The Role of Information in Bond Auction Markets

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June 6, 2025
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Abstract

This paper examines the occurrence of a debt crisis in a country that relies on raising bond revenue to finance its outstanding debt through bond auctions. This bond auction involves multiple lenders who do not observe fundamentals perfectly. Using a noisy rational expectation model, I demonstrate analytically that refinancing risk can arise due to information friction, even in cases where the fundamentals alone would not have led to a crisis. The high precision of information amplifies the sensitivity of the bond price, causing a discrepancy between default outcomes implied by the fundamentals and those driven by fundamental-irrelevant shocks. Pessimistic signals can trigger a country to default, even in scenarios with low-debt fundamentals. Conversely, optimistic signals can help a country avoid default, even when facing high-debt fundamentals. The paper highlights how information friction renders the rollover debt problem more vulnerable upon the work of Cole and Kehoe (2000).

Keywords: Noisy rational expectation; Primary market; Debt rollover; Refinancing risk; Default

JEL Classification Numbers: D44, D82, F34, H63

*Department of Macroeconomics and Financial policy, Korea Development Institute (KDI). *First version: September 2020,* Email: juyoungyang@kdi.re.kr. This is a revised version of one of the chapter of my dissertation at the University of Minnesota. I am indebted to Timothy Kehoe, Manuel Amador, and Marco Bassetto, for their valuable guidance and support. I would like to thank all the participants of the Trade workshop at the University of Minnesota, UMN-UW Graduate Workshop, 2024 SKKU SERI, and the 23rd SAET conference for their comments and suggestions.

1 Introduction

The primary market for sovereign bonds is a central mechanism through which governments raise funds, primarily to refinance maturing debt. Debt refinancing is therefore a principal motivation for issuing sovereign bonds in this market, which typically operates via auction. In such auctions, investors submit bids based on their assessments of expected profitability and default risk. Importantly, issues of information transparency are particularly salient in the auction process, where information availability can significantly influence bidding behavior and pricing. While the existing literature has largely focused on how information asymmetry in the primary market affects bond yields (Cole et al. (2018), Bassetto and Galli (2019), Cole et al. (2022), Cole et al. (2025)), relatively little attention has been given to its implications for debt refinancing, despite the fact that refinancing is a key aspect of sovereign debt management. This paper addresses this gap by examining the role of information asymmetry in debt rollover and refinancing risk in the context of primary sovereign bond markets.

In this paper, I analyze the role of information asymmetry in a setting where the government faces a debt rollover problem, and the exact refinancing needs are not publicly known. Each lender observes a private, noisy signal regarding the amount of debt that must be refinanced. The government issues a bond with face value $\bar{B}$[1], promising repayment in the next period provided that the bond revenue is sufficient to cover the outstanding debt; otherwise, the government defaults. Lenders participate in a bond auction, seeking to maximize their expected payoff by inferring the unknown debt level from both their private signals and the equilibrium bond price, which results from the aggregation of all bids. However, due to the presence of market noise, the equilibrium bond price does not perfectly reflect the underlying fundamentals, thus preserving information asymmetry in the market.

To investigate this setup, I construct a noisy rational expectation model and focus on a monotone and symmetric equilibrium, where all lenders have the same optimal bidding strategy, which monotonically decreases in the private signal. I examine how more precise information about the government's debt affects the equilibrium bond price and default outcome. I first find that having sufficiently precise information can lead to multiple equilibrium bond prices. Specifically, it generates multiple prices which clear the bond market with a fixed face value. This result is consistent with the findings of Hellwig et al. (2006) and Bassetto and Galli (2019). The limit case of the precise information, which is the case of perfect information, also has multiple equilibria: one where the market values the bond price

[1]Here, $\bar{B}$ is a constant. It is based on the assumption that the government is guaranteed to have resources as much as $\bar{B}$ in the next period, provided the government does not default. The bond is issued based on this fact, which lenders know.

low, the government defaults, and this verifies the market's evaluation; and the other where the market values the bond high, the government repays, and this also verifies the market's evaluation. This result aligns with the concept of multiple equilibria in a self-fulfilling crisis, as described by Calvo (1988) and Cole and Kehoe (2000).

Under a parameterization that delivers a unique equilibrium, I find the following main result: precise information makes the price schedule more sensitive to the market-clearing price, and this increased sensitivity can have an adverse (or advantageous) impact on low (or high) debt states.[2] A marginal lender with rational expectations can infer debt state and market noise from a market-clearing price, which is called a market signal. He evaluates a bond price based on this market signal. As the private signal becomes more precise, the marginal lender has a more concentrated understanding of the debt state, which causes the price schedule to react more sensitively to the market signal. The increased sensitivity implies that the bond price appreciates more with a good market signal and depreciates more with a bad market signal.

An important feature of this paper is that the marginal lender cannot fully separate fundamental (i.e., debt state) and fundamental-irrelevant noise [3] (i.e., market noise) from the market signal, even with highly precise information. The market signal is derived from the market-clearing bond price and is used by the lender to infer the debt state. There are some states where the debt level is low, but the marginal lender perceives it as a pessimistic market signal. Conversely, in some states, the debt level is high, but the marginal lender perceives it as an optimistic market signal. There is a discrepancy between the actual debt state and the inferred debt state based on the market signal.

Due to the combination of this discrepancy and the sensitive price schedule coming from precise information, there are cases where the debt level is low (or high) but perceived as a pessimistic (or optimistic) signal, resulting in a depreciated (or appreciated) price. This effect is significant enough that it can cause a government with low debt to default even though the debt level is relatively low. The government could have repaid it without the sensitive price schedule. On the other hand, the sensitive price schedule appreciates the bond price sufficiently for high-debt states, simply because the states are perceived as optimistic market signals. Consequently, high-debt states can repay, even though the actual debt level is relatively high, and it could have defaulted without the sensitive price schedule.

Certainly, a marginal lender is better able to distinguish true fundamentals and market

[2] I restrict the degree of information precision parameter to guarantee a unique market-clearing price. Then, I proceed with comparative statics for equilibrium bond price and default outcome with respect to the precision parameter.

[3] Fundamental-irrelevancy means the independency to the debt state or the fundamental. See section 2 for the detail.

noise from a market signal as the private signal becomes more precise (*accuracy effect*). However, price depreciation to pessimistic signals (*sensitivity effect*) dominates price appreciation due to the market signal more accurately revealing sound fundamentals. As a result, low-debt states are adversely affected by the high precision of information. Conversely, the opposite happens when high-debt states benefit from highly precise information. This is because the effect of price appreciation on an optimistic signal (*sensitivity effect*) dominates the effect of the market signal in more accurately revealing high debt states (*accuracy effect*).

Finally, I demonstrate that the main result is robust in an environment where there is a prior belief about the debt level. In an economy with a low prior belief, it is common knowledge that the debt level is likely to be low, so the marginal lender is more positive toward the market signal. Conversely, in an economy with a high prior belief, the marginal lender is more negative toward the market signal. This means that the same market signal can be perceived as an optimistic signal in a low prior economy, whereas it is interpreted as a pessimistic signal in a high prior economy. Furthermore, the price schedule is generally higher in a low prior economy compared to a high prior economy. A low prior economy has a higher average bond price and a higher probability of repayment compared to a high prior economy, regardless of the degree of information precision.

As the information becomes more precise, the probability of repayment in a low prior economy decreases compared to that of low precision, and the opposite occurs in a high prior economy. It is low-debt states that are negatively affected by high information precision (i.e., lower bond price and a higher likelihood of default), while high-debt states are positively affected by high information precision (i.e., higher bond price and a higher likelihood of repayment). Since a low prior economy is more likely to realize low debt states, it is adversely affected by high information precision, resulting in a lower average bond price and a lower probability of repayment. Conversely, a high prior economy is more likely to realize high debt states, leading to a higher average bond price and a higher probability of repayment with high information precision.

The rest of the paper proceeds as follows: first I document literature related to this paper. In Section 2, I describe the model and define the equilibrium of the model. In Section 3, I explain how I solve the model and present a bond price characterizing equation, which is the key equation of the paper. I also show the possibility of multiple market-clearing prices. In Section 4, I go through a numerical exercise to proceed with comparative statics on bond price and default outcome, having information precision as a parameter of interest. I present how information precision affects price schedule and market signal, after which the main result comes. Decomposition exercise into the accuracy and the sensitivity effect is conducted to analyze how information precision affects default outcome. In Section 5, I extend the model by adding prior belief. I present how prior belief can change the price schedule and show that the main result is robust to the extended model.

1.1 Related literature

This paper contributes to the literature on rollover risk and sovereign debt refinancing (Calvo (1988), Cole and Kehoe (2000), Conesa and Kehoe (2017), He et al. (2019), Lorenzoni and Werning (2019), Bocola and Dovis (2019), Aguiar et al. (2022)). Different from the literature on strategic default and sovereign default risk (see, for example, Aguiar and Gopinath (2006), Arellano (2008), Mendoza and Yue (2012), Aguiar and Amador (2014)), the literature on rollover risk is often related to self-fulfilling debt crises. This paper also examines an environment where default occurs due to insufficient bond revenue to finance outstanding debt. Among the sizable literature on self-fulfilling crises, this paper falls into the category of studies involving lenders with heterogeneous information. He et al. (2019), for example, examine the role of heterogeneously informed lenders in a country's rollover risk. They assumes that the country defaults due to insufficient bond revenue, and lenders decide where to invest among two countries of different sizes. This paper also assumes the same environment, where a low market valuation can trigger the country's default; however, I incorporate a more complex action for lenders. Specifically, lenders participate in a bond auction market, making price-contingent investment decisions rather than a binary choice of investing or not investing.

