The Role of Information in Bond Auction Markets

Juyoung Yang*

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Abstract

This paper examines the occurrence of a debt crisis in a country that relies on raising bond revenue to finance its outstanding debt through bond auctions involving multiple lenders who do not observe fundamental perfectly. Using a noisy rational expectation model, I demonstrate analytically that a debt crisis can occur due to information friction, even in cases where the fundamentals alone would not have led to a crisis. The high precision of information amplifies the sensitivity of the bond price, causing a discrepancy between default outcomes implied by the fundamentals and those driven by fundamental-irrelevant shocks. Pessimistic signals can trigger a country to default, even in scenarios with low default risk fundamentals. Conversely, optimistic signals can help a country avoid default, even when facing high default risk fundamentals. The paper highlights how information friction renders the rollover debt problem more vulnerable. This paper offers a new perspective on understanding self-fulfilling crises within the framework of rational agents with incomplete information, building upon the work of Cole and Kehoe (2000).

Keywords: Noisy rational expectation; Bond auction; Debt rollover; Default
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*Department of Economics, University of Minnesota, yang5877@umn.edu. I would like to thank my advisors, Timothy Kehoe, Manuel Amador, and Marco Bassetto, for their valuable guidance and support. I also would like to thank all the participants of the Trade workshop at the University of Minnesota and the UMN-UW Graduate Workshop for their comments and suggestions.

1 Introduction

Consider an environment where government has temporarily limited resources to finance its outstanding debt. This situation is expected to last for a short period, and the resources are guaranteed to return to their original level in the next period. This is a common knowledge among lenders. The government issues a bond and promises to repay it in the next period as long as bond revenue is sufficient to finance the debt. If the government fails to finance the debt, it defaults. Although lenders do not know the exact amount of the government's debt, each lender has an estimate of it. With this knowledge, lenders bid in bond auction to maximize their expected profit from the bond investment.

In this auction, the value of the auctioned item, i.e., the profit of bond investment, is endogenously determined, particularly by an equilibrium bond price in the auction. Equilibrium bond price plays a significant role as it not only determines the cost of bond investment but also influences bond revenue, default outcome, and the profit of bond investment. Moreover, it is the outcome of aggregating all bids in the bond auction market. It can be a another signal of the amount of the government's debt.

In this paper, I analyze the role of information in a setup where the government faces a rollover debt problem and the amount of debt is not publicly known. Each lender observes a private, noisy signal of the debt. The government issues a bond with a face value of $\bar{B}$[1], promising to repay it next period as long as bond revenue is enough to finance the debt. If not, the government defaults. Lenders bid in a bond auction market to maximize their expected payoff, inferring the unknown amount of debt from their private signal and the equilibrium bond price, which is determined by the aggregation of all bids in the market. However, the market has a noise, causing the equilibrium bond price not to reveal the fundamental perfectly.

To investigate this setup, I construct a noisy rational expectation model and focus on a monotone and symmetric equilibrium, where all lenders have the same optimal bidding strategy, which monotonically decreases in the private signal. I examine how more precise information about the government's debt affects the equilibrium bond price and default outcome.

I first find that having sufficiently precise information can lead to multiple equilibrium bond prices. Specifically, it generates multiple prices which clear the bond market with a fixed face value. This result is consistent with the findings of Hellwig et al. (2006) and Bassetto and Galli (2019). The limit case of the precise information, which is the case of

[1]The government is guaranteed to have resources as much as $\bar{B}$ in the next period, provided the government does not default. The bond is issued based on this fact, which lenders know.

perfect information, also has multiple equilibria: one where the market values the bond price low, the government defaults, and this verifies the market's evaluation; and the other where the market values the bond high, the government repays, and this also verifies the market's evaluation. This result aligns with the concept of multiple equilibria in a self-fulfilling crisis, as described by Calvo (1988) and Cole and Kehoe (2000).

Under a parameterization that delivers a unique equilibrium, I find the following main result: precise information makes the price schedule more sensitive to the market-clearing price, and this increased sensitivity can have an adverse (or advantageous) impact on low (or high) debt states.[2] A marginal lender with rational expectations can infer debt state and market noise from a market-clearing price, which is called a market signal. He evaluates a bond price based on this market signal. As the private signal becomes more precise, the marginal lender has a more concentrated understanding of the debt state, which causes the price schedule to react more sensitively to the market signal. The increased sensitivity implies that the bond price appreciates more with a good market signal and depreciates more with a bad market signal.

An important feature of this paper is that the marginal lender cannot fully separate fundamental (i.e., debt state) and fundamental-irrelevant noise (i.e., market noise) from the market signal, even with highly precise information. The market signal is derived from the market-clearing bond price and is used by the lender to infer the debt state. There are some states where the debt level is low, but the marginal lender perceives it as a bad market signal. Conversely, in some states, the debt level is high, but the marginal lender perceives it as a good market signal. There is a discrepancy between the actual debt state and the inferred debt state based on the market signal.

Due to the combination of this discrepancy and the sensitive price schedule coming from precise information, there are cases where the debt level is low (or high) but perceived as a bad (or good) signal, resulting in a depreciated (or appreciated) price. This effect is significant enough that it can cause a government with low debt to default even though the debt level is relatively low. The government could have repaid it without the sensitive price schedule. On the other hand, the sensitive price schedule appreciates the bond price sufficiently for high-debt states, simply because the states are perceived as good market signals. Consequently, high-debt states can repay, even though the actual debt level is relatively high, and it could have defaulted without the sensitive price schedule.

Certainly, a marginal lender is better able to distinguish true fundamentals and market

[2] I restrict the degree of information precision parameter to guarantee a unique market-clearing price. Then, I proceed with comparative statics for equilibrium bond price and default outcome with respect to the precision parameter.

noise from a market signal as the private signal becomes more precise. It is true that among a set of good signals, there are more states that are also fundamentally good. However, the effect of price depreciation to bad signals dominates the effect of the market signal in more accurately revealing good fundamentals. As a result, low-debt states are adversely affected by the high precision of information. Conversely, the opposite happens to bad fundamentals when high-debt states benefit from highly precise information. This is because the effect of price appreciation on a good signal dominates the effect of the market signal in more accurately revealing bad fundamentals.

Finally, I demonstrate that the main result is robust in an environment where there is a prior belief about the debt level. In an economy with a low prior belief, it is common knowledge that the debt level is likely to be low, so the marginal lender is more positive toward the market signal. Conversely, in an economy with a high prior belief, the marginal lender is more negative toward the market signal. This means that the same market signal can be perceived as a good signal in a low prior economy, whereas it is interpreted as a bad signal in a high prior economy. Furthermore, the price schedule is generally higher in a low prior economy compared to a high prior economy. A low prior economy has a higher average bond price and a higher probability of repayment compared to a high prior economy, regardless of the degree of information precision.

As the information becomes more precise, the probability of repayment in a low prior economy decreases compared to that of low precision, and the opposite occurs in a high prior economy. It is low-debt states that are negatively affected by high information precision (i.e., lower bond price and a higher likelihood of default), while high-debt states are positively affected by high information precision (i.e., higher bond price and a higher likelihood of repayment). Since a low prior economy is more likely to realize low debt states, it is adversely affected by high information precision, resulting in a lower average bond price and a lower probability of repayment. Conversely, a high prior economy is more likely to realize high debt states, leading to a higher average bond price and a higher probability of repayment with high information precision.

The rest of the paper proceeds as follows: first I document literature related to this paper. In Section 2, I describe the model and define the equilibrium of the model. In Section 3, I explain how I solve the model and present a bond price characterizing equation, which is the key equation of the paper. I also show the possibility of multiple market-clearing prices. In Section 4, I go through a numerical exercise to proceed with comparative statics on bond price and default outcome, having information precision as a parameter of interest. I present how information precision affects price schedule and market signal, after which the main result comes. In Section 5, I extend the model by adding prior belief. I present how prior belief can change the price schedule and show that the main result is robust to the extended model.

1.1 Related literature

Cole et al. (2018) set up the model where lenders have asymmetric information about a country's fundamentals and bids in bond auction market with rational expectation. They compare expected bond yields and yield volatility in uniform price and discriminatory price auction protocol. However, they assume exogenous default and it solely depends on a country's fundamentals. This paper assumes endogenous default and specifically default happens with not enough bond revenue. In my model, default occurs not necessarily in a bad fundamental but in not enough high bond price compared to debt level.

