in range(i):
            Sum=Sum+A[i][j]*k[j,:]
        k[i,:]=f(t+c[i]*h,x+h*Sum)
    F=x+h*np.dot(b,k)
    l=abs(h*np.dot(b_hat-b,k))
    if l>Tol:
        h=Fs*h*(Tol/l)**(1/(p+1))
        return h,list(x),0
    else:
        h=Fs*h*(Tol/l)**(1/(p+1))
        return h,list(F),1
```

[14]:
```
def Adaptive_Butcher_Stepper(f,t0,tf,y0,h,A,b,b_hat,c,p=2,Tol=1e-2,Fs=1):
    t=t0
    m=len(y0)
    sol=[y0]
    T=[t0]
    i=1
    while(t<tf):
        h,solt,s=Adaptive_Butcher(f,t,sol[i-1],h,A,b,b_hat,c,p,Tol,Fs)
        if s==1:
            sol=sol+[solt]
            t=t+h
            i=i+1
            T=T+[t]
    return sol,T
```

[15]:
```
A_DOPRI5=np.array([[0,0,0,0,0,0,0],[1/5,0,0,0,0,0,0],[3/40,9/40,0,0,0,0,0],[44/
 ↪45,-56/15,32/9,0,0,0,0],[19372/6561,-25360/2187,64448/6561,-212/
 ↪729,0,0,0],[9017/3168,-355/33,46732/5247,49/176,-5103/18656,0,0],[35/384.
 ↪,0,500/1113,125/192,-2187/6784,11/84,0]])
b_DOPRI5=np.array([35/384.,0,500/1113,125/192,-2187/6784,11/84,0])
b_hat_DOPRI5=np.array([5179/57600,0,7571/16695,393/640,-92097/339200,187/2100,1/
