Financial Time Series Analysis Using Wavelet Transform

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Introduction

- Financial time series data, like stock prices and interest rates, are complex and non-stationary. Traditional methods struggle to capture both time and frequency variations, but wavelets excel here.
- By providing a time-frequency representation, wavelets can detect trends, volatility, and anomalies across different time scales, making them ideal for financial analysis.
- This presentation explores how wavelets reveal insights in financial data that other techniques may miss.

Fourier Transforms

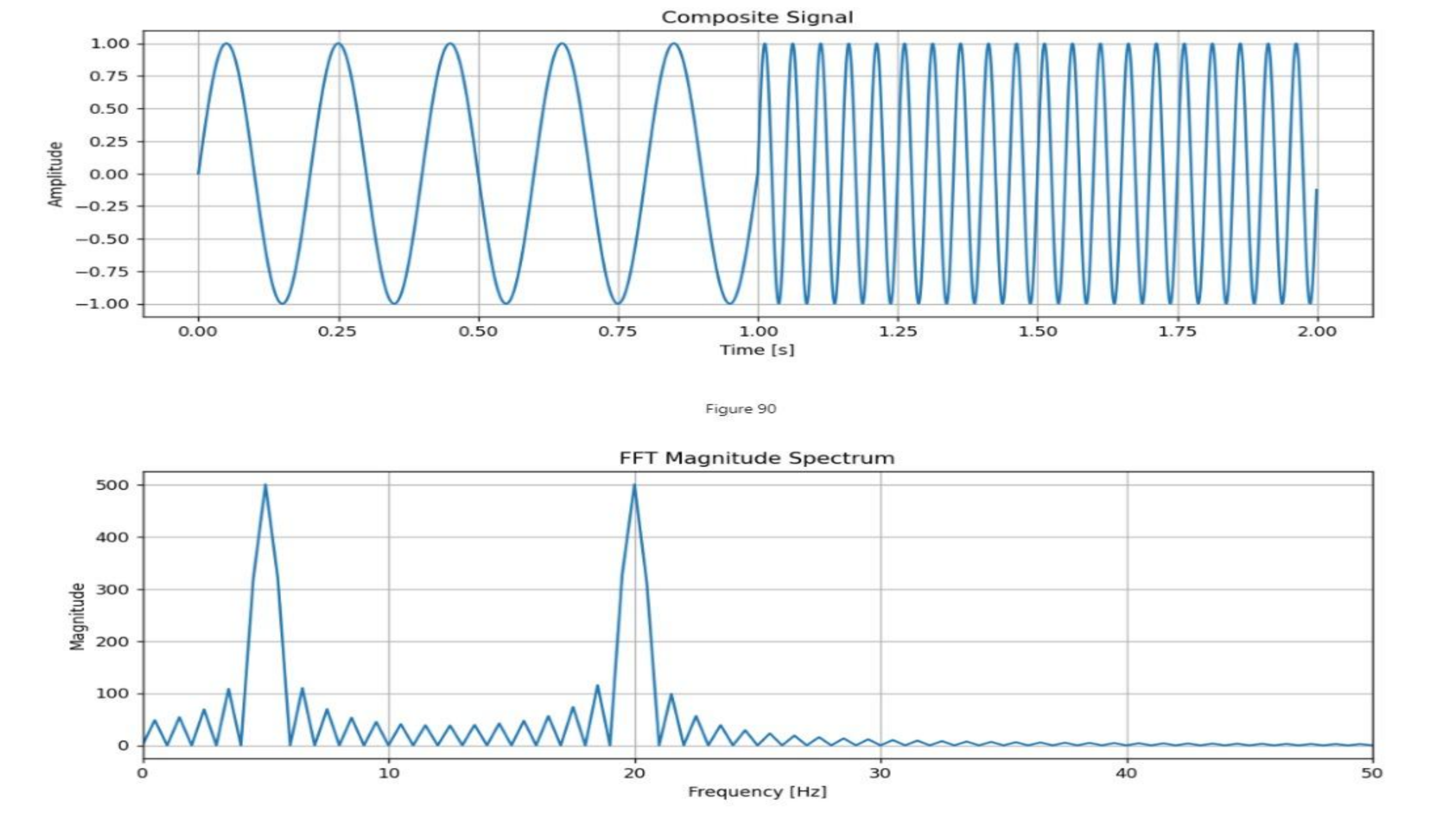
- Fourier Transform (FT) decomposes a signal into its constituent frequencies.
- It can be calculated using:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Where ***F(ω)*** is the frequency domain representation, and ***f(t)*** is the time domain signal.