Among the literature on sovereign default, the growing literature has been paying attention to the primary market of sovereign bonds (Brenner et al. (2009), Cole et al. (2018), Cole et al. (2022), Monteiro (2022), Cole et al. (2025)). Cole et al. (2018) establish a model where lenders have asymmetric information regarding a country's fundamentals and participate in a bond auction market with rational expectation. They compare the expected bond yields and yield volatility between uniform price and discriminatory price auction protocols. Their model assumes exogenous default which is solely determined by a country's fundamentals. In contrast, this paper assumes endogenous default, specifically occurring when bond revenue is insufficient. In my model, a default does not necessarily arise from bad fundamentals but rather when the bond price is not high enough compared to debt level.

This paper is also closely connected to a broad body of literature that examines noisy rational expectation model, proposed by Grossman and Stiglitz (1980) and Hellwig (1980). Among several applications, Hellwig et al. (2006) and Bassetto and Galli (2019) employ noisy rational expectation models to study situations involving endogenous currency and debt crisis, respectively. Both papers discover the possibility of multiple market-clearing price arising from endogenous currency regime or default choices, respectively. Bassetto and Galli (2019) further find that the price is more sensitive to market signal when lenders possess more precise information. This paper contributes to the existing literature by analyzing the default outcome in greater detail. Building on the insight regarding the sensitivity of price schedules, I examine how a sensitive price schedule can influence the default outcome and whether such sensitivity is advantageous in debt financing.

2 The Model with Bond Auction Market

I consider an environment where a government faces a rollover debt problem. If the government successfully finances its debt, it is guaranteed to have resources $\bar{B}$ in the next period. Based on this, the government issues a bond with a face value of parameter $\bar{B}$, and the bond revenue is utilized to finance the debt if feasible. However, the government must default if the bond revenue falls short compared to the debt level θ. The precise level of debt is not perfectly observed, and each lender receives a private noisy signal s_i that provides information about it. Lenders participate in the bond auction market and infer θ not only through their respective s_i but also by considering the bond price p using rational expectation. Additionally, there exists a market noise that is unrelated to θ, which leads to the limited revelation of the debt level from the bond price.

2.1 Players, Actions, and Payoffs

In the model, there are a government and a unit mass of lenders indexed by $i \in [0, 1]$. The government faces an outstanding debt, θ. If the debt is financed successfully, the government is guaranteed to receive an endowment $\bar{B}$ in the next period. It can be understood that the government has limited resources temporarily, and as long as the government does not default, it will surely return to the original resource level.

With this guarantee, the government issues a bond with face value $\bar{B}$ and raises bond revenue through a bond auction market. The government will repay its obligations only if the bond revenue exceeds the level of debt. Mathematically, this condition is expressed as

$$\theta \leq p\bar{B} \tag{1}$$

where p represents the bond price determined in the bond auction market. If $\theta > p\bar{B}$, it defaults on all its obligations, including newly issued debt. I assume $\bar{B}$ to be a parameter, which implies that the government follows a predetermined strategy in issuing bonds. It

Table 1: A lender's ex-post payoff under a marginal price, p

	Default	Repay
$b(p) = 1$	$-p$	$-p+1$
$b(p) = 0$	0	0

issues a fixed amount of bonds and adheres to the default/repayment decision as in (1).

The strategic players of interest are lenders participating in the bond auction market. Each lender submits a bid for bond investment, and if they win the auction, they commit to purchasing the quantity of bonds as specified in their bid. I denote b_i as the quantity of bonds that lender i promises to purchase upon winning the auction. For tractability, I assume that each lender is limited to purchasing one unit of bond auctioned by the government.[4] Consequently, $b_i \in \{0, 1\}$ for $i \in [0, 1]$. I define a bid as a price-contingent bond purchase schedule, denoted as $\{b(p)\}_p$, where $b(p)$ represents the quantity of bonds to be purchased at a given bond price p.

Each lender maximizes their expected payoff by strategically submitting a bid and investing in bonds accordingly. I assume lenders are risk-neutral and have linear utility. The lender's payoff is determined by whether they win the auction, and the auction rules are detailed in subsection 2.2. Additionally, the lender's payoff is contingent on the government's default. If the government repays, the lender will receive one unit of numeraire in the next period, assuming a lender invests one unit of bond. However, if the government defaults, the lender will not receive any returns on their investment. I assume zero discounting between periods.

2.2 Auction Protocol and Auction Market

There are mainly two auction protocols in sovereign bond auctions: uniform-price auction and discriminatory auction (or multiple-price auction). A uniform-price auction is where all auctioned items are executed at a single price, which is called a marginal price. Discriminatory auction is where auctioned items are executed at the price each bidder bids at. Brenner et al. (2009) categorize countries by the type of protocol they use for sovereign bond auctions. They document that Germany, France, Greece, and many other European countries use discriminatory auction protocol, and USA, South Korea, and Argentina use uniform-price auction protocol. I focus on uniform-price auction protocol in this paper.

The bond auction in the model proceeds as follows: Each lender simultaneously submits

[4] One unit of bond promises a claim to one unit of the numeraire conditional on government repayment.

their bid $\{b_i(p)\}$. The auctioneer collects all bids and determines a marginal price p at which the bond will be executed. Lenders who bid to purchase the bond at p, denoted as $\{i|b_i(p) = 1\}$, win the auction and are obligated to purchase the bond at a price p. Those who bid not to purchase the bond at p, denoted as $\{i|b_i(p) = 0\}$, lose the auction and enjoy the outside option. The payoff of the outside option is normalized to 0, meaning that the payoff of not investing in a bond is 0. In the next period, the lenders who purchased the bond will receive one unit of numeraire only if the government does not default. The lender's payoff under a marginal price p is summarized in Table 1.

In this auction, the value of the auctioned item, i.e., the profit of bond investment, is endogenously determined, particularly by an equilibrium bond price in the auction. Equilibrium bond price plays a significant role as it not only determines the cost of bond investment but also influences bond revenue, default outcome, and the profit of bond investment. Moreover, it is the outcome of aggregating all bids in the bond auction market. It can provide information about the amount of the government's debt θ, a true state that an individual cannot perfectly observe. To prevent the marginal price from fully revealing θ, I introduce a market noise as a form of noisy trader shock following Hellwig et al. (2006), Bassetto and Galli (2019). Specifically, I assume that noisy traders bid sufficiently high and always win the auction, demanding a portion of total bonds given by $\Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)$. Here, Φ is the cumulative standard normal distribution function, $\sigma > 0$ is a parameter, and the random variable $\mu \in \mathbb{R}$ represents noisy trader shock. This shock μ is independent of θ, meaning a fundamental-irrelevant shock. It follows a normal distribution with mean zero and precision $\alpha > 0$, $\mu \sim N(0, \frac{1}{\alpha})$. The parameter σ governs the concentrated impact of the market noise μ on the market. A low σ results in a dispersed demand from noisy traders within a given range of μ, whereas a high σ leads to a concentrated demand within the same range of μ.[5]

The marginal price p is determined to clear the bond market. Bond market clearing condition is as follows:

$$\int b_i(p)di + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B} = \overline{B} \tag{2}$$

2.3 Information Structure and Timing

I assume that lenders cannot observe θ. Instead, each lender privately observes a noisy signal of θ, denoted as $s_i = \theta + \epsilon_i$. Here, the random variable ϵ is drawn from a normal distribution with mean zero and precision $\beta > 0$, $\epsilon \sim N(0, \frac{1}{\beta})$. A natural interpretation of s is each lender's estimate of the fundamental (the level of debt that the government needs

[5]If $\sigma = \infty$, the demand by noisy traders is $\frac{\bar{B}}{2}$ regardless of the realization of μ shock.

to finance). A lender with a high s believes that the government owes a high level of debt, whereas a low s lender has an optimistic view of the debt situation.

The parameter β represents the precision of individual information regarding the fundamental, which is a crucial parameter in this paper. High β indicates that lenders are better informed and have a concentrated fundamental prediction. I assume that the number of lenders is sufficiently large, and the law of large numbers (LLN) applies. Consequently, the proportion of lenders with a signal lower than $x \in \mathbb{R}$ corresponds to the probability that the random variable s is lower than x. The prior of θ is uniform in $\mathbb{R}$ (improper uniform distribution) for tractability. However, in section 5, I generalize it to a more realistic distribution.

The timing in the model is as follows: In period 1, nature draws the values for $(\theta, \mu, \{\epsilon_i\}_{i\in[0,1]})$ from a given distribution, respectively. The government then issues a bond in the bond auction market. Each lender i privately observes their signal s_i and simultaneously submits a bid. The auction closes, and the market clearing price p is determined by (2). The auction winner is determined, and the bond purchase takes place accordingly. In period 2, the government observes θ, and the outcome of default or repayment is determined by (1). The lenders' payoffs are determined based on default/repayment outcome.

