deeplearning.ai

Optimization Algorithms

Mini-batch gradient descent

Batch vs. mini-batch gradient descent

X, Y $\qquad X^{\{t\}}, Y^{\{t\}}$.

Vectorization allows you to efficiently compute on m examples.

$$X = [\underbrace{x^{(1)}\ x^{(2)}\ x^{(3)}\ \dots\ x^{(1000)}}_{X^{\{1\}}\ (n_x, 1000)} \mid \underbrace{x^{(1001)}\ \dots\ x^{(2000)}}_{X^{\{2\}}\ (n_x, 1000)} \mid \dots \mid \underbrace{\dots\ x^{(m)}}_{X^{\{5,000\}}\ (n_x, 1000)}]$$

(n_x, m)

$$Y = [\underbrace{y^{(1)}\ y^{(2)}\ y^{(3)}\ \dots\ y^{(1000)}}_{Y^{\{1\}}\ (1, 1000)} \mid \underbrace{y^{(1001)}\ \dots\ y^{(2000)}}_{Y^{\{2\}}\ (1, 1000)} \mid \dots \mid \underbrace{\dots\ y^{(m)}}_{Y^{\{5,000\}}\ (1, 1000)}]$$

$(1, m)$

What if $m = 5,000,000$?

5,000 mini-batches of 1,000 each

Mini-batch t: $X^{\{t\}}, Y^{\{t\}}$.

$x^{(i)}$

$z^{[l]}$

$X^{\{t\}}, Y^{\{t\}}$.

Andrew Ng

Mini-batch gradient descent

1 step of gradient descent using $\underline{X^{\{t\}}, Y^{\{t\}}}$ (as if $m = 1000$)

$\underline{X, Y}$

repeat {

for $t = 1, \ldots, 5000$ {

Forward prop on $X^{\{t\}}$.

$$\left.\begin{aligned} Z^{[1]} &= W^{[1]} X^{\{t\}} + b^{[1]} \\ A^{[1]} &= g^{[1]}(Z^{[1]}) \\ &\vdots \\ A^{[L]} &= g^{[L]}(Z^{[L]}) \end{aligned}\right\} \text{Vectorized implementation (1000 examples)}$$

Compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{l} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \cdot 1000} \sum_{l} \|W^{[l]}\|_F^2$.

from $X^{\{t\}}, Y^{\{t\}}$.

Backprop to compute gradients wrt $J^{\{t\}}$ (using $(X^{\{t\}}, Y^{\{t\}})$)

$W^{[l]} := W^{[l]} - \alpha dW^{[l]}$, $b^{[l]} := b^{[l]} - \alpha db^{[l]}$

}

}

"1 epoch" — pass through training set.

