# Model-Based Advancements in Submersible Navigation: TDOA and Kalman Filtering Techniques


$Zizhan\ Tang^{1^*}$ $Yao\ Liu^{1^*}$ $Jiexi\ Liu^{1^*}$
University of Southern California, USA
{jliu3548, mtang410, jiexiliu@usc.edu}@usc.edu
* Equal Contribution



The paper presents a novel submersible safety assurance system for addressing safety concerns during underwater sightseeing tours of submersibles operated by MCMS company in the Ionian Sea. The system incorporates multiple mathematical models and employs methods such as TDOA (Time Difference of Arrival) localization and Kalman filtering to enhance real-time and accurate prediction of the submersible's movements, and to improve the success rate of search operations in case of unexpected loss of contact.

For locating the submersible, due to factors like observation noise, simple localization methods often yield significant errors in estimating the actual submersible position. Hence, the paper establishes a localization model based on acoustic signals transmitted from multiple known buoys and anchor points to continuously obtain real-time observed positions of the submersible through TDOA localization. Then, leveraging the localization model, the paper proposes a prediction model based on the Kalman filtering algorithm to optimally predict the submersible position, thereby achieving more precise prediction. Simulation results are provided to demonstrate the accuracy of the proposed model.

Furthermore, for preparing for search operations, the paper discusses and analyzes the search equipment that should be carried by the mother ship and search vessels, considering safety, efficiency, and feasibility, for conducting search operations in the event of unexpected loss of contact with the submersible.

Moreover, for searching a submersible that is loss of contact, the paper considers the operational dynamics of the submersible, and categorizes scenarios of unexpected loss of contact based on whether the submersible continues along a predetermined course and whether it encounters a loss of propulsion. Corresponding search deployment positions and search patterns are established using the prediction models integrated into the safety assurance system, and simulations validate the effectiveness of the proposed system in improving the success rate of search operations for lost submersibles.

Finally, to extrapolate the submersible safety assurance system, the paper addresses the adaptation and updates of the established models for different sightseeing areas or scenarios involving multiple submersibles concurrently, enhancing the versatility of the system for potential application in diverse settings in the future

# Contents

# 1 Introduction

The quest for underwater exploration has always been at the forefront of human endeavors, fueling innovations and advancements in marine technology. This paper introduces a novel challenge faced by Maritime Cruises Mini-Submarines (MCMS), a company poised to pioneer the next frontier of oceanic tourism. MCMS's mission to navigate the enigmatic depths of the Ionian Sea with tourists is predicated on ensuring paramount safety measures against communication loss and mechanical failures. In an environment as unpredictable as the deep sea, these measures become not just a necessity but a responsibility. The following sections elaborate on the background of the problem, a restatement of the challenge at hand, and provide an overview of our comprehensive approach to predict and secure the trajectory of these underwater vessels.

## 1.1 Problem Background

Venturing into the abyss requires not only courage and curiosity but also an assurance of safety and reliability. MCMS's submersibles offer a passage to the unseen parts of our world, yet their operation is accompanied by considerable risks. Unlike surface or land-based vehicles, a submersible's failure could lead it to the seafloor or leave it adrift in a state of neutral buoyancy, compounded by factors such as sea currents, varying densities, and the geographical contours of the seabed. The background of this issue is deeply rooted in the physics of submersible dynamics and the unpredictability of the marine environment. It is within this context that our problem is situated, necessitating a predictive model for the submersible's location over time, ensuring that regulatory standards are met and that explorers' safety is never compromised.

## 1.2 Problem Restatement

Our mandate is to develop a comprehensive model to predict the real-time location of MCMS's submersibles, addressing uncertainties in underwater navigation by determining critical data for transmission and identifying necessary communication equipment. Additionally, we must recommend cost-effective search gear for the host ship, tailored for swift emergency responses and propose a search strategy to expedite the discovery of a lost submersible, quantifying the likelihood of recovery over time. This model will also be adaptable for use with multiple submersibles in varied environments such as the Caribbean Sea, ensuring its broad applicability and scalability for MCMS's expanding operations.

## 1.3 Contributions

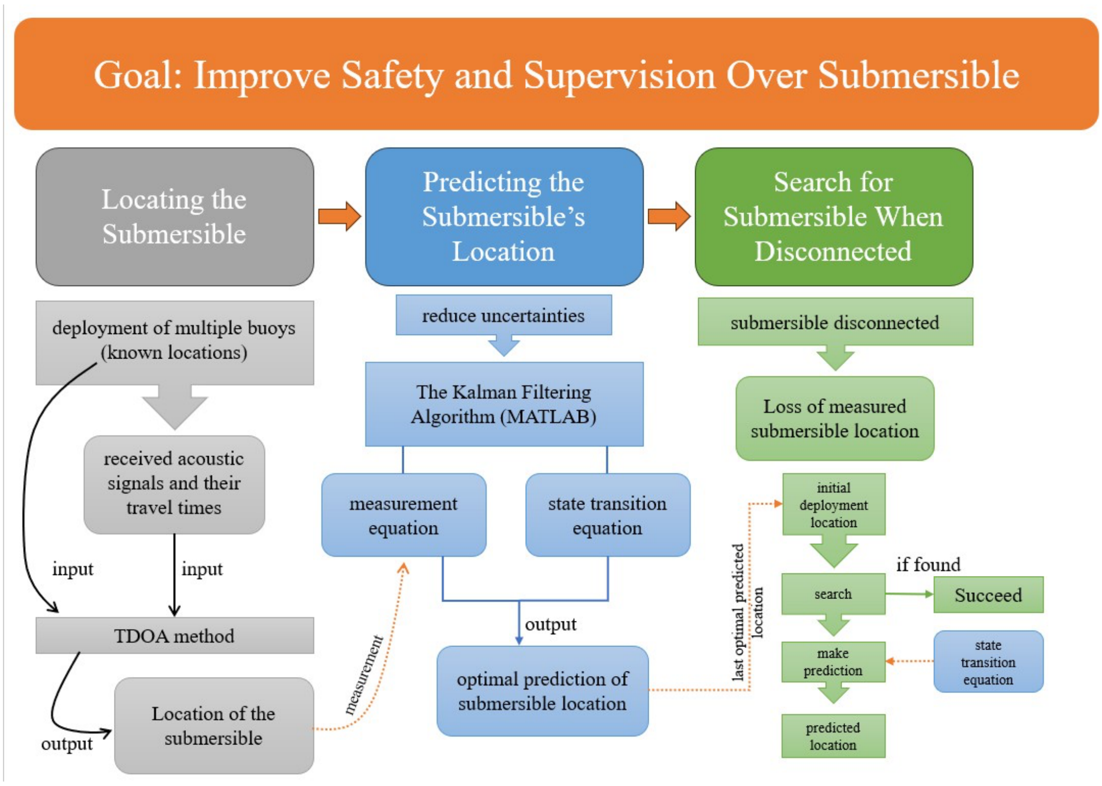



Figure 1: the general goal and system model

Throughout the paper, as shown in Figure 1, a novel submersible safety assurance system, comprising several mathematical models, is established. The contributions of the paper include:

1. A multi-buoy localization model using the Time Difference of Arrival (TDOA) method is developed. Through the model, the measured location of the submersible can be obtained. Then, a location prediction model based on Kalman filtering is developed. Through the model, the optimal prediction of the submersible can be obtained.
2. Instructions and suggestions of the choice of equipment that the host ship and search vessels should prepare are given after considering safety, efficiency, and feasibility. Such guidance can be used to assist real world submersible tour companies to improve safety.

