THE UNIVERSITY OF CALGARY

FACULTY OF SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

MIDTERM EXAMINATION

FALL 2018

CPSC 313 L01

November 8, 2018 **Time: 2 hours**

NAME: ____

Tutorial Section (please circle one):

T01	T02	T03	T04
(Mon 12:00)	(Mon 17:00)	(Wed 12:00)	(Wed 17:00)

Instructions:

- Answer all questions on the exam paper in the space provided.
- Show all your work.
- Use the last two pages to continue answers if you need more space, or as rough paper.
- **No aids are allowed.**
- Total marks: **85+1 bonus point**. Maximum attainable score: 86/85 (101.2 percent)

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Question	**Score**	**Out Of**
1. Multiple Choice Questions		5
2. True/False Questions		5
3. Definitions and Short Answer Questions		10
4. Decision Problems		8
5. NFAs and DFAs		20
6. Pumping Lemma for Regular Languages		5
7. Regular Expressions, DFAs, CFGs		14+1
8. Context-Free Grammars and Languages		8
9. Chomsky Normal Form		10
Total:		85

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1. [5 points] **Multiple Choice Questions**

   For each question, check exactly one answer.

   (a) [1 point] Which of the following statements is true about the transition function of an NFA over some alphabet Σ?
   - ○ It can take on the value $\emptyset$ (the empty set).
   - ○ For some states q, the empty string ε may be a valid input, i.e. $\delta(q, \varepsilon)$ is defined.
   - ○ Both these statements are true.
   - ○ Neither of these statements is true.

   (b) [1 point] Consider the language $L = \{w \in \{0,1\}^* \mid w \text{ has odd length and ends in } 01\ \}$. Which of the following is true?
   - ○ L is regular, but not context-free.
   - ○ L is context-free, but not regular.
   - ○ L is regular and context-free.
   - ○ L is neither regular nor context-free.

   (c) [1 point] Which of the following statements holds for the language $L = \{10^n1^{2n} \mid n \geq 0\}$?
   - ○ There exists a DFA that accepts it.
   - ○ There exists an NFA that accepts it.
   - ○ There exists a regular expression that describes it.
   - ○ None of the above is true.

   (d) [1 point] Which of the following statements is true about a regular language?
   - ○ Its complement is regular.
   - ○ Its Kleene closure is regular.
   - ○ Both these statements are true.
   - ○ Neither of these statements is true.

   (e) [1 point] Check the correct continuation of the statement "A context-free grammar G is ambiguous if and only if ..."
   - ○ every string in $L(G)$ has more than one parse tree in G.
   - ○ there exists a string in $L(G)$ that has more than one parse tree in G.
   - ○ every string in $L(G)$ has more than one derivation in G.
   - ○ there exists a string in $L(G)$ that has more than one derivation in G.

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2. [5 points] **True/False Questions**

   Answer every questions with **TRUE** or **FALSE**. No explanations are required.

   (a) [1 point]. Every finite language is regular.

   (b) [1 point] Every context-free language contains the empty string.

   (c) [1 point] The union of two context-free languages is context-free.

   (d) [1 point] No grammar in Chomsky Normal Form is ambiguous.

   (e) [1 point] The language of the regular expression $e = ((a \cup b)^* c(a \cup b)^* cc)^*$ over the alphabet $\Sigma = \{a, b, c\}$ contains the string $abccabcc$.

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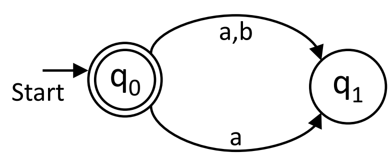
3. [10 points] **Definitions and Short Answer Questions**

   (a) [2 points] Formally define what it means for a problem to be a *decision problem*.

   (b) [2 points] Formally define the *Kleene closure* L^* of a language L over some alphabet Σ.

   (c) [2 points] Write down a regular expression for the language accepted by the following NFA over the alphabet $\Sigma = \{a, b\}$:



   (d) [2 points] Give (without proof or explanation) an example of a context-free language over the alphabet $\Sigma = \{0, 1\}$ that is not regular.

   (e) [2 points] Consider the grammar G over the alphabet $\Sigma = \{a, b\}$ with only one variable S and the rules $S \to aS \mid aSb \mid \varepsilon$. Give a set-theoretic description of $L(G)$.

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4. [8 points] **Decision Problems**

   (a) [4 points] For each of the decision problems stated below, give a set-theoretic description of the language of the problem.

      i. INPUT: a grammar G and a language L
         OUTPUT: "Yes" iff $L(G) = L$

      ii. INPUT: a DFA M and a positive integer n
         OUTPUT: "Yes" iff M has at least n states

   (b) [4 points] For each of the languages described below, state the decision problem which has that language.

      i. The language of all trees of height ≤ 10
         INPUT:

         OUTPUT:

      ii. The language $L = \{0^n 1^n \mid n \geq 0\}$ over the alphabet $\Sigma = \{0, 1\}$
         INPUT:

         OUTPUT:

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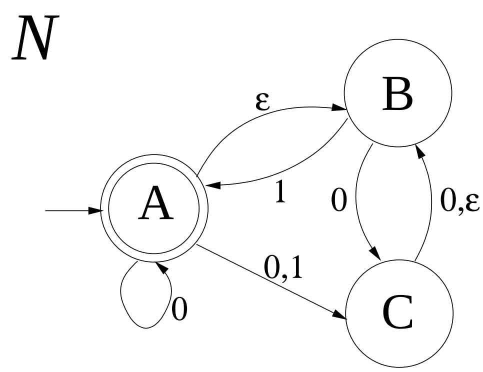
5. [20 points] **NFAs and DFAs**

   Consider the NFA N defined over the alphabet $\Sigma = \{0, 1\}$ with the following transition diagram.



