# MPS algorithms for optimization and time evolution:
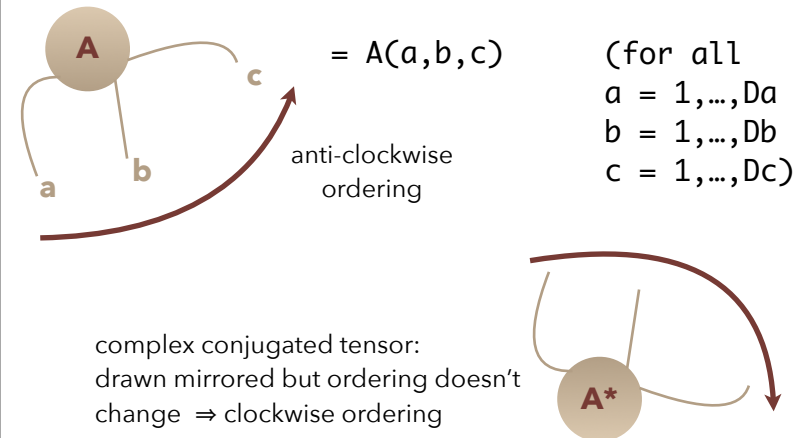## exercises

Jutho Haegeman
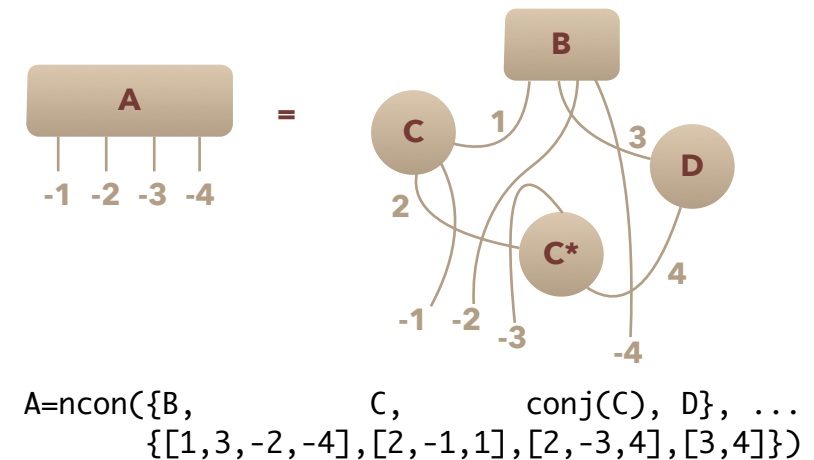
---

## Tensors / multi-dimensional arrays



$= A(a,b,c)$        (for all
$a = 1,…,Da$
$b = 1,…,Db$
$c = 1,…,Dc$)

anti-clockwise ordering

complex conjugated tensor:
drawn mirrored but ordering doesn't
change ⇒ clockwise ordering

---

## Tensor manipulations



(for all
$x = 1,…,Da*Db$
$c = 1,…,Dc$)

