CSE 31
Computer Organization

Lecture 6 – C Memory Management (3)
Integer Representations

Announcement

- Lab #3 this week
  - Due at 11:59pm on the same day of your next lab
  - You must demo your submission to your TA within 14 days
- Reading assignment
  - Chapter 1.1 – 1.3 of zyBook
    - Do all Participation Activities in each section
    - Access through **CatCourses**
    - Due Wednesday (2/20) at 11:59pm

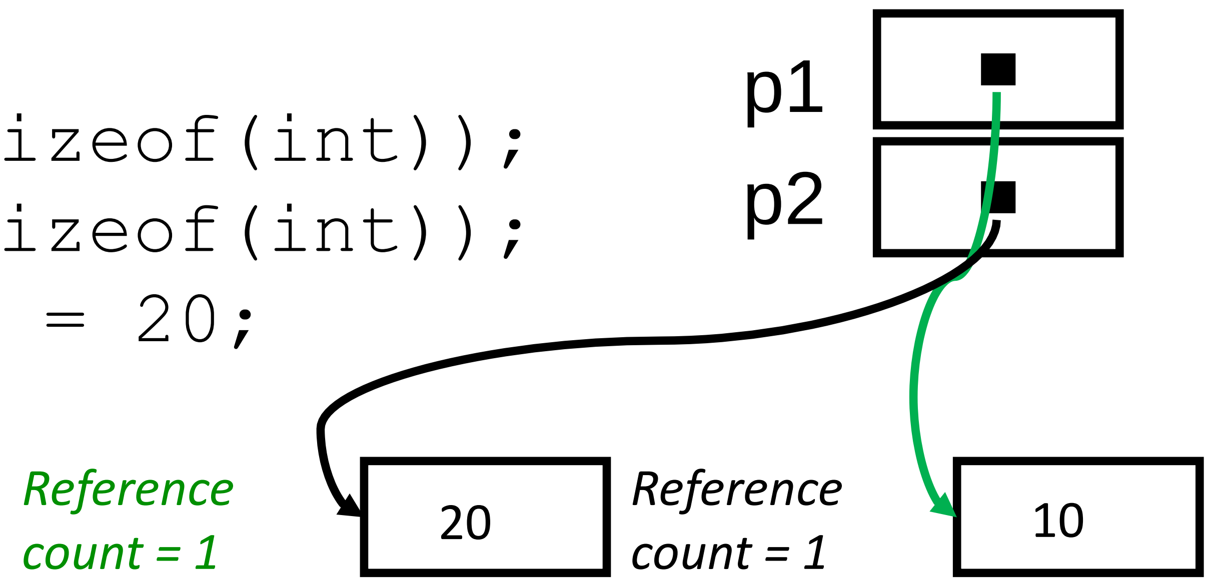
Automatic Memory Management

- Dynamically allocated memory is difficult to track
  - Why not track it automatically?
- If we can keep track of what memory is in use, we can reclaim everything else.
  - Unreachable memory is called garbage, the process of reclaiming it is called garbage collection.
- So how do we track what is in use?

Reference Counting Example

- For every chunk of dynamically allocated memory, keep a count of number of pointers that point to it.
  - When the count reaches 0, reclaim.

```
int *p1, *p2;
p1 = malloc(sizeof(int));
p2 = malloc(sizeof(int));
*p1 = 10; *p2 = 20;
```



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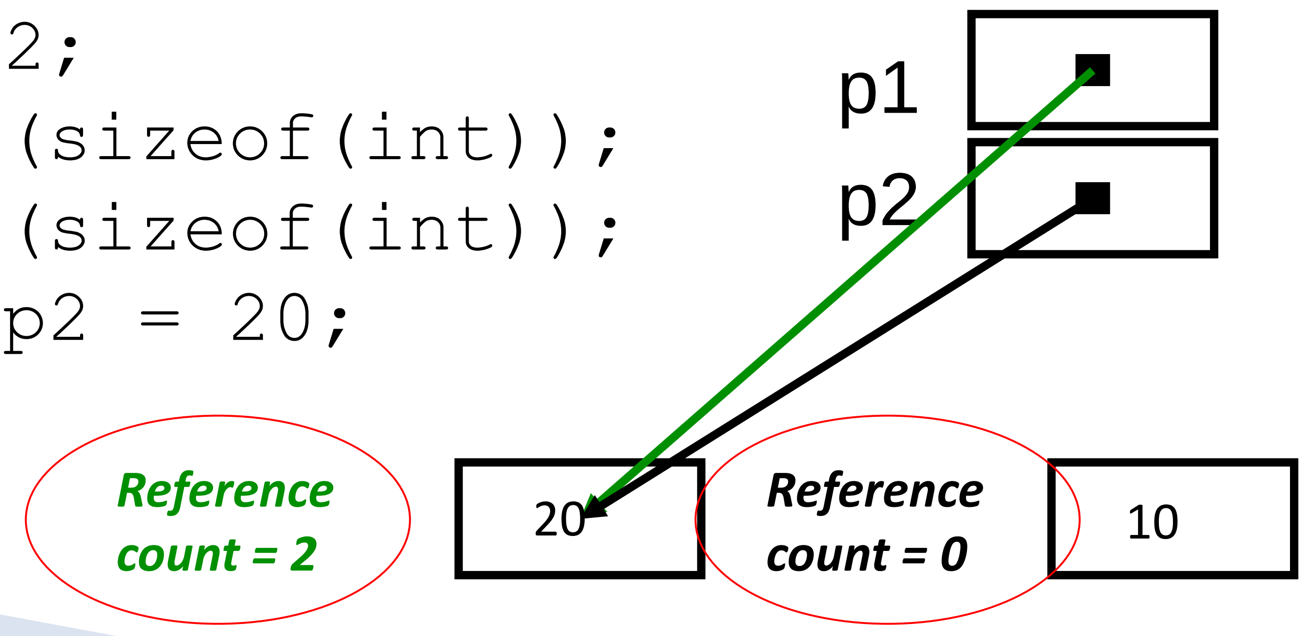


Scheme 2: Mark and Sweep Garbage Collection

- Keep allocating new memory until memory is exhausted, then try to find unused memory.
- Consider objects in a graph, chunks of memory (objects) are graph nodes, pointers to memory are graph edges.
  - Edge from A to B $\rightarrow$ A stores pointer to B
- Can start with the root set, perform a graph traversal, find all usable memory!
- 2 Phases:
  1. Mark used nodes
  2. Sweep free ones, returning list of free nodes