3. A search model based on the previous models is developed. Through the model, initial search deployment locations and optimized search patterns are recommended. Therefore, the utilization of the search model can be used to enhance the searching process.
4. The versatility of the system is discussed, allowing it to be adapted for various scenarios with adjustments. For example, the system can be deployed in different maritime regions, enhancing its versatility and practicality.
5. MATLAB simulations validated the correctness, robustness, and feasibility of the system.

## 1.4 Significance

The development and implementation of a model that ensures the safety of submersible vessels hold profound implications. Our work takes a multi-faceted approach, encompassing mathematical analyses, technological integration, and strategic planning. We will present methodologies that encompass the engineering of submersibles, the nature of the marine environment, and the capabilities of search and rescue operations. The significance of our work lies in its potential to enable safe and sustainable exploration of the final frontier on Earth—the depths of our oceans. Through our research and recommendations, we aim to uphold the highest standards of safety, thereby fostering a legacy of responsible and exhilarating underwater adventure.

## 1.5 Organization of the Sections

Section 1 of the paper introduces the research problem and outlines the objectives of the study. Section 2 discusses the system model, including its assumptions and requirements. Section 3 presents the localization model and provides simulation results of the TDOA algorithm. Section 4 focuses on the Prediction model developed using the Kalman Filtering Algorithm, along with simulation results of optimal predictions. In Section 5, the paper introduces the search model for disconnected submersibles and presents simulation results of search operations. Section 6 extrapolates the current system and discusses its versatility in various conditions. Finally, Section 7 provides a discussion of the established system and concludes the study results.

# 2 System Model Requirements

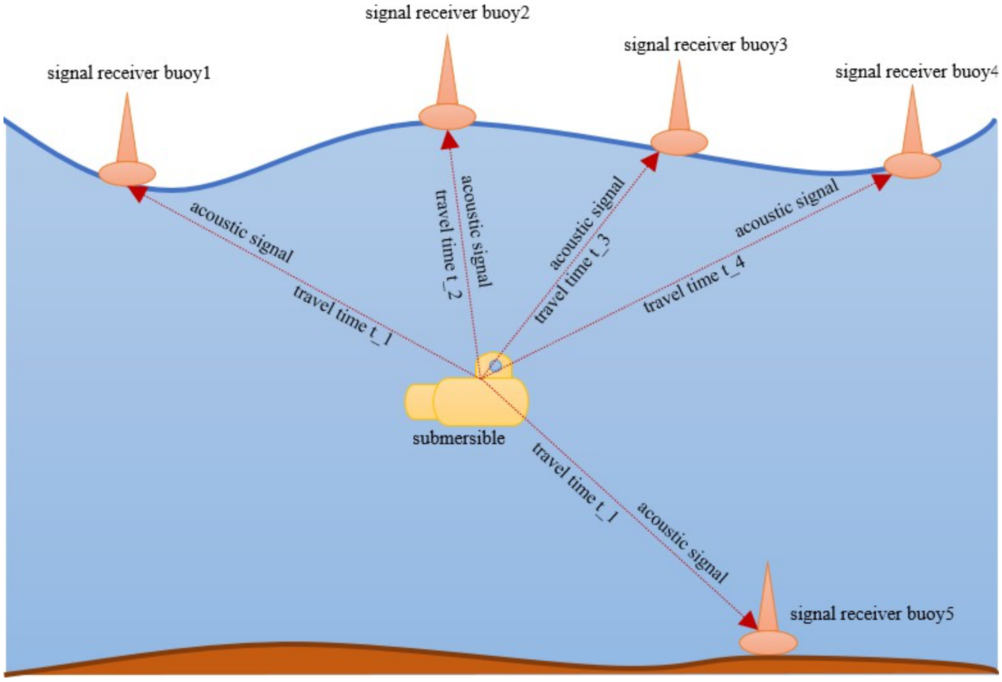



Figure 2: illustration of the localization system

## 2.1 The Submersible's Required Sensors and Data Collected

In order to implement the Kalman filtering algorithm as discussed above, the submersible needs to carry specific sensors and transmit necessary data to the host ship at designated time intervals. Additionally, the submersible needs to emit acoustic signals to designated buoys for calculating and obtaining the observed position coordinates of the submersible. The following are the sensors that the submersible needs to carry and their roles in constructing the prediction model.

1. **Timer:** The submersible requires a precise timer to transmit data to the host ship and send acoustic signals to buoys at predefined time intervals. Due to practical considerations, it is not assumed that the clocks of the host ship, submersible, and buoys remain synchronized indefinitely. Therefore, the submersible only needs to transmit data and send signals at the time intervals indicated by the precise timer, ensuring the acquisition of the submersible's observed position and the implementation of the Kalman filtering algorithm.

2. **Velocity sensors:** At predefined time intervals, the submersible needs to transmit its velocity in each direction to the host ship. Considering the submersible in a Cartesian coordinate system, it needs to detect the velocity along the X-axis, the velocity along the Y-axis, and the velocity along the Z-axis at each time interval.
3. **Engine acceleration sensors:** Similar to velocity sensors, the submersible also needs to transmit the acceleration provided by its engine in each direction at predefined time intervals.
4. **Ocean current acceleration sensors:** Similar to engine acceleration sensors, the submersible also needs to transmit the acceleration exerted by the ocean currents on the submersible in each direction at predefined time intervals.

## 2.2 Preparation of the Host Ship

In preparing for the potential loss of a submersible, it is essential for the host ship to be equipped with a suite of search equipment that not only enhances the probability of locating the vessel but also ensures the timely and efficient execution of rescue operations. Given the multifaceted nature of underwater search and rescue, the equipment must be selected based on its functionality, reliability, and cost-effectiveness, with considerations for availability, maintenance, readiness, and usage.

