Untitled

Probability

Random events

- probablity space: (Ω, S, P)
- measurement space: (Ω, S)
- sample space: Ω
- event space: S, σ-algebra/field
  - $S \neq \emptyset$
  - if $A \in S$, then $A^C \in S$
  - if $A_1, A_2 ... \in S$, then $A_1 \cup A_2 ... \in S$
- probability measure: $P : S \rightarrow R$, assigns a probability to every event in the event space.
- Bayes' rules: $P(B) = \frac{P(B|A)P(A)}{P(B)}$

Random Variables

- A **random variable** is a function $X : \Omega \rightarrow R$ such that, $\forall r \in R, \{a \in \Omega : X(a) \leq r\} \in S$
- Function of a Random Variable
  - Let $g : U \rightarrow R$ be a function, where $X(\Omega) \subseteq U \subseteq R$. Then, if $g \circ X :\rightarrow R$ is a random variable, we say that g is a function of X, and write $g(X)$ to denote the random variable $g \circ X$.
  - $g \circ X$

$g \circ f$	The *composition* of two functions. For functions $f : S \rightarrow T$ and $g : U \rightarrow V$, where $f(S) \subseteq U \subseteq T$, $g \circ f : S \rightarrow V$ is the function that first applies f and then applies g to its output: $\forall s \in S$, $(g \circ f)(s) = g(f(s))$.

  - Eg: $A = \{\omega \in : X(\omega) = 1\} = \{X = 1\}$, which means A is the event that X takes on the value of one
  - uppercase letters to denote random variables and lowercase letters to denote generic outcome

Support

- The set of values at which the PMF or PDF of a random variable is positive is called its support.

- For a random variable X with PMF/PDF f, the support of X is $Supp[X] = x \in R : f(x) > 0$.
    - The notion of the support of a random variable will be particularly important in Section 2.2.3, when we define conditional expectations, and in Part III, in which assumptions about support will comprise key elements of our approach to identification.

Bivariate relationship

- Equality of Random Variables
- Equality of Functions of a Random Variable
    - Let X be a random variable, and let f and g be functions of X. Then $g(X) = h(X) \Leftrightarrow \forall x \in X(\Omega), g(x) = h(x)$.
- Joint CDF: as in the univariate case, the joint CDF tells us everything about the behavior of X and Y.
- Discrete Marginal and Conditional Distributions
    - Marginal PMF
    - Conditional PMF
    - Multiplicative Law for PMFs (same for PDFs)
        - Let X and Y be two discrete random variables with joint PMF f. Then, $\forall x \in R$ and $\forall y \in Supp[Y]$,$f_{X|Y}(x|y)f_Y(y) = f(x, y)$.
- Jointly Continuous Random Variables
    - Joint PDF
    - Marginal PDF
    - Conditional PDF

Multivariate Generalizations

- This section outlines the generalizations of some of the above concepts to the case where we have more than two random variables.
- Random Vector
- Joint CDF (Multivariate Case)
- Joint PMF (Multivariate Case)
- Joint PDF (Multivariate Case)
- Marginal PMF/PDF (Multivariate Case)
- Conditional PMF/PDF (Multivariate Case)