- Efficient computation (using FFT) enables applications in signal processing, audio, and image compression.

Fourier Transform Example



Short-Time Fourier Transforms (STFT)

- STFT applies Fourier Transform on small segments of a signal, capturing frequency information over short time windows.
- It can be calculated using:

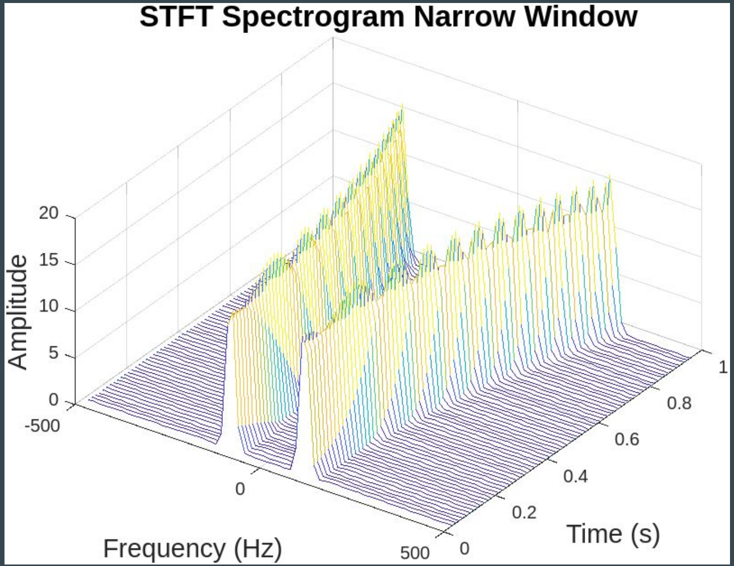
$$STFT\{f(t)\}(t,\omega) = \int_{-\infty}^{\infty} f(\tau) \cdot w(t-\tau) \cdot e^{-j\omega\tau}\, d\tau$$

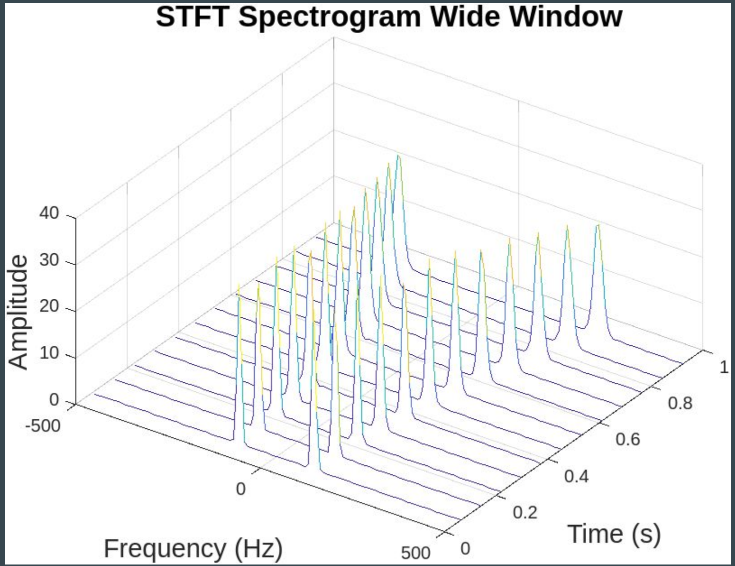
Where ***w(t)*** represents a window function that isolates a segment of the signal for localized frequency analysis.

- Useful for analyzing non-stationary signals where frequency content changes over time.