1. **Acoustic Positioning Systems:** A primary recommendation is the deployment of an ultra-short baseline (USBL) system, which utilizes acoustic signals to determine the position of the submersible. The USBL system has a transceiver mounted on the host ship that sends signals to a transponder on the submersible, which then replies, allowing the system to calculate the distance and direction to the submersible based on the time delay and angle of the returning signal. While USBL systems can be costly, their precision in deep-water localization justifies the investment. Regular maintenance and calibration are essential to ensure accuracy, and the system's ease of use contributes to its operational readiness. (Luo et al., 2020)
2. **Remotely Operated Vehicles (ROVs):** Equipping the host ship with ROVs can significantly enhance search capabilities. ROVs can be deployed to conduct visual searches and inspections in areas where it is too risky or deep for divers to operate. They can be fitted with cameras, sonars, and manipulator arms, making

them versatile tools for both search and rescue operations. The cost of ROVs varies widely based on their capabilities, but the investment in a mid-range model offers a balance between cost and functionality. ROVs require regular technical maintenance and skilled operators, thus training is a critical component of ensuring readiness. (Macreadie et al., 2018)

3. **Autonomous Underwater Vehicles (AUVs):** AUVs offer a complementary capability to ROVs. They can autonomously cover large areas of the ocean floor using side-scan or synthetic aperture sonar to create detailed images of large swathes of the seabed, which is invaluable in search missions. The higher cost of AUVs is offset by their extensive coverage and the reduced need for human intervention. However, AUVs do require significant maintenance and a specialized team to interpret sonar data. (Wynn et al., 2014)
4. **Satellite Communication:** To coordinate with land-based operations and other vessels, a robust satellite communication system is recommended for both the host and rescue vessels. While costly, satellite communications offer a reliable method of maintaining contact and coordinating efforts in the vast maritime environment.

# 3 Localization Model

## 3.1 Notations

$P_{ue} = [x_{ue}, y_{ue}, z_{ue}]^T$: Predetermined coordinates of the submarine at the time $t$

- $x_{ue}$: The predetermined coordinate for the submarine at time t along the x-axis
- $y_{ue}$: The predetermined coordinate for the submarine at time t along the y-axis
- $z_{ue}$: The predetermined coordinate for the submarine at time t along the z-axis

$\boldsymbol{I} = [\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}]$: The solution to Systems of the equation–computed location of submarine via TDOA

- $x$: The calculated coordinate for the submarine at time t along the x-axis
- $y$: The calculated coordinate for the submarine at time t along the y-axis
- $z$: The calculated coordinate for the submarine at time t along the z-axis

$r_0$ : the Euclidean distance between the reference station $s_0$ and submarine

$a_I$: Solution $Ax = C$

$b_I$: Solution $Ax = D$

$d_i$: the Euclidean distance between the station $\boldsymbol{s_i}$ and the submarine

$\Delta t_i$ : the time of difference arrival between $t_i$ and reference time

$t_0$

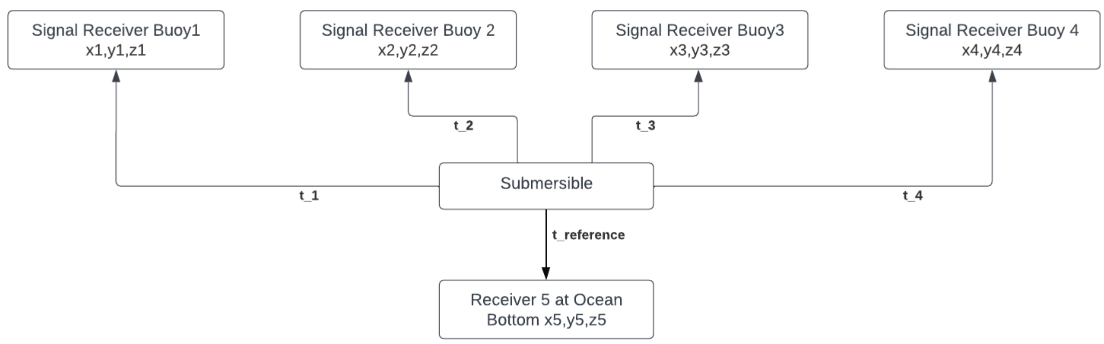
$\Delta t \ = [\Delta t_1, \Delta t_2, \Delta t_3, \Delta t_4]^T$: Time difference vector that

interpolates $\Delta t_i$ ; $i = 1,2,3,4$

$$k_i^{\,2} \ = \ x_i^{\,2} + y_i^{\,2} + z_i^{\,2}$$

## 3.2 Submersible Localization via Acoustics Transmission

Given the limited GPS localization for submarines underwater, we designated an algorithm via TDOA(Time of Difference Arrival) to compute the most precise location of the submersible. Taking advantage of the limited propagation speed of sound in water and the existence of acoustic signal transmitters and receivers, we deploy four surface buoys with known positions. The measured position of our submersible in this section plays an important role in the subsequent model in which we used Kalman Filtering to estimate the navigation behavior of the submarine. To achieve clock synchronization and ensure accurate positioning of the submarine, we added the fifth receiver placed at the bottom of the ocean as the clock for synchronization reference. In the submersible acoustic positioning, we use TDOA under the condition of equal sound velocity profile to simulate the position of the submersible position. Each buoy is equipped with an acoustic signal receiver for the signal emitted by the submarine transmitter every t-second.




We let the position of the station be $s_i \ = [x_i, \ y_i, \ z_i]^T$ where $i \ = \ 0,1,2,3,4$. For accurate 3D localization, the receivers are deployed so that at least one of the receiver stations has a distinct z-coordinate value. In calculating the TDOA, we choose a reference based on the time when the station $s_0$ receives the signal denoted as $t_0$. On top of that, the TDOA for the subsequent stations can be computed by taking the difference of their respective arrival time $t_0$, namely $\Delta t_i \ = \ t_i - t_0, i \ = \ 1,2,3,4$. In the simulation and modeling process, the TDOA of the signals received from the subsequent four stations can then be represented as $\Delta t_i,\ i = 1,2,3,4$. Due to the limitation on measuring actual data for time differences, we propose a preposition location for the submarine for calculating the time difference vector. We will demonstrate by the end of this section that the theoretical positioning calculated by our algorithm matches in precision with the preset location of the submersible. In real submersible sailing, our algorithm takes in the measured time of differences, synchronizes them to the same reference clock, and computes the location of the submersible.

