CPSC 331 — Solution for Question #2 on the Practice Midterm Test

This question also concerned the "Slytherin Number Computation" problem and the recursive `slytherin` algorithm that were shown on the first page of the supplement for this test.

You were asked to write a ***recurrence*** for the number $T_{\texttt{slytherin}}(\texttt{n})$ executed by the the algorithm `slytherin` when this executed with a nonnegative integer $\texttt{n}$ as input.

Recurrence: If $\texttt{n} = 0$ then two steps — at lines 1 and 2 — are executed. Thus $T_{\texttt{slytherin}}(0) = 2$.

If $\texttt{n} = 1$ then three steps — at lines 1, 3 and 4 — are executed. Thus $T_{\texttt{slytherin}}(1) = 3$.

Finally, if $\texttt{n} \geq 2$ then the execution of the algorithm includes three steps — at lines 1, 3, and 5 — along with recursive applications of the algorithm with inputs $\texttt{n} - 1$ and $\texttt{n} - 2$. The numbers of steps used by these recursive applications are (by definition) $T_{\texttt{slytherin}}(\texttt{n}-1)$ and $T_{\texttt{slytherin}}(\texttt{n}-2)$, respectively.

It follows that the desired recurrence is as follows: For every integer $\texttt{n}$ such that $\texttt{n} \geq 0$,

$$T_{\texttt{slytherin}}(\texttt{n}) = \begin{cases} 2 & \text{if } \texttt{n} = 0, \\ 3 & \text{if } \texttt{n} = 1, \\ T_{\texttt{slytherin}}(\texttt{n}-1) + T_{\texttt{slytherin}}(\texttt{n}-2) + 3 & \text{if } \texttt{n} \geq 2. \end{cases}$$

How Large is This Function? It is certainly *not* obvious, but the above recurrence can be used to prove that

$$T_{\texttt{slytherin}}(\texttt{n}) = F_{\texttt{n}} + 5F_{\texttt{n}+1} - 3$$

for every integer $\texttt{n} \geq 0$. As material from previous lecture supplements and tutorial exercises can be used to show, it follows from this that

$$T_{\texttt{slytherin}}(\texttt{n}) \in \Theta\left(\left(\tfrac{1+\sqrt{5}}{2}\right)^{\texttt{n}}\right),$$

so that $T_{\texttt{slytherin}}(\texttt{n})$ is exponential in $\texttt{n}$.