2.4 Strategy and Symmetric Equilibrium

Lender i's strategy is a function that maps the signal $s_i \in \mathbb{R}$ to the bid $\{b(p)\}_p$. In this analysis, I focus on a symmetric equilibrium where all lenders adopt the same equilibrium strategy. Henceforth, I denote the strategy as $b(s, p)$, which represents the quantity of bond to be purchased given a signal s at a marginal price p.[6] The lender chooses the optimal bidding strategy $b(s, p)$ to maximize the expected payoff from bond investment for all (s, p).

$$\max_b \mathrm{E}[(-pb + b)\mathbf{1}\{p\overline{B} \geq \theta\} + (-pb)\mathbf{1}\{p\overline{B} < \theta\}|s, p] \quad \forall(s, p)$$

The first part represents the payoff conditional on repayment, while the second part represents the one conditional on default. Given (s, p), the lender forms a posterior belief in θ, which is crucial for determining the default probability. The optimization problem can be reformulated as follows:

$$\max_b \mathrm{E}[-pb + b\mathbf{1}\{p\overline{B} \geq \theta\}|s, p] \quad \forall(s, p) \tag{3}$$

[6] It is worth noting that lenders are ex-ante identical but exhibit ex-post heterogeneity based on the signals they receive. However, since the focus is on symmetric equilibrium, the subscript i can be dropped.

An equilibrium of interest consists of lender's strategy $b(s,p)$ and lender's posterior belief $\hat{\theta}(s,p)$ where $b(s,p)$ maximizes the lender's expected payoff for all (p,s) and belief consistency holds. The equilibrium outcome of interest is the bond price p and resulting default outcome.

Definition 1 *A Symmetric Perfect Bayesian Equilibrium consists of bidding strategy $b(s,p)$, bond price function $p(\theta,\mu)$ and posterior belief $\hat{\theta}(s,p)$ such that*

(i) Given $\hat{\theta}(s,p)$, bidding strategy $b(s,p)$ solves lender's maximization problem as in (3)

(ii) Given $b(s,p)$, bond price p clears the bond market for all (θ,μ) as in (2)

(iii) posterior belief $\hat{\theta}(s,p)$ satisfies Bayes' rule

3 Solving the Model

3.1 Optimal Bidding Strategy

I define $\delta(s,p)$ as the expected repayment probability conditional on a marginal price p and the signal s. Then, the expected payoff from bidding $b(s,p)$ given (s,p) is

$$\delta(s,p)\{-b(s,p)p+b(s,p)\}+(1-\delta(s,p))\{-b(s,p)p\}=b(s,p)\{\delta(s,p)-p\}$$

Since the payoff is linear in $b(s,p)$, the optimal bidding strategy is in the form of cutoff strategy:

$$b(s,p)=\begin{cases}1, & \text{if} \quad \delta(s,p)>p\\ 1 \quad \text{or} \quad 0, & \text{if} \quad \delta(s,p)=p\\ 0, & \text{if} \quad \delta(s,p)<p\end{cases}$$

p is cost for investing the bond regardless of default outcome, whereas $\delta(p,s)$ is the expected benefit from investing the bond, and it is high as the lender expects the government repayment with high probability.

The expected repayment probability is influenced by three factors. First, it is affected by the signal s. A low s suggests a higher probability of a low θ. The lender anticipates a higher probability of government repayment at any p, leading to their willingness to invest in the bond. Second, the bond price p determines the bond revenue, $p\bar{B}$, which in turn impacts the default outcome. Given s and the corresponding posterior belief $\hat{\theta}$, a high p indicates a high bond revenue, suggesting a higher probability of government repayment. The reason p can affect the expected repayment probability and subsequent bond demand, based on a posterior belief, is because default outcomes depend on the market's valuation of the bond. This implies the possibility of self-fulfilling default: at a low p, lenders anticipate a low repayment probability and abstain from purchasing the bond. As a result, aggregate bond demand decreases, and the bond price p is set low, confirming the lenders' initial pessimistic expectations.

Lastly, the marginal price p can be another signal for the unobserved θ. Bond market clearing condition explains it clearly.

$$\int b_i(p, s_{i|\theta})di + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right)\overline{B} = \overline{B} \tag{4}$$

Note that the equation above represents a market clearing p, and a market clearing p should satisfy the equation. It is important to recognize that the equation includes states of the world, (θ, μ). The quantity of bonds demanded by the noisy trader is determined by μ, and θ determines the distribution of s_i, impacting the bidding strategies of all lenders and the overall demand for the bond. In essence, the market clearing equation establishes a relationship between the bond price p and the state (θ, μ). For a high p to be a market clearing price, one of two conditions must hold: either the fundamental is favorable (indicating a low θ), or the aggregate bond demand is bolstered by the noisy trader, despite an unfavorable fundamental (indicating a high θ and a high μ). With this inference, rational lenders can update their belief in θ accordingly.

3.2 Monotone Equilibrium

There will be many classes of equilibrium for this model, but I focus on a monotone equilibrium, where optimal bidding strategy $b(s, p)$ is monotonic decreasing in s. This means that in equilibrium, lenders bid to purchase the bond as they receive a low signal. The solution strategy will be first solving for the market clearing p and then verifying that the monotone strategy is indeed optimal.

In monotone equilibrium, the equilibrium bidding strategy is characterized as a signal cutoff $s^*(p)$ where a given p, $b(s, p) = 1$ for $s \leq s^*(p)$ and $b(s, p) = 0$ for $s > s^*(p)$. By definition, $s^*(p)$ has to satisfy $\delta(s^*(p), p) = p$. Also, equilibrium default outcome is characterized as a default cutoff $\theta^*(p)$ where a given p, the government defaults if and only if $\theta > \theta^*(p)$ and the government repays if and only if $\theta \leq \theta^*(p)$. By definition, $\theta^*(p)$ has to satisfy $p\bar{B} = \theta^*(p)$. From now on, I normalize $\bar{B} = 1$ so that the equation can be simplified as $p = \theta^*(p)$.

For a given cutoff $s^*(p)$, lenders' aggregate bond demand is $\int b_i(p, s_{i|\theta})di = prob\{s_i < s^*(p)|\theta\} = \Phi(\sqrt{\beta}(s^*(p) - \theta))$. For a state (θ, μ), p is a market clearing price if and only if $\Phi(\sqrt{\beta}(s^*(p) - \theta)) + \Phi\left(\frac{\mu}{\sqrt{\sigma}}\right) = 1$, or equivalently

$$s^*(p) = \theta - \frac{\mu}{\sqrt{\beta\sigma}} \equiv x \tag{5}$$

This equation mathematically shows how a state (θ, μ) is inferred from a market clearing price p. Given a p, lenders can infer that the unknown state (θ, μ) should satisfy (5) using the fact that a market clearing p satisfies equation (4) and the knowledge of the cutoff $s^*(p)$. I define the right-hand side of this equation as x, which represents a market signal about the state inferred from the market price p.

The market signal x is a public signal among lenders, since p is public knowledge, and every lender observes p. Another important feature is that the market signal cannot distinguish between θ and μ, even though it provides an unbiased estimate of θ. The market signal is informative in Bayesian' updating, allowing lenders to narrow down the entire state space to $\{(\theta, \mu)|\theta - \frac{\mu}{\sqrt{\beta\sigma}} = x\}$. However, it still cannot precisely determine the true (θ, μ). As mentioned, x is an unbiased estimate of θ, and it follows a normal distribution $x \sim N\left(\theta, \frac{1}{\beta\sigma\alpha}\right)$. The precision of x depends not only on β, α but also on σ. A higher value of σ concentrates the impact of a noisy trader within a given interval of μ, which has a similar effect to a higher precision of μ (represented by a higher value of α).

3.3 Characteristic Equation for Bond Price

Given x and its distribution, a lender i with its signal s_i updates posterior belief as follows:

$$\theta|s_i, x \sim N\left(\frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}, \frac{1}{\beta + \sigma\alpha\beta}\right).$$

For the marginal trader with its signal $s^*(p)$, posterior belief is

$$\theta|s^*(p), x \sim N\left(x, \frac{1}{\beta + \sigma\alpha\beta}\right).$$

The marginal lender is as optimistic as the market signal. Those with $s_i > x$ expect θ to have mean greater then x, meaning less optimistic, and $b(s_i, p) = 0$, whereas those with $s_i < x$ expects θ to be less then x on average, meaning more optimistic, and $b(s_i, p) = 1$. With this update, I can express the expected repayment probability for the lender i and the marginal lender as follows:

$$\delta(s_i, p) \equiv prob\{\theta \leq \theta^*(p)|s_i, p\} = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$$

.

$$\delta(s^*(p), p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - x\right)\right)$$

The equilibrium bond price p is characterized by the marginal lender having zero expected payoff of bond investment. Substituting $\theta^*(p)$ to p, the market clearing bond price p given x can be implicitly characterized, which is the key equation in this paper.

$$p = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(p - x\right)\right) \tag{6}$$

p is determined by x because the market signal cannot separate θ from x. The marginal lender can neither separate θ from x, and they decide market price p depending on his signal and market signal, x.