## 3.3 Algorithm Demo for Localization

By Chan's Algorithm, we construct linear equations $Ax = r_0C + D$ given by:

$$\begin{bmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \triangle t_1 \\ \triangle t_2 \\ \triangle t_3 \\ \triangle t_4 \end{bmatrix} \cdot r_0 + \begin{bmatrix} \left(k_1 - k_0 - t_1^2\right)/2 \\ \left(k_2 - k_0 - t_2^2\right)/2 \\ \left(k_3 - k_0 - t_3^2\right)/2 \\ \left(k_4 - k_0 - t_4^2\right)/2 \end{bmatrix} \ldots\ldots (3-1)$$

where $r_0$ denotes the Euclidean distance between the reference station $s_0$ and our submarine, namely, $r_0 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$ and $k_i^2 = x_i^2 + y_i^2 + z_i^2$. Solving for the position vector $[x, y, z]^T$, we obtain the position of the submarine predicted as a function of the position of the stations and the time difference vector. Applying the Cramer Rule, the solution to the systems of equation is represented as a linear combination of the solution to $Ax = C$ and $Ax = D$, namely:

$$\begin{cases} x = a_1 \cdot r_0 + b_1 \\ y = a_2 \cdot r_0 + b_2 \\ z = a_3 \cdot r_0 + b_3 \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3-2)$$

Because the position of the submarine is assumed to be unknown, the next step is to calculate $r_0$. Substituting the solution we obtained from the last step we have:

$$\left(a_1^2 + a_2^2 + a_3^2\right) \cdot r_0 + 2r_0(a_1b_1 + a_2b_2 + a_3b_3 - a_1x_0 - a_2y_0 - a_3z_0) + (x_0 - b_0)^2 + (y_0 - b_2)^2 + (z_0 - b_3)^2$$

Notably, this equation matched the form of a quadratic $\tilde{A}r_0^2 + \tilde{B}r_0^2 + \tilde{C}$ equation and thus we can rewrite it as:

$$\begin{cases} \tilde{A} = a_1^2 + a_2^2 + a_3^2 \\ \tilde{B} = a_1(b_1 - x_0) + a_2(b_2 - y_0) + a_3(b_3 - y_0) \\ \tilde{C} = (x_0 - b_1)^2 + (y_0 - b_2)^2 + (z_0 - b_3)^2 \end{cases} \quad \dots\dots\dots\dots\dots\dots (3-3)$$

By solving for the quadratic equation we have:

$$r_0 = \frac{\tilde{B} \pm \sqrt{\tilde{B^2} - 4\,\tilde{A}\,\tilde{C}}}{2\tilde{A}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3-4)$$

Note that erroneous solutions might occur when solving for $r_0$ and it is necessary to add observation stations.

## 3.4 Simulation Results:

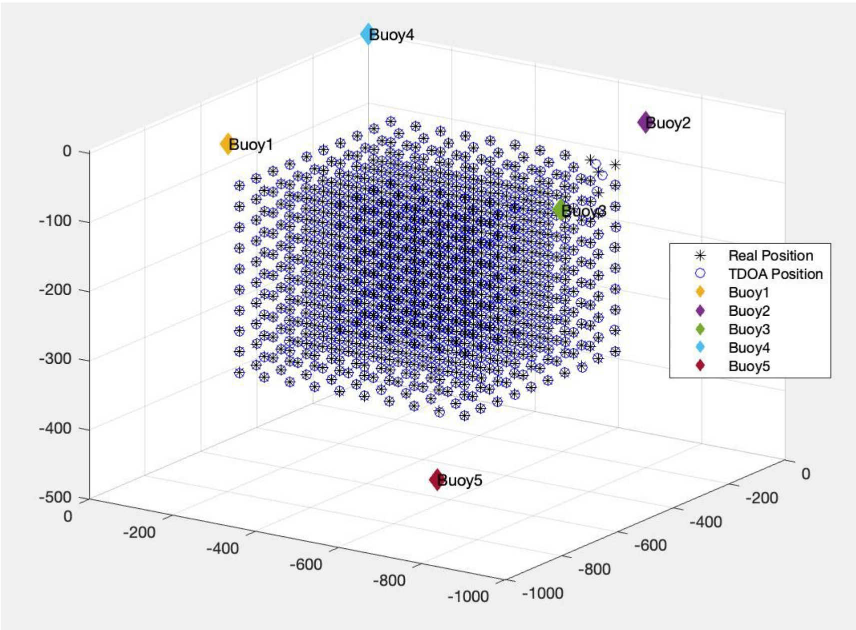



Figure 3: Localization Results for 1000 Test Points

We run the simulation of TDOA based on the following instantiations:

The coordinates to be stations are set to$[-800\ -200\ 3]$, $[-200\ -800\ 0]$, $[-800\ -1000\ 0]$, $[0\ 0\ 0]$, $[-500\ -500\ -500]$. We uniformly place a total of 1000 test points which represent predetermined coordinates of the submarines in space starting from coordinates $[100\ -100\ -50]$, with intervals of 60 $m$ along the x-axis, 60 $m$ along the y-axis, and 30 $m$ along the z-axis.

In Figure 3, the circles in blue represent our predetermined location of the submersible. Our algorithm computes the theoretical location through the time difference vector derived from the predetermined location. The asterisks in blue showcase our theoretical position of the submarine which aligns accurately with the predetermined location.

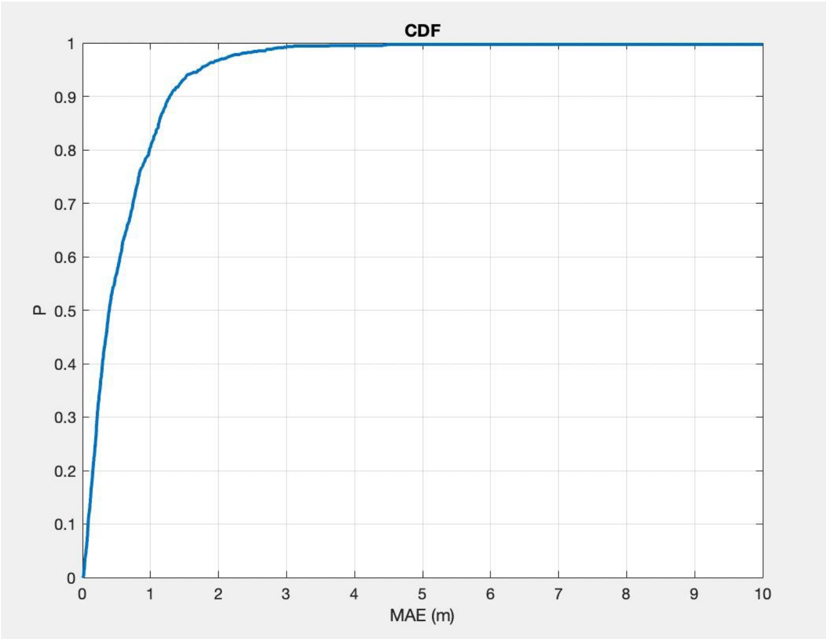



Figure 4: CDF of MAE for Submarine Localization

Figure 3 illustrates the Cumulative Distribution Function(CDF) of Mean Absolute Error(MAE) between the predetermined position of the submarines(test points) and the theoretical position given by our TDOA Algorithm.