$B = reshape(A, Da*Db, Dc)$

$B = permute(A, [2,1,3])$

---
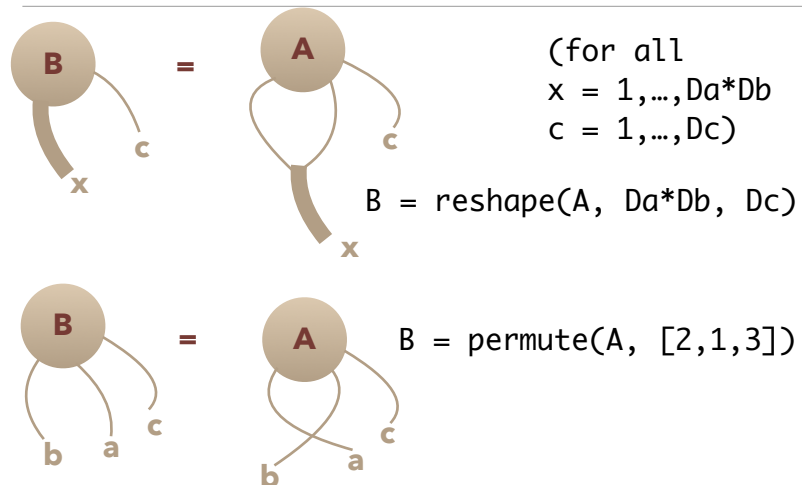
## Contracting tensors and tensor networks



```
A=ncon({B,          C,          conj(C), D}, ...
    {[1,3,-2,-4],[2,-1,1],[2,-3,4],[3,4]})
```
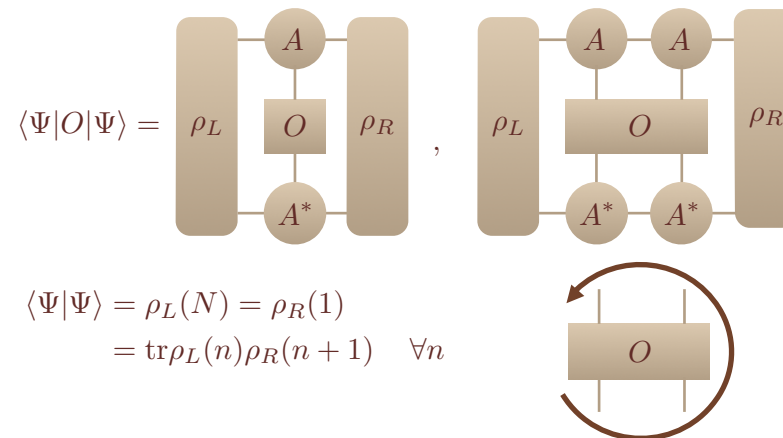
## MPS: evaluating expectation values

$$\rho_R(n) = \sum_{s=1}^{d} A^s(n)\rho_R(n+1)A^s(n)^{\dagger}, \qquad \rho_R(N+1) = 1$$

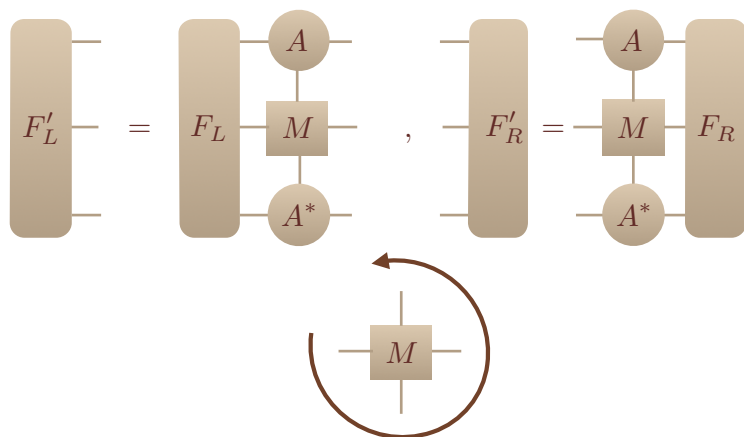$$\rho_L(n) = \sum_{s=1}^{d} A^s(n)^{\dagger}\rho_L(n-1)A^s(n), \qquad \rho_L(0) = 1$$
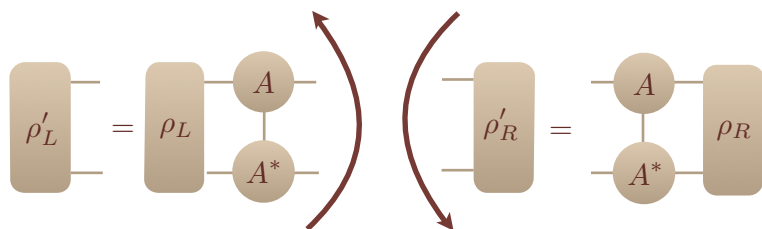
## MPS: evaluating expectation values

Local observables:

$$\langle\Psi|O|\Psi\rangle =$$

$$\langle\Psi|\Psi\rangle = \rho_L(N) = \rho_R(1)$$
$$= \mathrm{tr}\rho_L(n)\rho_R(n+1) \quad \forall n$$

## MPS: evaluating expectation values

Matrix product operators:

## MPS: DMRG and TDVP

Effective Hamiltonian

$$H_{\mathrm{eff}} = \qquad\qquad H_{0,\mathrm{eff}} =$$

$$H_{2,\mathrm{eff}} =$$