Short-Time Fourier Transforms (STFT)

- Dilemma of Resolution:
  - Narrow Window (Good Time Resolution) → Poor Frequency Resolution
  - Wide Window (Poor Time Resolution) → Good Frequency Resolution





Wavelet Theory

- Wavelets are functions that decompose signals into different scales and resolutions, capturing both time and frequency information.
- It can be calculated using:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a, b \in \mathbb{R}$$

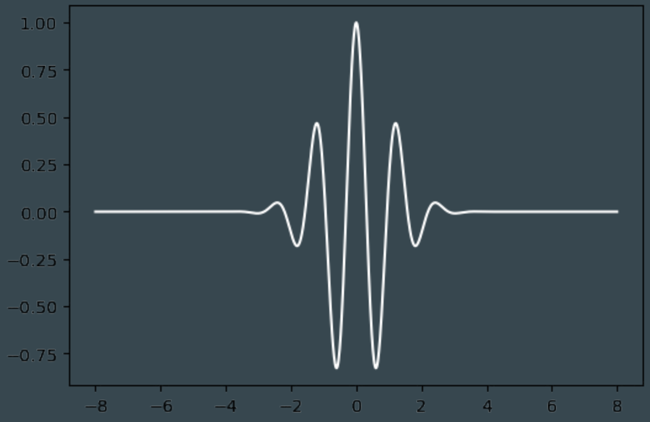
Where ***a*** and ***b*** are the Dilation and Translation parameters respectively.

- Wavelets are used for multi-resolution analysis, allowing for efficient signal representation, compression, and noise reduction in various applications such as image processing, audio analysis, and time-frequency signal analysis.

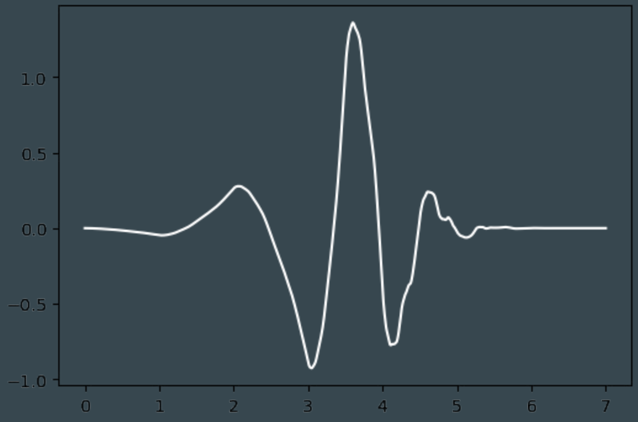
Wavelet Theory

- Mother wavelets are the fundamental wavelet functions used to generate a family of wavelets through scaling and translation, serving as the basis for multi-resolution analysis in signal processing.

Morlet Wavelet (morl)



Daubechies Wavelet (db4)



Continuous Wavelet Transforms

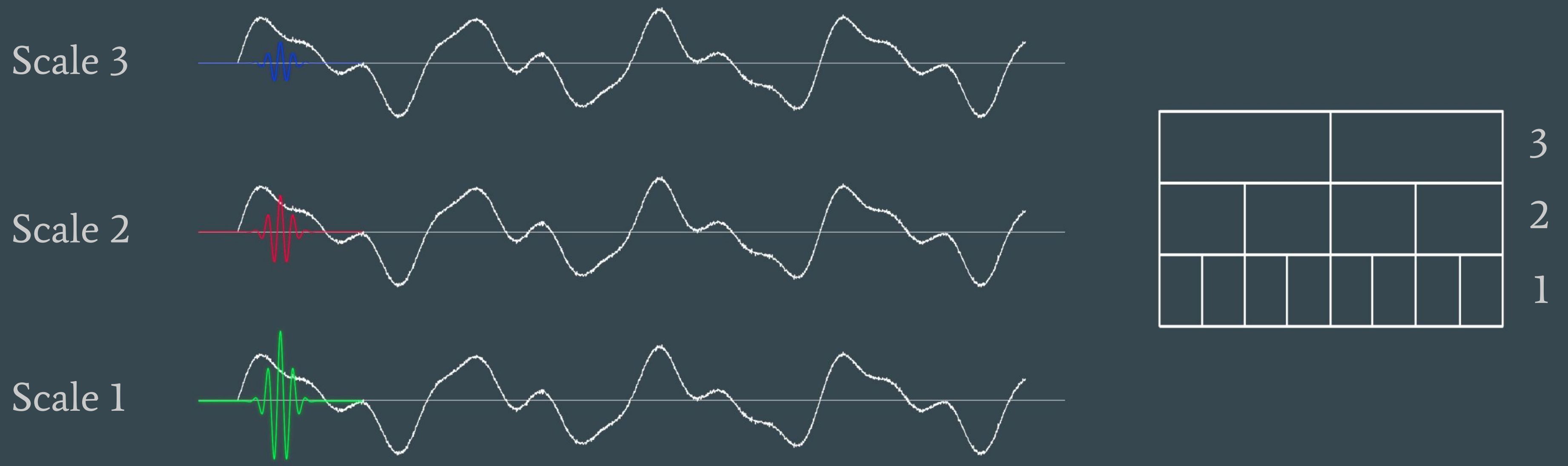
- The Continuous Wavelet Transform (CWT) analyzes signals by decomposing them into wavelets at various scales, providing a time-frequency representation.
- It is given by the equation:

$$CWT\{f(t)\}(\tau, s) = \int_{-\infty}^{\infty} f(t) \cdot \psi^* \left(\frac{t - \tau}{s} \right) dt$$

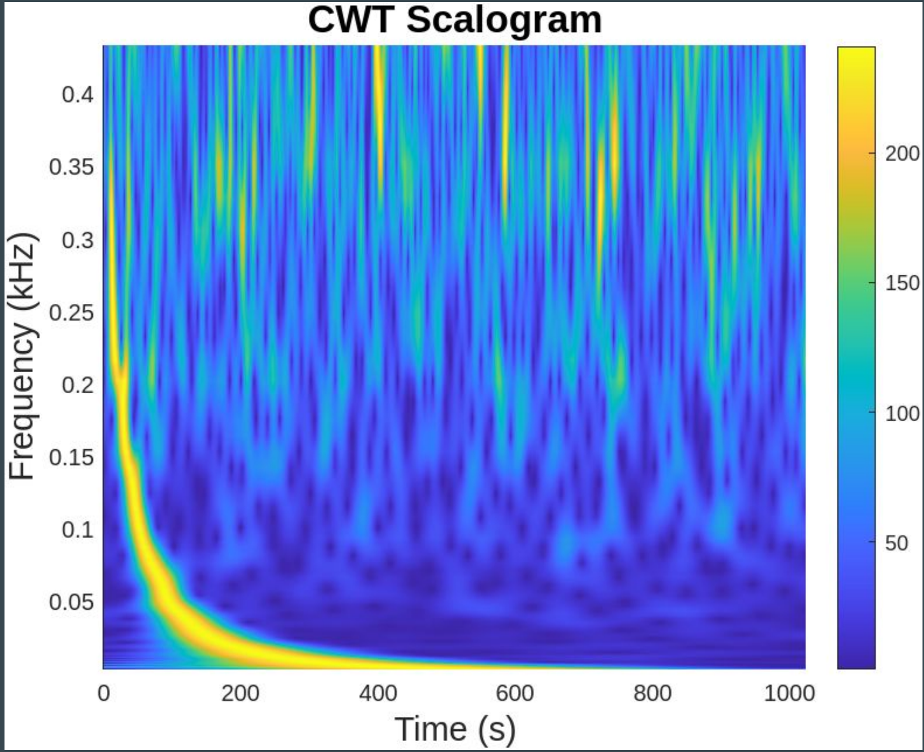
Where ψ^* is the complex conjugate of the mother wavelet.

- CWT provides detailed frequency information across different scales, making it suitable for non-stationary signals.