The CDF initiates at 0 on the y-axis and concludes at 1, portraying the complete spectrum of probabilities from 0% to 100%. At any given position along the x-axis, the corresponding y-value on the CDF signifies the likelihood that the MAE of a randomly chosen test point is either less than or equal to that specific MAE value. Based on this interpretation, we can infer that our measurements for the theoretical position of the submarine closely align with the predetermined expectations. As the CDF approaches 1, indicating near-certainty, the MAE remains less than or equal to $4\ m$, ensuring that with 100% confidence, the error of our result will be less than or equal to $4\ m$.

The position vector of the submarine measured in this section scaffolds for the estimation task of the navigation behavior at the next time t via Kalman Filtering. In the following section, we will illustrate how to integrate the measurement position.

# 4 Prediction Model

## 4.1 Introduction to the Model and the Kalman Filtering Algorithm

In order to achieve real-time positioning prediction for the submersible, a prediction model optimized using the Kalman filtering method was constructed. The Kalman filtering algorithm is a special case of Bayesian filtering, a powerful tool for estimating and predicting system states in the presence of uncertainty, widely applied in applications such as target tracking, navigation, and control. To reflect real-world conditions, noise was assumed to affect both observations and predictions in the model. Therefore, applying the Kalman filtering algorithm efficiently computes optimal estimates, also known as posterior estimates, enabling the host ship to more accurately ascertain the submersible's underwater positioning. The notations that will be used are as follows:

$\hat{x}_{\bar{k}}\left(\hat{x}_{\bar{t}}\right)$: The predicted value of the object given by the state transition equation

$\hat{x}_k\left(\hat{x}_t\right)$: The optimal value of the object after Kalman filtering

$x_k\left(x_t\right)$: The measured value of the object given by the locating model

$z_k\left(z_t\right)$: The measured value of the object given by the measure equation

$w$: The prediction noise of the Kalman filtering algorithm

$v$: The measurement noise of the Kalman filtering algorithm

$Q$: The standard deviation of the prediction noise

$R$: The standard deviation of the measurement noise

$P_k\left(P_t\right)$: The covariance of the optimal prediction and the measured value

$P_{\bar{k}}\left(P_{\bar{t}}\right)$: The covariance of the prediction given by the state transition equation and the measured value

$K_k$: The Kalman gain

$x^t$: The coordinate of the object along the x-axis at given time t

$y^t$: The coordinate of the object along the y-axis at given time t

$z^t$: The coordinate of the object along the z-axis at given time t

$v_x^{\ t}$: The velocity of the object along the x-axis at given time t

$v_y^{\ t}$: The velocity of the object along the y-axis at given time t

$v_z^t$: The velocity of the object along the z-axis at given time t

$a_x^t$: The acceleration of the object along the x-axis at given time t

$a_y^t$: The acceleration of the object along the y-axis at given time t

$a_z^t$: The acceleration of the object along the z-axis at given time t

$T$: The time interval

## 4.2 Principles of the Kalman Filtering Algorithm

The two fundamental assumptions of Kalman filtering are: firstly, that the state transition equation and observation equation of the object are linear and invariant; secondly, that both observation noise and prediction noise follow Gaussian distributions. To implement the Kalman filtering algorithm, it is necessary to obtain the state transition equation of the object. Using this equation and the object's posterior estimated value at time t, as well as the assumed noise influence, the object's prior estimated value at time $t$+1, or the prediction result, is calculated. In this instance, the state transition equation of the Kalman filtering algorithm is derived based on the data sent from the submersible to the host ship at time $t$, including the submersible's motion velocity and acceleration in all directions at time $t$, as well as the ocean current acceleration in all directions. This is combined with the optimal estimated position coordinates obtained through the Kalman filtering algorithm at time t and the assumed noise, to infer the submersible's position coordinates at time $t$+1.

In addition to the state transition equation, the implementation of the Kalman filtering algorithm also requires a measurement equation, obtained from actual observation data and the assumed noise influence to derive the observed state. In this example, the observation equation of the Kalman filtering algorithm is based on the positioning results obtained from the location model and the assumed noise addition. The location model, as previously mentioned, calculates the submersible's position coordinates by receiving acoustic signals from buoys.

After obtaining the state transition equation and the observation equation, the posterior estimate of the object at time $t$, derived through the Kalman filtering algorithm, can be computed. Firstly, the Kalman coefficients of the object at time $t$ are calculated based on existing data, resulting in intermediate results of the filtering calculation. Finally, the posterior or optimal estimate of the object at time t is calculated as the sum of the object's prior estimate at time $t$ and the product of the Kalman coefficient multiplied by the difference between the object's observation value at time $t$ and the object's prior estimate value at time $t$.

## 4.3 Implementation of the Kalman Filtering Algorithm

Prior to the performing the Kalman filtering algorithm to make optimized prediction of the submersible's location, the values of the following variables are given based on prior knowledge:

$Q$, the standard deviation of the prediction noise.

$R$, the standard deviation of the measurement noise.

$P0$, the initial covariance of the measured value and the predicted value.