Since this paper assumes an environment of rollover risk and default with not enough bond revenue, it shares the same sprite of self-fulfilling crises and multiple equilibrium with Calvo (1988) and Cole and Kehoe (2000). Among huge literature of self-fulfilling crises, this paper can be classified in one with heterogeneously informed lender. He et al. (2019) is one of the recent paper with heterogeneously informed lender in country's rollover risk. They assumes country defaults with not enough bond revenue and lenders chooses where to invest in between two countries different size. This paper assumes the same country's environment where low valuation from the market can lead to a country's default, however I assumes lenders have more complicated action. In my model, lenders buy bond through bond auction market and therefore their action is price contingent investment decision rather than binary choice of invest or not.

This paper is also related to another huge literature of noisy rational expectation equilibrium. Bassetto and Galli (2019) and Hellwig et al. (2006) use noisy rational expectation model to capture a situation with default risk and currency crisis each. Both papers find that multiple market clearing price can happen with endogenous default or endogenous currency regime choice. Bassetto and Galli (2019) also find that price is more sensitive to market signal as lender's information is more precise. My paper has a contribution in those papers in a sense that I analyze further to default outcome. Using the finding of sensitive price schedule, I analyze how sensitive price schedule can make change in default outcome and whether sensitive price schedule is good for low/high debt economy or not.

2 The model

I consider an environment where government faces a rollover debt problem. If government manages to finance debt, it guarantees to have resources $\bar{B}$ in the next period. Based on it, government issues bond with face value $\bar{B}$ and bond revenue is used to finance debt if possible. Government has to default with not enough bond revenue compared to θ. The debt θ is not perfectly observed and each lender has private noisy signal s_i of it. Lenders bids in bond auction market and they infer θ through not only s_i but also bond price p using rational expectation. There is market noise so that θ is not perfectly revealed from bond price.

2.1 Players, Actions, and Payoffs

In the model, there are a government and an unit mass of lenders, indexed by $i \in [0, 1]$. The government faces outstanding debt, θ, and if it is financed well, the government is guaranteed to have $\bar{B}$ in the next period. It can be understood as government has a limited resources temporarily and as long as no default, it gets back to the original resource level for sure. With this guarantee, government issues bond with face value $\bar{B}$ and it repay iff $\theta \leq p\bar{B}$. If $\theta > p\bar{B}$, it defaults on all its obligations including newly issued debt. I assume $\bar{B}$ is fixed and a parameter of the model so that government is a manual player; it issues a fixed amount of bond and it defaults manually.

Lenders are more strategic players. Bond is traded in bond auction market and each lender submits a bid to buy the bond. For tractability, I assume that each lender is limited to buy one unit of bond or not, $b_i \in \{0, 1\}$ where b_i is bond holding of lender i. I define a bid is a price contingent bond holding schedule, $\{b(p)\}_p$, where $b(p)$ is bond holding if bond price is p.

Payoff from a bid depends on what p is and whether government defaults or not. When bond price is determined as p and $b(p) = 1$, lender wins the auction and pay p for purchasing one unit of bond regardless of default decision. If government repays, lender will get payed one unit of numeraire in the next period. If government defaults, lenders gets nothing. I assume zero discounting between periods for lenders and outside options payoff is normalized as 0 so that payoff of no purchasing bond is 0 regardless of government default. Lender's payoff is summarized in Table 1.

	Default	Repay
$b(p) = 1$	$-p$	$-p+1$
$b(p) = 0$	0	0

Table 1: Lender's payoff of bidding when bond price is p

2.2 Auction Protocol and Auction Market

In sovereign bond auction market, there are mainly two auction protocol: uniform price auction and discriminatory auction. Uniform price auction is where all auctioned item is executed at a single price which is called as a marginal price. Discriminatory auction is where auctioned item is executed at the price each bidder bids at. Brenner et al. (2009) categorized countries into what type of protocol they use for sovereign bond auction. Based on their finding, Germany, France, Greece and most other European countries use discriminatory auction protocol and USA, South Korea, Argentina use uniform auction protocol. I focus on uniform price auction protocol in this model.

Bond auction goes on as follows : Each lender submits its bidding $b_i(p)$ simultaneously. Auctioneer collects all bidding and set a marginal price p at which bond is executed. For Lenders who bid to buy the bond at p, $\{i|b_i(p) = 1\}$, win the auction and have obligation to buy the bond at price p. Those who bid not to buy the bond at p, $\{i|b_i(p) = 0\}$, lose the auction and they lose nothing nor gain anything. The next period, bond holders will get repayed one unit of numeraire only when government does not default.

Marginal price p is determined to clear the bond price. Bond market clearing condition is as follows.

$$\int b_i(p)di + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B} = \overline{B} \tag{1}$$

In order to prevent bond price from fully reveal the fundamental, I put market noise μ as a form of noisy trader. Noisy traders bid sufficiently high so that they always win the auction and take $\Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)$ portion of total bond supply. $\mu \sim N(0, \frac{1}{\alpha})$ and it is independent from θ, meaning it is a pure fundamental-irrelevant market noise. σ is another parameter of the model and it governs how much concentrated impact random variable μ has to the market. High σ will make the level of the portion more concentrated with a fixed range of μ and low σ will make the level of the portion more dispersed with the same range of σ.

2.3 Information Structure and Timing

Each lender privately observes noisy signal of the fundamental. Lender i's private signal is $s_i = \theta + \epsilon_i$ where $\epsilon \sim N(0, \frac{1}{\beta})$. s_i can be interpreted as each lender's estimate of the fundamental. β is a key parameter in this paper, meaning the precision of individual information. High β means lenders are better informed of the fundamentals. I assume LLN applies with sufficiently large number of lenders so that mass of lenders with signal lower than x equals to probability that s is lower than x. The prior of θ is uniform in $\mathbb{R}$ (improper uniform distribution) for tractability, but I generalize it to $N(\theta_0, \frac{1}{\gamma})$ in the later section of the paper.

The timing is as followed : in period 1, the nature draws $(\theta, \mu, \{\epsilon_i\}_{i\in[0,1]})$. Government issues bond in the bond auction market. Each lender i observes privately its signal s_i and simultaneously bids on the bond auction market. Auction closes and market clearing price p is determined. Winner of the auction market is decided bond is executed accordingly. In period 2, government observes θ and default/repay outcome is determined automatically by comparing debt level and bond revenue. Lenders' payoff is determined accordingly based on default/repay outcome

2.4 Strategy and Symmetric Equilibrium

Lender's strategy is a function mapping from signal s_i to bidding $b(p)$. I focus on symmetric equilibrium where all lenders has the same strategy in equilibrium. From now on I denote strategy as $b(s_i, p)$ meaning bid with signal s_i at marginal price p. Lender chooses the optimal strategy $b(s_i, p)$ to maximize expected payoff

$$\mathrm{E}[(-pb_i(p) + b_i(p))\mathbf{1}\{p\overline{B} \geq \theta\} + (-pb_i(p))\mathbf{1}\{p\overline{B} < \theta\}|s_i, p] \tag{2}$$

The first chunk is payoff conditional on repay and the second one is conditional on default. The expected payoff is in other form as follows.

$$\mathrm{E}[-pb_i(p) + b_i(p)\mathbf{1}\{p\overline{B} \geq \theta\}|s_i, p]$$

Symmetric equilibrium consists of lender's strategy $b(s_i, p)$ and lender's belief on state where $b(s_i, p)$ maximizes expected payoff for all (p, s_i) and belief consistency holds. Equilibrium outcome that I am interested in is bond price for all states $p(\theta, \mu)$ and according default outcome for all states.

3 Solving the Model

3.1 Optimal Bidding Strategy

Define $\delta(p, s_i)$ as expected repay probability conditional on market price p and private signal s_i. Then expected payoff of bidding $b(p, s_i)$ at a given (p, s_i) is

$$\delta(p,s_i)\{-b(p,s_i)p+b(p,s_i)\}+(1-\delta(p,s_i))\{-b(p,s_i)p\}=b(p,s_i)(\delta(p,s_i)-p)$$

Since it is linear in $b(p,s_i)$, optimal bidding strategy is

$$b(p,s_i)=\begin{cases}1, & \text{if} \quad \delta(p,s_i)>p\\ 1 \quad \text{or} \quad 0, & \text{if} \quad \delta(p,s_i)=p\\ 0, & \text{if} \quad \delta(p,s_i)<p\end{cases}$$

p is cost for holding bond regardless of default or repay, whereas $\delta(p,s_i)$ is expected benefit for holding bond and it is higher as the lender expects repay with a higher probability.