   (a) [3 points] Give a formal description of N.

   (b) [2 points] Write down the ε-closures of the state sets

      i. $\{A\}$:

      ii. $\{A, C\}$:

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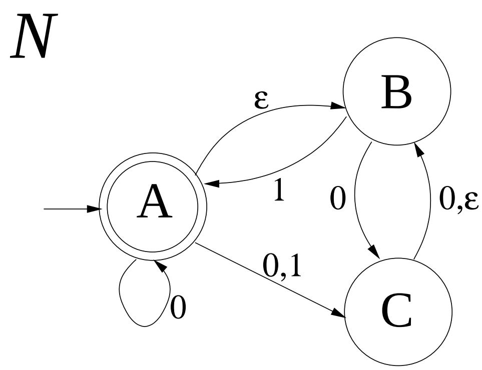
(c) [8 points] Give a formal description of a DFA M that is equivalent to N, (i.e. recognizes the same language as N). Continue on the next page if necessary. For your convenience, here is the diagram of the NFA N once more.



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(d) [2 points] Write down all the reachable states in your DFA M. (If you used the algorithm from class, there should only be two reachable states.)

(e) [3 points] Draw a diagram of M with all the non-reachable states removed.

(f) [2 points] What is the language accepted by M? Just state your answer, no proof required.

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6. [5 points] **Pumping Lemma for Regular Languages** Use the Pumping Lemma for regular languages to prove that the language

$$L = \{w0w \mid w \in \Sigma^*\}$$

over the alphabet $\Sigma = \{0, 1\}$ is not regular.

For your convenience, here is the Pumping Lemma for regular languages.

Pumping Lemma for Regular Languages. Let L be a regular language over some alphabet Σ. Then there exists a positive integer p (the *pumping length* of L) such that every string $s \in L$ of length at least p can be written as $s = xyz$ with strings $x, y, x \in \Sigma^*$ such that

1. $|xy| \leq p$;
2. $y \neq \varepsilon$;
3. $xy^iz \in L$ for all $i \geq 0$.

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7. [14 points plus 1 bonus point] **Regular Expressions, DFAs, Context-Free Grammars**
   Let L be the language of the regular expression $e = (a \cup b)^* c^*$ over the alphabet $\Sigma = \{a, b, c\}$.

   (a) [2 points] Give a set-theoretic description of L. You need not prove your answer correct.

   (b) [4 points] Draw a diagram of a DFA M with at most 3 states that accepts L.

   (c) [4 points plus 1 bonus point] Give a context-free grammar G such that $L(G) = L$. Carefully write down the variables and rules, and use the letter S for your starting variable. You need not prove your answer correct. *Solutions with at most 2 variables will receive one bonus point.* Continue your solution on the next page if necessary.

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(Continued from Problem 7 (c))

(d) [2 points] Give a left-most derivation for the string $w = abacc \in L$ using your grammar of part (d).

(e) [2 points] Draw a parse tree for the string $w = abc \in L$ using your grammar of part (d).

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8. [8 points] **Context-Free Grammars and Languages**

   Consider the context-free grammar $G = (V, \Sigma, R, S)$ with $V = \{S, A\}$, $\Sigma = \{0, 1\}$, and R contains the rules

$$\begin{aligned} S &\rightarrow 0A0 \mid 1A1 \mid \varepsilon \\ A &\rightarrow 0A \mid 1A \mid \varepsilon \end{aligned}$$

   Formally prove that

$$L(G) = \{w \in \Sigma^* \mid w \text{ starts and ends with the same symbol}\}.$$

   Continue your proof on the next page if necessary.

   (Note that $\varepsilon \in L$: it is vacuously true that ε starts and ends with the same symbol since it has no symbols.)

   You may use without proof the following:

   Lemma. Let $G = (V', \Sigma, R', A)$ where $V' = \{A\}$ and R' consists of the rules

$$A \rightarrow 0A \mid 1A \mid \varepsilon \ .$$

   Then $L(G') = \Sigma^*$.

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(Continued from Problem 8)

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9. [10 points] **Chomsky Normal Form** Consider the grammar $G = (V, \Sigma, R, S)$ where $V = \{S, A, B\}$, $\Sigma = \{a, b\}$, and R contains the following rules:

$$\begin{array}{rcl} S & \to & AB \mid A \\ A & \to & aBb \mid \varepsilon \\ B & \to & aB \mid bB \mid a \mid b \end{array}$$

Convert G to Chomsky Normal Form. Carefully describe all the steps you are taking, which new variables and/or rules you are introducing, which rules you are removing or replacing, etc.

Note that S does not appear on the right-hand side of any rule, so you can (and should) skip step 1 of the Chomsky Normal Form conversion algorithm discussed in class.

Continue your work on the next page(s) if necessary.

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(Continued from Problem 9)

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