1 Emprical Simulation

1.1 Estimation of Correlation Coefficient and Confidence Interval (CI) Estimation

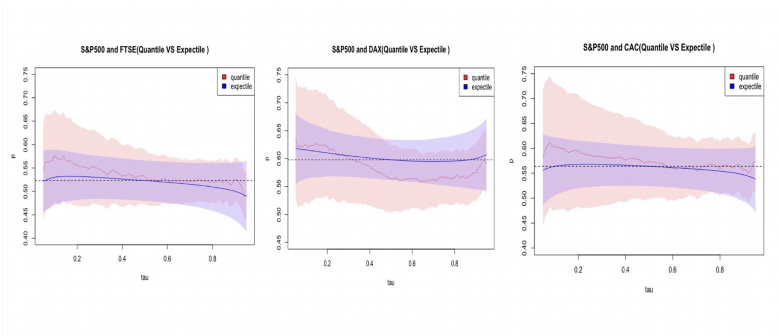


Figure 1: The sample quantile correlation coefficients $\rho_\tau^{(q)}$,95confidence interval (CI) for $\rho_\tau^{(q)}$

The figure above visualizes the estimated values and 95% confidence intervals for $\rho_\tau^{(q)}$ and $\rho_\tau^{(e)}$ based on $\tau = 0.05, 0.06, ..., 0.95$. In the process of estimating the standard deviation of $\rho_\tau^{(q)}$, the estimation method for the conditional density function utilized f_{BH}. Upon examining the figure, it is noticeable that for lower τ values of $\rho_\tau^{(q)}$, the correlation coefficient values are larger. This signifies stronger correlation in the left-tail region compared to other parts. Our expectile also demonstrates a similar trend to quantiles, presenting a somewhat smoother pattern. However, there is a significant difference in the lengths of confidence intervals between $\rho_\tau^{(q)}$ and $\rho_\tau^{(e)}$. Due to the necessity of the conditional density function estimation process, determining the confidence intervals for $\rho_\tau^{(q)}$ is more complex and time-consuming. Moreover, $\rho_\tau^{(e)}$ calculations are more straightforward and come with the advantage of shorter confidence intervals.

1.2 The interpretation of the quantile correlation coefficient

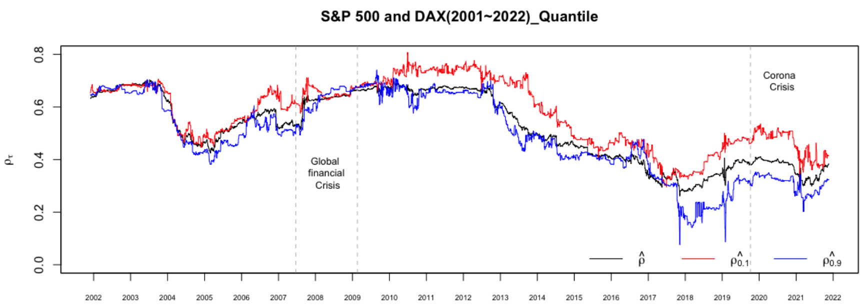


Figure 2: The sample quantile correlation coefficients $\rho_{0.1}^{(q)}$ and $\rho_{0.9}^{(q)}$ and the Pearson correlation ρ of the pairs of stock price returns based on 500 day rolling window samples

In the study by Choi and Shin (2022), a 500-day rolling window technique was employed to analyze the $\rho_\tau^{(q)}$ of the log return values of two stock indices (S&P 500 and DAX), during the period from 01/03/2000 - 11/30/2017. Furthermore, the period from 08/01/2007 - 03/31/2009, was designated as the global financial crisis. Examining the $\rho_\tau^{(q)}$, it can be observed that prior to the financial crisis, both the $\rho_{0.1}^{(q)}$ and $\rho_{0.9}^{(q)}$ exhibited similar trends to the Pearson correlation coefficient. However, post the financial crisis, the order of correlation coefficients changed, $\rho_{0.1}^{(q)} > \rho_{0.9}^{(q)}$. This implies that during periods of low quantiles(bad markets), the correlation between the two stock indices increases, whereas during high quantiles(good markets), the correlation is lower compared to bad markets. In other words, it can be concluded that the correlation between the two stock index is higher when the market conditions are bad. This interpretation aligns with the findings presented in the paper by Choi and Shin (2022). Our research initially verifies the validity of this interpretation and subsequently extends the analysis to include data up to 2022, exploring the impact of stock index correlations due to the COVID-19 pandemic.

Our own research extended this analysis to include data up to 2022 and investigated the impact of the COVID-19 pandemic, using 11/17/2019 as a reference point. Applying the 500-day rolling window, we observed a decreasing trend in $\rho_\tau^{(q)}$ since the baseline. During the COVID-19 pandemic, the correlation coefficients for bad market scenarios exhibited significantly higher values compared to previous situations. Notably, these values showed a noticeable gap when compared to other correlation coefficients. This observation aligns with the idea that during intense market shocks like the COVID-19-induced market crash, there was a widespread downturn in stock indices across different countries, suggesting an elevated correlation during bad market scenarios. Moreover, as the extreme market conditions began to stabilize, the gap between $\rho_\tau^{(q)}$ gradually normalized.