Expected repay probability is affected in three ways : first, by the private signal, s_i. As lenders receive smaller s_i, it implies θ is also more likely to be small. They expect government is more likely to repay at a given p and therefore they are more willing to buy the bond. Second, p determines bond revenue $p\bar{B}$ and it affects default/repay outcome. At a given s_i and a corresponding posterior θ belief, higher p implies bond revenue is also higher, meaning government is more likely to repay. The fact that p can affect expected repay probability and further bond demand with a given posterior θ belief is because default outcomes depend on how market value its bond. It implies that self-fulfilling default could happen; at a low p, lenders expect low repay probability and do not buy the bond. Aggregate bond demand is low and market price p is set as a low level and it verifies initial lender's pessimistic belief.

Lastly, p can be another signal for unobserved θ. Bond market clearing condition explains it more clearly.

$$\int b_i(p,s_{i|\theta})di+\Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B}=\overline{B} \tag{3}$$

Note that p that satisfies the above equation is market clearing p and market clearing p should satisfy the above equation; it is the characteristic equation for market clearing p. Also note that the equation has a given state of the world, (θ,μ). μ determines the portion of total bond supply that noisy trader gets and θ determines the distribution of s_i, affecting lender's bidding strategy and the aggregated demand of the bond. In short, market clearing equation relates bond price and a state (θ,μ). In order for a high p to be a market clearing p, it should be either the case where the fundamental is good (θ is low) or the case where aggregate bond demand is exaggerated by large noisy trader (μ is high) though the fundamental is bad (θ is high). Rational lenders can infer (θ,μ) from observed p so that they can infer either two (θ,μ) must be the true state given high p. With this inference of state, they will update expected repay probability accordingly.

3.2 Monotone Equilibrium

There will be many classes of equilibrium of this model and I assume monotone equilibrium where optimal $b(s_i, p)$ is monotonic decreasing in s_i. With this assumption, I solve for market clearing p and verify that monotone strategy is optimal in the end.

In monotone equilibrium, equilibrium strategy is characterized as signal cutoff $s^*(p)$ where for a given p, $b(s_i, p) = 1$ for $s_i \leq s^*(p)$ and $b(s_i, p) = 0$ for $s_i > s^*(p)$. By definition, $s^*(p)$ has to satisfy $\delta(s^*(p), p) = p$. Also, equilibrium default outcome is characterized as default cutoff $\theta^*(p)$ where for a given p, default if and only if $\theta > \theta^*(p)$ and repay if and only if $\theta \leq \theta^*(p)$. By definition again, $\theta^*(p)$ has to satisfy $p\bar{B} = \theta^*(p)$. From now on, I normalize $\bar{B} = 1$ so that the equation can be simplified as $p = \theta^*(p)$.

With this equilibrium strategy, I go back to market clearing condition. For a given cutoff $s^*(p)$, aggregate bond demand from lenders is $\int b_i(p, s_{i|\theta})di = prob\{s_i < s^*(p)|\theta\} = \Phi(\sqrt{\beta}(s^*(p) - \theta))$. p is a market clearing price in a state (θ, μ) if and only if $\Phi(\sqrt{\beta}(s^*(p) - \theta)) + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right) = 1$, or equivalently

$$s^*(p) = \theta - \frac{\mu}{\sqrt{\beta\sigma}} \equiv x \tag{4}$$

The equation (4) mathematically shows how (θ, μ) is inferred from market clearing p. At a given p, with the fact that market clearing p satisfies equation (3) and the knowledge of $s^*(p)$, lenders can infer that the unknown state (θ, μ) should satisfy equation (4). I define right-hand side as x, meaning a market signal about the state conveyed by market price p. Since p is public knowledge among all lenders, x is also a public signal among lenders. Another important feature of market signal x is that although it is unbiased signal of θ, it cannot separate between θ and μ. It means a given x can narrow down the entire state set into $\{(\theta, \mu)|\theta - \frac{\mu}{\sqrt{\beta\sigma}} = x\}$, however it still cannot pin down the true (θ, μ). As mentioned, x is unbiased estimator of θ and $x \sim N\left(\theta, \frac{1}{\beta\sigma\alpha}\right)$. The precision of x depends not only β, α but also σ. High σ makes the impact of noisy trader more concentrated for a given interval of μ, having the same impact with high precision of μ.

3.3 Characteristic Equation for Bond Price

Given x and its distribution, lender i with s_i updates posterior belief of θ as the follows.

$$\theta|s_i, x \sim N\left(\frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}, \frac{1}{\beta + \sigma\alpha\beta}\right)$$

For marginal trader with signal $s^*(p)$, posterior belief of θ is

$$\theta | s^*(p), x \sim N\left(x, \frac{1}{\beta + \sigma\alpha\beta}\right)$$

Marginal lender is as optimistic as the market signal. Those with $s_i > x$ expects θ to have its mean less then x, meaning less optimistic, and $b(s_i, p) = 0$. On the other side, those with $s_i < x$ expects θ to have its mean greater then x, meaning more optimistic, and $b(s_i, p) = 1$. With this update, $\delta(s_i, p) = prob\{\theta \leq \theta^*(p) | s_i, p\} = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$ and for marginal lender, $\delta(s^*(p), p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\,(\theta^*(p) - x)\right)$. Characteristic equation for market clearing p is from indifference condition for marginal lender. Substituting $\theta^*(p)$ to p, the key equation in this paper is as follows. Market clearing p at a given state x is implicitly characterized as

$$p = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\,(p - x)\right) \tag{5}$$

p determining not by θ only but by x naturally comes from the fact that market signal cannot separate θ from x. Marginal lender neither can separate θ from x and he decide market price p depending on what his signal and market signal, x, is.

Left is verifying that monotone strategy is the optimal strategy. It is straightforward given the fact that posterior repay belief is strictly decreasing. Recall $\delta(s_i, p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$. For $s_i < s^*(p)$, $\delta(s_i, p) > \delta(s^*(p), p) = p$ by indifference condition of marginal lender. Therefore, $\delta(s_i, p) > p$ and $b_i(s_i, p) = 1$. The similar logic applies to $s_i > s^*(p)$ and for all p also.

3.4 Multiple Market Clearing Prices

It is possible that there are multiple p satisfying equation (5), meaning a possibility of multiple market clearing prices. Figure 1 shows it graphically. It plots right-hand side of equation (5) as a function of p with different precision of marginal lender's posterior belief, $\beta + \beta\sigma\alpha$. It shows that high precision case has three fixed points, meaning three market clearing prices and low precision case has an unique market clearing price. Multiple bond prices with high enough precision can be related to equilibrium in perfect information case. Consider $\theta \in [0, 1]$ and it is a common knowledge. The common knowledge version of the model has two equilibrium; one equilibrium where market values the bond low and $p = 0$, government defaults and it verifies the initial evaluation $p = 0$. The other equilibrium is where market values the bond high and $p = 1$, government repays and it verifies the initial evaluation $p = 1$. These two equilibrium can be seen as a limit case of equilibrium prices in high enough precision, one with very low p and the other with very high p.