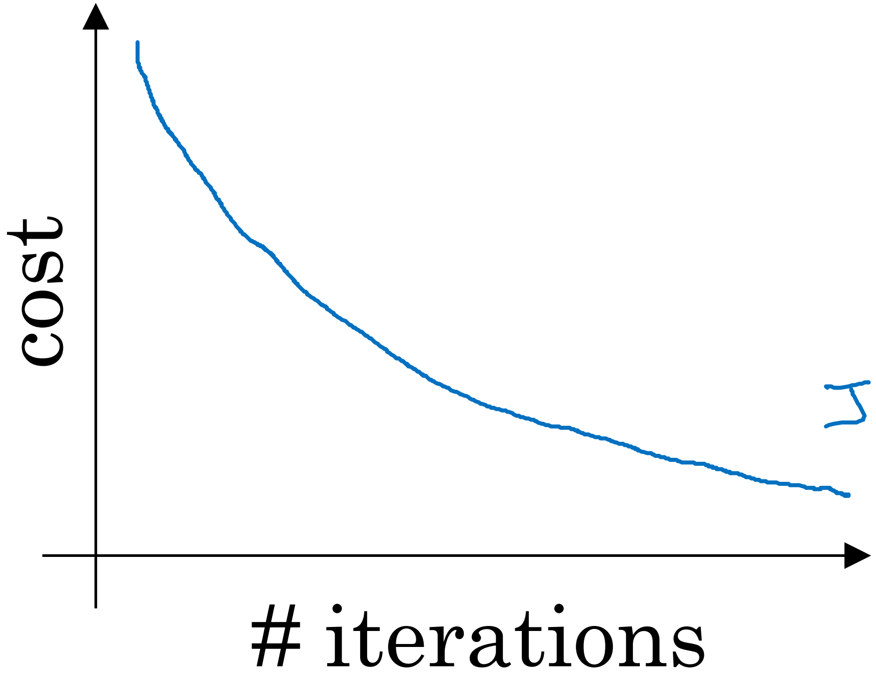
deeplearning.ai

Optimization Algorithms

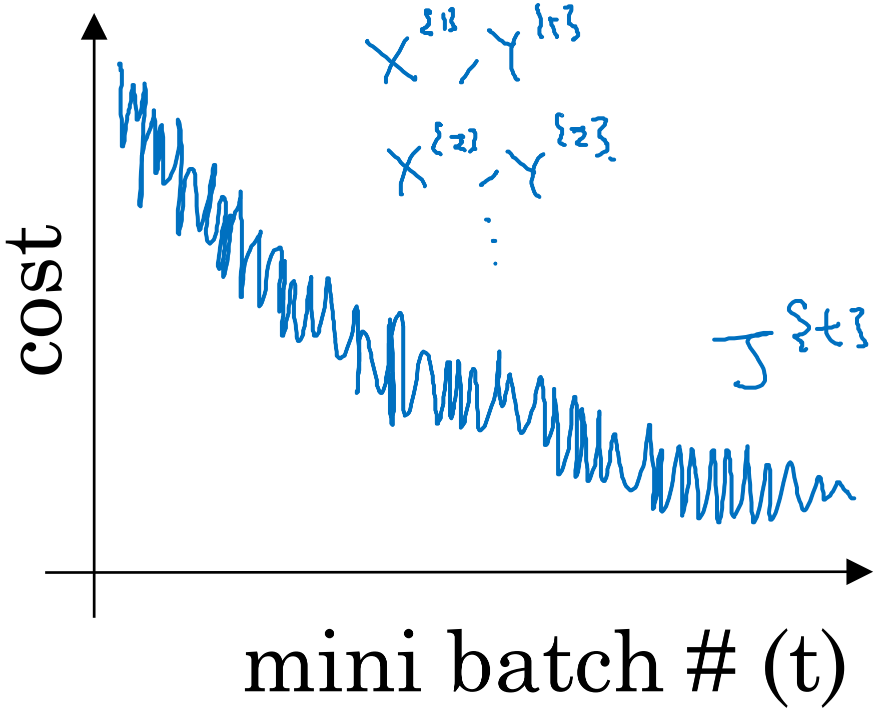
Understanding mini-batch gradient descent

Training with mini batch gradient descent

Batch gradient descent



Mini-batch gradient descent



Plot $J^{\{t\}}$ computed using $\underline{X^{\{t\}}, Y^{\{t\}}}$

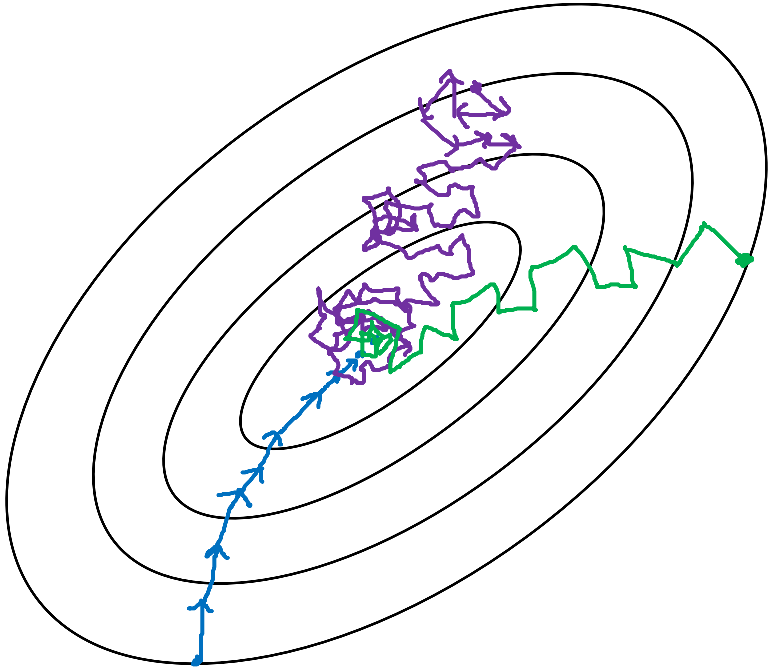
Andrew Ng

Choosing your mini-batch size

→ If mini-batch size = m : Batch gradient descent. $(X^{\{1\}}, Y^{\{1\}}) = (X, Y)$.

→ If mini-batch size = 1 : Stochastic gradient descent. Every example is it own mini-batch.
$(X^{\{1\}}, Y^{\{1\}}) = (x^{(1)}, y^{(1)})$ … $(x^{(2)}, y^{(2)})$ mini-batch.

In practice: Somewhere in-between 1 and m



Stochastic gradient descent
↓
Lose speedup from vectorization

In-between (mini-batch size not too big/small)
↓
Fastest learning.
- Vectorization. (~1000)
- Make progress without processing entire training set.

