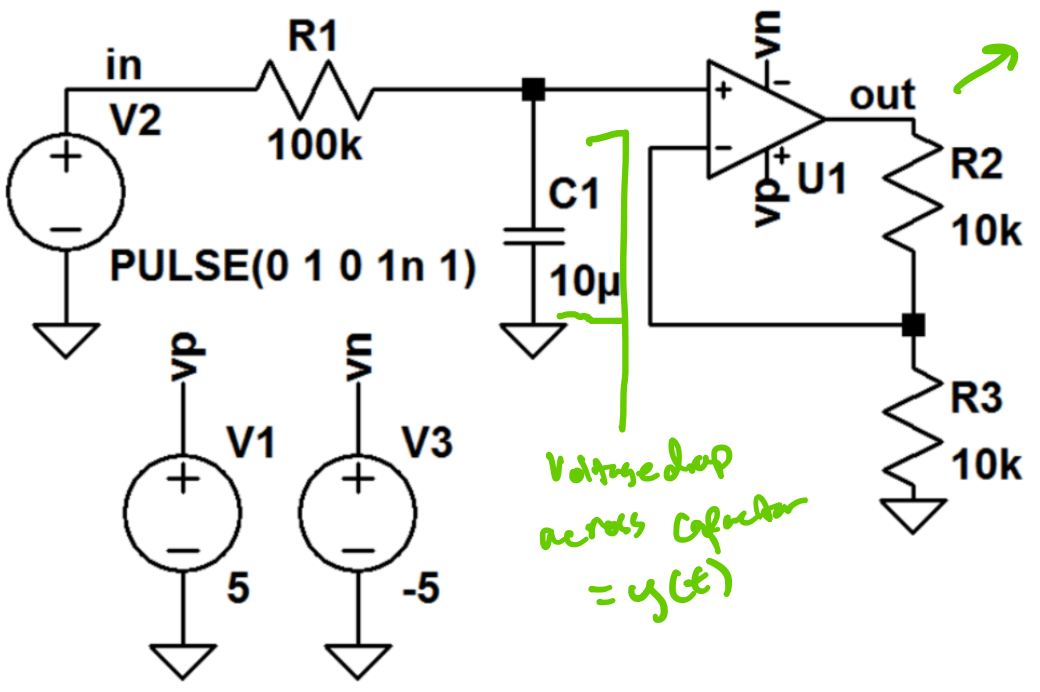
Deriving Step, Ramp, and Impulse Response

Monday, September 2, 2019 2:09 PM



$\rightarrow y_{op}(t)$

Voltage drop across capacitor $= y(t)$

$A_v = \frac{R_2}{R_3} + 1$

Gain = 2

Kirchoff Loop

$x(t) = iR_1 + y(t)$

where $y_{op}(t) = 2 \cdot y(t)$

solve for unit step response

$i = C\frac{dv}{dt}$

$x(t) = RC\frac{dv}{dt} + y(t)$

$\frac{dv}{dt} + \frac{1}{\tau_c} y(t) = \frac{1}{\tau_c} x(t)$

$\mathcal{L}^{-1}\{\frac{dv}{dt} + \frac{1}{\tau_c} y(t)\}$

$= [sY(s) + y(0)] + \frac{1}{\tau_c} Y(s) = \frac{1}{\tau_c}\mathcal{L}\{x(t)\}$

$\uparrow$ 0

$sY(s) + \frac{1}{\tau_c} Y(s)$

$\tau_c Y(s) = \frac{\mathcal{L}\{x(t)\}}{s + \frac{1}{\tau_c}}$

$x(t) = u(t)$

$\mathcal{L}\{u(t)\} = \frac{1}{s}$

$\mathcal{L}^{-1}\{\frac{1}{s(s-a)}\} = \frac{1}{a}(e^{ax} - 1)$

$a = -\frac{1}{\tau_c}$

$\tau_c Y(s) = \frac{1}{s(s + \frac{1}{\tau_c})}$

$\tau_c \mathcal{L}^{-1}\{Y(s)\} = -\tau_c(e^{-\frac{t}{\tau_c}} - 1)$

$y_{step}(t) = (1 - e^{-t/\tau_c}) \cdot u(t)$

$y_{op\,step}(t) = 2[1 - e^{-t/\tau_c}) \cdot u(t)$

$h_{op}(t) = \frac{d}{dt} y_{op\,step}(t)$

$= \frac{d}{dt} 2(1 - e^{-t/\tau_c})$

$= -2 \cdot \frac{d}{dt} e^{-t/\tau_c}$

$= \frac{2}{\tau_c} e^{-t/\tau_c} \cdot u(t)$

$y_{op\,ramp}(t) = \int y_{op\,step}(t)$

$= 2\int (1 - e^{-t/\tau_c}) \, dt$

$= 2[t + \tau_c e^{-t/\tau_c} \Big|_0^T$

$= 2(t + \tau_c e^{-t/\tau_c})$