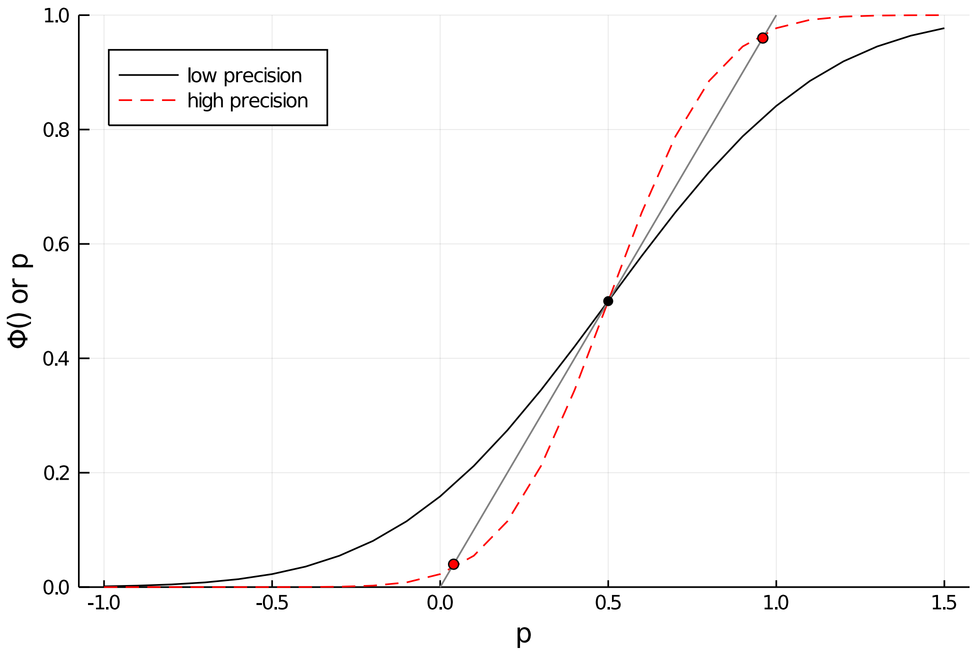

Figure 1: Equation (2) with different precision values

This paper is more focused on comparative statics in unique price equilibrium. From now on, I restrict parameter sets so that the precision is low enough to guarantee unique price. Computationally, I define unique price is attained at a given parameter (β, σ, α) if fixed points with different initial guess are equivalent within a certain tolerance level for all $x \in R$. More specifically, for a given value (β, σ, α), I set $0.5, 0.0, 1.0$ as different initial guesses, compute a fixed point for each initial guess and as long as all fixed points are equivalent within tolerance level $1e-6$ for x grid $\in [0.0, 1.0]$, I denote (β, σ, α) guarantees unique price. Numerically, it turns out (β, σ, α) guarantees unique price as long as $\beta + \beta\sigma\alpha$ < 2.439 and I proceed numerical exercise with parameter sets that satisfy the unique price guaranteeing condition.

4 Numerical Exercise

4.1 Parameterization

For numerical exercise, I set θ grid in interval $[0.0, 1.0]$ because it is relevant values for default outcome to depend on $p \in [0.0, 1.0]$. For μ grid, I set it symmetric around 0. Given the fact that mean of $\mu = 0$, $\mu > 0$ is interpreted as high noisy demand shock than the average and $\mu < 0$ as low noisy demand shock than the average. In a nutshell, $\mu < 0$ will be a

Parameter	value
σ	0.7
α	0.1
Low β	1.0
High β	2.0

Table 2: Parameter values

favorable market noise, making market signal x more likely to be a good signal for a given θ. Whether x is interpreted as a good or a bad signal crucially determines whether high precision disadvantages default outcome or not. The distribution of (θ, μ) is constructed conditionally on the truncated intervals.

Parameter values for (β, σ, α) is assigned as Table 2. In order to analyze the impact of information precision to default outcome, I especially focus on β, the precision of private signal, as a variable for comparative statics. The following numerical exercise and the main result comes from comparing default outcome in different β economies, fixing other parameters' value.

4.2 Price Schedule and the Role of β

After guaranteeing unique price, Figure 2 plots p as a function of x with different β values. High precision plot is with high β and low precision plot is with low β. First observation is p is low in high x regardless of β. market signal x, though it has some biases from market noise, in general tells what level θ is. High x means market predicts θ is also likely to be high. Marginal lender responses to it and devalues p accordingly regardless of β.

Second observation is that with high β price schedule responses more sensitively to x. Graphically it means higher concavity or convexity compared to low β case. Intuitively high β means market signal as well as private signal is a more precise estimate of θ. Marginal lender's posterior belief of θ is, therefore, more concentrated and posterior repay probability changes more dramatically by a small change of x.

Lastly, sensitive price schedule with high β reacts depending on what value x is. Sensitive price function does not necessarily appreciate p for all x, but only when x is low. When x is high, on the other hand, price function with high β gives lower p than what price function with low β does. The different responses happen around the inflection point and I define the infection point as market signal cutoff x^*. It is market's criteria for interpreting whether x is a good signal or a bad signal. In Figure 2, $x^* = 0.5$ and $x < 0.5$ is interpreted as a good market signal. Given this good signal, sensitive price schedule appreciate p more than low β price schedule, $p(x)$ with high $\beta > p(x)$ with low β for any $x < 0.5$. The reverse happens to $x > 0.5$ which is interpreted as a bad market signal. To a bad signal, sensitive price schedule depreciates more and p gets lower. Essentially, whether high precision is good or not depends on whether market interprets x as a good signal or not. Mathematically it depends on $x < x^*$ or $x > x^*$ and I will keep this comparison as I proceed numerical exercises.

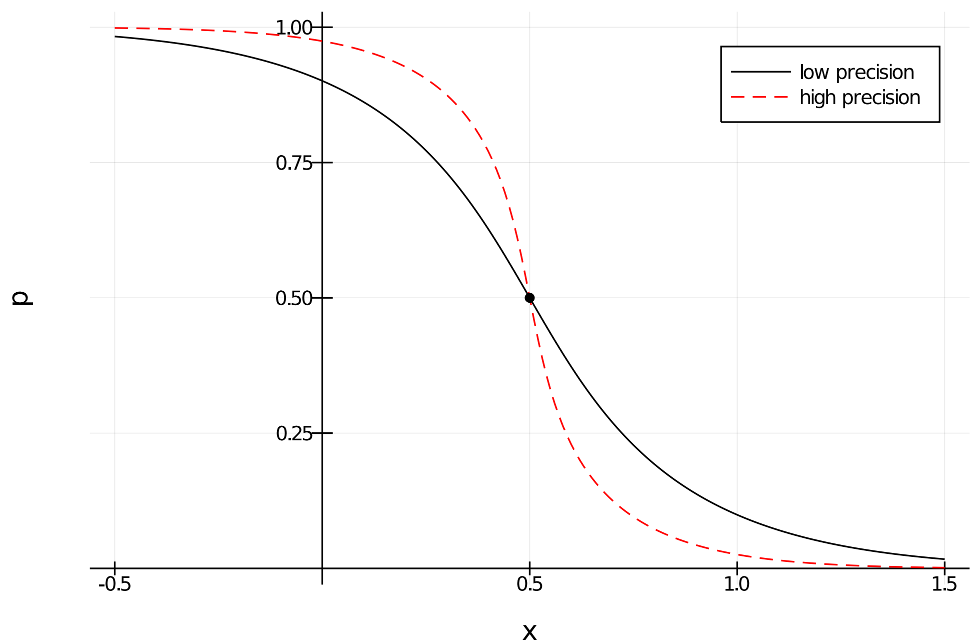

Figure 2: p as a function of x with different β

4.3 Market Signal x and the Role of β

So far I have shown high β makes price schedule more sensitive to x and it reacts differently to good signals and bad signals. In this section, I show how high β affects market signal x for a given θ. In short, higher β is, less deviated market signal from the true θ. Mathematically, $|x-\theta| = \left|\frac{\mu}{\sqrt{\beta}\sigma}\right|$, so high β makes low deviation. Intuitively, as private signal is more precise, x also becomes more precise indicator for true θ because x is constructed by aggregating all private information in the bond auction market.