Batch gradient descent (mini-batch size = m)
↓
Too long per iteration

Andrew Ng

Choosing your mini-batch size

If small trng set: Use batch gradient descent.
($m \leq 2000$)

Typical mini-batch sizes:

$\rightarrow \underbrace{\underset{2^6}{64}, \underset{2^7}{128}, \underset{2^8}{256}, \underset{2^9}{512}}$ $\quad \underline{1024}$ 2^{10}

Make sure mini-batch fit in CPU/GPU memory.
$X^{\{t\}}, Y^{\{t\}}$

Andrew Ng

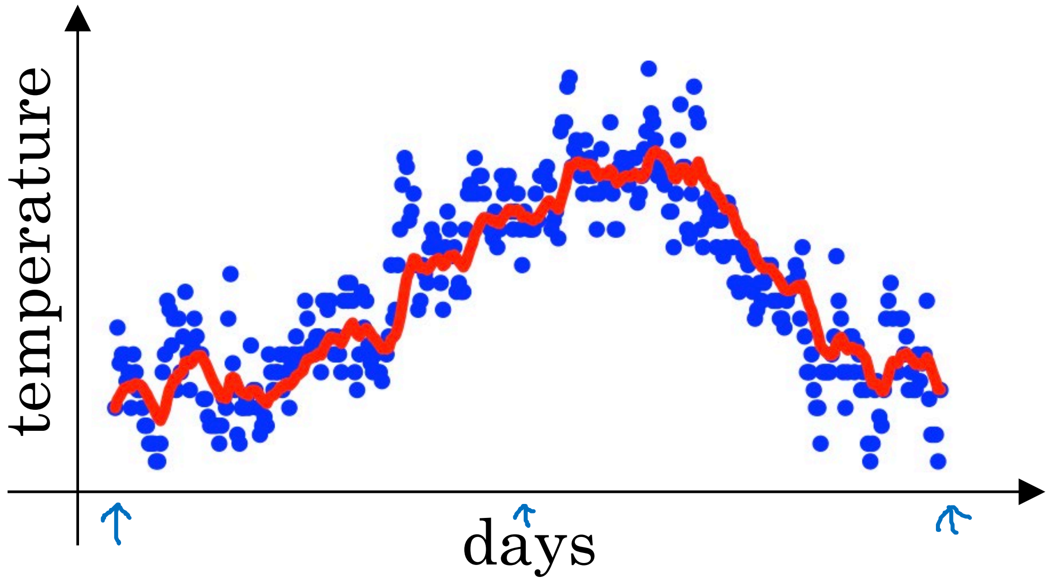
deeplearning.ai

Optimization Algorithms

Exponentially weighted averages

Temperature in London

$\theta_1 = 40°F$ $4°C$ ←
$\theta_2 = 49°F$ $9°C$
$\theta_3 = 45°F$ $\vdots$
$\vdots$
$\theta_{180} = 60°F$ $15°C$
$\theta_{181} = 56°F$ $\vdots$
$\vdots$



$V_0 = 0$
$V_1 = 0.9\,V_0 + 0.1\,\theta_1$
$V_2 = 0.9\,V_1 + 0.1\,\theta_2$
$V_3 = 0.9\,V_2 + 0.1\,\theta_3$
$\vdots$

$$\boxed{V_t = 0.9\,V_{t-1} + 0.1\,\theta_t}$$

Andrew Ng

Exponentially weighted (moving) averages

$$V_t = \beta V_{t-1} + (1-\beta)\theta_t \leftarrow$$

$\beta = 0.9$: ≈ 10 days' temperature.

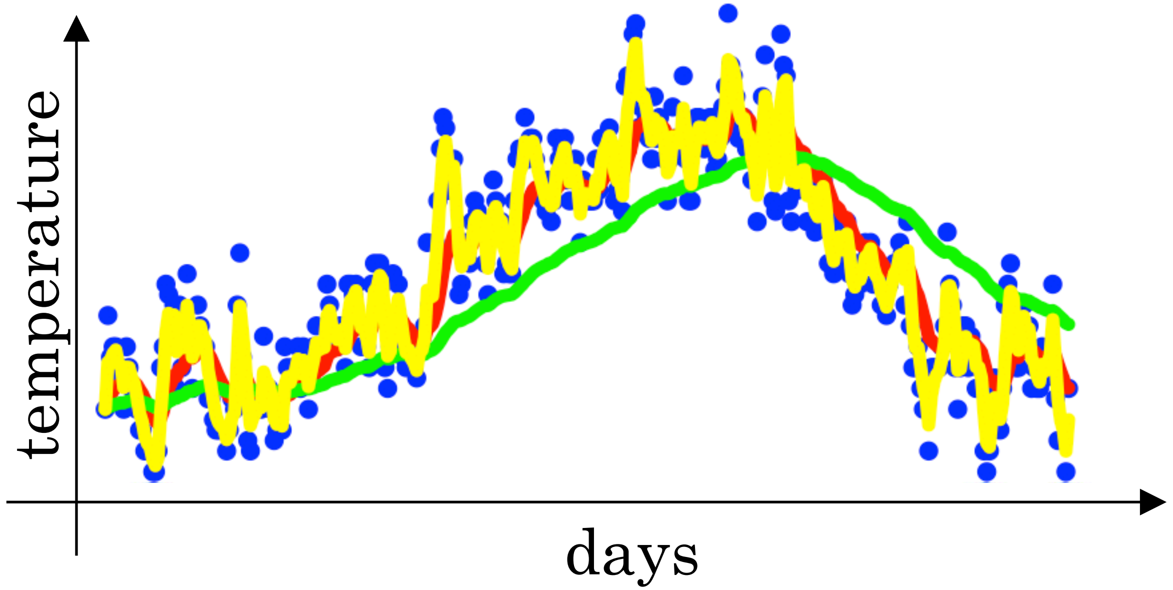
$\beta = 0.98$: ≈ 50 days

$\beta = 0.5$: ≈ 2 days

V_t as approximately average over

$\rightarrow \approx \frac{1}{1-\beta}$ days' temperature.