Then, the Kalman filtering algorithm can be performed as indicated below:

First, given known state transition equation (predicted value):

$$\hat{x}_k^- = A\hat{\mathbf{x}}_{k-1} + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{w}_k \quad \text{(4-1)}$$

Second, given known measurement equation (measured value):

$$z_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{v}_k \quad \text{(4-2)}$$

Then, the covariance of predicted value and measured value can be calculated:

$$P_k^- = \boldsymbol{A}P_{k-1}\boldsymbol{A}^T + Q \quad \text{(4-3)}$$

Next, the Kalman gain can be calculated given the result of equation (4-3):

$$K_k = \frac{P_k^- \boldsymbol{H}^T}{\boldsymbol{A}P_{k-1}\boldsymbol{A}^T + R} \quad \text{(4-4)}$$

In addition, the covariance of the optimal predicted value and the measured value can be calculated:

$$P_k = \left(\boldsymbol{I} - K_k\boldsymbol{H}\right)P_k^- \quad \text{(4-5)}$$

Finally, given the result of equation (3), the optimal predicted value can be calculated:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + K_k\left(z_k - H\hat{\mathbf{x}}_k^-\right) \quad \text{(4-6)}$$

Based on our assumptions that the submersible moves with velocity v and acceleration a, the sum of the engine acceleration and the ocean current acceleration, over the time interval T, as shown in Figure 4, the predicted location of the submersible at time t can be calculated by：

$$\begin{bmatrix} x^t \\ y^t \\ z^t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x^{t-1} \\ y^{t-1} \\ z^{t-1} \end{bmatrix} + \begin{bmatrix} T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \cdot \begin{bmatrix} v_x^{\ t-1} \\ v_y^{\ t-1} \\ v_z^{\ t-1} \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2} & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 \\ 0 & 0 & \frac{T^2}{2} \end{bmatrix} \cdot \begin{bmatrix} a_x^{\ t-1} \\ a_y^{\ t-1} \\ a_z^{\ t-1} \end{bmatrix} \qquad \text{.........(4-7)}$$

Therefore, the state transition equation is given as:

$$\begin{bmatrix} x^t \\ y^t \\ z^t \\ v_x^{\ t} \\ v_y^{\ t} \\ v_z^{\ t} \\ a_x^{\ t} \\ a_y^{\ t} \\ a_z^{\ t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 & \frac{T^2}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 & 0 & \frac{T^2}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & T & 0 & 0 & \frac{T^2}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x^{t-1} \\ y^{t-1} \\ z^{t-1} \\ v_x^{\ t-1} \\ v_y^{\ t-1} \\ v_z^{\ t-1} \\ a_x^{\ t-1} \\ a_y^{\ t-1} \\ a_z^{\ t-1} \end{bmatrix} + \boldsymbol{w}_{p_t} \qquad \text{.........................(4-8)}$$

Let the predicted value calculated by the state transition equation be $\hat{x}_t^-$ . Since the prediction model of the submersible is based on time intervals, t is used to denote the sequence of data instead of k in the original equations. Then, the measurement function is given as:

$$\begin{bmatrix} \hat{x}^t \\ \hat{y}^t \\ \hat{z}^t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x^t \\ y^t \\ z^t \end{bmatrix} + \boldsymbol{v}_{\mathbf{t}} \qquad \text{..............................(4-9)}$$

Let the measured value calculated by the state transition equation be $\hat{x}_t$. Now, with the object's state transition equation and measurement equation known, a MATLAB model containing a Kalman filtering function meeting the requirements can be used for simulation.

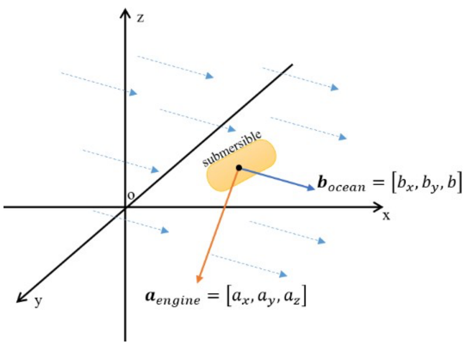



Figure 5: illustration of mechanical analysis of the submersible under ocean current

## 4.5 Simulation Results

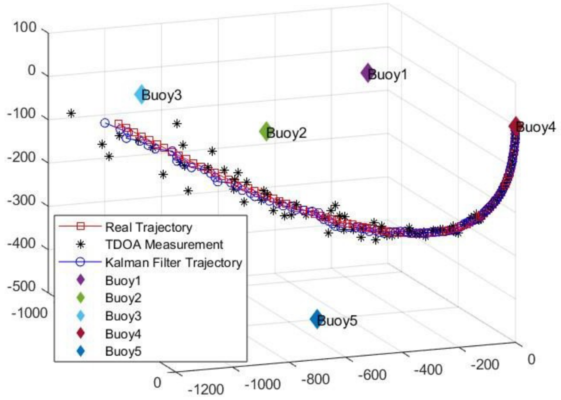



Figure 6: simulation result of the prediction model

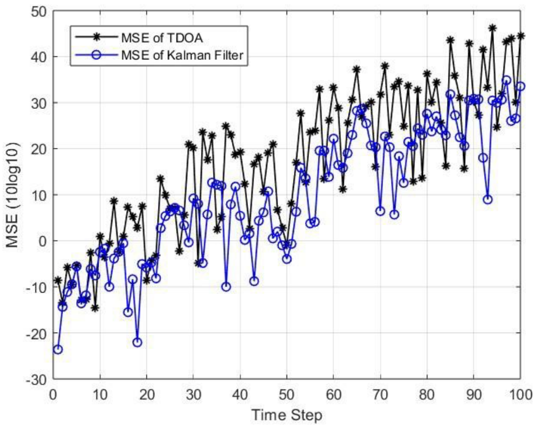



Figure 7: MSE calculation of prediction model

When simulating the performance of the Prediction Model, the locations of the buoy stations are set to [-800, -200, 3], [-200, -800, 0], [-800, -1000, 0], [0, 0, 0], and [-500, -500, -500]. The test points are uniformly placed in space, starting from coordinates (-100, -100, -50) with intervals of 60m along the x-axis, 60m along the y-axis, and 30m along the z-axis. In total, 1000 test points are placed. The time interval is set to be 10s. The starting position of the submersible is [0, 0, 0]. The velocity vector of the submersible is set to [-0.001, -0.002, -1], and the engine acceleration vector of the submersible is set to [-0.001, -0.001, 0.0018]. The acceleration vector of the ocean is set to [-0.001, -0.001, 0]. After the simulation, Figure 6 and Figure 7 are obtained.

Figure 6 illustrates the simulated real trajectory, the predicted locations of the submersible provided by the TDOA measurement, and the optimal predictions returned by the Kalman filtering algorithm. The simulation results depicted in Figure 6 reveal that as time progresses, the disparity between the actual position and the TDOA measurement increases rapidly. Conversely, the optimal results obtained through the prediction model demonstrate superior performance, indicating that the Kalman filtering algorithm effectively mitigates the negative effects of noise.

Figure 7 demonstrates that the Mean Square Error (MSE) generated by the prediction model tends to be smaller than that of the TDOA measurement model. This finding further suggests that the Kalman filtering algorithm yields a more accurate prediction of the submersible's location.

# 5 Search Model

## 5.1 Disconnection

When a submarine fails to transmit data to the host ship and send acoustic signals to buoys for positional observation at predefined intervals, it can be deemed disconnected. During the search for a disconnected submarine, predictive models based on Kalman filtering algorithms can be employed to forecast the submarine's trajectory, and suggest initial points of deployment, aiding in the search process. Firstly, it is necessary to identify different scenarios of the status of the submarine after disconnection.

