Prof. Dr. I. F. Sbalzarini
TU Dresden, 01187 Dresden, Germany

Exercise 13

Release: 01.02.2021
Due: 08.02.2021

Question 1: Solution of PDEs

Consider the PDE for advection equation $u_t + cu_x = 0$. Assuming that, we are only allowed to Fourier transform along x, i.e.

$$\hat{u}(\xi, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,t) e^{-i\xi x} dx$$

a) Formulate the analytical solution for $u(x,t)$ given the initial data for $\hat{u}(\xi, 0)$.

b) Repeat the same analysis for diffusion equation $u_t = Du_{xx}$.

HINT : Take the fourier transform of the respective equations

Question 2: Semi-analytical solution

Given the PDE for heat conduction,

$$u_t = u_{xx} - cos(2\pi x), \quad x \in \mathbb{R}$$

with boundary conditions $u(t,0) = u(t,1) = 0$ and Initial conditions $u(0,x) = 0$.

Approximate the solution by applying the Method of Lines with the implicit Euler procedure for time integration. Formulating the method and then calculate an example with $h = 0.01, t_f = 10$.

Question 3: Programming Task

Consider the poisson equation

$$\begin{aligned} \Delta u(x,y) = 1 \quad & (x,y) \in \Omega := (-1,1) \times (-1,1) \\ u = 0 \quad & \text{at } \partial\Omega \end{aligned}$$

a) Discretize the problem with 5-point FD stencil.

b) Solve the linear system and plot the solution.