Mark and Sweep

- Graph traversal is relatively easy to implement recursively

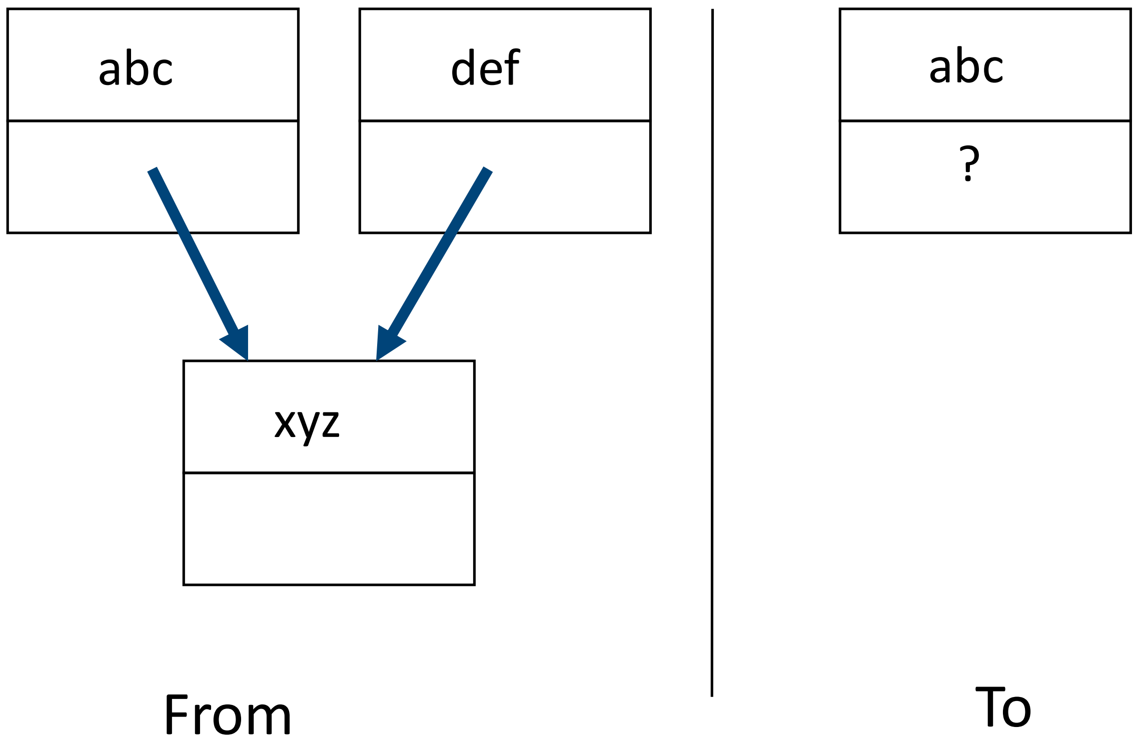
```
void traverse(struct graph_node *node) {
    /* visit this node */
    foreach child in node->children {
       traverse(child);
    }
}
```

- But with recursion, state is stored on the execution stack.
  - Garbage collection is invoked when not much memory left
- As before, we could traverse in constant space (by reversing pointers)

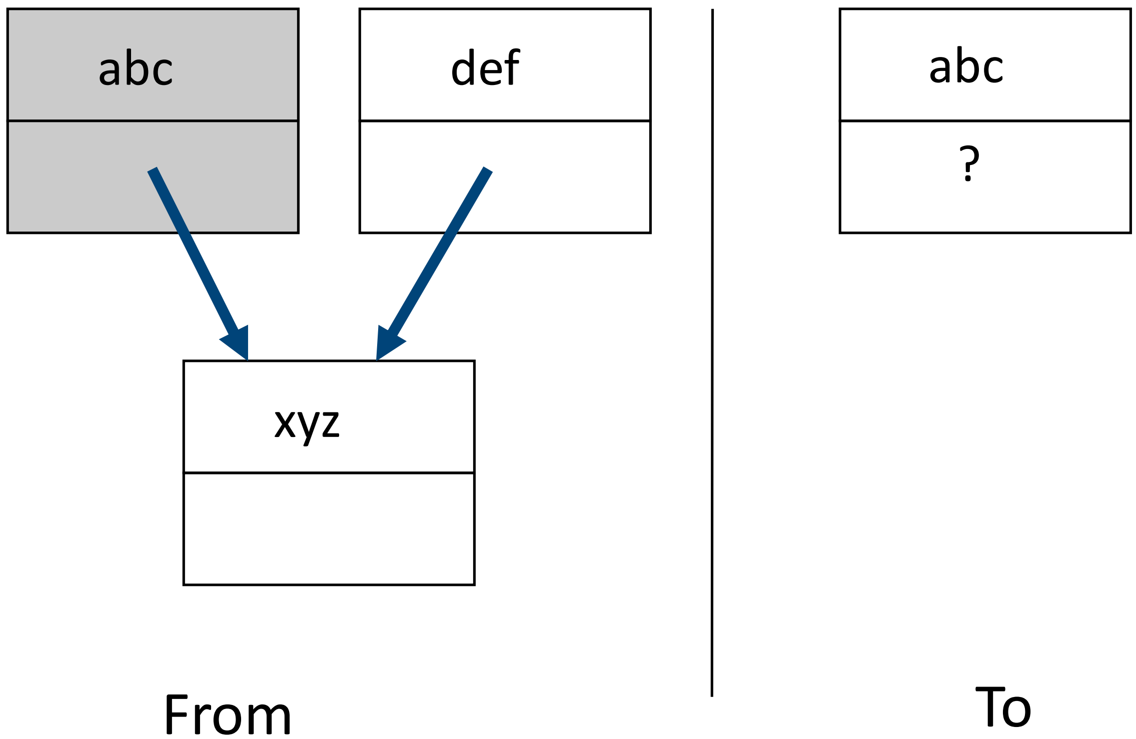
Scheme 3: Copying Garbage Collection

- Divide memory into two spaces, only one in use at any time.
- When active space is exhausted, traverse the active space, copying all objects to the other space, then make the new space active and continue.
  - Only reachable objects are copied!
- Use “forwarding pointers” to keep consistency
  - Simple solution to avoiding having to have a table of old and new addresses, and to mark objects already copied

