

# Bringing Serendipity Methods to Computational Practice in Firedrake

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An abstract about Firedrake and FEM here.

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## 1 INTRODUCTION

## 2 BACKGROUND ON SERENDIPITY AND TRIMMED SERENDIPITY ELEMENTS

### 2.1 2D Elements

- (1) Scalar (classical = Arnold-Awanou =  $S_r\Lambda^0(\mathbb{R}^2)$ )
- (2) Vector Serendipity (BDM = Arnold-Awanou =  $S_r\Lambda^1(\mathbb{R}^2)$ )
- (3) Vector Trimmed Serendipity (Arbogast-Correa = Gillette-Kloefkorn =  $S_r^-\Lambda^1(\mathbb{R}^2)$ )
- (4) Direct (Arbogast-Tao / Arbogast-Correa)

2.1.1 *Scalar (classical = Arnold-Awanou =  $S_r\Lambda^0(\mathbb{R}^2)$ ).*

2.1.2 *Vector Serendipity (BDM = Arnold-Awanou =  $S_r\Lambda^1(\mathbb{R}^2)$ ).*

2.1.3 *Vector Trimmed Serendipity (Arbogast-Correa = Gillette-Kloefkorn =  $S_r^-\Lambda^1(\mathbb{R}^2)$ ).*

2.1.4 *Direct (Arbogast-Tao / Arbogast-Correa).*

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## 2.2 3D Elements

- (1) Scalar (classical = Arnold-Awanou =  $\mathcal{S}_r \Lambda^0(\mathbb{R}^3)$ )
- (2) Vector serendipity (Arnold-Awanou =  $\mathcal{S}_r \Lambda^1(\mathbb{R}^3)$  and  $\mathcal{S}_r \Lambda^2(\mathbb{R}^3)$ )
- (3) Vector trimmed serendipity (Gillette-Kloefkorn =  $\mathcal{S}_r^- \Lambda^1(\mathbb{R}^3)$  and  $\mathcal{S}_r^- \Lambda^2(\mathbb{R}^3)$ )

## 3 BUILDING CAPACITY FOR SERENDIPITY ELEMENT TYPES IN FIREDRAKE

Description of which elements are now available in Firedrake and how to call them.

## 4 TIMING EXPERIMENTS

Compare tensor product to serendipity and direct serendipity as applicable

- (1) Wall clock time
- (2) Static condensation time
- (3) matvec time
- (4) KSP / solver time

### 4.1 Timing: 2D Elements

### 4.2 Timing: 3D Elements

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## 5 DEGREES OF FREEDOM AND MEMORY EXPERIMENTS

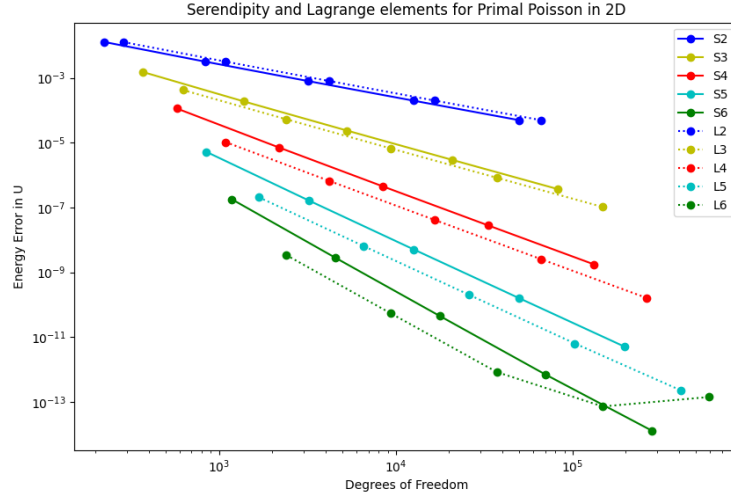
### 5.1 DOFs and Memory: 2D Elements

### 5.2 DOFs and Memory: 3D Elements

## 6 ACCURACY AND CONVERGENCE RATE EXPERIMENTS

### 6.1 Error: 2D Elements

Fig. 1. Degrees of Freedom vs Energy Error analysis of Serendipity and Lagrange  $L^2$  elements.



### 6.2 Error: 3D Elements

Fig. 2. Degrees of Freedom vs Error analysis of  $S^-(\text{Div})$  and RTCF elements for a mixed Poisson PDE, with exact solution  $\sin(\pi x)\sin(\pi y)$  !

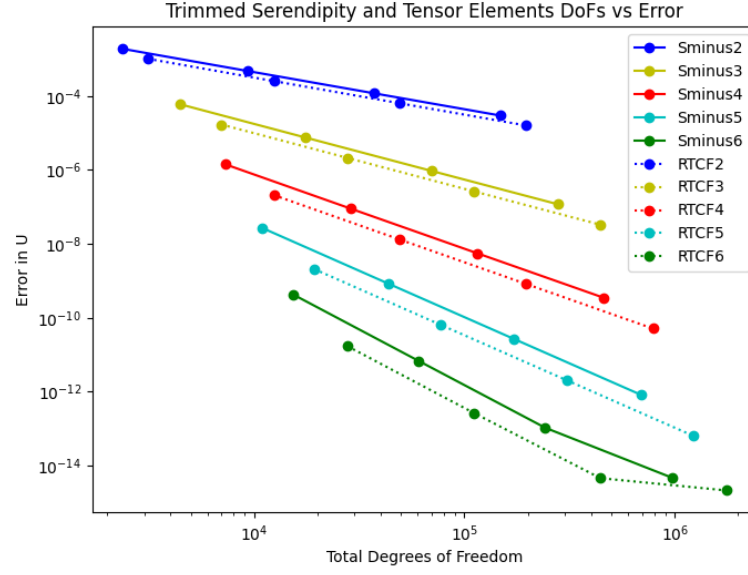


Fig. 3. Degrees of Freedom vs Error analysis of projection using  $S^-(\text{Curl})$  and RTCE.

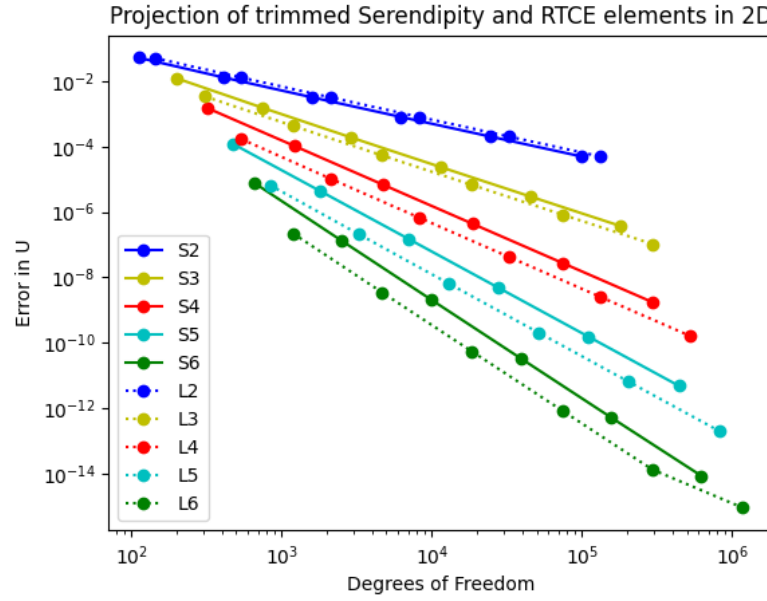


Fig. 4. Preliminary results for testing a mixed Poisson problem in 3D, using  $S^-$  (Div) and NCF elements.