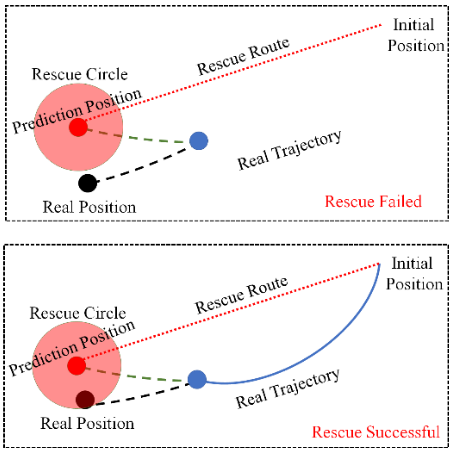



Figure 8: illustration of the searching process

## 5.2 Scenario One: No Mechanical Failures

This scenario refers to the cases that the submarine loses connection but does not encounter any mechanical failures apart from the communication device, indicating that it is still navigating normally but unable to communicate with the host ship or buoys. In this case, further classification is needed based on whether the submarine has a predefined navigation plan.

### 5.2.1 Scenario One (a): Predetermined Plan

If the submarine has a predetermined navigation plan, such as following a fixed sightseeing route exploring the bottom of the Ionian Sea, where the submarine's velocity and engine-acceleration at each moment can be considered as pre-planned. Therefore, the future velocity and acceleration of the submarine can be assumed to be consistent with the predefined plan. The last obtained data before disconnection is used to calculate the posterior optimal estimate of the submarine's last connected position through Kalman filtering, and this estimate is used to calculate the predicted position of the submarine at the next time step, where the submersible will be disconnected. However, as the observed position of the submarine cannot be obtained after disconnection, there will be no posterior update of the submarine's position. Instead, the prior predicted position is used as the optimal estimate, and the next position prediction is made based on the data provided by the predefined plan, repeating this process.

### 5.2.2 Scenario One (b): Unplanned

If the submarine lacks a predetermined navigation plan, such as going on a free-sightseeing of the sunken shipwrecks, the velocities and engine-provided accelerations at the next time step are unknown. In this case, a model needs to be built to predict the submarine's sailing velocity and engine-acceleration based on the last obtained data before disconnection, and the predictive data of the submarine after disconnection is obtained through the new model. Then, with the generated predictive data, similar to Scenario 1a, the position of the submarine continues to be predicted without observed positions.

## 5.3 Scenario Two: Loss of Propulsion

This scenario refers to the cases that the submarine is disconnected and encounters mechanical failures, such as engine power loss or fuel depletion. It is possible that the submarine encounters an accident and becomes disconnected after reporting loss of propulsion to the host ship. Although it is impossible to continuously obtain the velocity and acceleration data of the submarine at predefined time intervals, the submarine's engine-acceleration can be assumed to be zero due to propulsion loss, and it becomes a random walk model, where the submarine has a known initial velocity and only moves under the influence of acceleration caused by ocean currents. Consequently, the velocity and acceleration data of the disconnected submarine undergoing random walks are obtained, followed by position predictions similar to that in Scenario One.

## 5.4 Simulation Results

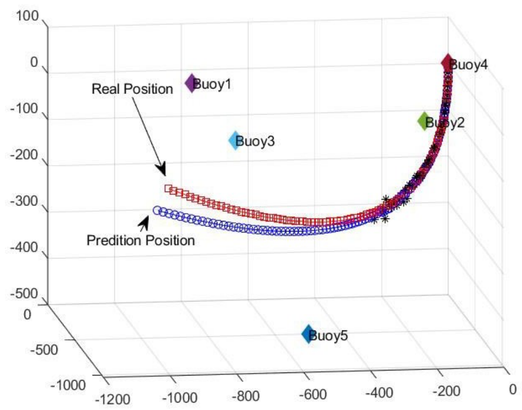



Figure 9: simulation of search process when t = 50

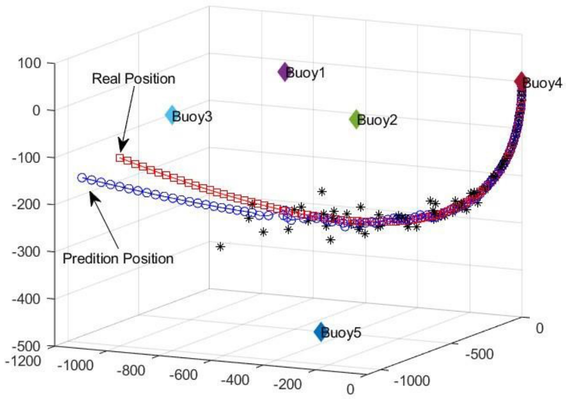



Figure 10: simulation of search process when t = 80

Similar to the simulation of the prediction Model, when simulating and analyzing the probability of successfully finding the disconnected submersible as a function of the time, the locations of the buoy stations are set to [-800, -200, 3], [-200, -800, 0], [-800, -1000, 0], [0, 0, 0], and [-500, -500, -500]. The test points are uniformly placed in space, starting from coordinates (-100, -100, -50) with intervals of 60m along the x-axis, 60m along the y-axis, and 30m along the z-axis. In total, 1000 test points are placed. The time interval is set to be 10s. The starting position of the submersible is [0, 0, 0]. The velocity vector of the submersible is set to [-0.001, -0.002, -1], and the engine acceleration vector of the submersible is set to [-0.001, -0.001, 0.0018]. The acceleration vector of the ocean is set to [-0.001, -0.001, 0]. After the simulation, Figure 9 and Figure 10 are obtained.

Figure 9 simulates the search process when the submersible disconnects with host ship at time $t = 50$, and Figure 10 simulates the search process when the submersible disconnects at time $t = 80$. From the figures, it can be observed that although the search model can generate a predicted trajectory similar to the real trajectory, which indicates that the search pattern suggested is indeed practical, the disparity between the predicted location and the real submersible location tends to increase as $t$ increases. Consequently, the probability of finding the submersible is likely to decrease as $t$ increases.

# 6 Extrapolate

Expanding our model to encompass the diverse environment of the Caribbean Sea and manage multiple submersibles simultaneously presents a complex, yet feasible challenge. To integrate the unique oceanographic characteristics of the Caribbean, we'll adjust our Kalman Filter for the region's specific acoustic properties.