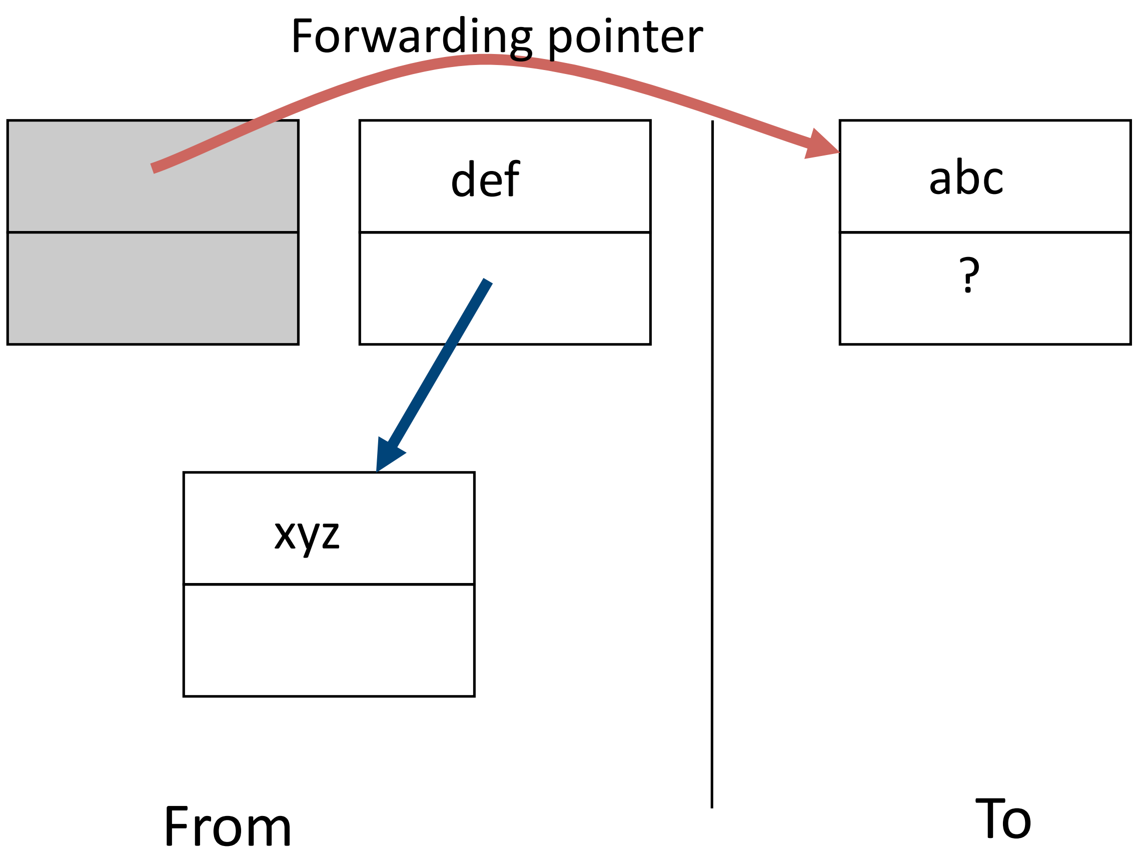
Forwarding Pointers: 1st copy "abc"



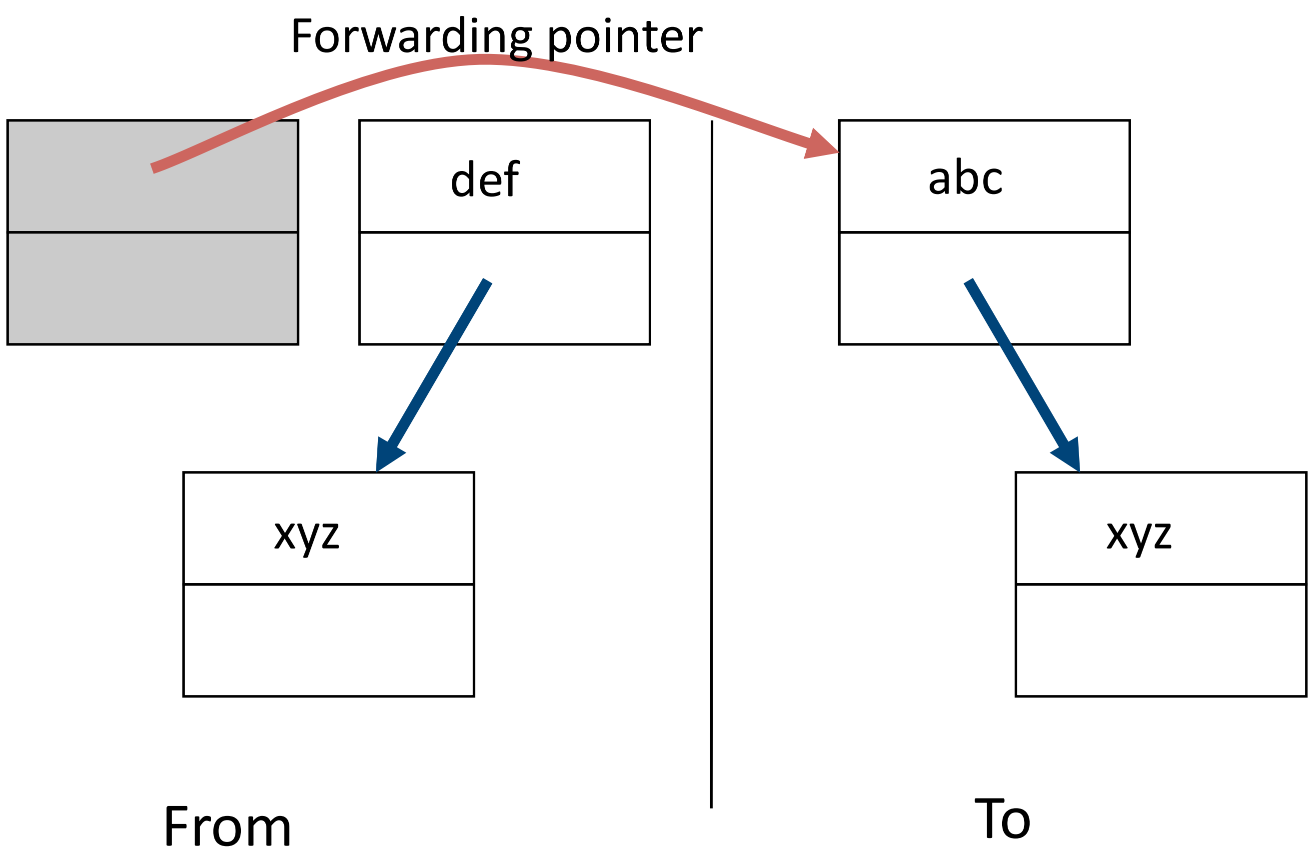
Forwarding Pointers: leave ptr to new abc