1.3 Comparison Between Quantile and Expectile

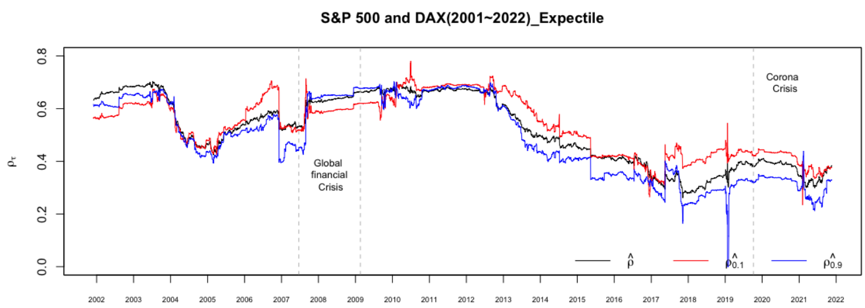


Figure 3: The sample Expectile correlation coeffficients $\rho_{0.1}^{(e)}$ and $\rho_{0.9}^{(e)}$ and the Pearson correlation ρ of the pairs of stock price returns based on 500 day rolling window samples

In the study, the authors examined the behavior of Expectile correlation coefficients in comparison to Quantile correlation coefficients during different market conditions. They utilized a 500-day rolling window technique and analyzed the correlation of logarithmic return values between the S&P 500 and DAX indices. The analysis focused on two distinct periods: before and after the global financial crisis, which occurred between 08/01/2007 - 03/31/2009. It was observed that, in general, Expectile correlation coefficients exhibited a smoother pattern compared to Quantile correlation coefficients. However, the overall trend of $\rho_\tau^{(e)}$ closely resembled that of $\rho_\tau^{(q)}$. Notably, the gap between them remained consistent in both periods, prior to and after the financial crisis. This suggests that our $\rho_\tau^{(e)}$ can inherit the interpretations and properties of $\rho_\tau^{(q)}$. During the global financial crisis, $\rho_{0.9}^{(q)} > \rho > \rho_{0.1}^{(q)}$ pattern emerged as time progressed. This observation emphasizes that in the aftermath of the global financial crisis, the correlation between the two indices was more pronounced in situations characterized by the $\rho = 0.9$(good market).

1.4 Confindence Interval for Quantile and Expectile

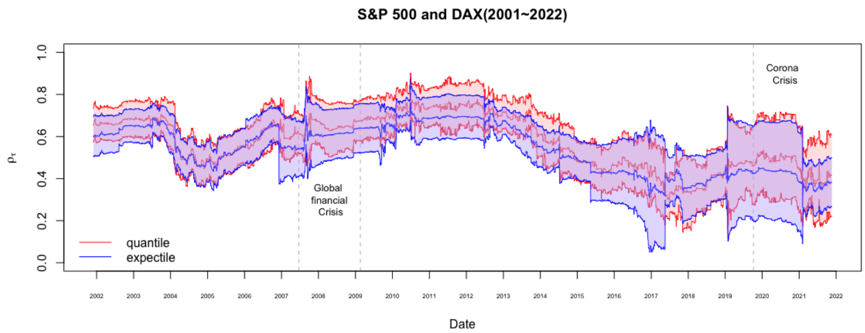


Figure 4: $\rho_{0.1}^{(e)}$ and $\rho_{0.1}^{(q)}$, 95% pointwise confidence interval(CI)

The figure illustrates the results obtained from utilizing a 500-day rolling window within the context described earlier. In each interval, the estimated standard deviation was calculated, and a 95% confidence interval was visualized. This graph corresponds to the situation where $\tau = 0.1$. In the process of estimating the conditional density for $\rho_\tau^{(q)}$, a procedure involving approximately 12 hours of estimation was undertaken to determine the standard deviation of the $\rho_\tau^{(q)}$. In contrast, the standard deviation value for the $\rho_\tau^{(e)}$ was readily calculable due to the absence of the need for a conditional density estimation process.The depicted figure further demonstrates that the $\rho_\tau^{(e)}$ exhibits a smoother trend compared to the $\rho_\tau^{(q)}$. The lengths of the confidence intervals are similar for both coefficients. Notably, the $\rho_\tau^{(e)}$ values appear slightly lower than $\rho_\tau^{(q}$.