Figure 3 shows it graphically. I plot x as a function of θ with a given μ, the blue for $\mu < 0$ and the red for $\mu > 0$. Dash line is low β case and solid line is high case. I plot true as a black line also. The figure shows that solid line, X with high β, is closer to the black line, true θ, compared to dash line, X with low β, for all θ realization and in both μ cases. The

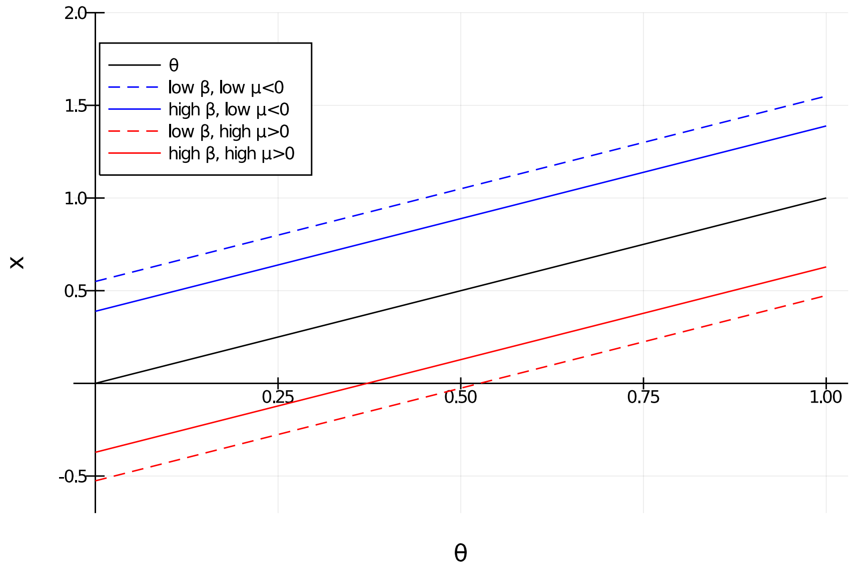

Figure 3: Sign effect of μ to x

difference between $\mu < 0$ case and $\mu > 0$ case is that x is in general low in $\mu > 0$ whereas x is in general high in $\mu < 0$. It means even with the same level of θ, x is interpreted as a good signal with high demand case and a bad signal with low demand case. It is due to the fact that lenders can infer true state from market price as a market signal but they cannot infer θ and μ separately. With the same value of x, it could come from low θ with low demand shock but it could also come from high θ with high demand shock. Therefore, whether high information precision is good or bad also depends on whether the economy has low or high demand shock because it is the demand shock that affects x being generally a good signal or not.

Figure 4 plots x with the same positive sign of μ but with different magnitude. As larger magnitude the demand shock is, x has larger deviation from true θ and more powerful the effect of high β reducing those deviation is. The figure shows that the reduction between blue solid line and blue dash line is larger than that of small magnitude of μ.

4.4 The Main Result

With observations from the previous sections, I present the main result of this paper. In a nutshell, precise information surprisingly makes relatively low θ state default whereas it makes relatively high θ state repay. With low precision, those relatively low θ state could have repayed and relatively high θ state could have defaulted. It might be a counter-intuitive result but it implies that there always exists states susceptive to market noise and the change

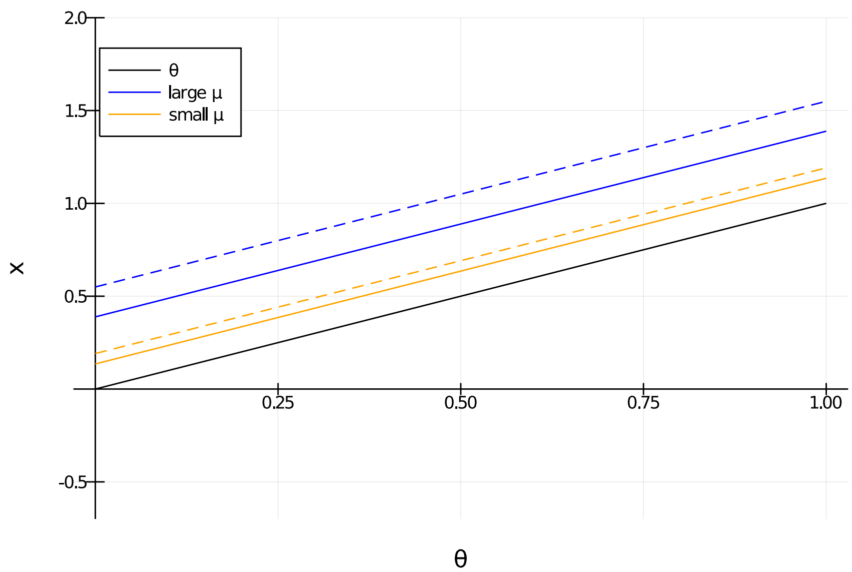

Figure 4: Magnitude effect of μ to x

in default outcome happens in those susceptive states combined with sensitive price schedule.

Figure 5 shows how x and according $p(x)$ is formed across θ with a given $\mu > 0$. Positive noisy demand makes x a good signal in general and $x < \theta$ for all θ. However, only those $x < 0.5$ are interpreted as a good signal by the market and according $\theta < 0.765$ benefit from sensitive price schedule which appreciates more to those good signals. Right panel of Figure 5 shows θ in good signal region has higher p under high β. To see how appreciated p affects default outcome, compare p and θ using 45 degree line in right panel of Figure 5. $\theta \in [0.625, 0.735]$ is the state where price under low β is lower and it defaults however price under high β is higher than θ so that it repays under high β. It is also shown in the left panel of Figure 7. It is not necessarily because θ itself is low and it has less default risk, but because θ is not too extremely high so that it can be interpreted as a good signal with positive noisy demand. Information precision makes price response to the good signal sufficiently so that good signal near the cutoff, even though it has fairly high debt level, gets benefit from it.

It is important to note that high β has trade off effect but sensitive price schedule dominates the other effect, having repay outcome in fairly high level of θ in the end. Other than high precision makes price schedule more sensitive, high precision makes market signal more accurate to true θ. As shown the left panel of Figure 5, high precision moderates the positive market noise and x in high $\beta > x$ in low β for all θ. It actually has bad effect to p and if price schedule were as sensitive as low β case, it would have had a lower p for all θ as shown in the right panel of Figure 5. Accordingly there would have had more default as shown in left panel of Figure 7. However, sensitive price schedule dominates higher x effect

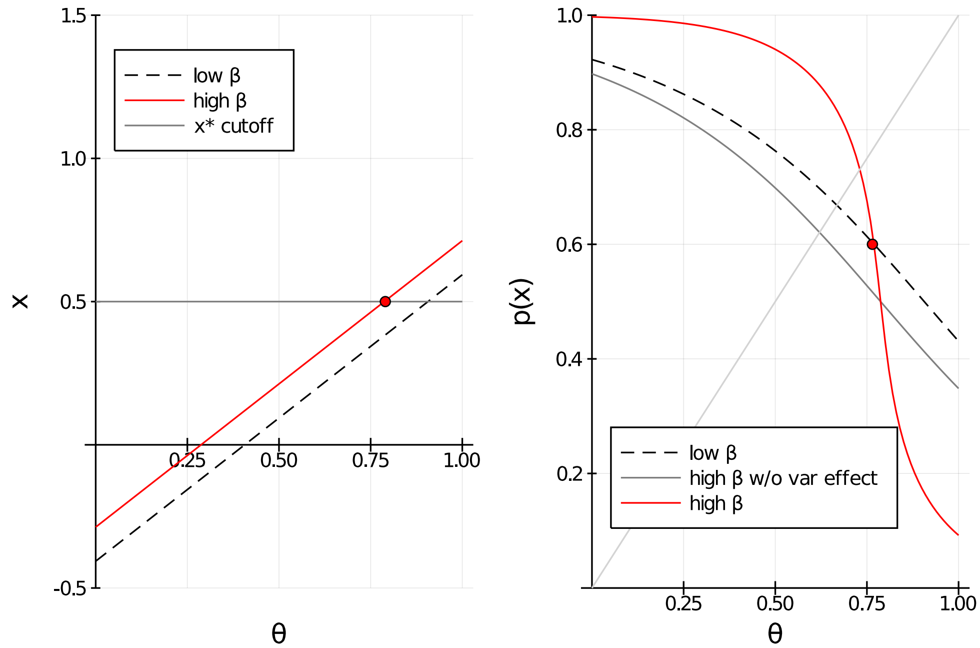

Figure 5: x and $p(x)$ in positive μ case

and even though market signal is not as much as good compared to low β, there are θ states that benefit from high precision.