$$\frac{1}{1-0.98} = 50$$



Andrew Ng

deeplearning.ai

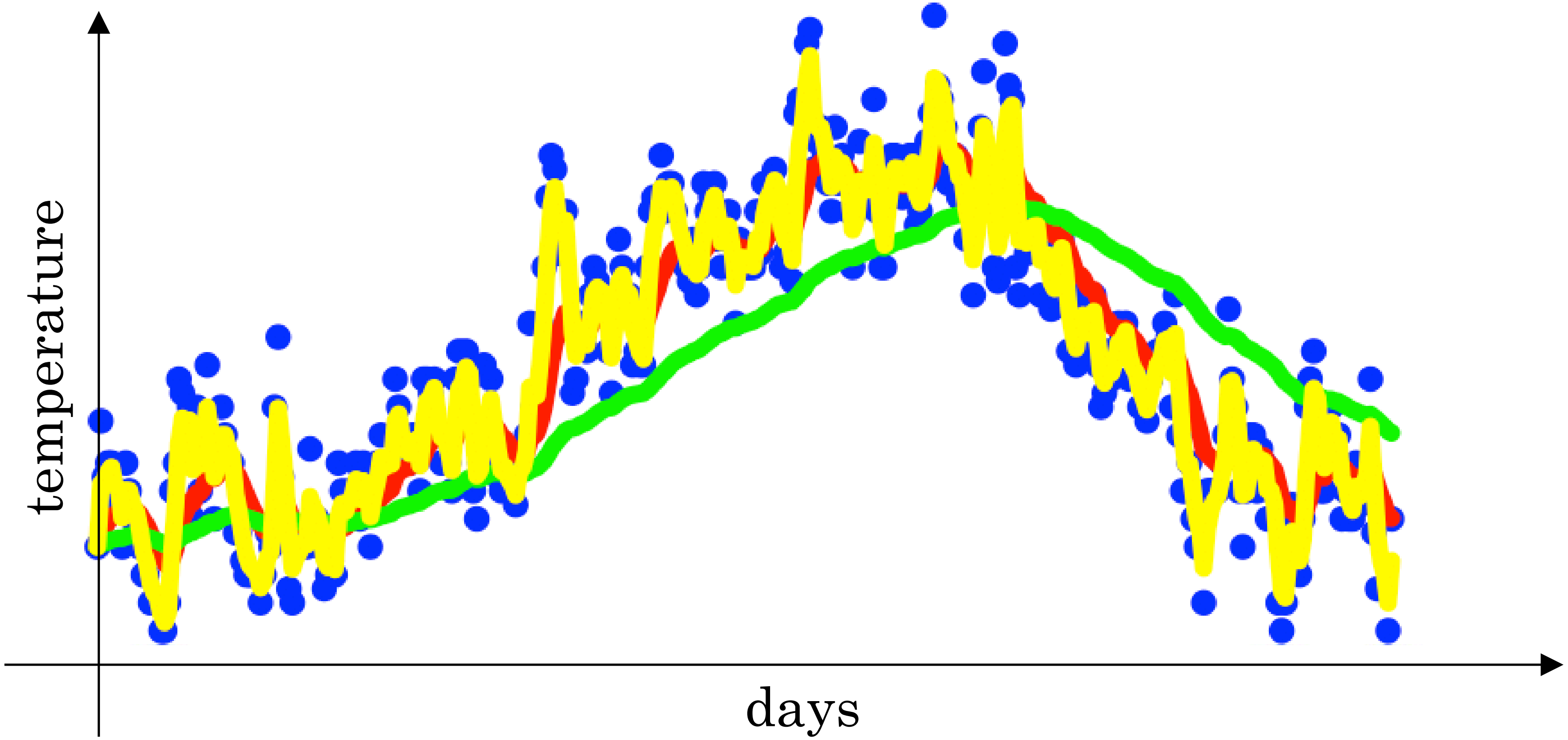
Optimization Algorithms

Understanding exponentially weighted averages

Exponentially weighted averages

$$\boxed{v_t = \beta v_{t-1} + (1-\beta)\theta_t}$$

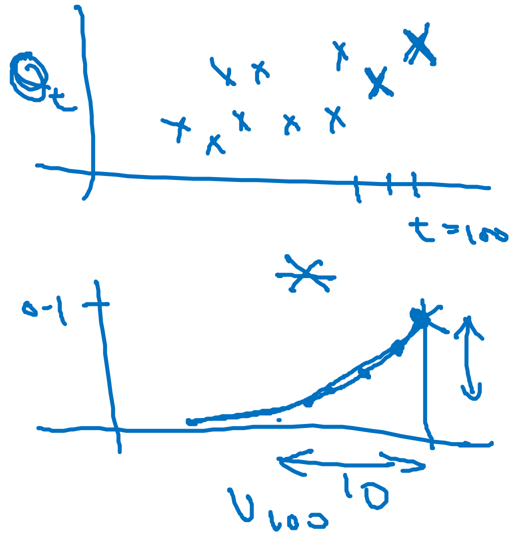
$\beta = 0.9 \qquad 0.98 \qquad 0.5$



Andrew Ng

Exponentially weighted averages

$$v_t = \beta v_{t-1} + (1-\beta)\theta_t$$



$$\approx \frac{1}{1-\beta}$$

$$\varepsilon = 1-\beta$$

$$v_{100} = 0.9 v_{99} + 0.1\theta_{100}$$

$$\boxed{v_{99} = 0.9 v_{98} + 0.1\theta_{99}}$$

$$\boxed{v_{98} = 0.9 v_{97} + 0.1\theta_{98}}$$

$$\cdots$$

$$0.1\,\theta_{98} + 0.9\,v_{97}$$

$$v_{100} = 0.1\,\theta_{100} + 0.9\,\cancel{v_{99}}\left(0.1\,\theta_{99} + 0.9\,\cancel{v_{98}}\right)$$

$$= 0.1\,\theta_{100} + 0.1 \times 0.9 \cdot \theta_{99} + 0.1\,(0.9)^2\,\theta_{98} + 0.1\,(0.9)^3\,\theta_{97} + 0.1\,(0.9)^4\,\theta_{96} + \cdots$$

$$0.9^{10} \approx 0.35 \approx \frac{1}{e}$$

$$\underbrace{(1-\varepsilon)}_{0.9}{}^{1/\varepsilon} = \frac{1}{e}$$

$$0.98\ ?$$

$$\varepsilon = 0.02 \rightarrow 0.98^{50} \approx \frac{1}{e}$$

Andrew Ng

Implementing exponentially weighted averages

$$v_0 = 0$$
$$v_1 = \beta v_0 + (1-\beta)\,\theta_1$$
$$v_2 = \beta v_1 + (1-\beta)\,\theta_2$$
$$v_3 = \beta v_2 + (1-\beta)\,\theta_3$$
$$\ldots$$