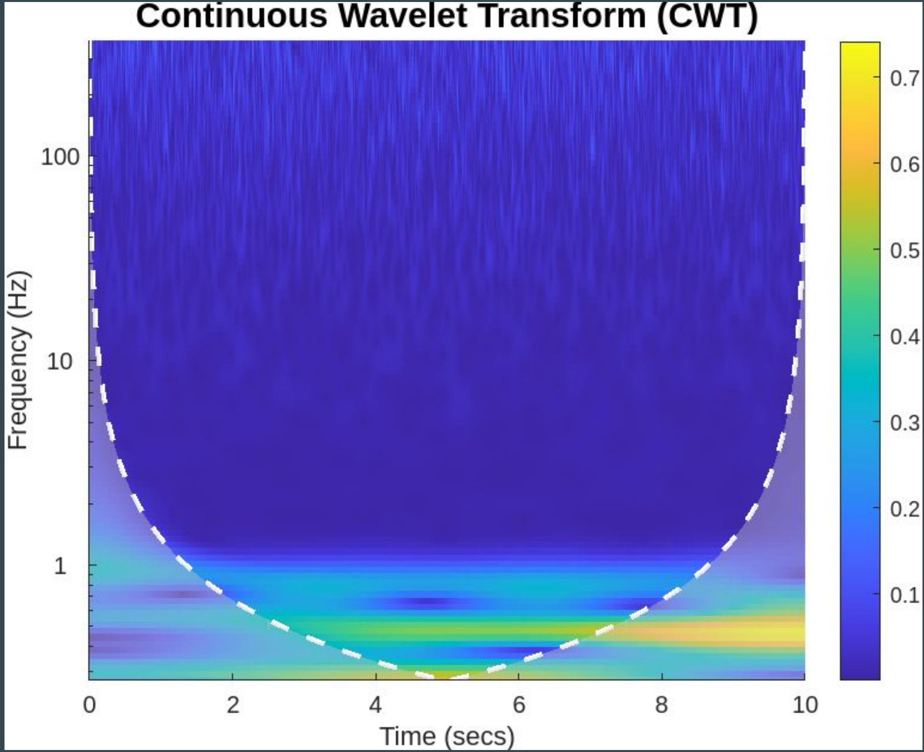
Continuous Wavelet Transforms



Continuous Wavelet Transforms Example



Continuous Wavelet Transforms Example



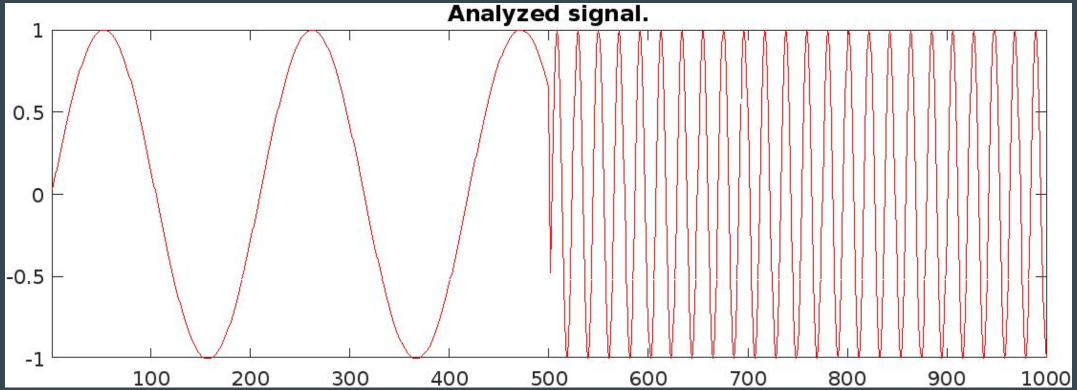
Discrete Wavelet Transforms

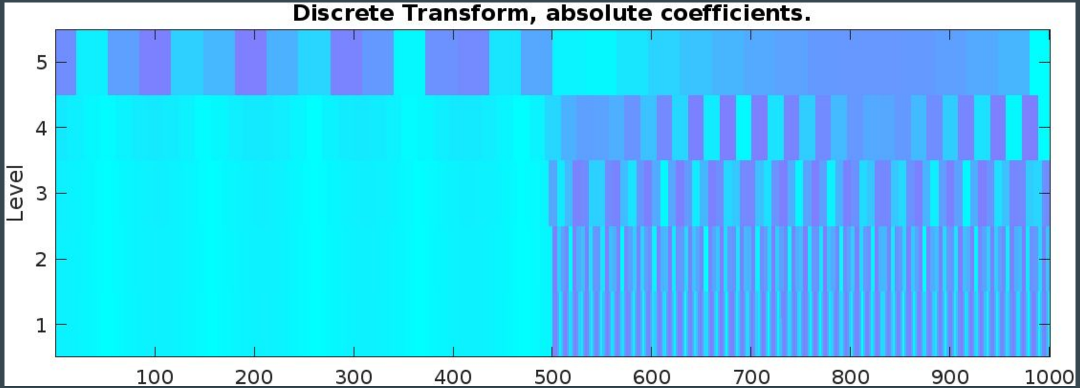
- The Discrete Wavelet Transform (DWT) decomposes a discrete signal into a set of wavelet coefficients, allowing for efficient multi-resolution analysis and signal representation.
- It is given by the equation:

$$DWT\{f[n]\} = \sum_{m} \sum_{k} f[m] \cdot \psi_{j,k}[n]$$

- DWT provides a hierarchical representation of signals at different resolutions, making it useful for various applications such as compression and noise reduction.

