Biostat 212a Homework 5

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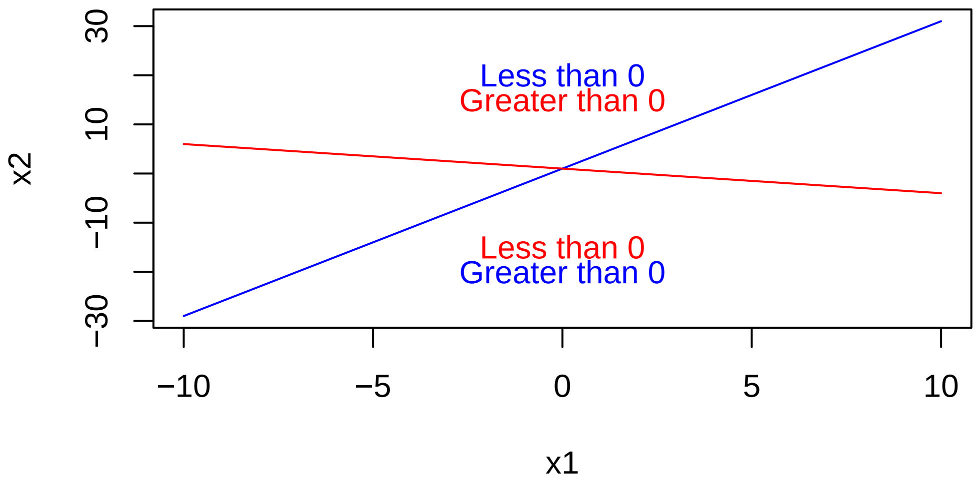
Table of contents

0.1 ISL Exercise 9.7.1 (10pts)

This problem involves hyperplanes in two dimensions. (a) Sketch the hyperplane $1 + 3X1 - X2 = 0$. Indicate the set of points for which $1+3X1 - X2 > 0$, as well as the set of points for which $1 + 3X1 - X2 < 0$. (b) On the same plot, sketch the hyperplane $-2 + X1 + 2X2 = 0$. Indicate the set of points for which $-2 + X1 + 2X2 > 0$, as well as the set of points for which $-2 + X1 + 2X2 < 0$.

```
x1 <- -10:10
x2 <- 1 + 3 * x1
plot(x1, x2, type = "l", col = "blue")
text(c(0), c(-20), "Greater than 0", col = "blue")
text(c(0), c(20), "Less than 0", col = "blue")
lines(x1, 1 - x1/2, col = "red")
text(c(0), c(-15), "Less than 0", col = "red")
text(c(0), c(15), "Greater than 0", col = "red")
```

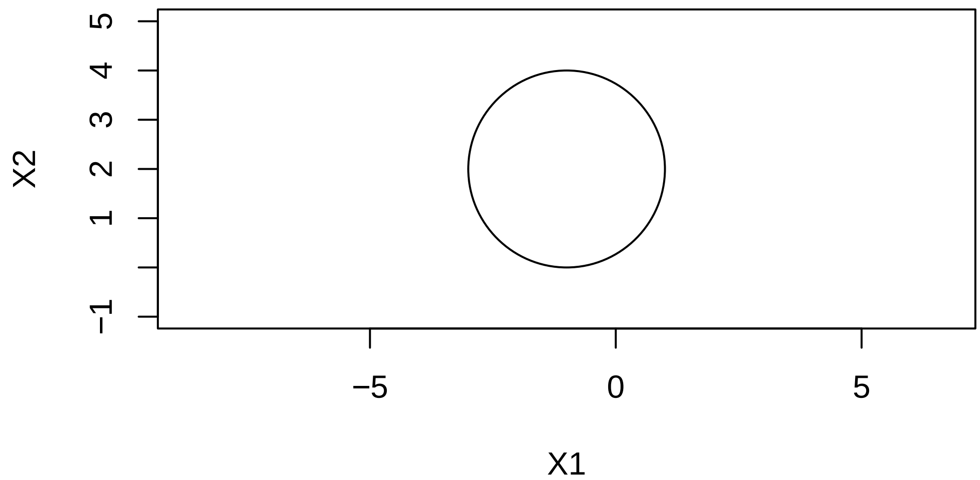


0.2 ISL Exercise 9.7.2 (10pts)

2. We have seen that in p = 2 dimensions, a linear decision boundary takes the form 0 + 1X1 + 2X2 = 0. We now investigate a non-linear decision boundary.

