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Exercise 2

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Question 1: LU decomposition and roundoff

Solve the Linear system Ax = y using LU decomposition

$$A = \begin{pmatrix} 0.005 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

a) exactly

b) by rounding all the values of L and U to 2 decimal places after comma

c) by first swapping the rows of A and y and rounding them as in b)

Explain the difference between the solution obtained in b) and the exact value obtained from a). Compute the solution with the MATLAB command **lu**

Question 2: LU Decomposition

Determine the LU decomposition of the matrix

$$\mathrm{A} = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

and thus solve the linear equation system $Ax = b$ for $b = (1, 2, 3)^T$. Verify your results with MATLAB command **lu**

Question 3: Iterative schemes

a) Can you briefly explain the difference between Direct and Iterative methods for solution of the linear system $Ax = b$.

b) For an iterative scheme given by $x^{k+1} = \mathrm{T}x^k + \mathrm{c}$, where T is the iteration matrix, x^k is the solution at k iteration and c is a column vector

   i) What is the sufficient condition for convergence of the iterative scheme ?

   ii) What is the sufficient and necessary condition for convergence of the iterative scheme ?

   iii) See if matrix A in Problem 2) is diagonally dominant and also compute the spectral radius of the same.

Question 4: Gaussian elimination in computer

Write a MATLAB/python code for solving the linear system $Ax = b$ using Gaussian elimination with partial pivoting. Test your code for linear system in Problem 2.