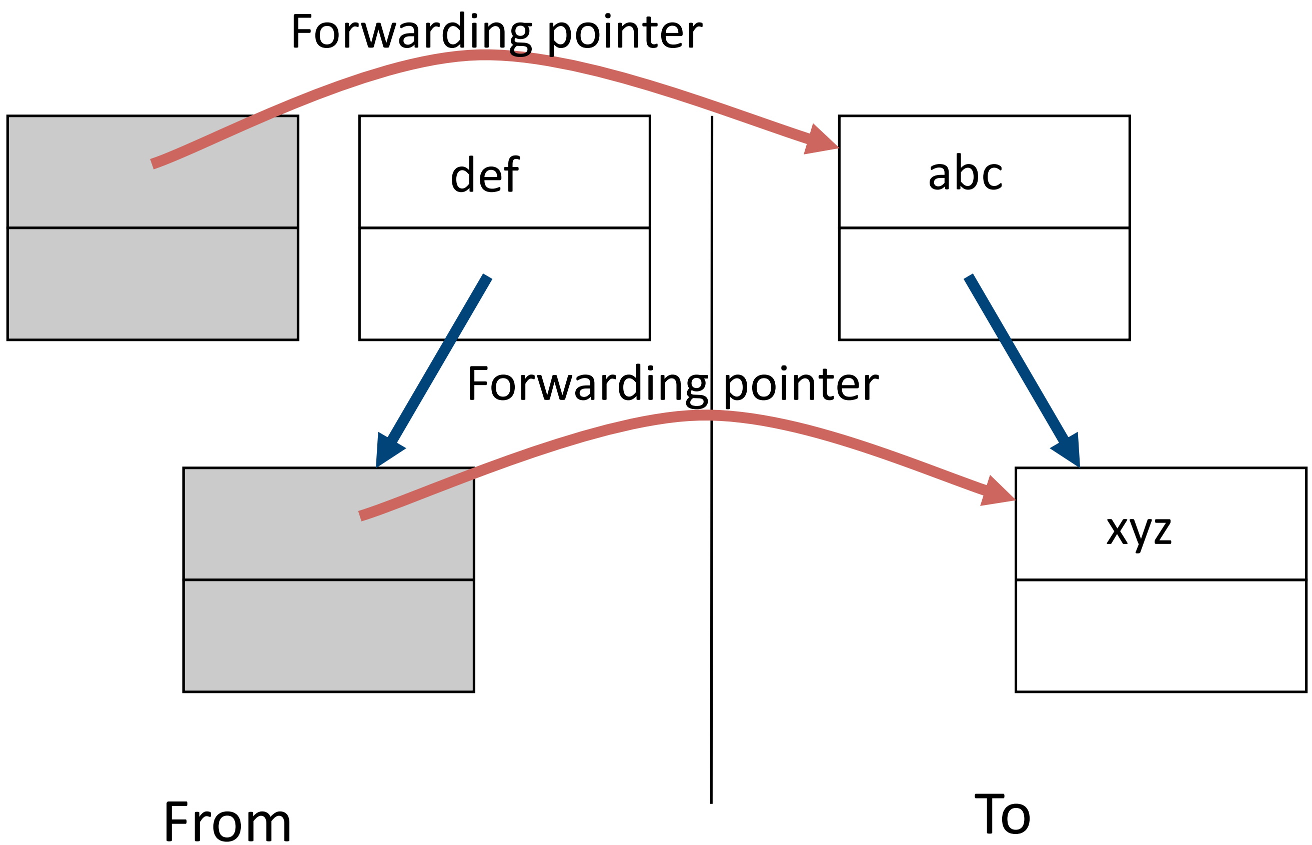
Forwarding Pointers : now copy "xyz"