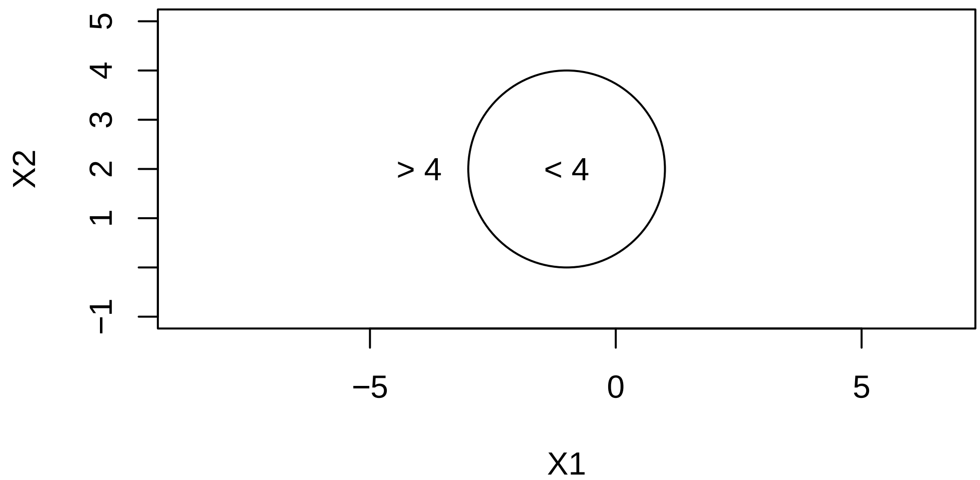
(a) Sketch the curve (1+X1)^2 +(2−X2)^2 =4.

```
plot(NA, NA, type = "n", xlim = c(-4, 2), ylim = c(-1, 5), asp = 1, xlab = "X1", ylab = "X2")
symbols(c(-1), c(2), circles = c(2), add = TRUE, inches = FALSE)
```



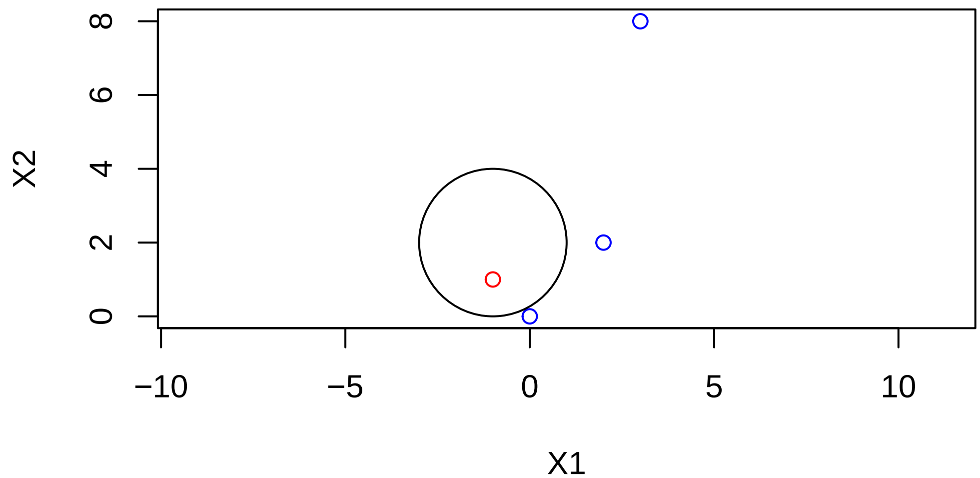
(b) On your sketch, indicate the set of points for which $(1+X1)\hat{}2 + (2-X2)\hat{}2 > 4$, as well as the set of points for which $(1+X1)\hat{}2 + (2-X2)\hat{}2 \ \ 4$.

```
plot(NA, NA, type = "n", xlim = c(-4, 2), ylim = c(-1, 5), asp = 1, xlab = "X1", ylab = "X2")
symbols(c(-1), c(2), circles = c(2), add = TRUE, inches = FALSE)
text(c(-1), c(2), "< 4")
text(c(-4), c(2), "> 4")
```



(c) Suppose that a classifier assigns an observation to the blue class if and to the red class otherwise. To what class is the observation (1+X1)^2 +(2−X2)^2 >4, (0, 0) classified? (−1, 1)? (2, 2)? (3, 8)?

```
plot(c(0, -1, 2, 3), c(0, 1, 2, 8), col = c("blue", "red", "blue", "blue"),
    type = "p", asp = 1, xlab = "X1", ylab = "X2")
symbols(c(-1), c(2), circles = c(2), add = TRUE, inches = FALSE)
```



(d) Argue that while the decision boundary in (c) is not linear in terms of X1 and X2, it is linear in terms of X1, X12, X2, and X2 .

Expend the equation of the decision boundary :

$$(1+X_1)^2 + (2-X_2)^2 = 4$$

$$1+2X_1+X_1^2+4-4X_2+X_2^2=4$$

$$X_1^2+X_2^2+2X_1-4X_2+1=0$$

$\Rightarrow$ which is linear in terms of X_1, X_1^2, X_2 and X_2^2.