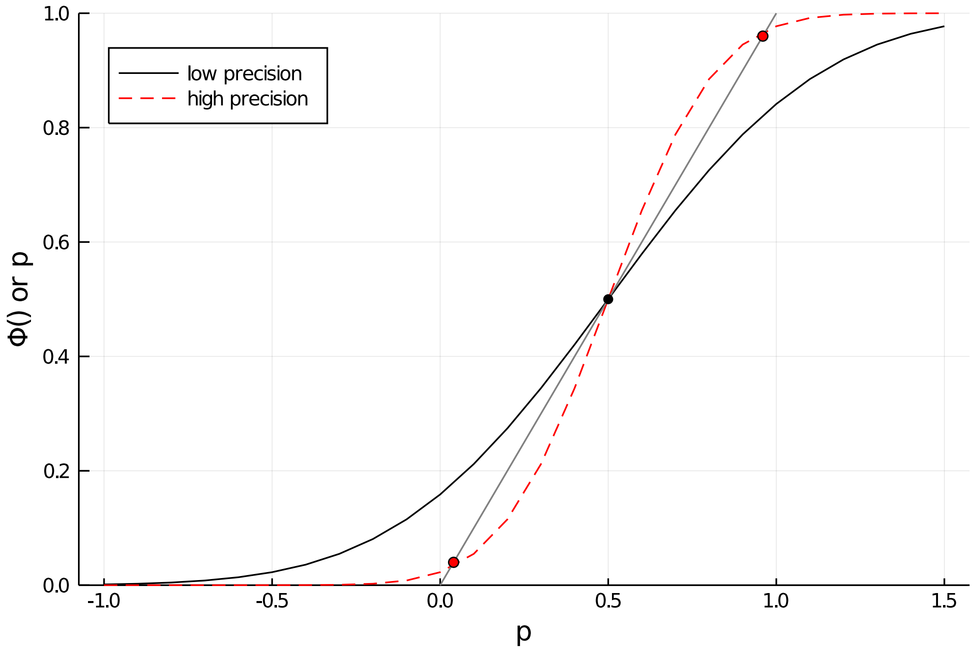
What remains is verifying that the monotone strategy is the optimal strategy. It is straightforward given the fact that the expected repayment probability is strictly decreasing in the signal. Recall $\delta(s_i, p) = \Phi\left(\sqrt{\beta + \sigma\alpha\beta}\left(\theta^*(p) - \frac{\beta s_i + \sigma\alpha\beta x}{\beta + \sigma\alpha\beta}\right)\right)$. Given p, and for $s_i < s^*(p), \delta(s_i, p) > \delta(s^*(p), p) = p$ by the indifference condition of the marginal lender. Therefore, $\delta(s_i, p) > p$ and $b_i(s_i, p) = 1$. The similar logic applies to $s_i > s^*(p)$.

3.4 Multiple Market Clearing Prices

There could be multiple p satisfying (6), implying a possibility of multiple market clearing prices. Figure 1 shows it graphically by plotting the right-hand side of (6) as a function of p with a various precision of marginal lender's posterior belief, $\beta + \beta\sigma\alpha$.

It shows that the high precision case has three fixed points, meaning three market clearing prices, and the low precision case has a unique market clearing price. Multiple bond prices with high enough precision can be related to multiple equilibria under perfect information. Consider $\theta \in [0, 1]$, and the value of θ is a common knowledge. The common knowledge version of the model has two equilibria; one equilibrium where the market values the bond low and $p = 0$, the government defaults, and it verifies the initial evaluation of $p = 0$. The other equilibrium is where the market values the bond high and $p = 1$, the government repays, and it verifies the initial evaluation of $p = 1$. These two equilibria can be seen as a limit case of equilibrium prices in high precision, one with very low p and the other with very high p. The relationship between the multiplicity and parameter values is explained in details in Appendix A.

Figure 1: Equilibrium bond prices with different precision values



Note: This plot is for illustrating equilibrium bond prices with different precision parameters β. For a given p, I plot the right-hand side of (6) as a function of p. Parameter values are assigned as Table 2. The black solid line is for low β, and the red dash line is for high β. The gray line is a 45-degree line, so that the intersections are the fixed point of (6), or bond prices that satisfies the equilibrium condition.

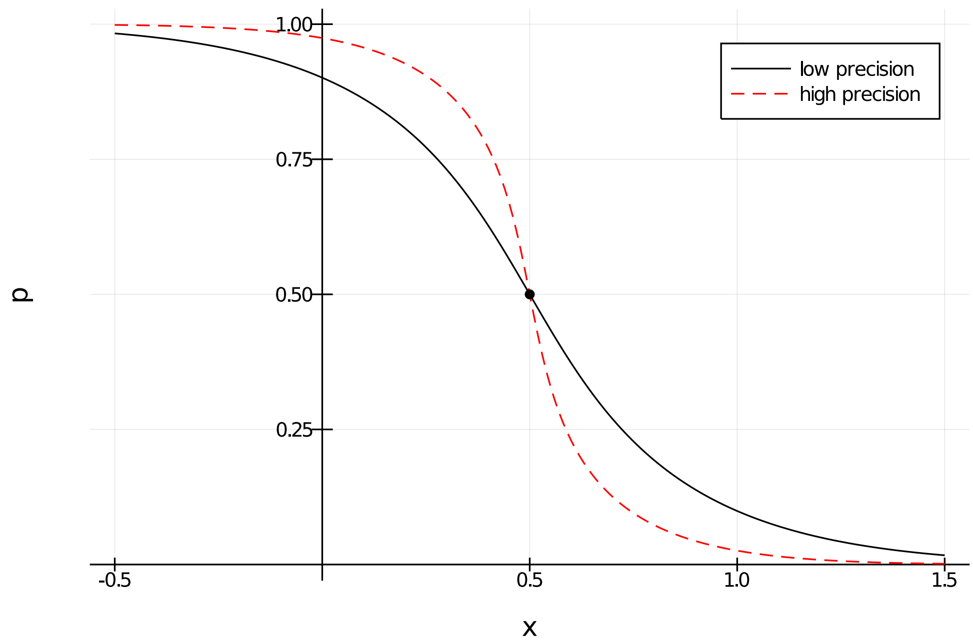
Table 2: Parameter values

Parameter	value
σ	0.7
α	0.1
Low β	1.0
High β	2.0

4 Numerical Exercise

Parameter values for (β, σ, α) are assigned as shown in Table 2. To analyze the role of information in the rollover-debt problem, I specifically focus on β, which represents the precision of the private signal. I compare high and low β economies while fixing the values of other parameters. In the high β economy, lenders' private signals are centered around the true θ, allowing the marginal lender to have accurate information about the fundamental. I am interested in observing how the bond price and default outcome are affected by an

Figure 2: Equilibrium bond price schedule as a function of market signal



Note: This figure depicts the equilibrium bond price as a function of market signal, x, for different values of information precision. The black solid line is for low β, and the red dash line is for high β. Parameter values are assigned as Table 2. For a given x, the equilibrium bond price is the fixed point of (6).

increase in the precision of private signals. Further discussion on how I construct grid points and how to solve the equilibrium can be found in Appendix B.

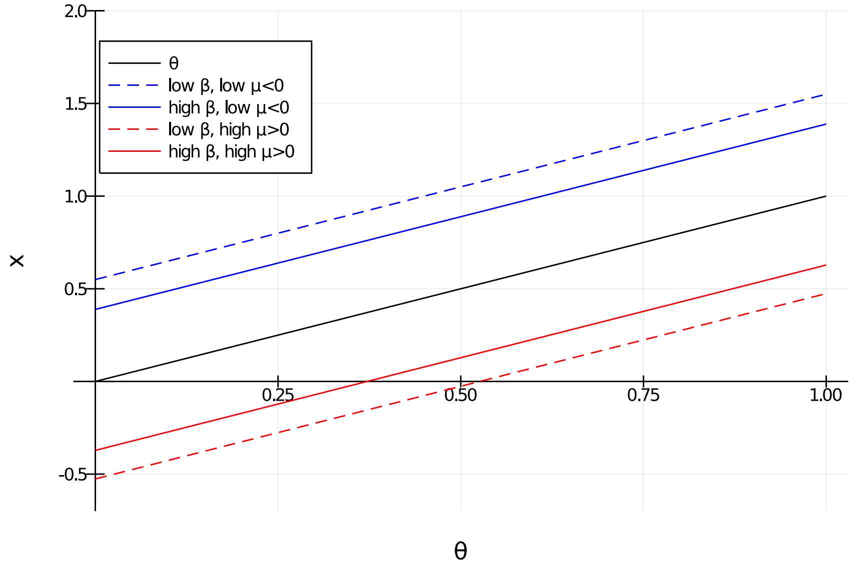
4.1 Price Schedule and Market Signal

Figure 2 plots the equilibrium bond price p as a function of market signal x. The high precision plot uses high β, and the low precision plot uses low β.

The first observation is that both price schedules are decreasing in x, implying that the bond is priced low with a high value of market signal. Even though the market signal is affected by market noise, it indicates the level of θ on average. A high x means that the market predicts θ is likely to be high. The marginal lender expects a higher probability of default, and devalues the bond accordingly.