Forwarding Pointers: leave ptr to new xyz

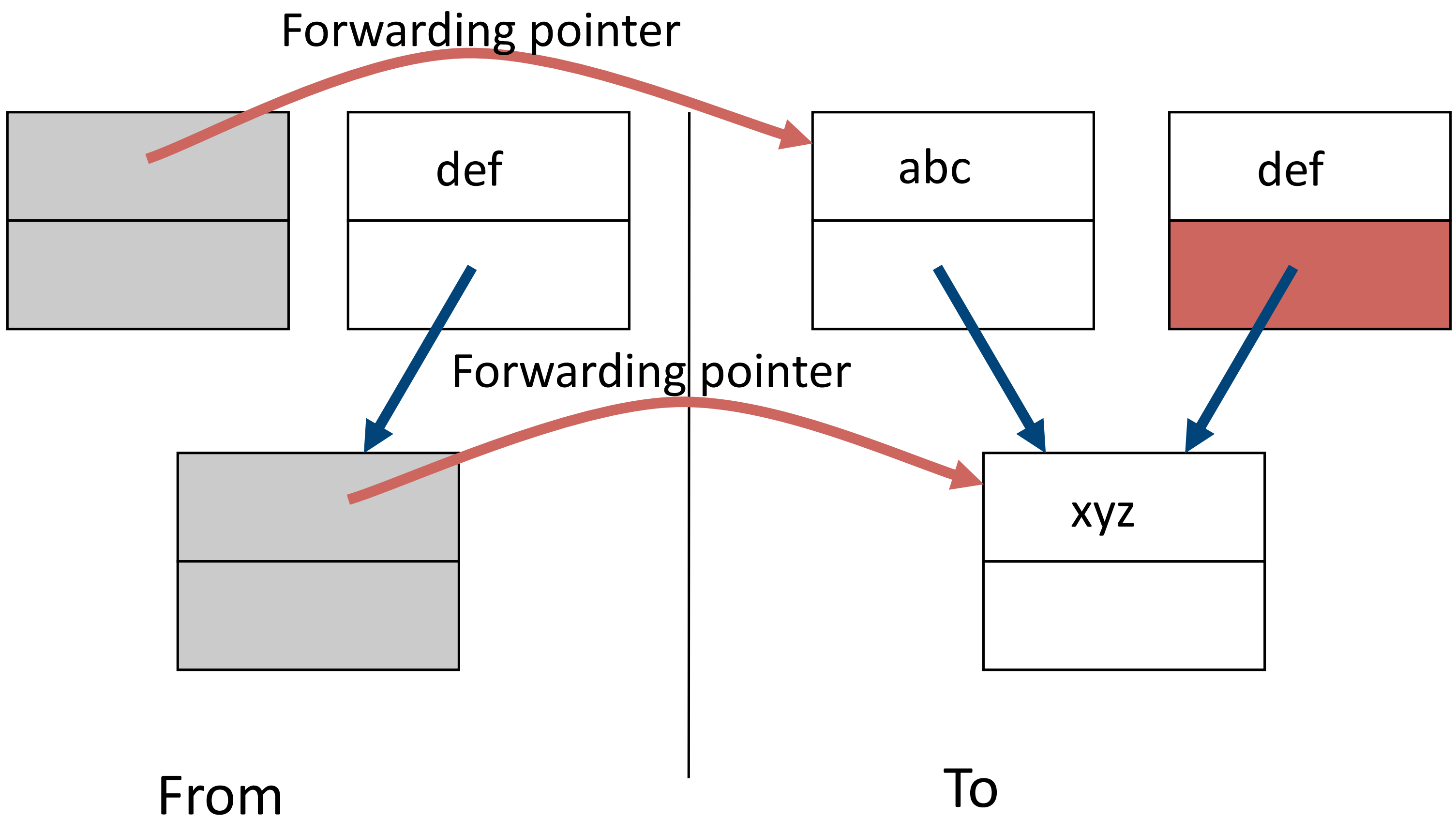


Forwarding Pointers: now copy “def”



Since xyz was already copied,
def uses xyz’s forwarding pointer
to find its new location

Forwarding Pointers



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Summary

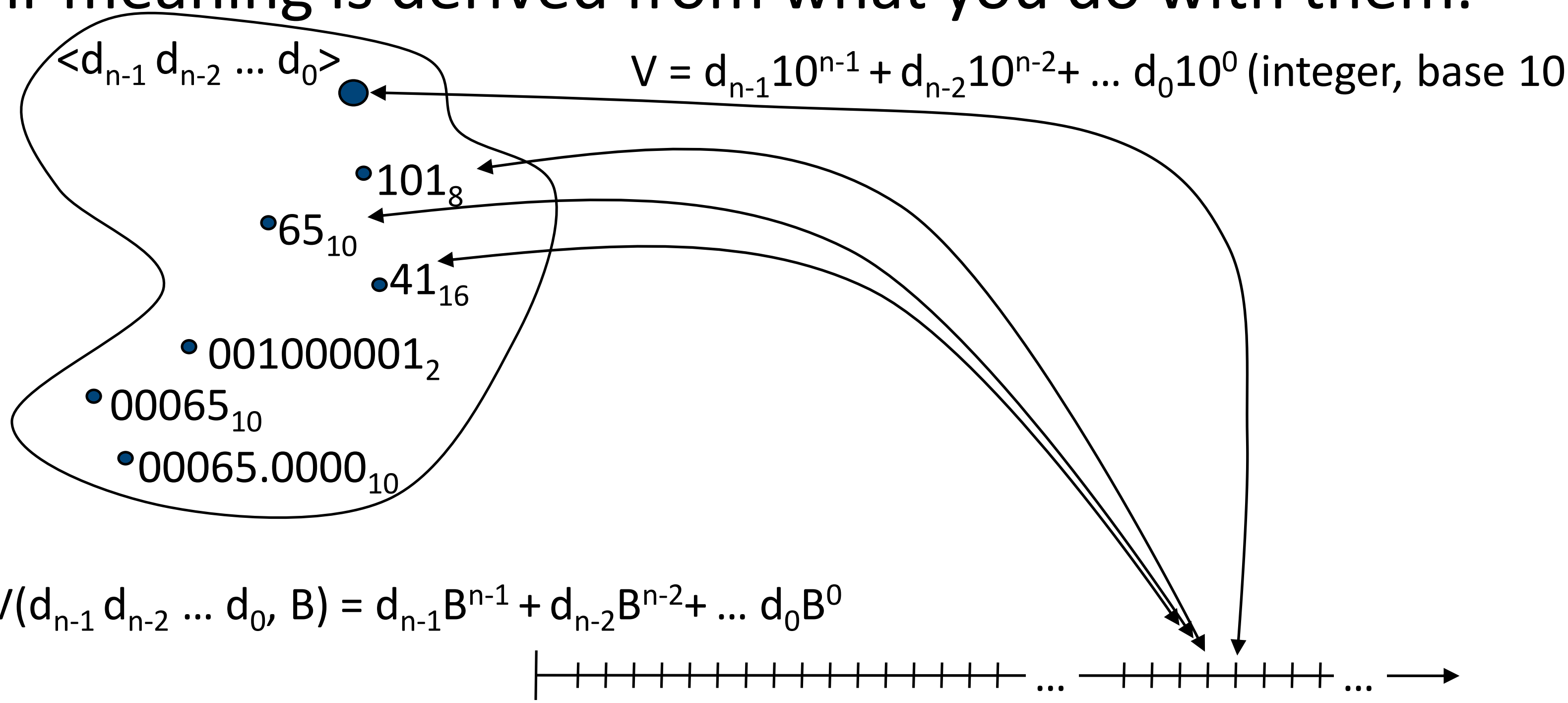
- Several techniques for managing heap via malloc and free: best-, first-, next-fit
  - 2 types of memory fragmentation: internal & external; all suffer from some kind of frag.
  - Each technique has strengths and weaknesses, none is definitively best
- Automatic memory management relieves programmer from managing memory.
  - All require help from language and compiler
  - Reference Count: not for circular structures
  - Mark and Sweep: complicated and slow, works
  - Copying: Divides memory to copy good stuff

Number Representations

- What do these numbers mean?
  - 101
  - 0101
- Depends on what representation!

Representation and Meaning

- Objects are represented as collections of symbols (bits, digits)
- Their meaning is derived from what you do with them.



Representation (how many bits?)

- Characters?
  - 26 letters → 5 bits ($2^5 = 32$)
  - upper/lower case + punctuation → 7 bits (in 8) ("ASCII")
  - standard code to cover all the world's languages → 8,16,32 bits ("Unicode") `www.unicode.com`
- Logical values?
  - 0 → False, 1 → True
- Colors? Ex: *Red (00)* *Green (01)* *Blue (11)*
- Remember: N bits → at most 2^N things

How many bits to represent π ?

a) 1
b) 9 (π = 3.14, so that's 011 . 001 100)
c) 64 (Since modern computers are 64-bit machines)
d) Every bit the machine has!
e) ∞

We are going to learn how to represent floating point numbers later!