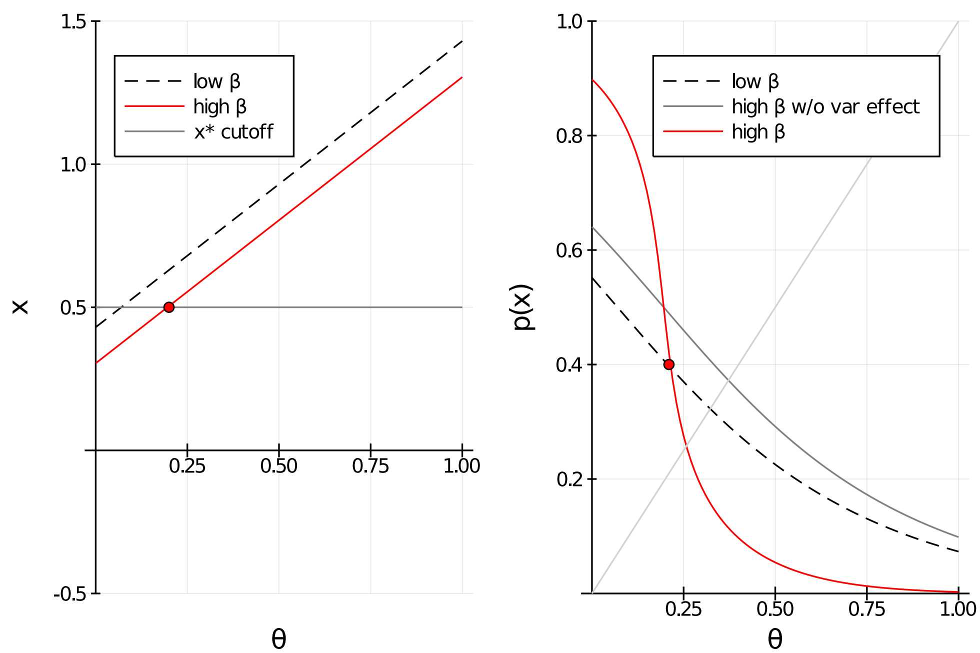

Figure 6: x and $p(x)$ in negative μ case

The exact opposite happens in negative noisy demand shock case in Figure 6. Negative noisy demand makes x a bad signal in general and $x > \theta$ for all θ. However, only those $x > 0.5$ are interpreted as a bad signal and according $\theta > 0.235$ disadvantages from sensitive price schedule which depreciates more to those bad signals. From the right panel of Figure 6, $\theta \in [0.265, 0.375]$ is the state where price under low β is higher and it repays however price under high β is lower than θ so that it defaults under high β. The change in repay outcome is shown in the right panel of Figure 7. Again, the state get changed to default not necessarily because θ itself has high value and it has high default risk but because θ is not too extremely low so that it can be interpreted as a bad signal with negative noisy demand. The reason why it happens is because market cannot separate repay relevant fundamental θ from market signal x and there exists some θ that itself is a good fundamental but are seen as a bad signal by the market with sufficient market noise.

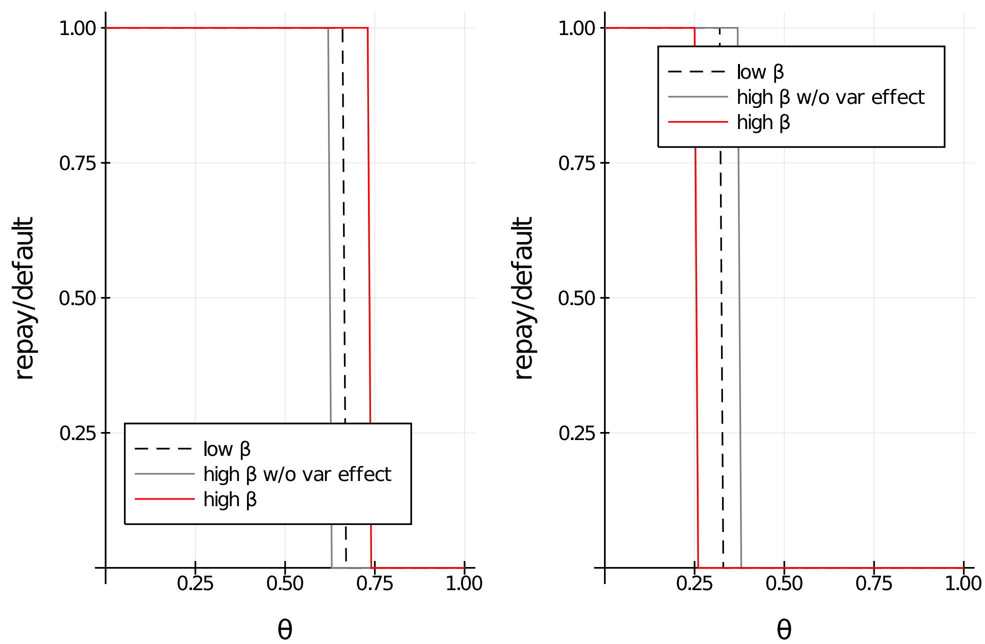

Figure 7: Default outcome in positive (left) and negative (right) μ

5 Extension : Add a Prior on the Debt Level

The model has θ prior as $\theta \sim U(R)$ and it has no notion of mean of θ. It is convenient for tractability but a bit far from being realistic. There is a country with low debt level on average and also a country with high debt level on average. Analytically no prior means market price solely depends on market signal x. I extend the model into the one where prior of θ is a common knowledge and lenders updates posterior belief of θ based on not only market signal x and its private signal s_i, but also prior belief.

5.1 The Model with Prior

I assume the economy has prior on θ as $\theta \sim N(\theta_0, \frac{1}{\gamma})$, where γ is precision of θ distribution and θ_0 is prior mean. It is a common knowledge. Almost all parts of analysis hold as same except for lender s_i's posterior belief. It becomes $\theta|_{s_i,x,\theta_0} \sim N\left(\frac{\gamma\theta_0+\beta s_i+\beta\sigma\alpha x}{\gamma+\beta+\beta\sigma\alpha}, \quad \frac{1}{\gamma+\beta+\beta\sigma\alpha}\right)$. For a marginal lender, it becomes

$$\theta|_{s^*(p),x,\theta_0} \sim N\left((1-\tau)\theta_0 + \tau x, \quad \frac{1}{\gamma+\beta+\beta\sigma\alpha}\right)$$

where $\tau = \frac{\beta+\beta\sigma\alpha}{\gamma+\beta+\beta\sigma\alpha}$. Adding prior gives additional information of θ and lenders form posterior belief not only based on x and s_i, but also θ_0. Closely looking at the mean, it is a weighted average of prior mean θ_0, private signal driven mean s_i, and market driven means x, which are weighted at its precision. As before, marginal lender is as optimistic as the market and he has his private signal as same as market signal, $s^*(p) = x$. Those whose $s_i > x$ are less optimistic than marginal lender and those whose $s_i < x$ are more optimistic and they are the ones who buy the bond.

Bond price determined by marginal lender is

$$p = \Phi\left(\sqrt{\gamma+\beta+\beta\sigma\alpha}\,(p-(1-\tau)\theta_0 - \tau x)\right) \tag{6}$$

Again, price is a fixed point of equation (6). Sufficiently high precision induces multiple prices still holds here and I adjust parameter values accordingly to guarantee unique price. The specific values are listed in Table 3. Furthermore, I do comparative statics with regard to different value of θ_0 and I set low prior case $\theta_0 = 0.1$ representing low debt country and high prior case with $\theta_0 = 0.9$ representing high debt country.

5.2 Price Schedule and the Role of prior

Even though with additional parameters, p is still a function of x and Figure 8 plots it with different precision and prior values. As in the main model, price schedule is a decreasing function of x. It also has the inflection point around where sensitive price schedule has a different response. The way θ_0 affects price schedule is it changes x^*. Recall that in the main model, $x^* = 0.5$. It turns out that the model with $\theta_0 = 0.5$ has the same $x^* = 0.5$ and price schedules with low and high β. The model without prior can be understood as the model

Parameter	value
σ	0.5
α	0.1
γ	0.3
Low β	1.0
High β	2.0
Low θ_0	0.1
High θ_0	0.9

Table 3: Parameter values in the model with prior

with $\theta_0 = 0.5$. The red graphs represents price schedules with low θ_0 and its $x^* > 0.5$. The fact the the economy has a low prior makes market have more generous signal criteria.