Discrete Wavelet Transform Example





Understanding the Market

Market Indicators

Indicator is a mathematical calculation or model applied to price, volume, or open interest data to provide insight into market trends, momentum, volatility, or other significant aspects of the market.

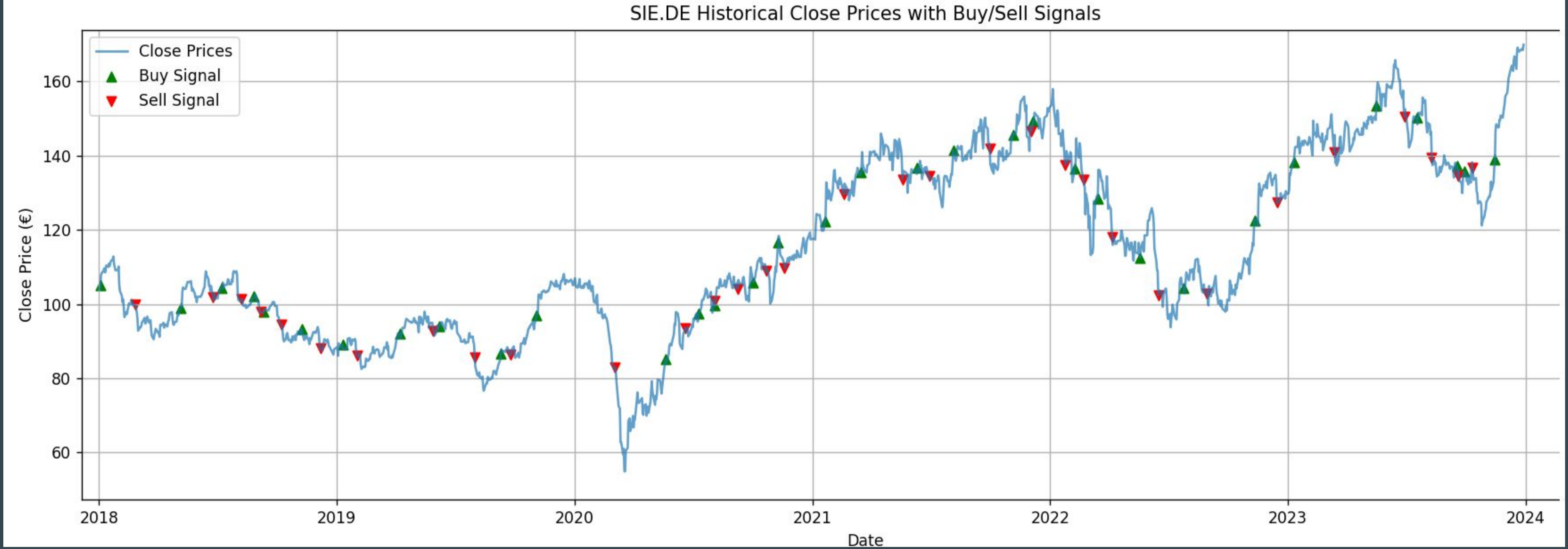
Some Famous Indicators :

- Moving Average Convergence Divergence (MACD)
- Relative Strength Index (RSI)
- Volume Weighted Average Price (VWAP)

We can use Wavelets as a potential Indicator!

Siemens closing price stock data

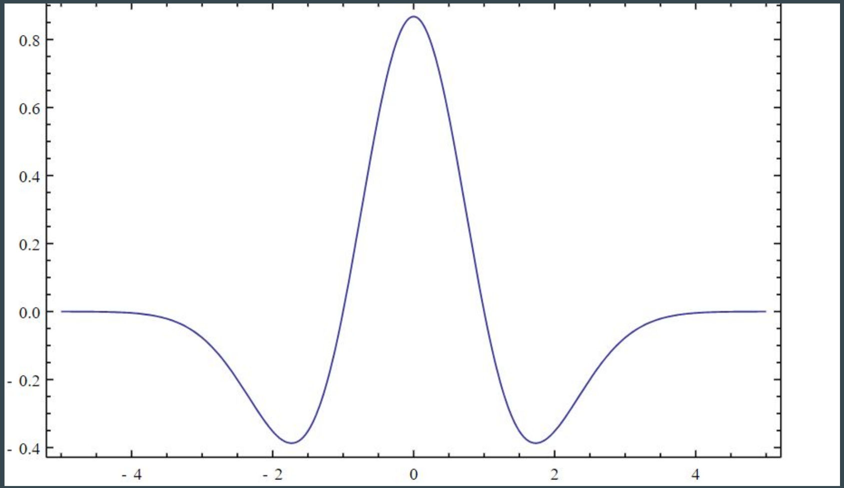
Threshold = 0 and wavelets from scale 1 to 14 were considered.