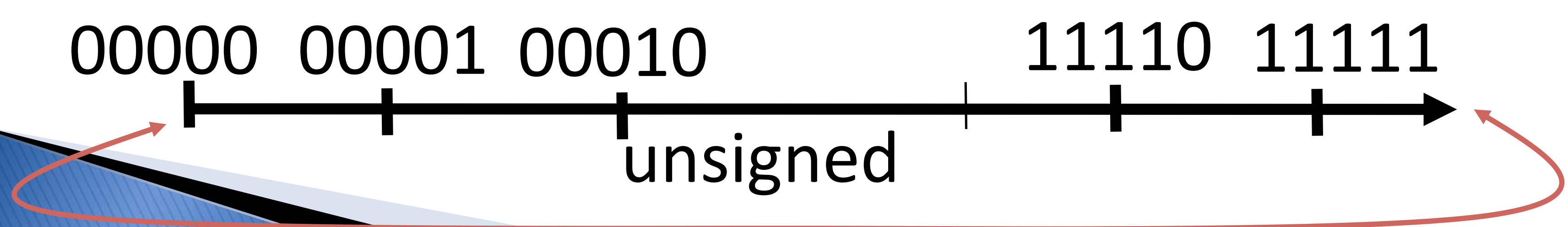
What to do with representations of numbers?

- Just what we do with numbers!
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them
- Example: 10 + 7 = 17
  - …so simple to add in binary that we can build circuits to do it!
  - subtraction just as you would in decimal
  - Comparison: How do you tell if X > Y ?

```
    1  1
    1  0  1  0
+   0  1  1  1
-------------------------
 1  0  0  0  1
```

What if too big?

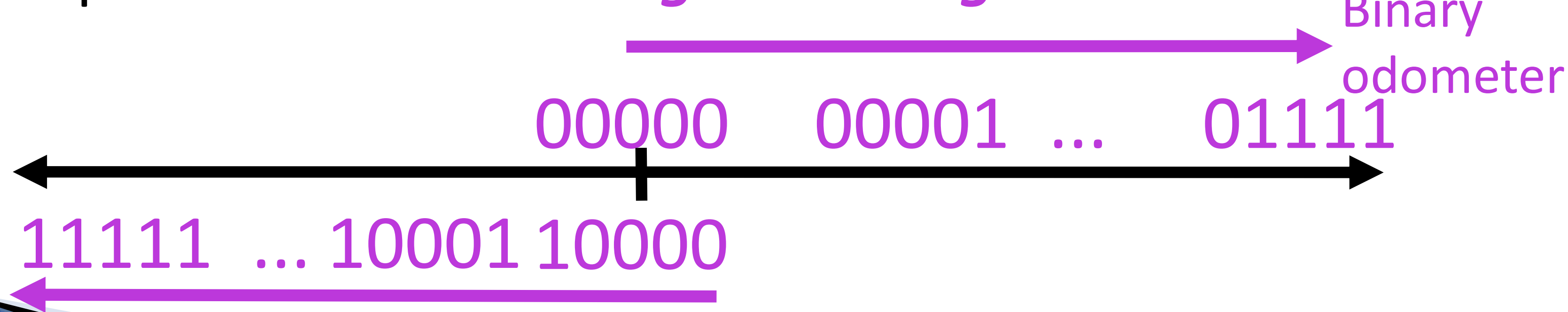
- Binary bit patterns above are simply representatives of numbers. Strictly speaking they are called “numerals”
- Numbers really have an ∞ number of digits
  - with almost all being same (00…0 or 11…1) except for a few of the rightmost digits
  - Just don’t normally show leading digits
- If result of add (or -, *, /) cannot be represented by these rightmost HW bits, ***overflow*** is said to have occurred.



Negative Numbers

- So far, ***unsigned numbers***

- Obvious solution: define leftmost bit to be sign!
  - 0 → + , 1 → –
  - Rest of bits can be numerical value of number
- Representation called ***sign and magnitude***

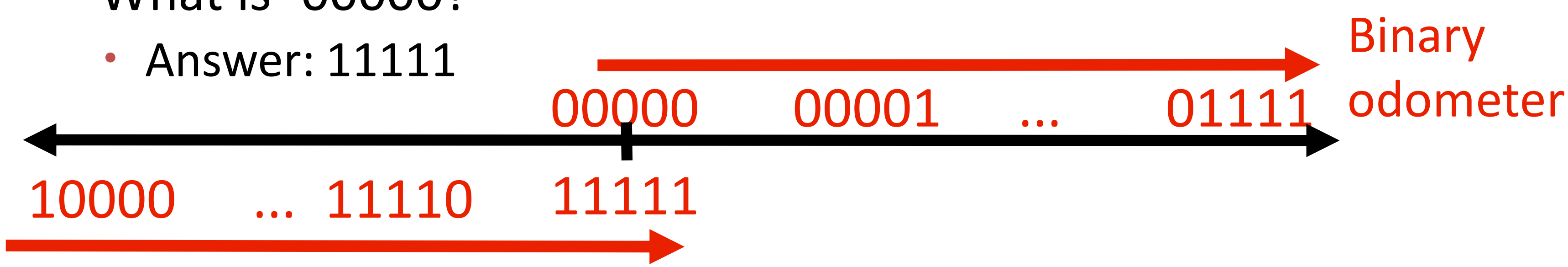


Shortcomings of Sign Magnitude?

- Arithmetic circuit complicated
  - Special steps depending whether signs are the same or not
- Also, two zeros
  - 0x00000000 = $+0_{ten}$
  - 0x80000000 = -0_{ten}
  - What would two 0s mean for programming?
- Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases!
- Therefore sign and magnitude abandoned

Another try

- Complement the bits
  - Example: $7_{10} = 00111_2 \quad -7_{10} = 11000_2$
  - Called ***One's Complement***
  - Note: positive numbers have leading 0s, negative numbers have leadings 1s.
  - What is -00000?
    - Answer: 11111



  - How many positive numbers in N bits? 2^{N-1}
  - How many negative numbers? 2^{N-1}

Shortcomings of One's complement?

