Bringing Serendipity Methods to Computational Practice in Firedrake

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An abstract about Firedrake and FEM here.

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1 INTRODUCTION

2 BACKGROUND ON SERENDIPITY AND TRIMMED SERENDIPITY ELEMENTS

2.1 2D Elements

(1) Scalar (classical = Arnold-Awanou = $\mathcal{S}_r\Lambda^0(\mathbb{R}^2)$)
(2) Vector Serendipity (BDM = Arnold-Awanou = $\mathcal{S}_r\Lambda^1(\mathbb{R}^2)$)
(3) Vector Trimmed Serendipity (Arbogast-Correa = Gillette-Kloefkorn = $\mathcal{S}_r^-\Lambda^1(\mathbb{R}^2)$)
(4) Direct (Arbogast-Tao / Arbogast-Correa)

2.1.1 Scalar (classical = Arnold-Awanou = $\mathcal{S}_r\Lambda^0(\mathbb{R}^2)$).

2.1.2 Vector Serendipity (BDM = Arnold-Awanou = $\mathcal{S}_r\Lambda^1(\mathbb{R}^2)$).

2.1.3 Vector Trimmed Serendipity (Arbogast-Correa = Gillette-Kloefkorn = $\mathcal{S}_r^-\Lambda^1(\mathbb{R}^2)$).

2.1.4 Direct (Arbogast-Tao / Arbogast-Correa).

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2.2 3D Elements

(1) Scalar (classical = Arnold-Awanou = $\mathcal{S}_r\Lambda^0(\mathbb{R}^3)$)
(2) Vector serendipity (Arnold-Awanou = $\mathcal{S}_r\Lambda^1(\mathbb{R}^3)$ and $\mathcal{S}_r\Lambda^2(\mathbb{R}^3)$)
(3) Vector trimmed serendipity (Gillette-Kloefkorn = $\mathcal{S}_r^-\Lambda^1(\mathbb{R}^3)$ and $\mathcal{S}_r^-\Lambda^2(\mathbb{R}^3)$)

3 BUILDING CAPACITY FOR SERENDIPITY ELEMENT TYPES IN FIREDRAKE

Description of which elements are now available in Firedrake and how to call them.

4 TIMING EXPERIMENTS

Compare tensor product to serendipity and direct serendipity as applicable

(1) Wall clock time
(2) Static condensation time
(3) matvec time
(4) KSP / solver time

4.1 Timing: 2D Elements

4.2 Timing: 3D Elements

5 DEGREES OF FREEDOM AND MEMORY EXPERIMENTS

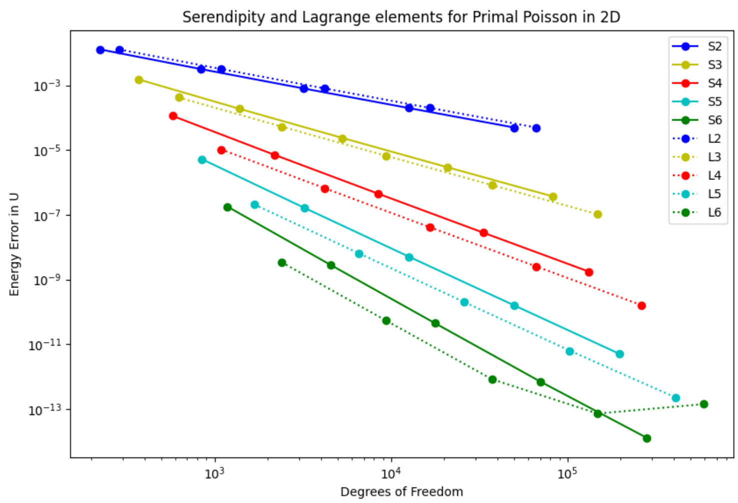
5.1 DOFs and Memory: 2D Elements

5.2 DOFs and Memory: 3D Elements

6 ACCURACY AND CONVERGENCE RATE EXPERIMENTS

6.1 Error: 2D Elements

Fig. 1. Degrees of Freedom vs Energy Error analysis of Serendipity and Lagrange L^2 elements.



6.2 Error: 3D Elements

Fig. 2. Degrees of Freedom vs Error analysis of S^{-}(Div) and RTCF elements for a mixed Poisson PDE, with exact solution $sin(\pi x)sin(\pi y)$!

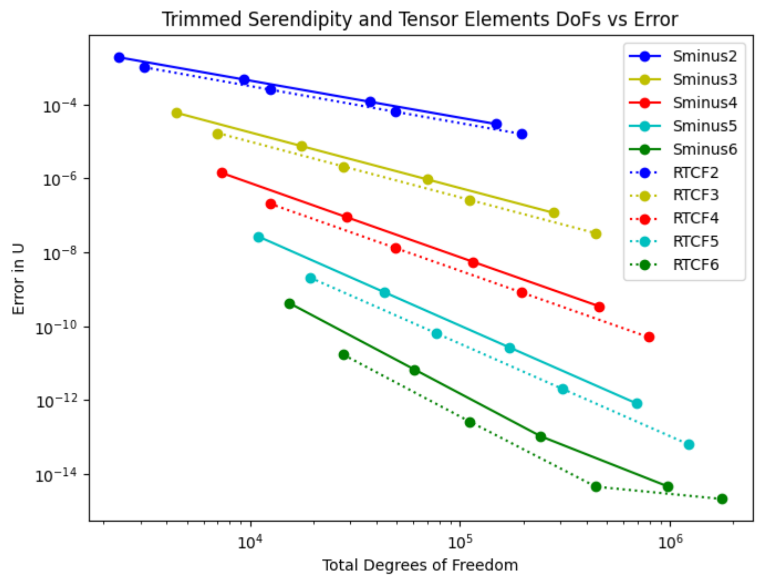


Fig. 3. Degrees of Freedom vs Error analysis of projection using S^{-}(Curl) and RTCE.

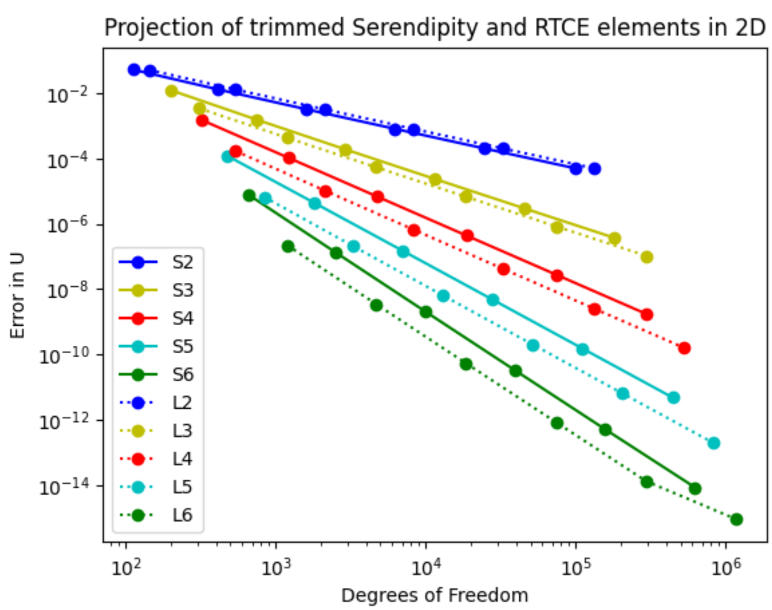


Fig. 4. Preliminary results for testing a mixed Poisson problem in 3D, using $\mathcal{S}^{-}(\mathrm{Div})$ and NCF elements.