The second observation is that with high β, the price schedule responds more sensitively to x. This is graphically shown as a higher degree of concavity/convexity under high β. The market signal is from aggregating all private signals through auction bidding, and more precise private signals under high β lead to the market signal being a more precise estimate of θ. The marginal lender's posterior belief is more concentrated on the true θ, and the

Figure 3: Sign effect of noisy traders shock to market signal



Note: This plot depicts the market signal, x, as a function of debt state, θ, for two value of μ and for two values of information precision, β. For a given μ and parameter values, for each θ level, the market signal is calculated by satisfying (5). The solid lines are for the high β, and the dash lines are for the low β. The blue lines are for a negative μ, and the red lines are for a positive μ. Parameter values are assigned as Table 2.

posterior repayment probability changes more dramatically by a small change in x. I refer to this relationship as the *sensitivity effect*.

Lastly, a sensitive price schedule with high β reacts depending on what the value of x is. Sensitive price function does not necessarily appreciate p for all x, but only when x is low. When x is high, on the other hand, is lower under price function with high β compared to p with low β. The different responses happen around the inflection point, and I define the infection point as market signal cutoff x^*. It is the market's criteria for interpreting whether x is a optimistic signal or a pessimistic signal. In Figure 2, $x^* = 0.5$, and $x < 0.5$ is interpreted as an optimistic market signal. Given this optimistic signal, the sensitive price schedule appreciates p more than the low β price schedule: $p(x)$ with high $\beta > p(x)$ with low β for any $x < 0.5$. The reverse happens when $x > 0.5$, which is interpreted as a pessimistic market signal. To a pessimistic signal, the sensitive price schedule depreciates more, and p is lower. Essentially, whether high precision is advantageous to debt rollover depends on whether the market interprets x as an optimistic signal or not. Mathematically it depends on $x < x^*$ or $x > x^*$. In conclusion, information precision makes the price schedule sensitive to market signal and the schedule reacts in a different way to optimistic and pessimistic signals.

Next, I examine how the precision of information influences market signal, conditional on the level of debt. Specifically, as β increases, the market signal deviates less from the true value of θ. Formally, the deviation is given by $|x - \theta| = \left|\frac{\mu}{\sqrt{\beta}\sigma}\right|$, indicating that a higher β leads to a smaller deviation. Intuitively, when private signals are more precise, the aggregate market signal x, which synthesizes all private information in the bond auction market, serves as a more accurate indicator of the true θ. I refer to this relationship as the *accuracy effect*.

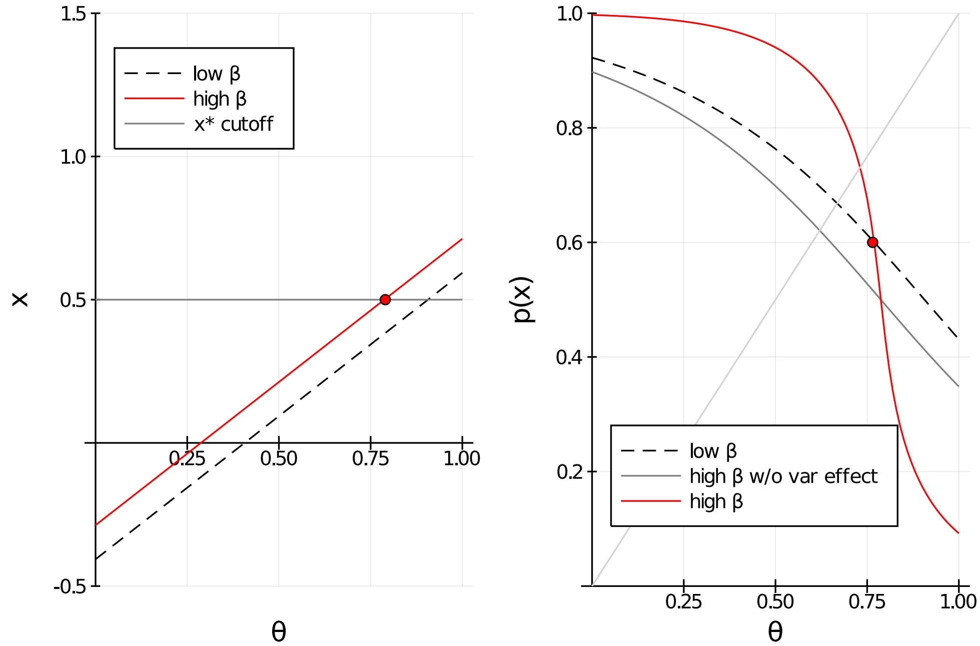
Figure 3 shows it graphically. For a fixed value of noisy trader shock, I plot the values of market signal across realized values of the level of debt. I use two values for μ: a positive value (blue lines) and a negative value (red lines). The dash lines are low β cases, and the solid lines are the high β cases. I plot the realized value of θ as a reference. The figure shows that the solid line, X with high β, is closer to the black line, true θ, compared to dash line, X with low β, for all θ realization and in both μ cases. The difference between $\mu < 0$ case and $\mu > 0$ case is that x is in general low in $\mu > 0$ whereas x is in general high in $\mu < 0$. It means even with the same level of θ, x is interpreted as an optimistic signal with high demand case and a pessimistic signal with low demand case. It is due to the fact that lenders can infer true state from market price as a market signal but they cannot separately infer θ and μ. With the same value of x, it could come from low θ with low demand shock but it could also come from high θ with high demand shock. Therefore, whether high information precision is optimistic or pessimistic depends on the magnitude of the demand shock, as it is the demand shock that determines whether x indicates an optimistic signal. In Appendix C, I add how different value of μ but the same sign affect the market signal.

4.2 Default Outcomes and Information Precision

Building on the observations from the previous section, I present the main result of this paper. Increased information precision induces default by governments with relatively low debt levels, while it encourages repayment by governments with relatively high debt levels. In contrast, under conditions of less precise information, states with relatively low θ may have chosen to repay, and those with relatively high θ may have defaulted. This outcome arises because there is always a set of states that are particularly susceptible to market noise, and changes in default outcome occur within these sensitive states combined with a sensitive price schedule.

Figure 4 illustrates how the market signal and the corresponding price schedule are determined across realized debt levels for a fixed noisy trader shock, specifically when $\mu > 0$. A positive noisy trader shock indicates that noisy traders exhibit relatively high demand for bonds. Consequently, the marginal lender's estimate of the debt level is lower than the

Figure 4: Market signal and equilibrium bond price in a positive noisy trader shock case



Note: This figure illustrates the market signal, x, and according equilibrium bond price, $p(x)$, for a given value of μ. Here, a positive value of μ is given, which implies high noisy trader shock. The black dash lines are for low β, and the red solid lines are for high β. Parameter values are assigned as Table 2. The left panel depicts the market signal as a function of θ by satisfying (5), and the market signal cutoff $x^* = 0.5$ is indicated as a horizontal gray line. The right panel depicts the equilibrium bond price as a function of debt state, θ. The price is obtained by using (6) and the corresponding market signal, x, for each debt state, θ, obtained as in the left panel. The gray line is obtained by for each debt state, θ, solving for p in (6) using low β value for β. For each θ, I use the same x value that I used for plotting high β (red solid) line.

actual value, indicating $E(\theta|s \cdot x) = x < \theta$ for all θ. However, only those market signals below the cutoff, $x < x^* = 0.5$, are interpreted as optimistic by the market, and the corresponding $\theta < 0.765$ benefit from a sensitive price schedule that responds more strongly to optimistic signals. The right panel of Figure 4 demonstrates that, within the optimistic signal region, bond prices are higher under high information precision. To assess how price appreciation affects default outcomes, one can compare p and θ using the 45-degree line in the right panel of Figure 4. The debt region $\theta \in [0.625, 0.735]$ represents cases where, under low information precision, the marginal lender assigns a low value to the bond and the government would default; however, with high information precision, bond prices are sufficiently high to enable the government to repay.

This is also illustrated in the left panel of Figure 5. The result does not stem simply from a low debt level or intrinsically lower default risk. Rather, it arises because the marginal lender, influenced by a positive noisy trader shock, infers the debt level to be lower than its true value. Information precision enhances the marginal lender's responsiveness to market signals interpreted as optimistic. Consequently, even when the government carries a relatively high debt burden, bonds are highly valued as long as the market perceives the debt status optimistically. The resulting high bond revenue enables the government to finance its outstanding obligations, thereby averting default.

