CSE 31
Computer Organization

Lecture 7 – Integer Representations (2)
MIPS Assembly Language

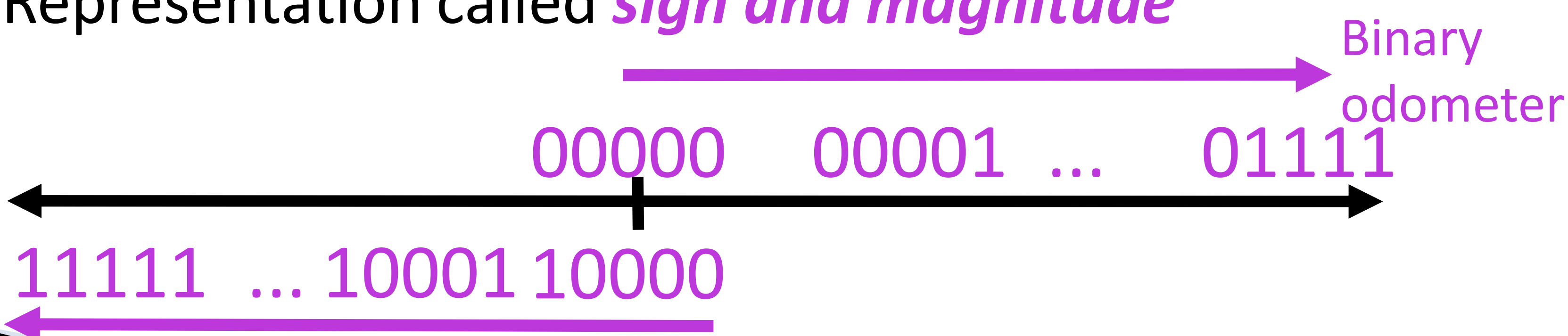
Announcement

- No Lecture next Monday (Holiday)
  - We do have labs next week
  - Extended due date for Monday lab.
- Lab #3 this week
  - Due at 11:59pm on the same day of your next lab
  - You must demo your submission to your TA within 14 days
- HW #1 out this Friday in CatCourses
  - Due Monday (2/25) at 11:59pm
- Reading assignment #1
  - Chapter 1.1 – 1.3 of zyBook
    - Do all Participation Activities in each section
    - Access through **CatCourses**
    - Due Wednesday (2/20) at 11:59pm
- Reading assignment #2
  - Chapter 2.1 – 2.9 of zyBook
    - Due Wednesday (2/27) at 11:59pm

Negative Numbers

- So far, ***unsigned numbers***

- Obvious solution: define leftmost bit to be sign!
  - 0 → + , 1 → –
  - Rest of bits can be numerical value of number
- Representation called ***sign and magnitude***

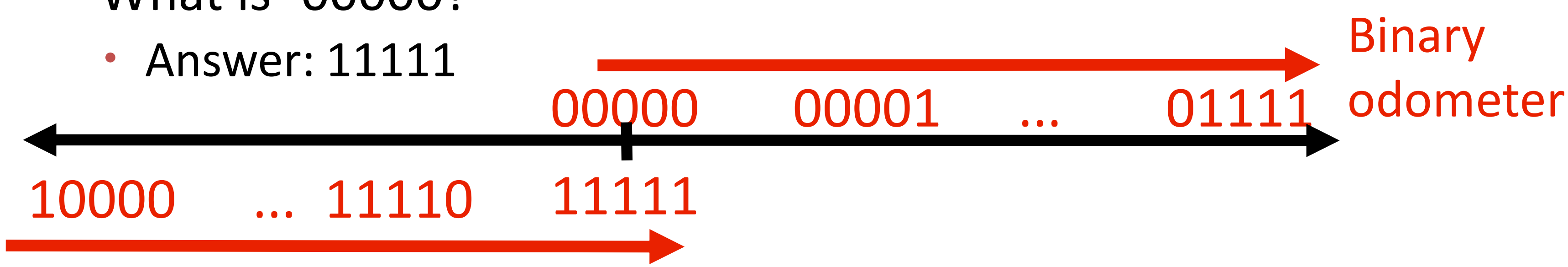


Shortcomings of Sign Magnitude?