$V_\theta := 0$

$V_\theta := \beta V + (1-\beta)\,\theta_1$

$V_\theta := \beta V + (1-\beta)\,\theta_2$

$\vdots$

$V_\theta = 0$

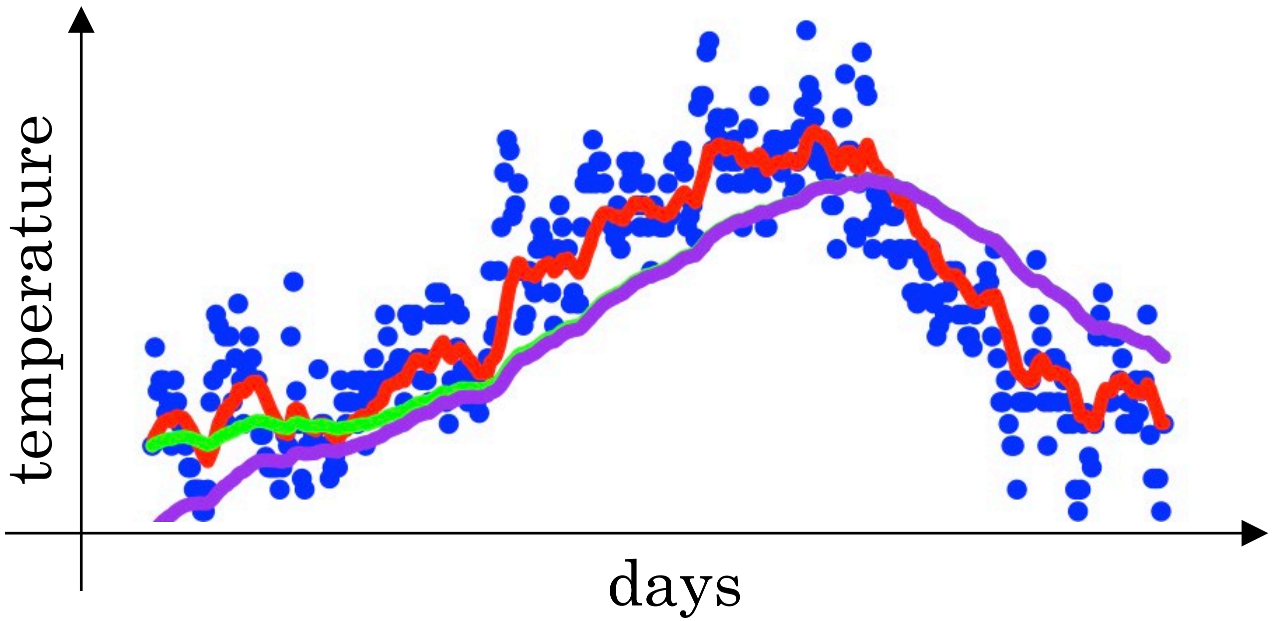
Repeat {

Get next θ_t

$V_\theta := \beta V_\theta + (1-\beta)\,\theta_t$

}

Bias correction



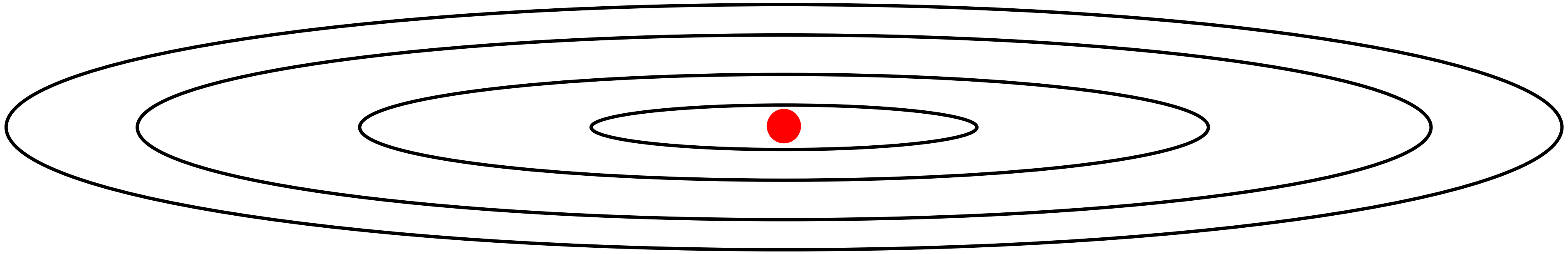
$$v_t = \beta v_{t-1} + (1-\beta)\theta_t$$

deeplearning.ai

Optimization Algorithms

Gradient descent with momentum

Gradient descent example



deeplearning.ai

Implementation details

On iteration t:

Compute dW, db on the current mini-batch

$$v_{dW} = \beta v_{dW} + (1-\beta)dW$$

$$v_{db} = \beta v_{db} + (1-\beta)db$$

$$W = W - \alpha v_{dW}, \;\; b = b - \alpha v_{db}$$

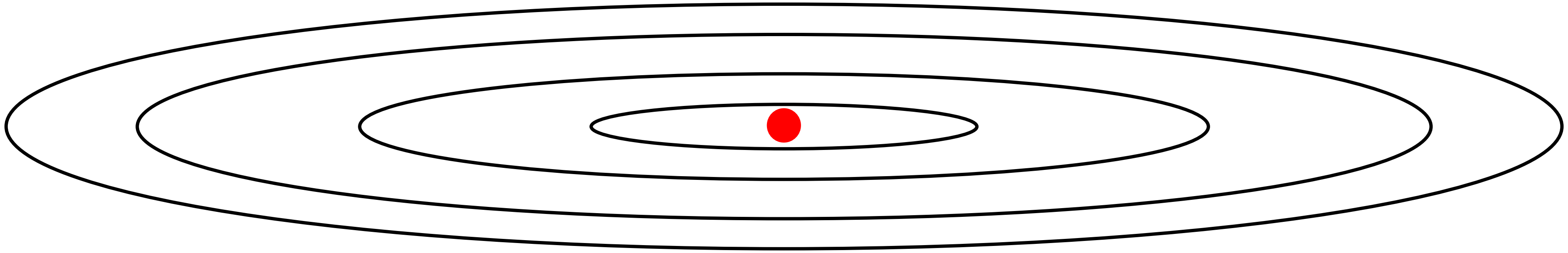
Hyperparameters: α, β $\qquad$ $\beta = 0.9$

deeplearning.ai

Optimization Algorithms

RMSprop

RMSprop



deeplearning.ai

Optimization Algorithms

Adam optimization algorithm

Adam optimization algorithm

```
yhat = np.array([.9, 0.2, 0.1, .4, .9])
```

Hyperparameters choice:

Adam Coates

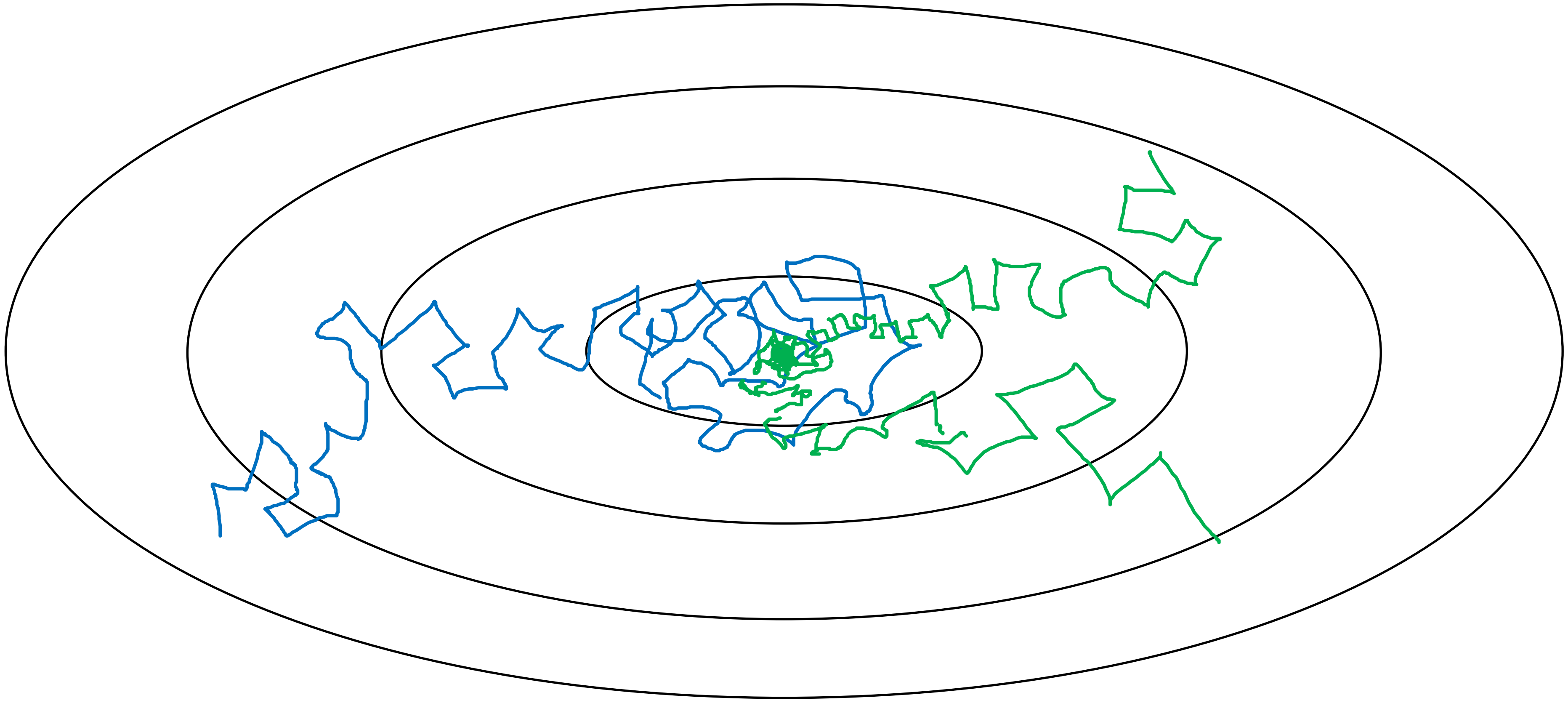
deeplearning.ai

Optimization Algorithms

Learning rate decay

Learning rate decay

Slowly reduce α



Andrew Ng

Learning rate decay

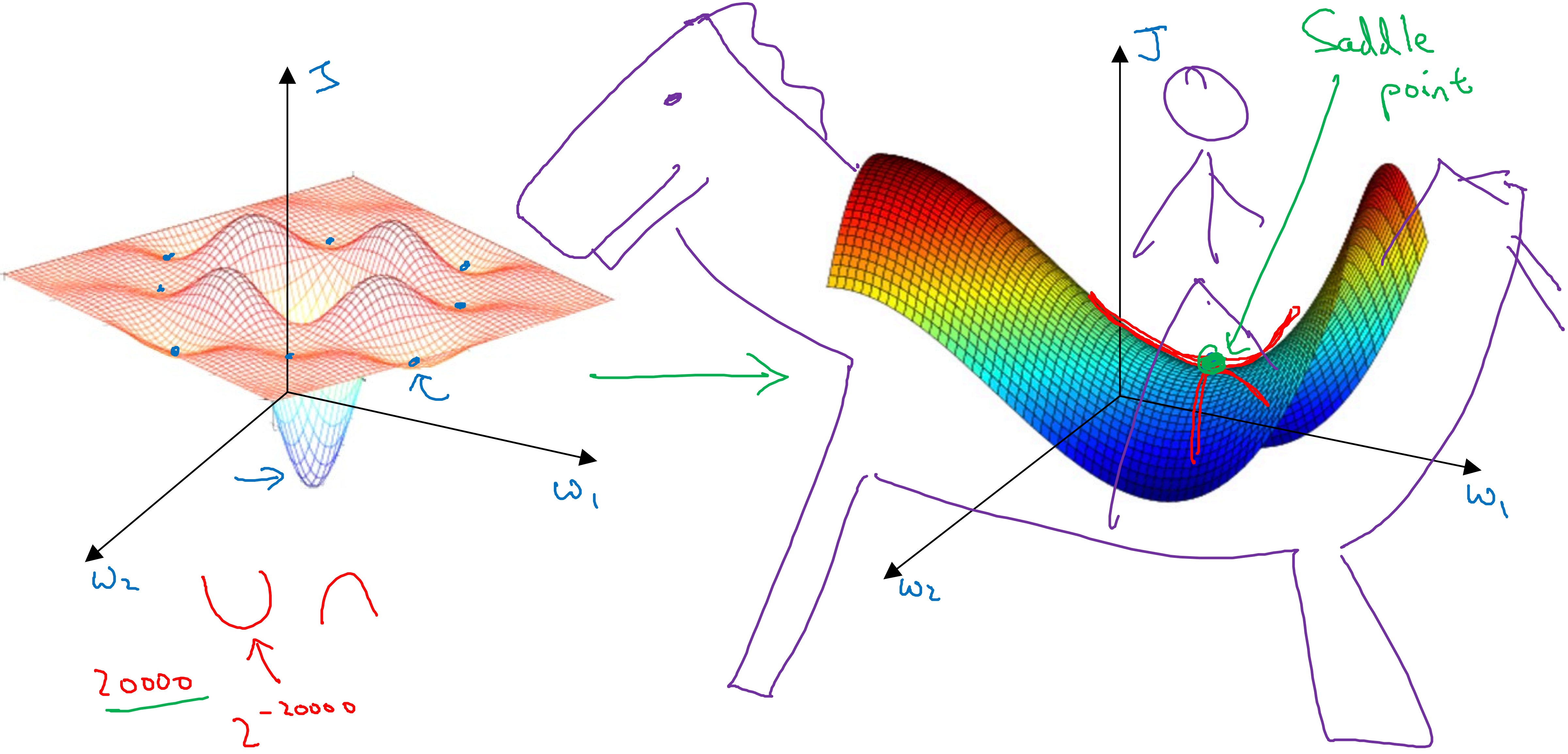
Other learning rate decay methods