Ricker Wavelet - Used for our Analysis

- Also known as the "Mexican Hat" wavelet, it's effective for analyzing frequency content of the stock market data.
- It is given by:

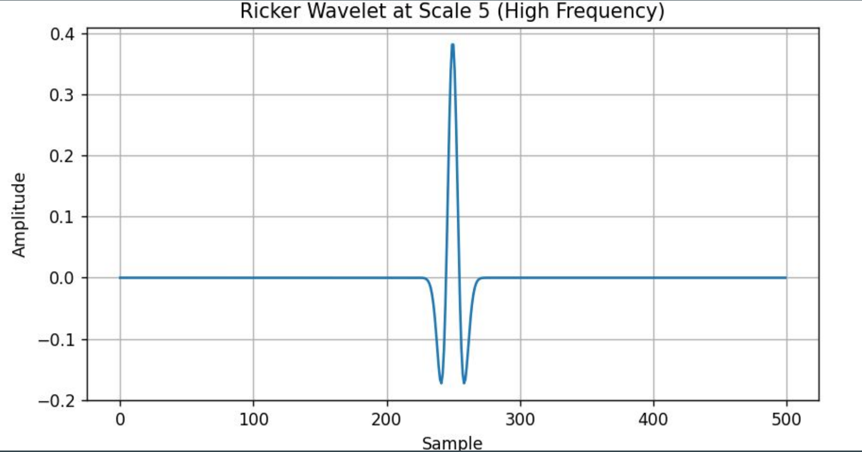
$$\psi(t) = \frac{2}{\sqrt{3\sigma}\pi^{1/4}}\left(1 - \left(\frac{t}{\sigma}\right)^2\right)e^{-\frac{t^2}{2\sigma^2}}$$

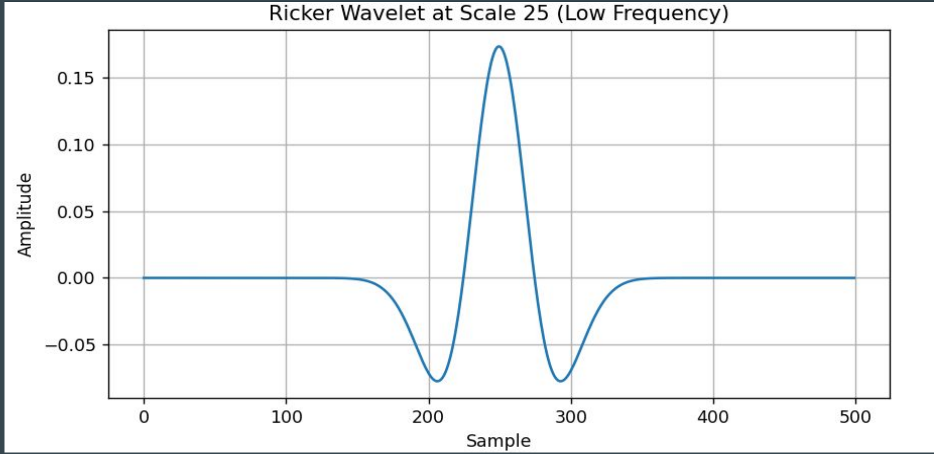


The Essence of Wavelet Transform

-> Smaller scale wavelets capture higher frequency component of the signal, which effectively gets **short term** trend in stock prices.

-> Larger scale wavelets capture low frequency component of the signal which gets captures **long term** trend in stock prices.