- Arithmetic circuit complicated
  - Special steps depending whether signs are the same or not
- Also, two zeros
  - $0x00000000 = +0_{ten}$
  - $0x80000000 = -0_{ten}$
  - What would two 0s mean for programming?
- Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases!
- Therefore sign and magnitude abandoned

Another try

- Complement the bits
  - Example: $7_{10} = 00111_2 \quad -7_{10} = 11000_2$
  - Called ***One's Complement***
  - Note: positive numbers have leading 0s, negative numbers have leadings 1s.
  - What is -00000?
    - Answer: 11111



  - How many positive numbers in N bits? 2^{N-1}
  - How many negative numbers? 2^{N-1}

Shortcomings of One's complement?

- Arithmetic is less complicate than sign & magnitude.
- Still two zeros
  - $0x00000000 = +0_{ten}$
  - $0xFFFFFFFF = -0_{ten}$
- Although used for a while on some computer products, one's complement was eventually abandoned because another solution was better.

Standard Negative # Representation

- Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.
  - Solution! For negative numbers, complement, then add 1 to the result
- As with sign and magnitude, & one's complement, leading 0s → positive, leading 1s → negative
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1...1111 is -1, not -0
- This representation is ***Two's Complement***
- This makes the hardware simple!

In C: short, int, long long, intN_t (C99) are all signed integers.

Two's Complement Formula

- Can represent positive and negative numbers in terms of the bit value times a power of 2:

  $d_{31} \times -(2^{31}) + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$

- Example: 1101_{two}

  $= 1 \mathrm{x} -(2^3) + 1 \mathrm{x} 2^2 + 0 \mathrm{x} 2^1 + 1 \mathrm{x} 2^0$

  $= -2^3 + 2^2 + 0 + 2^0$

  $= -8 + 4 + 0 + 1$

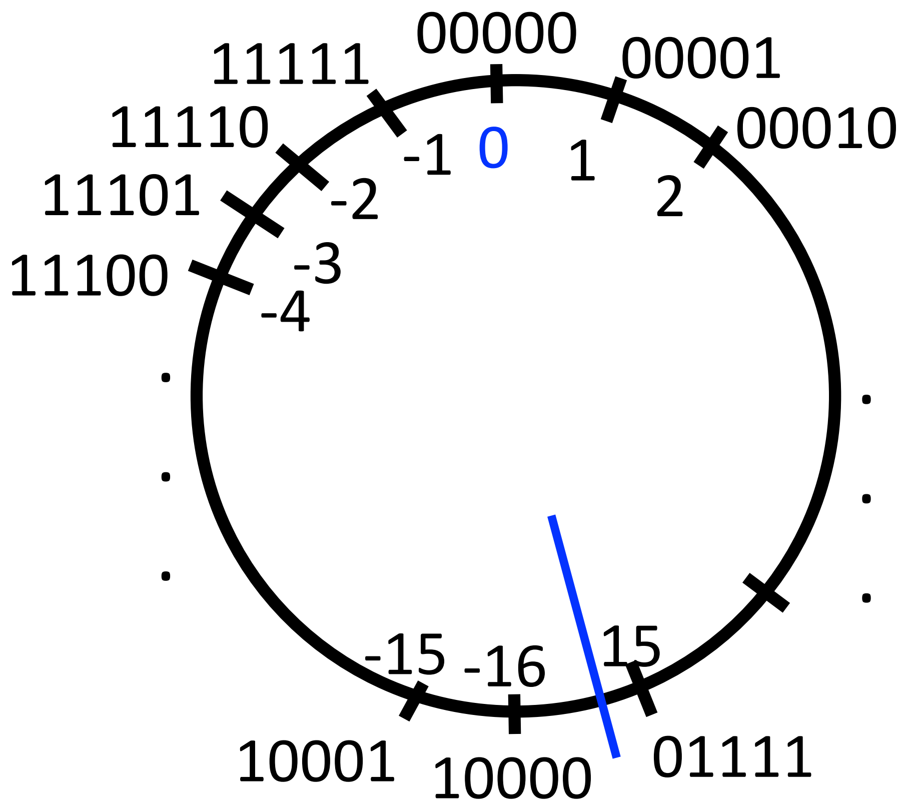
  $= -8 + 5$

  $= -3_{ten}$

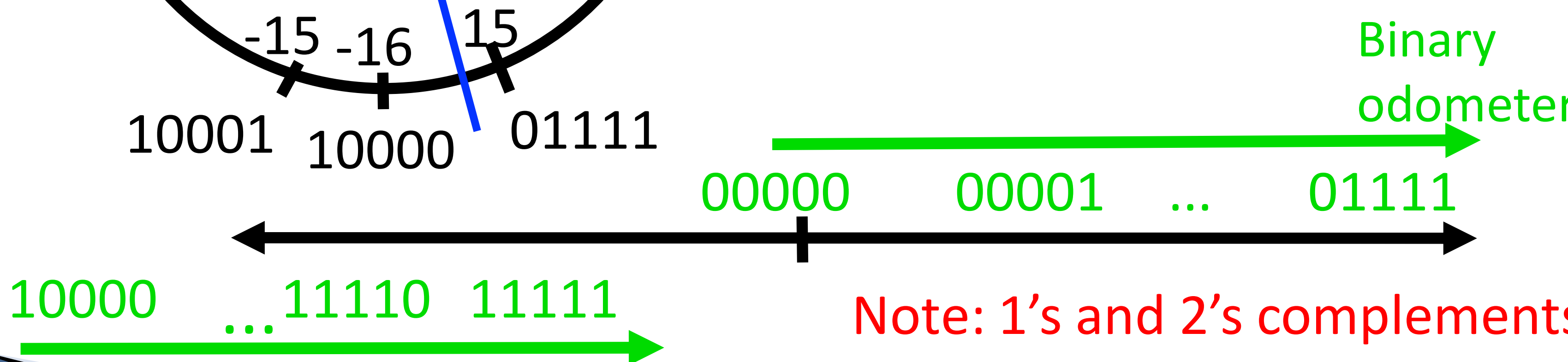
Example: -3 to +3 to -3:

x :	1101_{two}	(-3)
x':	0010_{two}	
+1:	0011_{two}	(3)
()':	1100_{two}	
+1:	1101_{two}	(-3)

2’s Complement Number “line”: N = 5

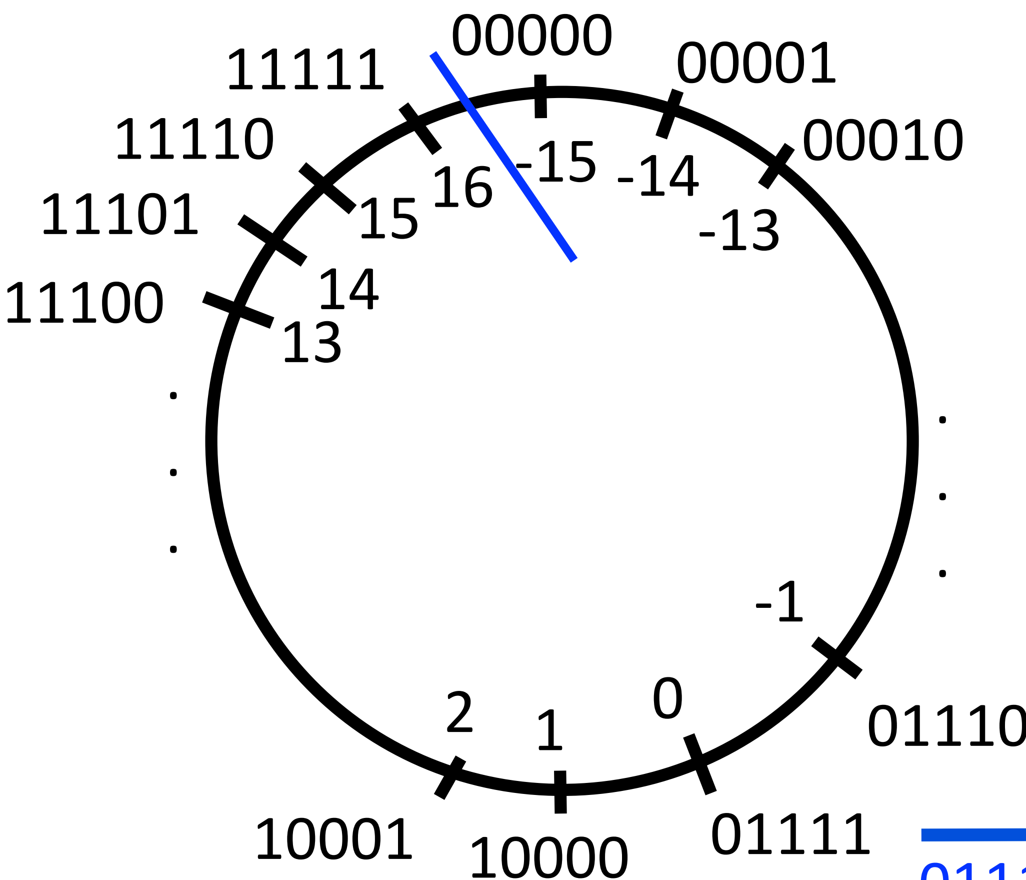


- 2^{N-1} non-negatives
- 2^{N-1} negatives
- one zero
- how many positives?
  - $2^{N-1} – 1$