 ↪40])
c_DOPRI5=np.array([0,1/5.,3/10.,4/5,8/9,1,1])
```

[16]:
```
def Sigmoid(t,y):
    x=y[0]
    dx=0.01*(1-x)*x
    return [dx]
```

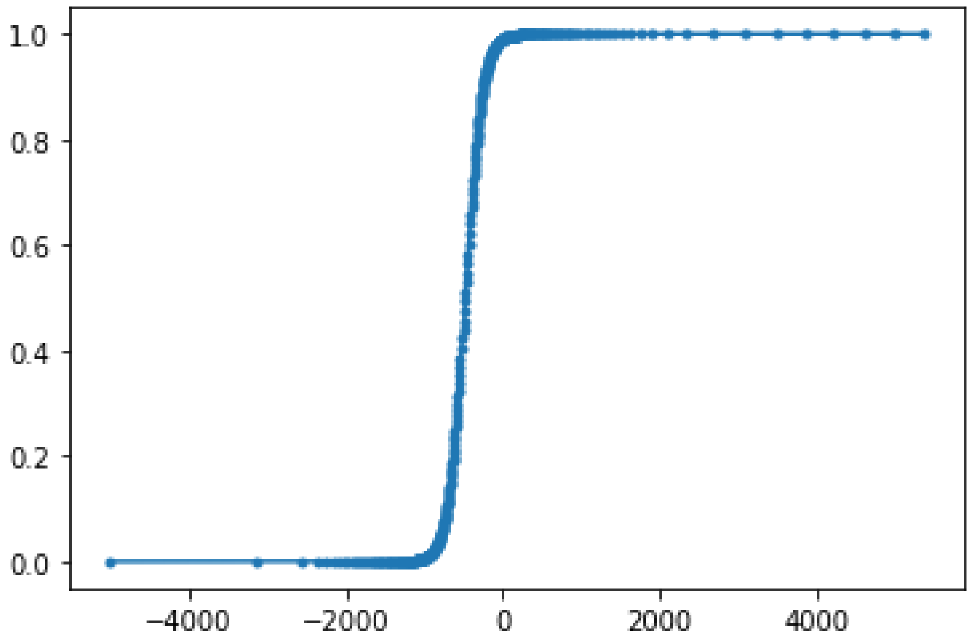
[17]:
```
sig_sol,T=Adaptive_Butcher_Stepper(Sigmoid,-5000,5000,[1e-08],1,A_DOPRI5,b_DOPRI5,b_hat_DOPRI5
 ↪8)
```

[18]:
```
print("Number of steps Taken:",len(T))
```

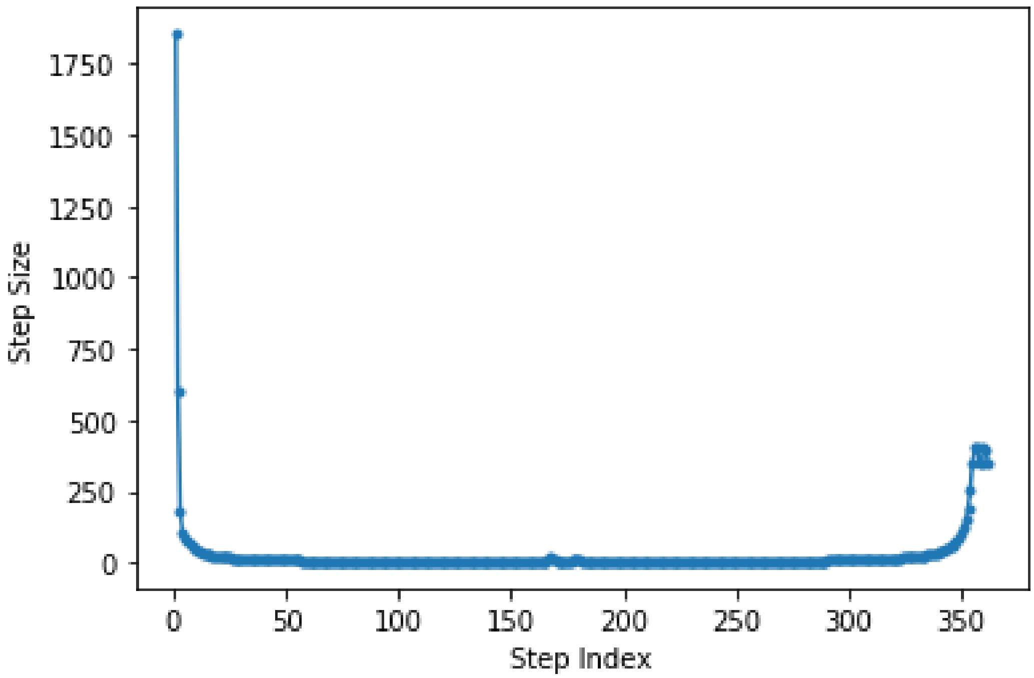
```
Number of steps Taken: 362
```

[ ]:
```
```

[19]:
```
plt.plot(T,sig_sol,'.-')
plt.show()
```



[20]:
```
#Plot of Stepindex vs Stepsize
StepSize=[]
for i in range(1,len(T)):
    StepSize=StepSize+[T[i]-T[i-1]]
plt.plot(np.linspace(1,362,361),StepSize,'.-')
plt.xlabel('Step Index')
plt.ylabel('Step Size')
plt.show()
```



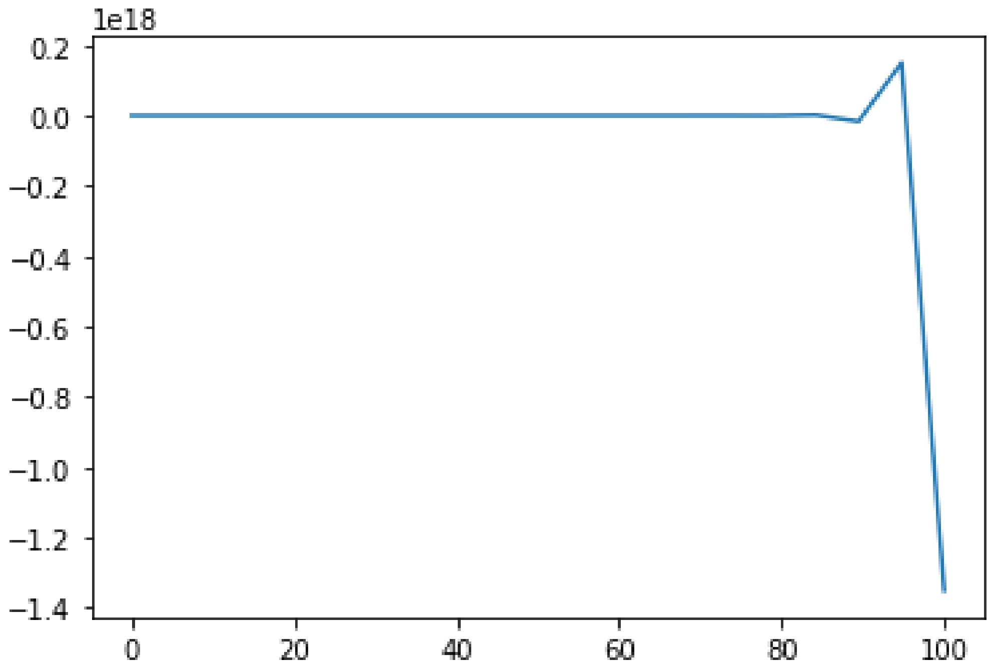
[21]:
```
def Explicit_euler(ode_system,y0,tf,h=0.01,t0=0):
    n=int((tf-t0)/h)
    m=len(y0)
    sol=np.zeros([n,m])
    sol[0]=y0
    for i in range(1,n):
        sol[i]=sol[i-1]+h*ode_system(sol[i-1])
    return sol
```

[24]:
```
def Ode(x):
    x1,x2=x
    dx1=-1001*x1+999*x2
    dx2=999*x1-1001*x2
