Wavelet transform

Continuous wavelet transform of frequency breakdown signal.
Used symlet with 5 vanishing moments.

The **wavelet transform** is a time-frequency representation of a signal. For example, we use it for noise reduction, feature extraction or signal compression.

Wavelet transform of continuous signal is defined as

$$[W_\psi f]\,(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi^* \left(\frac{t-b}{a}\right) dt\,,$$

where

- ψ is so called mother wavelet,
- a denotes wavelet dilation,
- b denotes time shift of wavelet and
- $*$ symbol denotes complex conjugate.

In case of $a = {a_0}^m$ and $b = {a_0}^m kT$, where $a_0 > 1$, $T > 0$ and m and k are integer constants, the wavelet transform is called discrete wavelet transform (of continuous signal).

In case of $a = 2^m$ and $b = 2^m kT$, where $m > 0$, the discrete wavelet transform is called dyadic. It is defined as

$$[W_\psi f]\,(m,k) = \frac{1}{\sqrt{2^m}} \int_{-\infty}^{\infty} f(t)\psi^* \left(2^{-m}t - kT\right) dt\,,$$

where

- m is frequency scale,
- k is time scale and
- T is constant which depends on mother wavelet.

It is possible to rewrite dyadic discrete wavelet transform as

$$[W_\psi f]\,(m,k) = \int_{-\infty}^{\infty} f(t) h_m \left(2^m kT - t\right) dt\,,$$

where h_m is impulse characteristic of continuous filter which is identical to ${\psi_m}^*$ for given m.

Analogously, dyadic wavelet transform with discrete time (of discrete signal) is defined as

$$y_m[n] = \sum_{k=-\infty}^{\infty} f[k] h_m[2^m n - k].$$