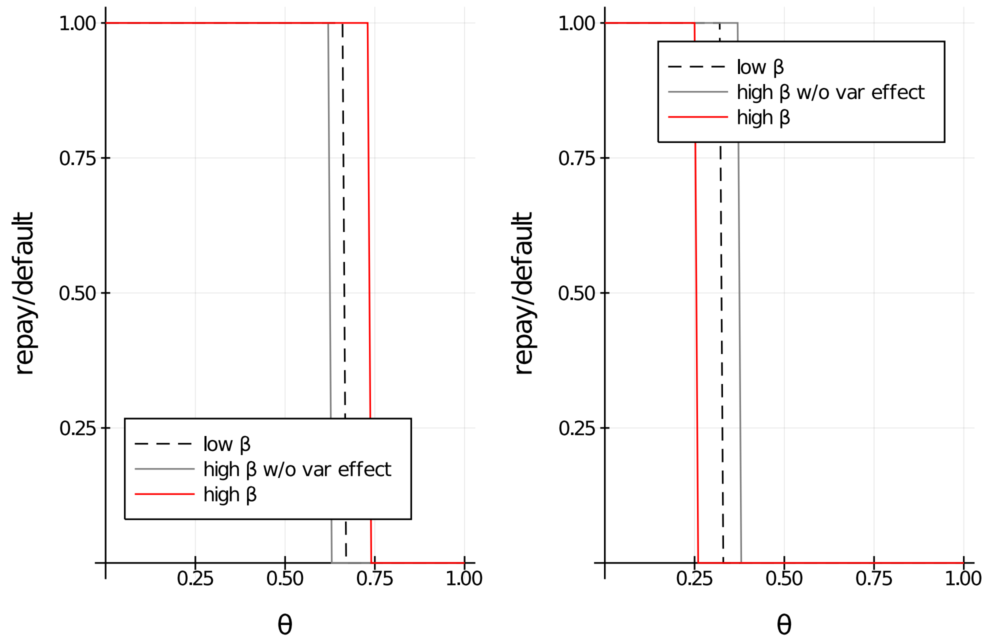
It is important to note that high information precision has a trade-off. While greater information precision increases the sensitivity of the price schedule, leading to bond price appreciation in some high-debt states (*sensitivity effect*), it also allows the market signal to more accurately reveal the true debt state (*accuracy effect*). However, the effect of increased price sensitivity tends to dominate, resulting in bond revenues that exceed the debt level in states of fairly high indebtedness, thereby preventing government default. The decomposition exercise in subsection 4.3 illustrates how information precision influences default outcomes through these two channels: the sensitivity effect and the accuracy effect.

As shown the left panel of Figure 4, high information precision moderates the impact of positive market noise, causing the market signal to be perceived as less optimistic across all debt levels compared to the low precision case. This, in turn, has a negative effect on the equilibrium bond price. If the price schedule were as sensitive as in the low precision scenario, the equilibrium would feature low bond prices for all debt states, as depicted in the right panel of Figure 4. Consequently, the incidence of default would also increase, as illustrated in the left panel of Figure 5. Nevertheless, the heightened sensitivity of the price schedule under high precision outweighs the less optimistic market signal. Thus, despite the more moderate market signal, there exist debt states in which the government ultimately benefits from high information precision.

Similarly, under a negative noisy trader shock, $\mu < 0$, high information precision can lead to government default in certain debt states where the government is unable to roll over its debt. In these cases, default occurs because the market perceives the debt status pessimistically. This outcome arises because the market cannot fully distinguish the fundamental debt status, θ, and the market signal, x. As a result, there exist states where θ may reflect sound fundamentals, yet the market, influenced by sufficient noise, interprets the signal as unfavorable. In Appendix C, I illustrate the dynamics that arise under a negative noisy demand shock.

Default outcome in each state (θ, μ) can be determined for the low and high β. Figure 6 illustrates how the default outcome are affected as information precision increases. Specifically, the unshaded area represents states where the default outcome remains unchanged as β increases from a low to a high level. Within this unshaded area, states in the upper contour of the shaded area are characterized by relatively low θ (debt levels) and high μ level (demand shocks), consistently succeeding in debt financing regardless of the level of

Figure 5: Default outcome in positive (left) and negative (right) noisy trader shock

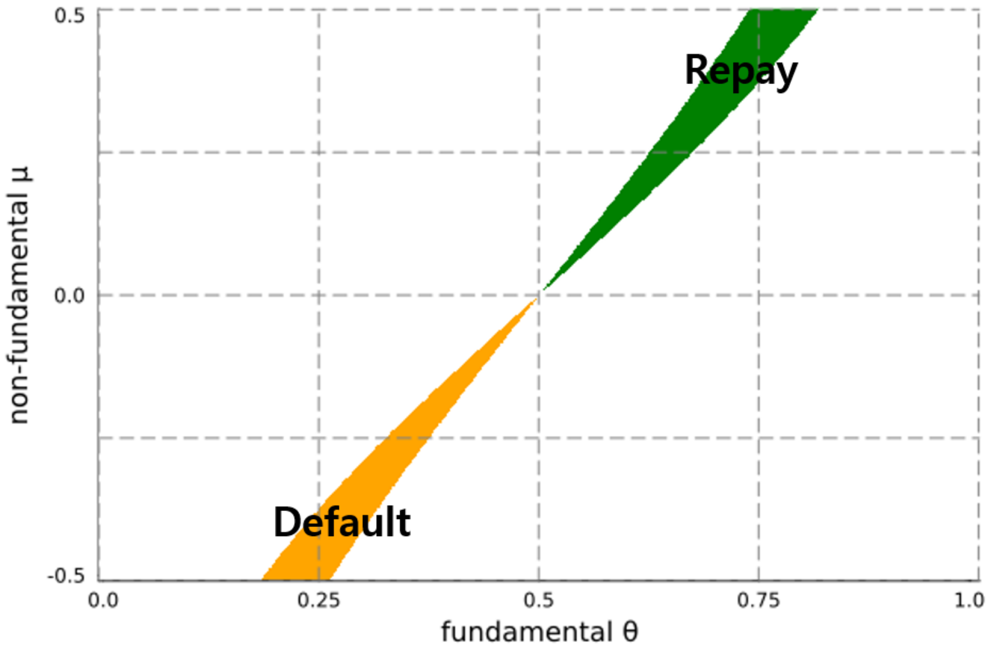


Note: This figure illustrates the government's default outcome in equilibrium for each debt state, θ, for two values of noisy trade shock, μ. The left panel is for a given $\mu < 0$, which implies low noisy trader shock, and the right one is for a given $\mu > 0$, which implies high noisy trader shock. The value 1.0 represents the government repayment, and the value 0.0 represents the government default. The default outcome satisfies (1) with $\bar{B} = 1$. The black dash lines are for low β, and the red solid lines are for high β. Parameter values are assigned as Table 2. The gray lines is obtained similarly as in Figure 4.

information precision. Conversely, states in the lower contour, with relatively high θ and low μ, experience default regardless of information precision.

Of particular interest is the shaded region, which highlights states where the default outcome changes as precision increases. As β rises from the low to the high level, states with moderately high debt levels but relatively strong demand shocks (green area) are able to avoid default. In contrast, states with moderately low debt levels but relatively weak demand shocks (orange area) experience a shift: while these states would not have defaulted under low information precision, they become unable to do so as information precision increases, resulting in default. This demonstrates how increased information precision can have heterogeneous effects on default outcomes depending on the combination of debt levels and demand shocks.

Figure 6: Change in default outcome under higher information precision



Note: This figure illustrates how default outcome has changed as information precision increases in a given state space. The orange set of state indicates default occurs as information precision increases, whereas the green set of state indicates repayment occurs as information precision increases. Parameter values are assigned as Table 2.

4.3 Decomposing the impact of information precision

The influence of information precision on default outcomes operates through two channels: the general responsiveness of the price schedule and the informativeness of the market signal, as depicted in Equation 7. The former is referred to as the *sensitivity effect*, while the latter is described as the *accuracy effect*. The accuracy effect captures the extent to which the market signal accurately reflects the true debt status under high information precision, thereby reducing the influence of noisy trader shocks. This effect is advantageous for successful debt rollover when noisy trader shocks are negative and demand shocks are relatively low. However, high information precision makes debt rollover more difficult in the presence of positive noisy trader shocks. As a result, default is more likely to occur in high-debt states under high information precision. In such situations, the marginal lender's estimate of the debt level closely matches the true value, even when the demand shock is relatively low, leading to lower bond prices and a higher occurance of default.

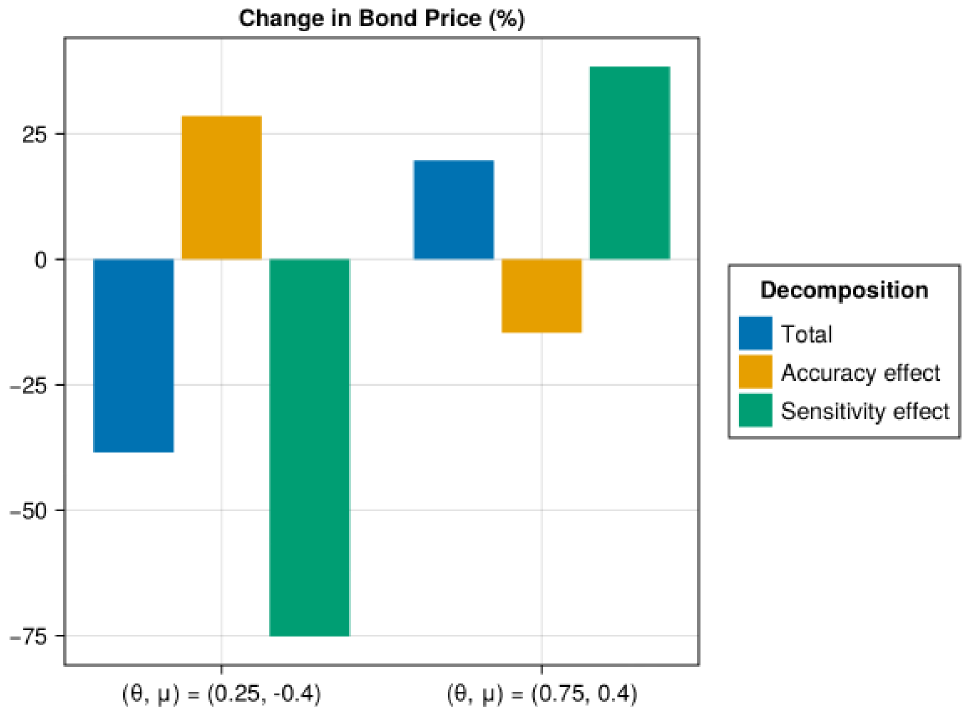
$$\frac{dDefault}{d\beta} = \frac{\partial Default}{\partial P}\left(\underbrace{\frac{\partial P}{\partial \beta}}_{\substack{\text{Sensitivity}\\ \text{effect}}} + \frac{\partial P}{\partial x}\underbrace{\frac{\partial x}{\partial \beta}}_{\substack{\text{Accuracy}\\ \text{effect}}}\right) \tag{7}$$