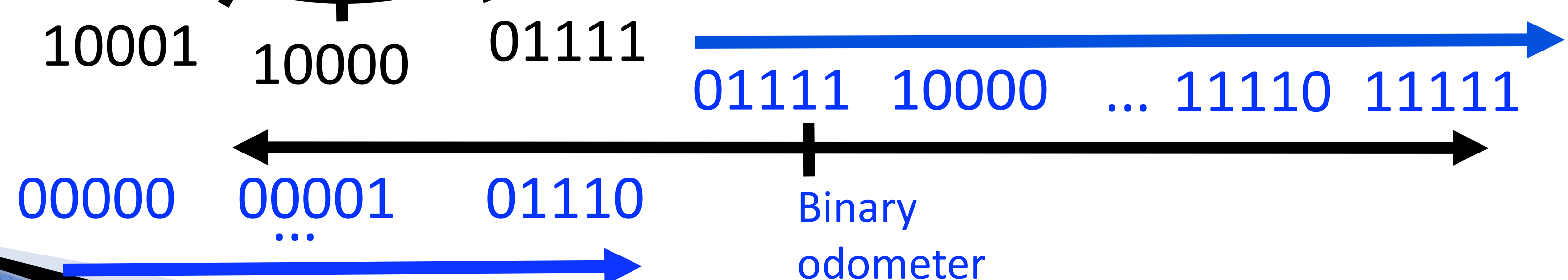


Note: 1’s and 2’s complements are used to represent negative numbers only!

Bias Encoding: N = 5 (bias = -15)



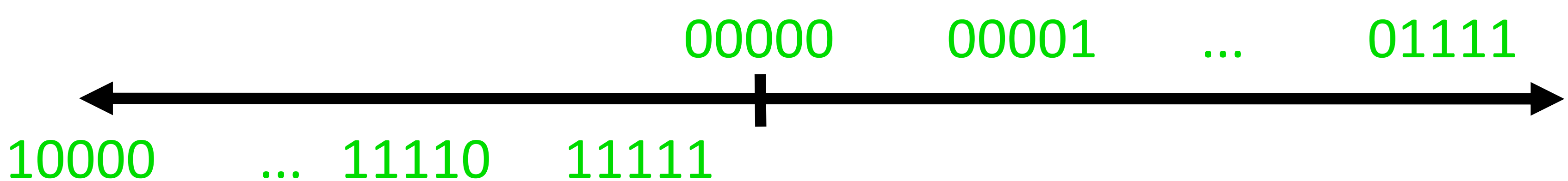
- Want 00… to represent the smallest number
- value = unsigned - bias
- Bias for N bits = $2^{N-1} – 1$
- one zero
- how many positives?
  - 2^{N-1}
  - (more than 2's complement)



Summary

- We represent "things" in computers as particular bit patterns:
  - N bits → 2^N things
- Different integer encodings have different benefits; 1s complement and sign/mag have most problems.
- unsigned (C99's `uintN_t`) :

- 2's complement (C99's `intN_t`): universal, learn it!



- Overflow: numbers ∞ ; computers finite → errors!

Floating Point Numbers

- How best to represent: 2.75_{10}?
  - 2s Complement (but shift binary pt)
  - Bias (but shift binary pt)
  - Combination of 2 encodings
  - Combination of 3 encodings
  - We can't

Shifting ***binary point*** means "divide" number by some power of 2.

$$11_{10} = 1011.0_2 \rightarrow \quad 10.110_2 = (11/4)_{10} = 2.75_{10}$$

Representation of Fractions

“Binary Point” like decimal point signifies boundary between integer and fractional parts:

Example 6-bit representation:

xx.yyyy

2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

$10.1010_2 = 1x2^1 + 1x2^{-1} + 1x2^{-3} = 2.625_{10}$

If we assume “**fixed binary point**”, range of 6-bit representations with this format:

0 to 3.9375 (almost 4)

Fractional Powers of 2

i	2^{-i}	
0.	1.0	1
1.	0.5	1/2
2.	0.25	1/4
3.	0.125	1/8
4.	0.0625	1/16
5.	0.03125	1/32
6.	0.015625	
7.	0.0078125	
8.	0.00390625	
9.	0.001953125	
10.	0.0009765625	
11.	0.00048828125	
12.	0.000244140625	
13.	0.0001220703125	
14.	0.00006103515625	
15.	0.000030517578125	