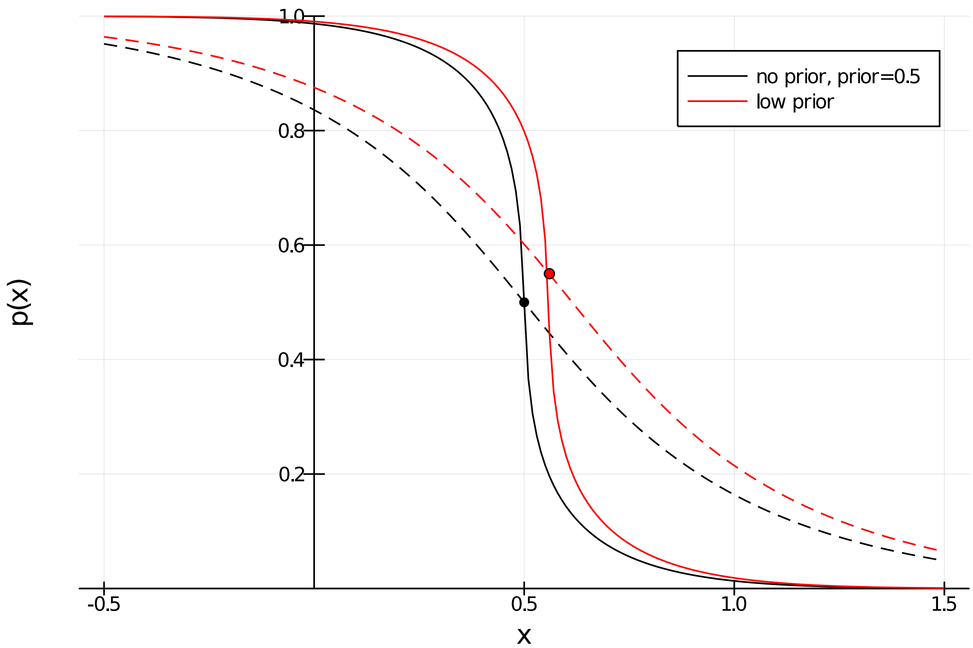

Figure 8: Price schedule with no prior and low θ_0

Figure 9 plots price schedule with low and high prior. Again, it has main features but different x^*. High prior has lower x^* and it means market is much harsher to good signal to high prior economy. The same value of x can be interpreted as a bad signal to high prior economy but a good signal to low prior economy.

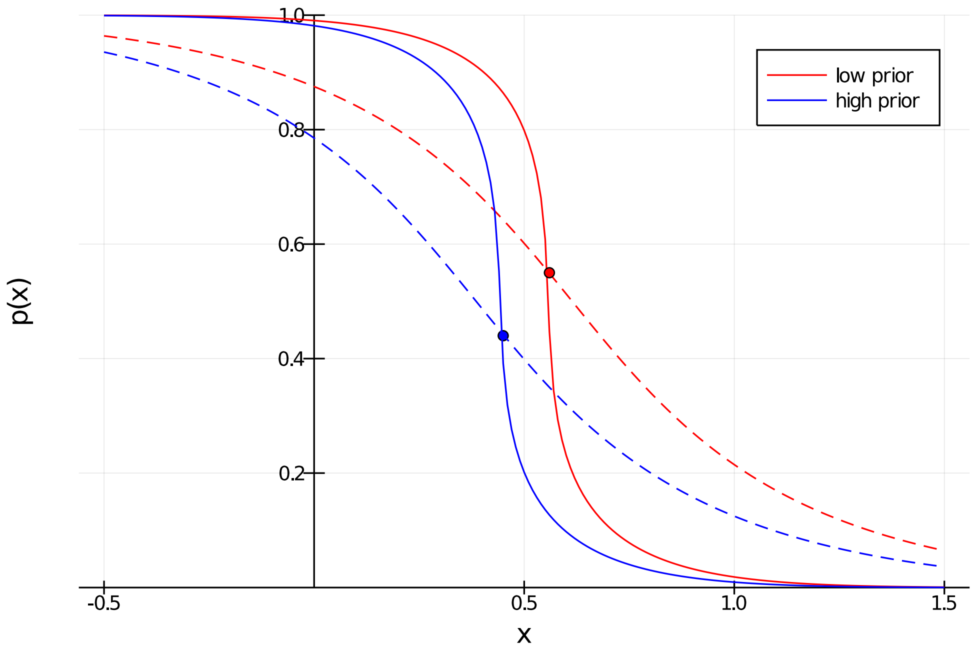

Figure 9: Price schedule with low and high θ_0

5.3 The Main Result with Prior

With the observation, I do the same exercise. I compare how price and default outcome changes as β gets high between low θ_0 and high θ_0 case. The result is robust in the model with prior and it can be extended to the conclusion that the economy with low debt on average has higher default probability with more precise information. This could be seen as counter-intuitive but it is because low debt country is more likely to have states where itself is low debt level but market interprets the state as a bad signal. Sensitive price schedule with precise information is a disadvantage to those states.

Figure 10 and Figure 11 shows it graphically. It shows for a given $\mu < 0$ (Figure 10) and $\mu > 0$ (Figure 11), how market signal x and according $p(x)$ is determined for each θ. The red represents low θ_0 economy and the blue represents the high one. Note that x is independent from any θ_0 for all states (θ, μ). It is the realization of θ and according s_i's distribution that determine what x is and it has nothing to do with θ's prior distribution. Both low θ_0 and high θ_0 economy has, therefore, the same value of x for a given state (θ, μ). What makes two economies differ is the way how the market interpret the same x. Low θ_0 economy has higher x^* and it has larger range of x interpreted as a good signal whereas high θ_0 economy, it has narrowed range of good signal. It is due to the fact that low θ_0 has a lower prior and it happens regardless of μ realization. It is shown in the left panel of Figure 10 and Figure 11.