The second channel—the sensitivity effect—is also noteworthy. The sensitivity effect implies that the bond prices rise when the market signal is perceived as optimistic and fall when it is perceived as pessimistic. High information precision is beneficial in scenarios where the market signal is sufficiently optimistic (i.e., $x < x^*$). Conversely, high information precision discourages debt rollover in states where $x > x^*$. In high-debt states, the market signal is more likely to be perceived as pessimistic under high information precision compared to the low precision case. Nevertheless, with a sufficiently large positive noisy trader shock, the market signal can be optimistic enough to warrant higher bond prices under high information precision. This sensitivity effect may facilitate successful debt rollover, depending on the extent of bond price appreciation.

Information precision can influence default outcomes when its two effects operate in opposite directions. Of particular importance is the sensitivity effect, which typically dominates and drives changes in default outcomes. Figure 7 decomposes the change in bond price into these two effects for given states. The left panel represents a state the orange area, where higher information precision leads the government to default, while the right panel represents a state from the green area, where higher information precision enables the government to repay.

In the case depicted in the left panel, the bond price in the low β economy is 0.338, which is sufficient to finance the debt level ($\theta = 0.25$). However, under high β, the bond price declines to 0.208, making it insufficient to cover the debt obligation. Although the debt level itself is relatively low, higher information precision enables the market signal to more accurately reflect the true debt status, which should, in principle, increase the bond price. This is reflected as a positive accuracy effect on the change in bond price in the left panel, indicating that the accuracy channel contributes to better debt financing. Nonetheless, the negative sensitivity effect is larger in magnitude, resulting in an overall decline in the equilibrium bond price under high precision. This outcome arises because the demand shock is sufficiently negative, causing the market signal—reflecting both the debt state and demand shock—to be perceived pessimistically. The sensitivity effect amplifies this pessimistic perception, leading to significant bond underpricing. In this parameterization,

Figure 7: Default outcome in positive (left) and negative (right) noisy trader shock



Note: This figure illustrates the government's default outcome in equilibrium for each debt state, θ, for two values of noisy trade shock, μ. The left panel is for a given $\mu < 0$, which implies low noisy trader shock, and the right one is for a given $\mu > 0$, which implies high noisy trader shock. The value 1.0 represents the government repayment, and the value 0.0 represents the government default. The default outcome satisfies (1) with $\bar{B} = 1$. The black dash lines are for low β, and the red solid lines are for high β. Parameter values are assigned as Table 2.

the adverse impact of the sensitivity effect outweighs the benefits of improved accuracy, ultimately causing default even in a low-debt state.

The right panel illustrates the opposite scenario, in which the sensitivity effect enables a high-debt state to successfully meet its debt obligations. In this case, the true debt level is relatively high ($\theta = 0.75$), and the bond price in the low β economy, $p = 0.661$, is insufficient to cover the debt. However, under high β, the bond price rises to 0.791, which is adequate for repayment. This increase in bond price is primarily driven by a strong positive sensitivity effect, which outweighs the negative accuracy effect. A substantial positive demand shock further contributes to this shift in the default outcome, causing the market signal to be perceived as optimistic even in the presence of high debt. The price schedule responds sufficiently to the optimistic signal under high information precision, resulting in successful debt rollover even in a high-debt state.

5 Extension: a Normal Distribution for Prior Belief

The model assumes that θ is drawn from an improper uniform distribution over the real line, and it has no notion of the mean of θ. It is a convenient assumption for tractability but a bit far from being realistic. Some governments maintain a low debt level, whereas other countries struggle from high indebtedness. The current specification of the prior distribution implies that the equilibrium bond price solely depends on what value the market signal turns out to be. From now on, I extend the common prior distribution to a normal distribution, so that those parameters governing this distribution jointly affect the equilibrium bond price, along with the value of market signal. In conclusion, I show that the main result is robust to this richer prior specification.

5.1 Model with Prior Belief

I assume that θ is now drawn from a normal distribution as follows: $\theta \sim N(\theta_0, \frac{1}{\gamma})$, where γ is the precision of θ distribution and θ_0 is a prior mean, which is common knowledge among all lenders. Almost all parts of the analysis hold as the same except for posterior belief. For lender i, their posterior belief is now

$$\theta|_{s_i,x,\theta_0} \sim N\left(\frac{\gamma\theta_0 + \beta s_i + \beta\sigma\alpha x}{\gamma + \beta + \beta\sigma\alpha}, \quad \frac{1}{\gamma + \beta + \beta\sigma\alpha}\right).$$

For the marginal lender, their posterior belief becomes

$$\theta|_{s^*(p),x,\theta_0} \sim N\left((1-\tau)\theta_0 + \tau x, \quad \frac{1}{\gamma + \beta + \beta\sigma\alpha}\right)$$

where $\tau = \frac{\beta+\beta\sigma\alpha}{\gamma+\beta+\beta\sigma\alpha}$. Adding prior gives additional information of the debt state, and lenders update posterior belief based on θ_0, the average value the debt level tends to be, as well as based on based on the market signal and the private signal. The mean of the posterior is determined as an average of the prior mean θ_0, the private signal s_i, and the market signal x, weighted by their precision respectively.

As before, in equilibrium, the private signal that the marginal lender receives has the same value as the market signal. This means that the marginal lender anticipates the debt level as same as the level that the market signal implies. Those lender whose private signal is higher than the market signal, $s_i > x$, are less optimistic than how the marginal lender is. Those whose signal is lower than the market signal, $s_i < x$, are more optimistic, and they are the ones who bid to purchase bonds.

The following equation is a characteristic equation for the equilibrium bond price:

$$p = \Phi\left(\sqrt{\gamma + \beta + \beta\sigma\alpha}\,(p - (1-\tau)\theta_0 - \tau x)\right) \tag{8}$$

Again, the equilibrium bond price is a fixed point of (7). Sufficiently high precision inducing multiple prices still holds here, and I adjust parameter values accordingly to guarantee a unique price. The specified values are listed in Table 3. I set a low prior case $\theta_0 = 0.1$ representing the low-debt country, and a high prior case with $\theta_0 = 0.9$ representing the high-debt country.

Table 3: Parameter values in the model with prior belief

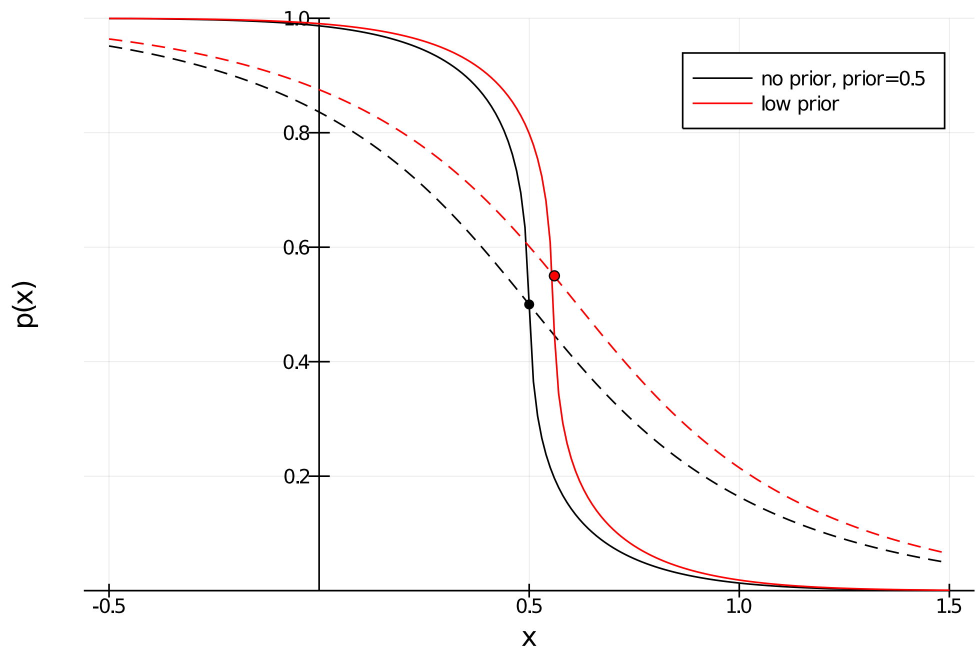
Parameter	value
σ	0.5
α	0.1
γ	0.3
Low β	1.0
High β	2.0
Low θ_0	0.1
High θ_0	0.9

5.2 Main Results

Similar to section 4, the equilibrium bond price is a function of the market signal. In Figure 8, various cases of price schedules are plotted under different precision levels and prior means. As in the benchmark case, the price schedule is a decreasing function of market signal. It also has an inflection point where a sensitive price schedule with high precision responds differently to the market signal: around which the bond price is higher or lower compared to a less sensitive case with low precision.