deeplearning.ai

Optimization Algorithms

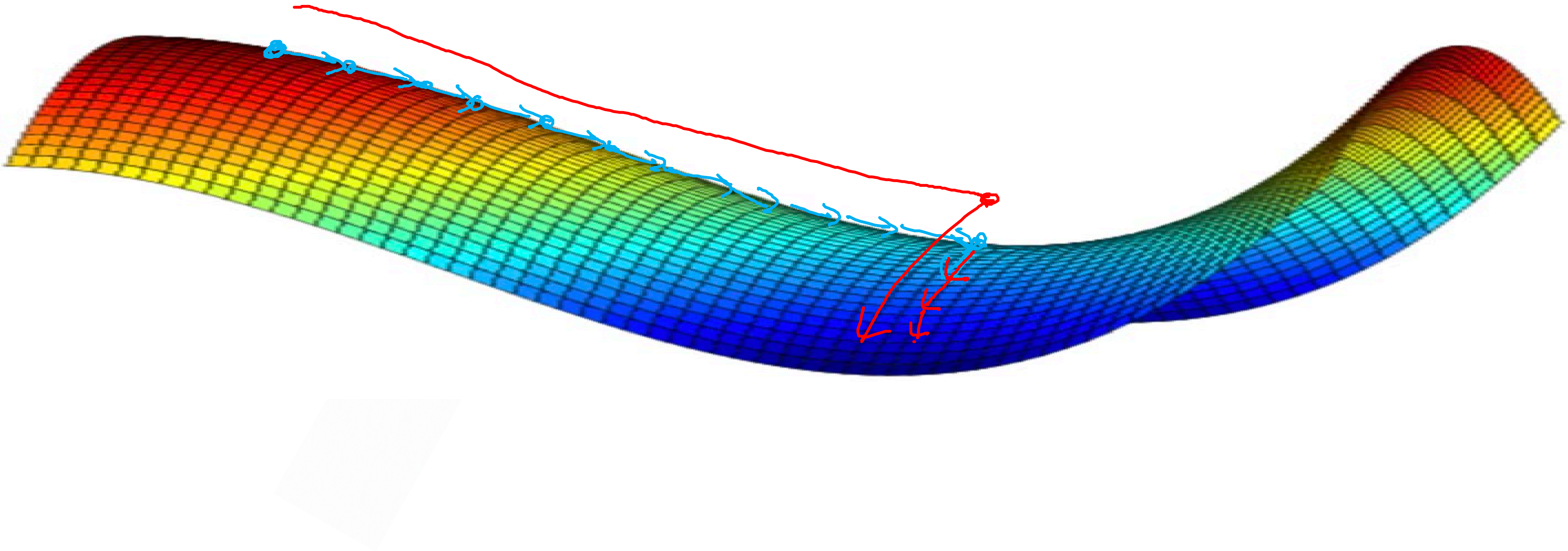
The problem of local optima

Local optima in neural networks



Andrew Ng

Problem of plateaus



- Unlikely to get stuck in a bad local optima
- Plateaus can make learning slow