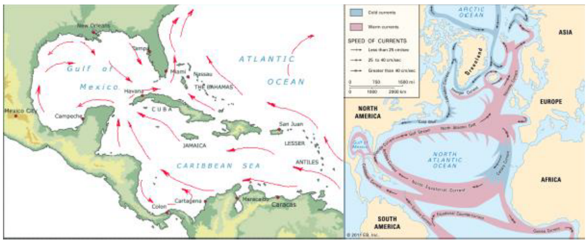



Figure :(*Map of Caribbean Sea Currents* | *Pirates & Zombies*, n.d.)

The Caribbean Current is a strong oceanic current that travels westward through the Caribbean Sea, heads north via the Yucatán Channel, and turns east through the Straits of Florida, forming the Florida Current. This warm current emerges from the confluence of the North Equatorial Current and the Guiana Current. It moves at an average speed ranging between 38 and 43 centimeters per second, carrying approximately 27,500,000 cubic meters of water per second. (The Editors of Encyclopedia Britannica, 1998)

We can effectively expand our model to the Caribbean Sea by specifically updating the acceleration parameter in our prediction model, and thus the search model. This adjustment is crucial to accurately reflect the distinct dynamics of the Caribbean's marine environment, which may differ significantly from the Ionian Sea. By placing sensors and fine-tuning the acceleration parameter, our model will be equipped to predict and adapt to the unique underwater conditions and movement patterns found in the Caribbean or any regions of interests, thereby enhancing its accuracy and reliability in these new contexts.

# 7 Discussion & Conclusion

In conclusion, our models have addressed the critical safety concerns associated with the operation of submersibles in the challenging marine environment. By integrating Kalman Filtering into the host system, we have proposed a comprehensive solution to predict and secure the trajectory of the submersible, particularly in scenarios involving communication loss and mechanical failures. Through the development of the trilateration localization model, Kalman filtering framework, and search & rescue model we have demonstrated the potential to mitigate risks and enhance safety standards for oceanic tourism. Particularly, we encountered limitations in directly measuring actual data for time differences, prompting the need for a preposition location for the submarine to calculate the time difference vector. Through this approach, we aim to demonstrate the precision and reliability of our algorithm in determining the submarine's theoretical positioning. Furthermore, our recommendations for search maximum likelihood and rescue equipment, including acoustic positioning systems, remotely operated vehicles, and autonomous underwater vehicles, offer practical solutions to expedite response efforts in emergencies. By fostering collaboration between mathematics, control system engineering, and strategic planning, our work strives to enable safe and sustainable exploration of the oceanic depths, ushering in a new era of responsible and exhilarating underwater adventure.

# 8 Reference

Luo, Q., Yan, X., Chen, J., Chen, Y., & Luo, Z. (2020). An Ultra-Short Baseline Underwater Positioning System with Kalman Filtering. *Sensors*, *21*(1), 143. https://doi.org/10.3390/s21010143

Macreadie, P. I., McLean, D. L., Thomson, P., Partridge, J. C., Jones, D. O., Gates, A. R., Benfield, M. C., Collin, S. P., Booth, D. J., Smith, L., Techera, E., Skropeta, D., Horton, T., Pattiaratchi, C., Bond, T., & Fowler, A. M. (2018). Eyes in the sea: Unlocking the mysteries of the ocean using industrial, remotely operated vehicles (ROVs). *Science of the Total Environment*, *634*, 1077–1091. https://doi.org/10.1016/j.scitotenv.2018.04.049

*Map of Caribbean Sea Currents | Pirates & Zombies*. (n.d.). https://pirates.missiledine.com/index.php/map-of-caribbean-sea-currents/

The Editors of Encyclopaedia Britannica. (1998, July 20). *Caribbean Current | Warm, saline & Nutrient-Rich*. Encyclopedia Britannica. https://www.britannica.com/place/Caribbean-Current

Wynn, R. B., Huvenne, V. A., Bas, T. L., Murton, B. J., Connelly, D. P., Bett, B. J., Ruhl, H. A., Morris, K. J., Peakall, J., Parsons, D. R., Sumner, E. J., Darby, S. E., Dorrell, R. M., & Hunt, J. E. (2014). Autonomous Underwater Vehicles (AUVs): Their past, present and future contributions to the advancement of marine geoscience. *Marine Geology*, *352*, 451–468. https://doi.org/10.1016/j.margeo.2014.03.012

# 9 Memo

A creative Greek firm called Maritime Cruises Mini-Submarines (MCMS) has created a cutting-edge submersible system that can explore the depths of the Ionian Sea in safety. Our in-depth research describes a painstakingly crafted safety framework that guarantees the best standards for tourist submersible operations. This paper, which urges the government to provide permission to start these novel and fascinating tourism endeavors, outlines our conclusions and suggestions.

**1. Submersible Localization and Prediction Model:**

Our system employs advanced Time Difference of Arrival (TDOA) localization and Kalman filtering algorithms, enabling real-time, accurate tracking of the submersible. This method effectively addresses uncertainties in underwater navigation, crucial in the unlikely event of communication loss or mechanical defects. By continuously receiving data from strategically positioned buoys and employing sophisticated algorithms, we can predict the submersible's location with exceptional precision.

**2. Enhancing Safety and Reducing Uncertainties:**

The submersible is outfitted with cutting-edge sensors that communicate critical data to the host ship, including velocity, engine acceleration, and ocean current acceleration, in order to increase safety and reduce uncertainty. The accuracy of our prediction model is greatly enhanced by this continuous data flow, guaranteeing quick and efficient action in emergency situations.

**3. Search and Rescue Preparedness:**

In the rare event of a submersible loss, our host ship is equipped with cutting-edge search equipment, including Acoustic Positioning Systems, Remotely Operated Vehicles (ROVs), and Autonomous Underwater Vehicles (AUVs). These tools, chosen for their effectiveness and cost-efficiency, enhance our ability to swiftly locate and recover the submersible, thereby ensuring passenger safety and regulatory compliance.

**4. Probabilistic Search Model:**

Our report details a probabilistic search model, utilizing the location predictions to define initial search areas and patterns. This model is dynamic, adjusting search strategies based on ongoing data, thereby maximizing the probability of locating a lost submersible promptly.

The creative strategy used by MCMS raises the bar for underwater tourist safety. Our plans and models ensure strong reaction capacities in improbable negative situations, greatly augmenting the safety of submerged tourism. This project not only promises visitors a one-of-a-kind experience, but it also has the potential to boost Greece's tourism sector and demonstrate our country's leadership in nautical innovation.

We respectfully request the Greek Government's approval to commence operations, confident in our ability to provide a safe, unforgettable experience for tourists, while adhering to the highest safety standards.