- Arithmetic is less complicate than sign & magnitude.
- Still two zeros
  - 0x00000000 = $+0_{ten}$
  - 0xFFFFFFFF = -0_{ten}
- Although used for a while on some computer products, one's complement was eventually abandoned because another solution was better.

Standard Negative # Representation

- Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.
  - Solution! For negative numbers, complement, then add 1 to the result
- As with sign and magnitude, & one's complement, leading 0s → positive, leading 1s → negative
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1...1111 is -1, not -0
- This representation is ***Two's Complement***
- This makes the hardware simple!

In C: short, int, long long, intN_t (C99) are all signed integers.

Two's Complement Formula

- Can represent positive and negative numbers in terms of the bit value times a power of 2:

  $d_{31} \times -(2^{31}) + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$

- Example: 1101_{two}

  $= 1x\text{-}(2^3) + 1x2^2 + 0x2^1 + 1x2^0$

  $= -2^3 + 2^2 + 0 + 2^0$

  $= -8 + 4 + 0 + 1$

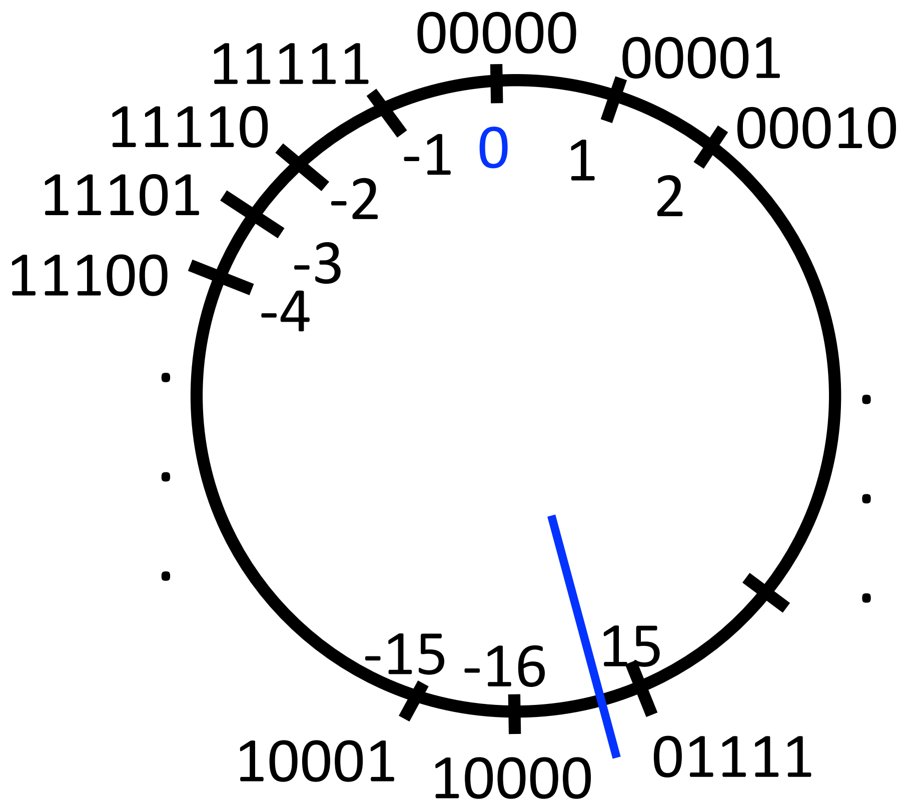
  $= -8 + 5$

  $= -3_{ten}$

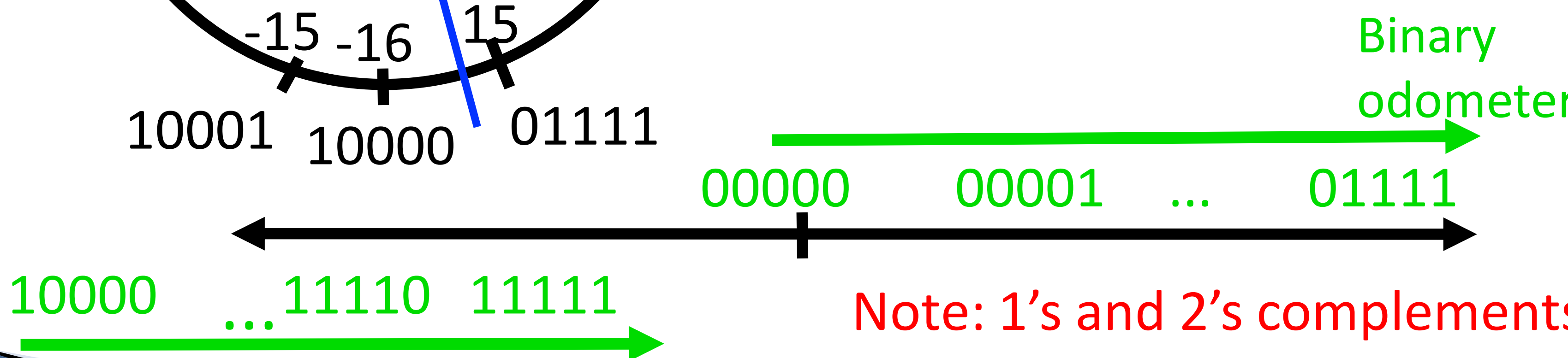
Example: -3 to +3 to -3:

x :	1101_{two}	(-3)
x':	0010_{two}	
+1:	0011_{two}	(3)
()':	1100_{two}	
+1:	1101_{two}	(-3)