    return np.array([dx1,dx2])
```

[25]:
```
%%time
t0=0
tf=100
h=5
sol=Explicit_euler(Ode,[1,1],tf,h)
```

```
CPU times: user 237 µs, sys: 81 µs, total: 318 µs
Wall time: 243 µs
```

[26]:
```
n=int((tf-t0)/h)
T=np.linspace(t0,tf,n)
plt.plot(T,sol[:,0])
plt.show()
```



[ ]:
```

```

[27]:
```
def Adams(ode_system,y0,f,tf,h=0.01,t0=0):
    x1=y0+h/12*(23*f[0]-16*f[1]+5*f[2])
```

[36]:
```
def PreyP(x):
    x1,x2=x
    dx1=x1-x1*x2
    dx2=-10*x2+x1*x2
    return np.array([dx1,dx2])
def PreyP_odeint(t,x):
    x1,x2=x
    dx1=x1-x1*x2
    dx2=-10*x2+x1*x2
    return np.array([dx1,dx2])
```

[37]:
```
def Adams_Solver(ode_system,f,tf,h=0.01,t0=0):
    n=int((tf-t0)/h)
    m=len(f[0])
```

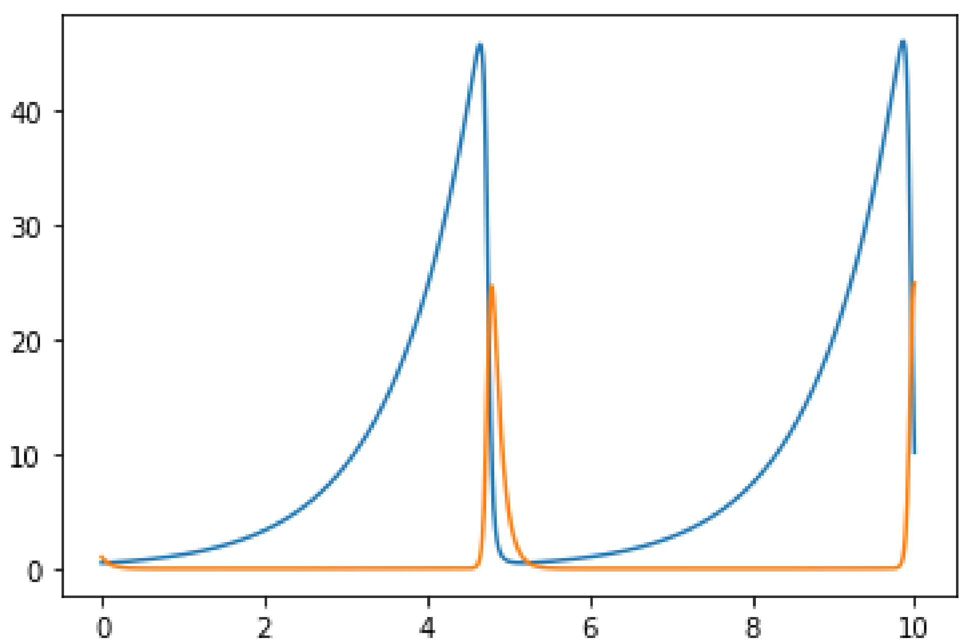
```
    sol=np.zeros([n,m])
    sol[0]=f[0]
    sol[1]=f[1]
    sol[2]=f[2]
    for i in range(3,n):
        sol[i]=sol[i-1]+h/
↪12*(23*ode_system(sol[i-1])-16*ode_system(sol[i-2])+5*ode_system(sol[i-3]))
    return sol
```

[38]:
```
f=np.array([[0.5,1],[0.50023020652423,0.90937363770619],[0.50089337004375,0.