0.3 Support vector machines (SVMs) on the `Carseats` data set (30pts)

Follow the machine learning workflow to train support vector classifier (same as SVM with linear kernel), SVM with polynomial kernel (tune the degree and regularization parameter

C), and SVM with radial kernel (tune the scale parameter γ and regularization parameter C) for classifying `Sales<=8` versus `Sales>8`. Use the same seed as in your HW4 for the initial test/train split and compare the final test AUC and accuracy to those methods you tried in HW4.

```
# Load necessary libraries
library(e1071)  # For SVM
library(caret)  # For evaluation metrics
library(ISLR2)
library(pROC)
library(ggplot2)
library(lattice)

# Set seed for reproducibility
set.seed(2)

# Convert Sales into binary classification
Carseats$High <- factor(ifelse(Carseats$Sales > 8, "Yes", "No"))

# Remove Sales column
Carseats <- subset(Carseats, select = -Sales)

# Train-test split (50-50)
train_idx <- sample(1:nrow(Carseats), nrow(Carseats) / 2)
Carseats.train <- Carseats[train_idx, ]
Carseats.test <- Carseats[-train_idx, ]
High.test <- Carseats.test$High
```

```
# Train SVC (linear kernel)
svc_model <- svm(High ~ ., data = Carseats.train, kernel = "linear", cost = 1, scale = TRUE)

# Prediction and evaluation
svc_pred <- predict(svc_model, Carseats.test)
confusion_matrix_svc <- confusionMatrix(svc_pred, High.test)

# Display accuracy
print(confusion_matrix_svc$overall["Accuracy"])
```

```
Accuracy
   0.915
```

```
# Extract decision values and ensure it's a numeric vector
svc_pred_prob <- attr(predict(svc_model, Carseats.test, decision.values = TRUE), "decision.va

# Compute ROC and AUC
svc_roc <- roc(High.test, svc_pred_prob)
print(svc_roc$auc)
```

```
Area under the curve: 0.9763
```

```
# Tune hyperparameters for polynomial kernel
set.seed(2)
tune_poly <- tune(svm, High ~ ., data = Carseats.train, kernel = "polynomial",
                  ranges = list(cost = c(0.1, 1, 10), degree = c(2, 3, 4)))

# Best model
best_poly_model <- tune_poly$best.model
poly_pred <- predict(best_poly_model, Carseats.test)
confusion_matrix_poly <- confusionMatrix(poly_pred, High.test)

# Accuracy and AUC
print(confusion_matrix_poly$overall["Accuracy"])
```

```
Accuracy
   0.855
```

```
poly_pred_prob <- attr(predict(best_poly_model, Carseats.test, decision.values = TRUE), "deci
poly_roc <- roc(High.test, poly_pred_prob)
print(poly_roc$auc)
```

```
Area under the curve: 0.9245
```

```
# Tune hyperparameters for radial kernel
set.seed(2)
tune_radial <- tune(svm, High ~ ., data = Carseats.train, kernel = "radial",
                    ranges = list(cost = c(0.1, 1, 10), gamma = c(0.01, 0.1, 1)))

# Best model
best_radial_model <- tune_radial$best.model
radial_pred <- predict(best_radial_model, Carseats.test)
confusion_matrix_radial <- confusionMatrix(radial_pred, High.test)
```

```r
# Accuracy and AUC
print(confusion_matrix_radial$overall["Accuracy"])
```

```
Accuracy
   0.895
```

```r
radial_pred_prob <- attr(predict(best_radial_model, Carseats.test, decision.values = TRUE), '
radial_roc <- roc(High.test, radial_pred_prob)
print(radial_roc$auc)
```

```
Area under the curve: 0.9697
```

```r
# Compile accuracy and AUC into a dataframe
results <- data.frame(
  Model = c("SVC (Linear)", "SVM (Polynomial)", "SVM (Radial)"),
  Accuracy = c(confusion_matrix_svc$overall["Accuracy"],
               confusion_matrix_poly$overall["Accuracy"],
               confusion_matrix_radial$overall["Accuracy"]),
  AUC = c(svc_roc$auc, poly_roc$auc, radial_roc$auc)
)

print(results)
```

```
             Model Accuracy       AUC
1     SVC (Linear)    0.915 0.9763155
2 SVM (Polynomial)    0.855 0.9245186
3     SVM (Radial)    0.895 0.9697251
```

0.4 Bonus (10pts)

Let

$$f(X) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p = \beta_0 + \beta^T X.$$

Then $f(X) = 0$ defines a hyperplane in $\mathbb{R}^p$. Show that $f(x)$ is proportional to the signed distance of a point x to the hyperplane $f(X) = 0$.