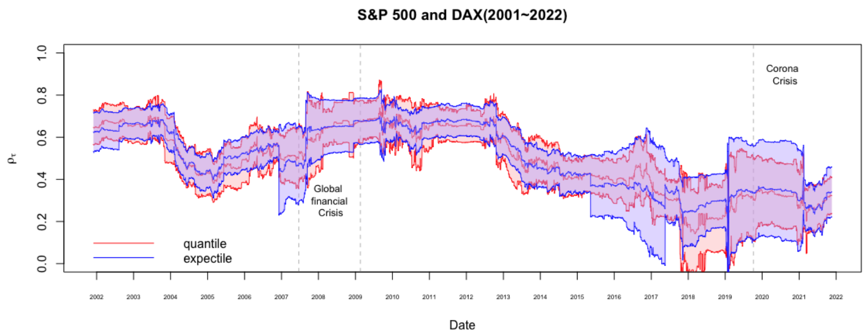
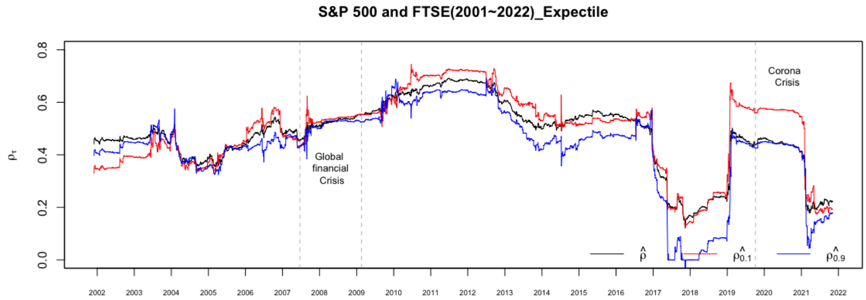
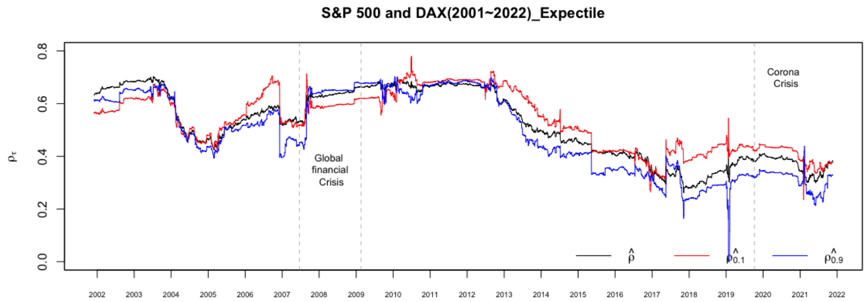


Figure 5: $\rho_{0.9}^{(e)}$ and $\rho_{0.9}^{(q)}$, 95% pointwise confidence interval(CI)

The figure depicts a scenario with the same context as before, but in this case, $\tau = 0.9$. Within this situation, the process of estimating the standard deviation value for the $\rho_\tau^{(q)}$ involved an estimation process of approximately 8.54 hours, as part of the conditional density estimation for Quantile. In contrast, the standard deviation value for $\rho_\tau^{(e)}$ was easily and quickly calculated due to the absence of the need for a conditional density estimation process. This enabled the efficient visualization of confidence intervals within a shorter timeframe. When $\tau = 0.9$, the figure illustrates that $\rho_\tau^{(e)}$ tends to be higher than $\rho_\tau^{(q)}$. Upon comparing the two scenarios, a general pattern emerges: in unfavorable market conditions (bad market), where τ values are smaller, $\rho_\tau^{(e)} < \rho_\tau^{(q)}$. Conversely, in favorable market conditions (good market), $\rho_\tau^{(e)} > \rho_\tau^{(q)}$.

1.5 Comparison with Various Stock Indices





In the preceding cases where we compared the S&P with the DAX , we aimed to examine the validity of our approach. Extending our analysis to include a comparison between the FTSE and the CAC40, we sought to further validate our methodology. Across all three scenarios, specifically during financial crises, we observed a consistent pattern where the $\rho_{0.9}^{(q)}$ reverses its relationship with the $\rho_{0.1}^{(q)}$. Notably, all three scenarios displayed a similar phenomenon of correlation coefficients sharply decreasing during the COVID-19 situation.

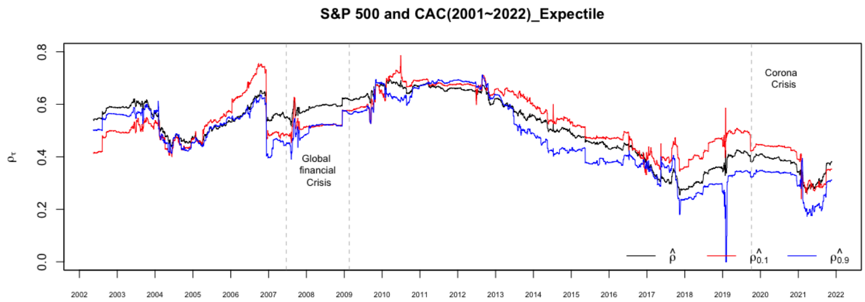


Figure 6: $\rho^{(e)}_{0.1}$ and $\rho^{(q)}_{0.1}$and the Pearson correlation τ of pairs of stock price returns based on 500 day rolling window samples