2’s Complement Number “line”: N = 5

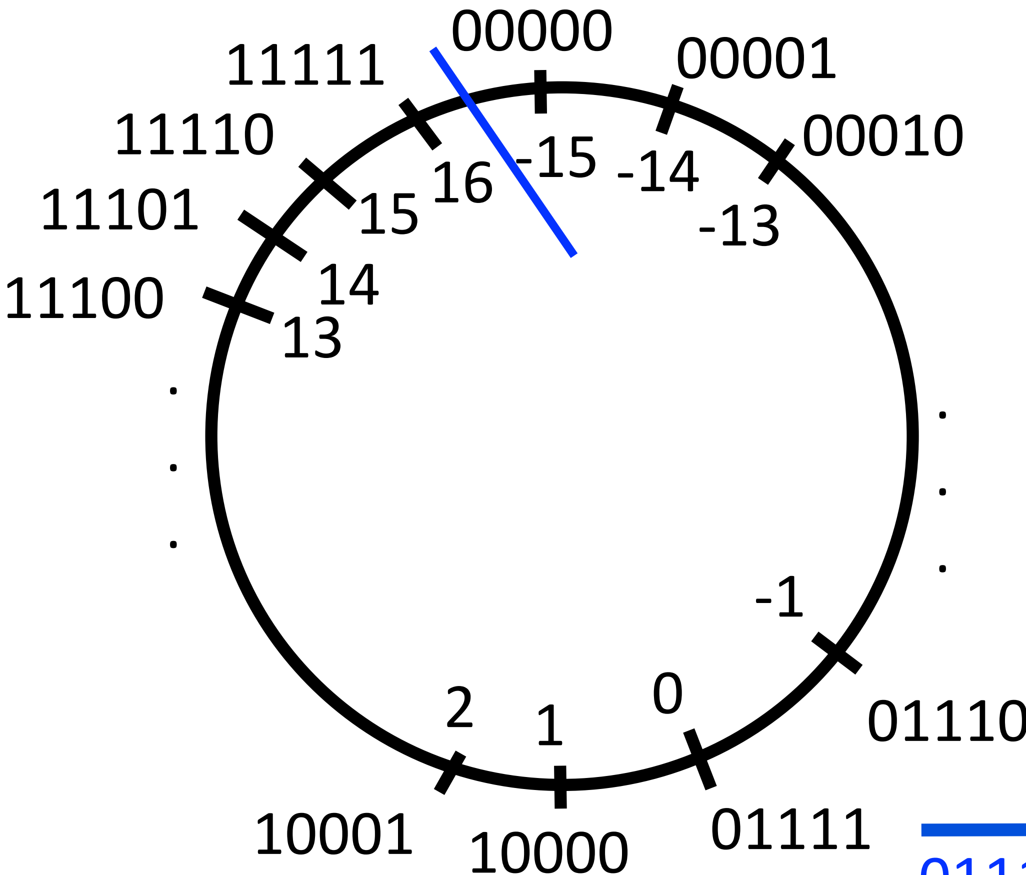


- 2^{N-1} non-negatives
- 2^{N-1} negatives
- one zero
- how many positives?
  - $2^{N-1} – 1$

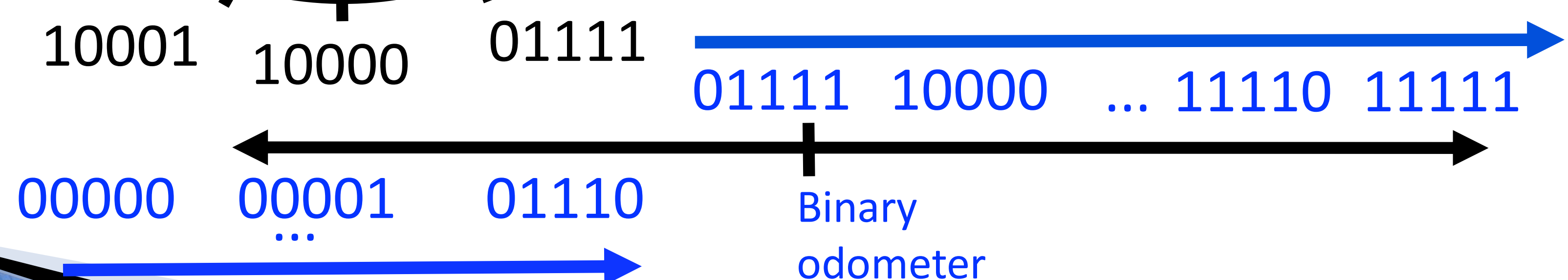


Note: 1’s and 2’s complements are used to represent negative numbers only!

Bias Encoding: N = 5 (bias = -15)



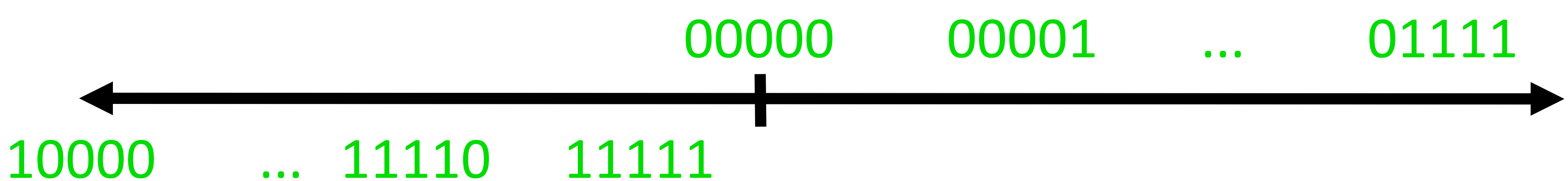
- Want 00… to represent the smallest number
- value = unsigned - bias
- Bias for N bits = $2^{N-1} – 1$
- one zero
- how many positives?
  - 2^{N-1}
  - (more than 2’s complement)



Summary

- We represent "things" in computers as particular bit patterns:
  - N bits → 2^N things
- Different integer encodings have different benefits; 1s complement and sign/mag have most problems.
- unsigned (C99's `uintN_t`) :

- 2's complement (C99's `intN_t`): universal, learn it!



- Overflow: numbers ∞ ; computers finite → errors!

Floating Point Numbers

- How best to represent: 2.75_{10}?
  - 2s Complement (but shift binary pt)
  - Bias (but shift binary pt)
  - Combination of 2 encodings
  - Combination of 3 encodings
  - We can't

Shifting ***binary point*** means "divide" number by some power of 2.

$11_{10} = 1011.0_2 \rightarrow \quad 10.110_2 = (11/4)_{10} = 2.75_{10}$

Representation of Fractions

"Binary Point" like decimal point signifies boundary between integer and fractional parts:

Example 6-bit representation:

xx.yyyy

2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

$10.1010_2 = 1x2^1 + 1x2^{-1} + 1x2^{-3} = 2.625_{10}$

If we assume "**fixed binary point**", range of 6-bit representations with this format:

0 to 3.9375 (almost 4)

Fractional Powers of 2

i	2^{-i}	
0.	1.0	1
1.	0.5	1/2
2.	0.25	1/4
3.	0.125	1/8
4.	0.0625	1/16
5.	0.03125	1/32
6.	0.015625	
7.	0.0078125	
8.	0.00390625	
9.	0.001953125	
10.	0.0009765625	
11.	0.00048828125	
12.	0.000244140625	
13.	0.0001220703125	
14.	0.00006103515625	
15.	0.000030517578125	