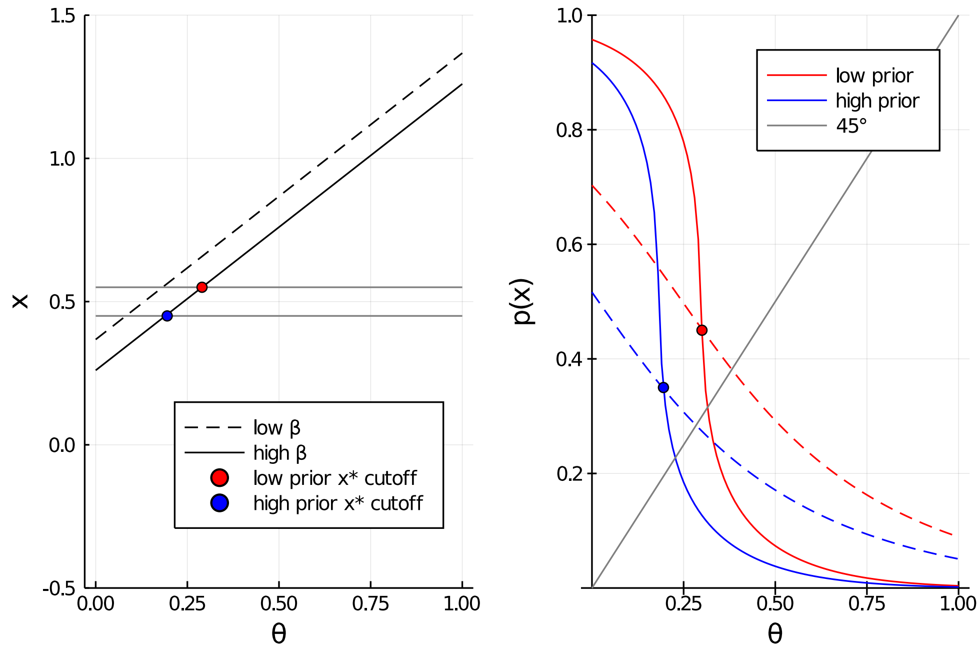

Figure 10: x and $p(x)$ for a given $\mu < 0$

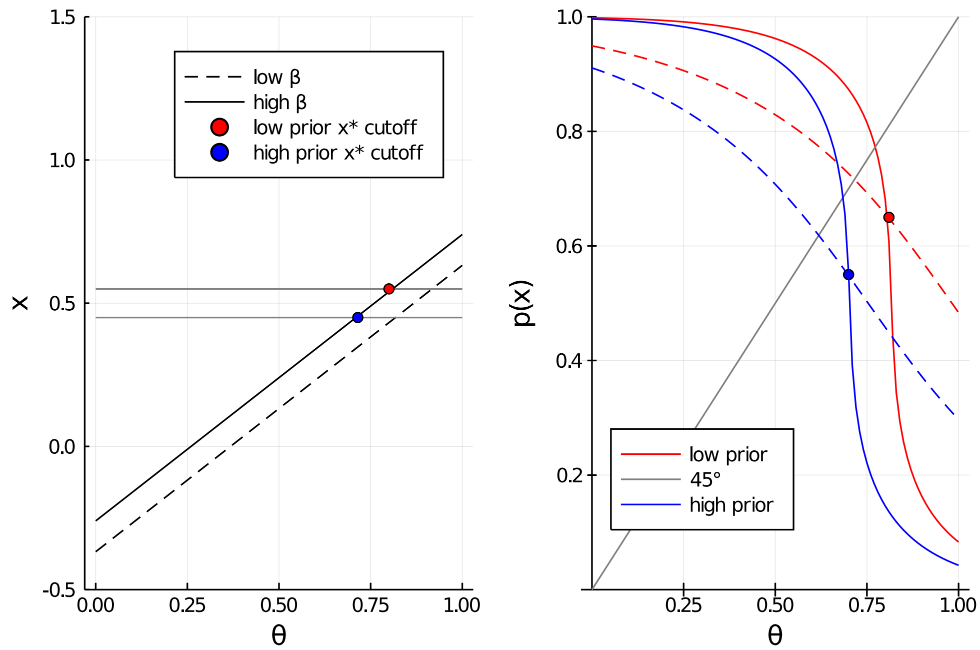

Figure 11: x and $p(x)$ for a given $\mu > 0$

Given this different interpretation, price reacts accordingly. It is shown in the right panel of Figure 10 and Figure 11. Note that low prior economy has higher p than high prior economy for all θ and regardless of μ being positive or negative. It is a combined result of price schedule of low prior being high in general and low prior having a larger range of good signal. Not only that low prior economy benefits from sensitive price schedule in a larger θ range since it has a larger range of good signal. However, regarding default outcome, the same thing happens as before. It seems high β is better for low prior economy in general so far, however, closer look to default outcome concludes differently. Low prior economy still has a θ range where it gets disadvantages from sensitive price schedule and have lower p. Sensitive price schedule devalues a bad signal sufficiently so that $\theta \in [0.315, 0.380]$ gets to default with high β where it used to repay with low β. $\theta \in [0.230, 0.285]$ is a counterpart in high prior economy. Both happens because as before θ is not low enough so that it is interpreted as a bad signal with a negative market noise and p largely depreciates. On the other hand in positive market noise case (Figure 11), the opposite happens and $\theta \in [0.715, 0.775]$ changes to repay in low prior economy and it is $\theta \in [0.620, 0.685]$ for high prior economy.

Figure 12 shows repay/default outcome in each $\mu < 0$ (left panel) and $\mu > 0$ (right panel) case. Repay is valued as 1 and default as 0. Similar to the model without prior, it is low θ state where it changes to default from repay as higher β and it is high θ state where it changes to repay from default as higher β. It happens in both low and high θ_0 economy. The difference is that since low prior economy is more generous to good signal, market generally values p higher to low prior economy than high prior economy. As a result, it has larger θ range of repay compared to high prior economy.

I calculate repay probability, defined as weighted average of repay outcome in all states (θ, μ) weighted by probability, and the result is in Table 4. The number can be seen as low in general, but it should be seen given the fact that repay probability with neutral prior ($\theta_0 = 0.5$) is 0.5. The table shows that low prior economy gets lower repay probability with higher information precision whereas high prior economy gets benefits from it. The reason is as follows. It is states (θ, μ) with relatively low θ that get p depreciated by sensitive price schedule and change to default from repay. Reversely, it is states (θ, μ) with relatively high θ that get p appreciated by sensitive price schedule and change to repay from default. I set μ grid symmetric so that positive market noise can realize as equal likely as negative market noise. In neutral prior or no prior case, both disadvantageous and advantageous event from high precision are equally likely to happen, cancel out each other and have no change in repay probability with high precision. Low prior economy is more likely to have relatively low θ and, although it is a good fundamental, it is more venerable to be interpreted as a bad signal with negative market noise and get changed to default with high precision. On the other hand, high prior economy is more likely to have relatively high θ and it has a change to be interpreted as a good signal with positive market noise and when that happens,

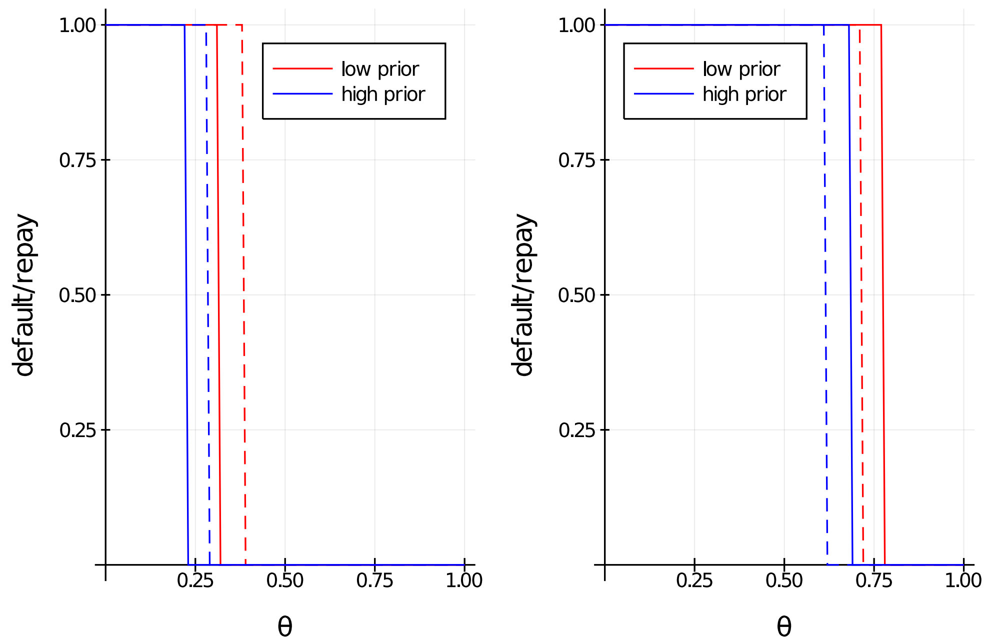

Figure 12: Default outcome with $\mu < 0$ (left) and $\mu > 0$ (right)

	low β	High β
Low prior (θ_0=0.1)	0.5500	0.5438
High prior (θ_0=0.9)	0.4499	0.4561
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

Table 4: Repay probability

it can get benefit from precise information since it appreciates p sufficiently so that it has higher p than θ even though θ itself is relatively high. However, note that low prior economy has higher repay probability than high prior economy in both low and high information precision systems. Market has more generous good signal criteria to low prior economy due to the fact that the economy has a good fundamental in general. However, market cannot distinguish the relevant fundamental and market noise so that price response and default outcome accordingly can happen adversely to a good fundamental economy.

I also calculate average bond price, defined as weighted average of p in all states (θ, μ) weighted by probability, and Table 5 shows the result. Low prior economy has higher average bond price than high prior economy in both low precision and high precision case. It shows that market appreciate a fact θ is more likely to be low and fundamentally good in general. However, as in Table 4, low prior economy has lower average price in high precision case.

	low β	High β
Low prior (θ_0=0.1)	0.5585	0.5501
High prior (θ_0=0.9)	0.4414	0.4498
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

Table 5: Average bond price

Low θ state is more likely to get adversely affect by precise information and p get depreciated by sensitive price schedule and low prior economy is more likely to have those states realized.

6 Conclusion

This paper explores the impact of precise information on default outcomes through bond prices. The key finding is that precise information increases the sensitivity of the price schedule to the market signal. However, this heightened sensitivity can have adverse consequences for states with low debt levels. The reason behind this is that the market struggles to distinguish between fundamental information and irrelevant market noise within the market signal. As a result, states that are fundamentally good with low debt levels are more susceptible to negative market noise compared to states with bad fundamentals. Consequently, fundamentally good states are more likely to be perceived as a bad market signal, prompting the sensitive price schedule to devalue the bond price. This devaluation effect is substantial enough to push a government with low debts into default, despite the state itself being fundamentally sound and capable of repayment without the sensitive response.

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