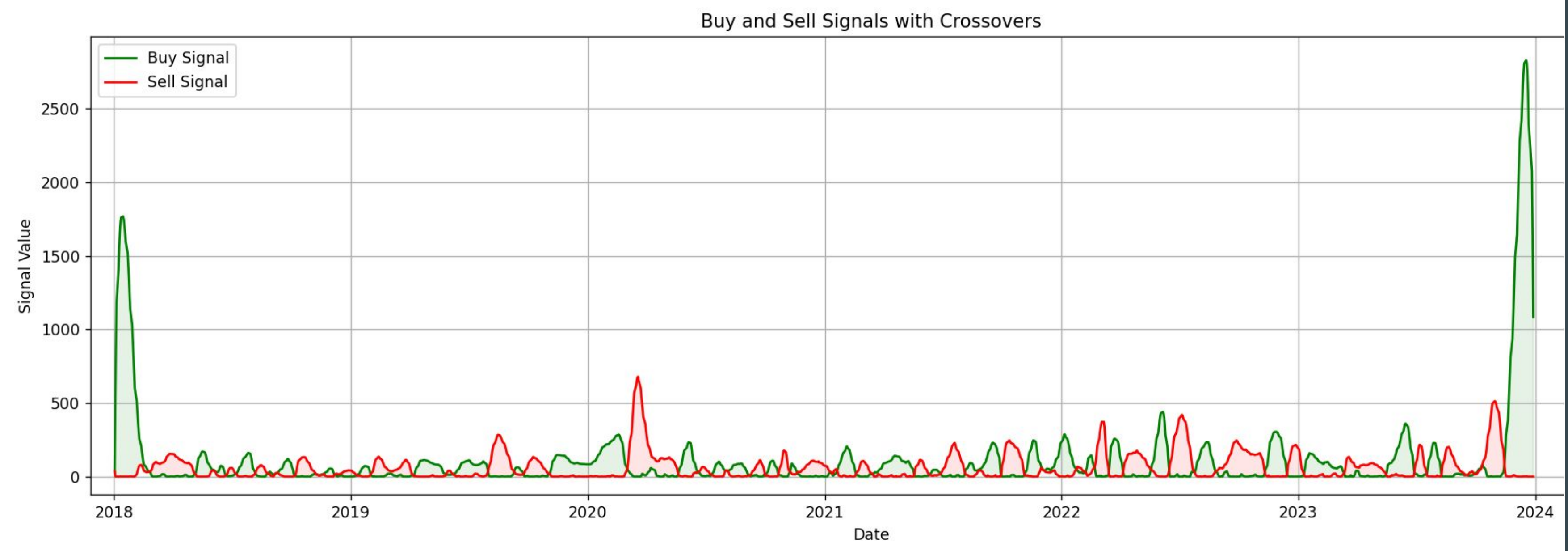


Intuition

- A higher value of CWT coefficient got using Ricker wavelet tends to show an upward tread relative to a lower CWT coefficient, but again that depends on the scale at which we are using it.
- *Show Animation*

Buy, sell signal and crossover

- Buy signal tells the extend to buy the stock and sell signal denotes the extend to sell the stock.
- When a buy signal crosses over sell signal we should change our position to buy and vice versa if sell signal cross-over buy signal.



How buy and sell signal is calculated?

Day	Close Price
1	100
2	102
3	101
4	103
5	105
6	104
7	106

Day	Scale = 2 Coefficient	Scale = 5 Coefficient	Buy Signal	Sell Signal
1	0	-4	0 + 0 = **0**	0 + 4= **4**
2	+1.0	-0.5	1.0 + 0 = **1.0**	0 + 0.5 = **0.5**
3	-1.5	-0.5	0 + 0 = **0**	1.5 + 0.5 = **2**
4	+1.2	+1.5	1.2 + 1.5 = **2.7**	0 + 0 = **0**
5	+1.0	-1.5	1.0 + 0 = **1**	0 + 1.5 = **1.5**
6	-0.5	+1.0	0 + 1.0 = **1.0**	0.5 + 0 = **0.5**
7	+1.0	+1.5	1.0 + 1.5 = **2.5**	0 + 0 = **0**

Hyper - parameters

Hyperparameters are tunable parameters that are adjusted to achieve the best model performance. In our case it includes :

- Type of wavelet to choose
- Threshold for calculating buy and sell signal
- Scales of the wavelets to consider
- Weightage to each of coefficients of different scales while calculating the buy/sell signal.

Challenges

- For different stocks we need to consider different hyperparameters and to find the best hyper parameter it is computationally intensive
- Financial time series are non-stationary, with statistical properties that change over time. Makes it difficult to select fixed parameters that are effective throughout the dataset.
- Edge Effects

THANK YOU