↪82696413439848]])
```

[39]:
```
%%time
t0=0
tf=10
h=0.01
sol=Adams_Solver(PreyP,f,tf,h)
```

```
CPU times: user 18.5 ms, sys: 3.41 ms, total: 21.9 ms
Wall time: 18.8 ms
```

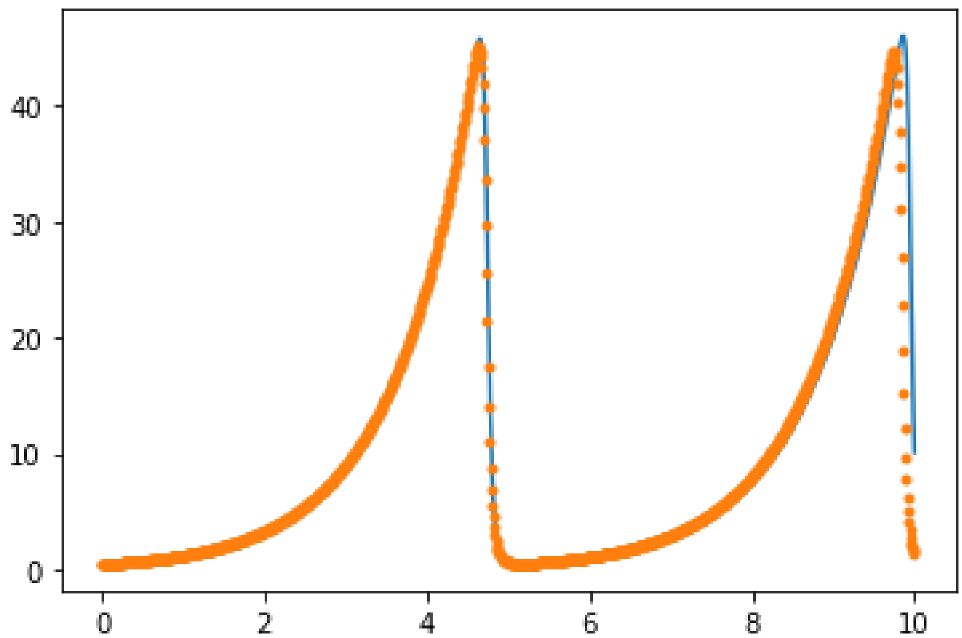
[40]:
```
n=int((tf-t0)/h)
T=np.linspace(t0,tf,n)
plt.plot(T,sol[:,0])
plt.plot(T,sol[:,1])
plt.show()
```



[41]:
```
from scipy.integrate import RK45
```

[43]:
```
sol1 =solve_ivp(PreyP_odeint, [0, 10], [0.5,1], RK45)
sol2 =odeint(PreyP_odeint,[0.5,1],T,tfirst=True)
```

[44]:
```
plt.plot(T,sol[:,0])
#plt.plot(T,sol[:,1],)
#plt.plot(sol1.t,sol1.y[0],'--')
#plt.plot(sol1.t,sol1.y[1],'--')
plt.plot(T,sol2[:,0],'.')
#plt.plot(T,sol2[:,1],'.')
plt.show()
```



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