The prior mean θ_0 affects the price schedule by influencing the market signal cutoff. When $\theta_0 = 0.5$, the market signal cutoff is $x^* = 0.5$, and the price schedule is the same as in the benchmark case. Additionally, the market signal cutoff in the benchmark model is the same as in the $\theta_0 = 0.5$ case. This suggests that the benchmark model can be interpreted as an extension of the model with $\theta_0 = 0.5$. The schedule with the low θ_0 shows that its market signal cutoff is higher than 0.5, or $x^* > 0.5$. Incorporating Figure C.3 from Appendix C, where I compare the price schedules with low and high θ_0, I can generalize the finding. The market signal cutoff with the high θ_0 is lower than that with the low θ_0. In other words, as

Figure 8: Price schedule of the extended model with low θ_0



Note: I plot the price schedule as a function of the market signal, x. Dash lines are under low precision of private signal (low β), and solid lines are under high precision of private signal (high β). Black lines correspond to the benchmark economy with improper uniform distribution, and red lines correspond to the economy with normal distribution prior with low mean (low θ_0). Parameter values are assigned as in Table 3.

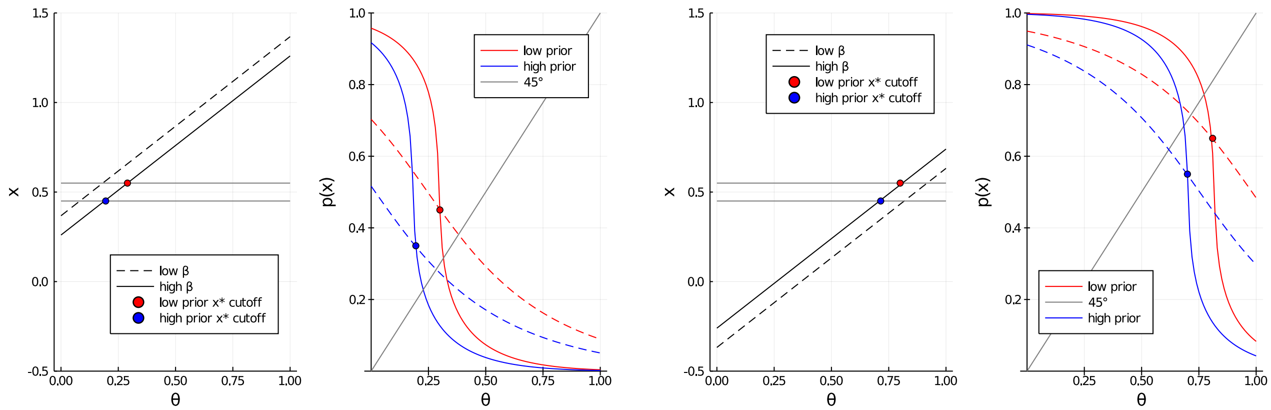
the prior mean θ_0 decreases, the market signal cutoff x^* increases. This finding implies that, for a given x, the market tends to infer a more favorable fundamental in an economy with low prior mean compared to an economy with high prior mean.

With the observation, I do the same exercise. I compare how price and default outcome changes as β gets high between the low θ_0 and high θ_0 case. The result is robust in the extended version of the model, and it can be extended to the conclusion that the economy with the low prior mean has a higher default probability with precise information. In this economy, the government tends to have states where its debt level is low, but due to noisy trader shock, the market infers the state as high default risk fundamental. A sensitive price schedule with precise information is disadvantageous to those states.

Note that for any states (θ, μ), the value of the market signal is the same in both low and high θ_0 economies. It is the realized value of θ and the distribution of s_i that determine what x is; this is unrelated to the prior distribution of θ. In both economies, x is determined by the measure of lenders who receive $s_i|\theta < s^*(p)$, and the conditional distribution of s is independent of the value of θ_0.

Figure 9: market signal and price schedule for a given noisy trader shock

(a) low noisy trader ($\mu < 0$) (b) high noisy trader ($\mu > 0$)



Note: This plot depicts the market signal and the according equilibrium price for each debt state, θ, for two different values for noisy trader shock, μ. The panel (a) represents the case of low noisy trader, $\mu < 0$, and the panel (b) represents the case of high noisy trader, $\mu > 0$. For each panel, the left plot is the market signal as a function of debt state, and the right plot is equilibrium price as a function of debt state. In the left plot, market signal cutoffs, x^*, for each low and high prior mean economy are represented as horizontal gray lines. The red lines represent the low prior mean economy with low θ_0, and the blue lines represent the high prior one. The solid lines represent high precision economy, and the dash lines represent low precision economy. Parameter values are assigned as Table 3.

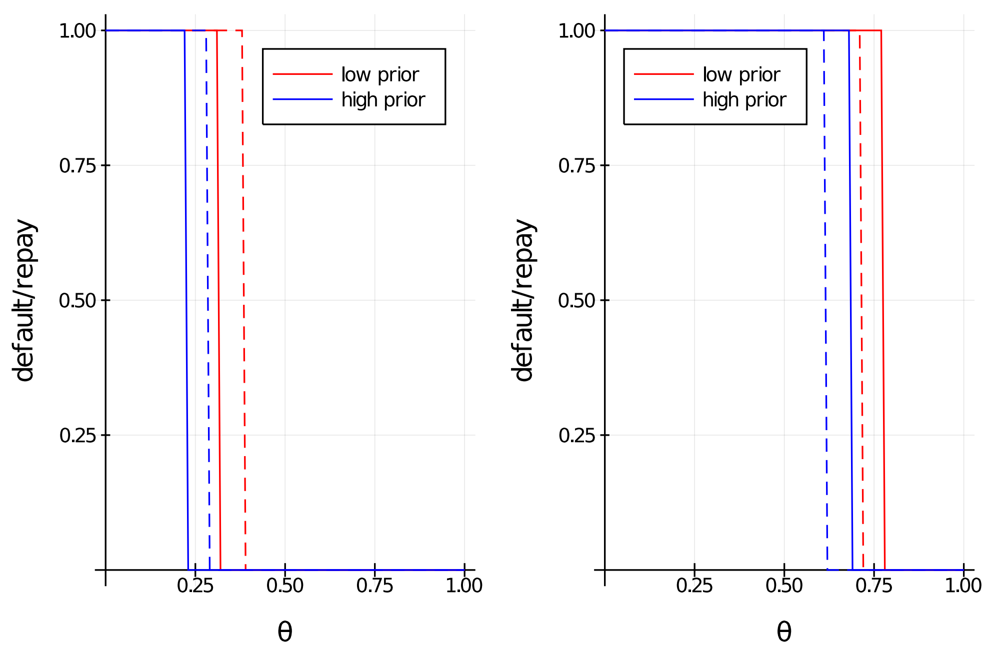
Both the low θ_0 and the high θ_0 economies, therefore, have the same value of x for a given state (θ, μ). What distinguishes the two economies is how the market interprets the same x. The low θ_0 economy has a higher x^* and a wider range of interpreting x as a favorable signal. On the other hand, the high θ_0 economy has a narrower range of favorable signals. This distinction arises from the fact that the low θ_0 economy has a lower prior, and it occurs regardless of the realization of μ.

Given this different interpretation, equilibrium bond price reacts accordingly, as shown in Figure 9. Note that the low prior mean economy has a higher p than the high prior mean economy for all θ and regardless of μ being positive or negative. This is a combined result of the price schedule of the low prior being high in general and low prior having a larger range of good signal. Not only that low prior economy benefits from sensitive price schedule in a larger θ range since it has a larger range of good signal.

The low prior economy still has a θ range where it is disadvantages from a sensitive price schedule and has lower p. The sensitive price schedule devalues unfavorable signals sufficiently so that the government with $\theta \in [0.315, 0.380]$ ends up default with high precision where it would not have defaulted with low precision. In the high prior economy, the government type of $\theta \in [0.230, 0.285]$ would default with higher information precision. Both happen because, as before, θ is not low enough to be interpreted as a favorable signal with negative market noise and p depreciates sufficiently. On the other hand, in the positive market noise case as shown in (b) of Figure 9, the opposite happens. $\theta \in [0.715, 0.775]$ changes to repay in low prior economy, and it is $\theta \in [0.620, 0.685]$ for the high prior economy.

Figure 10 shows the repayment/default outcome for a given $\mu < 0$ (left panel) or $\mu > 0$ (right panel) case. Similar to section 4, it is low θ states where high precision results the government default. Conversely, it is high θ states where high precision allows the government to rollover its debt. It happens in both the low and the high θ_0 economy. The difference is that since the low prior economy is more generous to favorable signals, the market generally values the bonds higher for the low prior economy than the high prior economy. As a result, the low prior economy has larger a θ range of repay compared to what the high prior economy has.

Figure 10: Default outcome in prior economy: positive (left) and negative (right) noisy trader



Note: This figure illustrates the government's default outcome in equilibrium for each debt state, θ, for two values of noisy trade shock, μ, and two different prior mean, θ_0. The left panel is for a given $\mu < 0$, which implies low noisy trader shock, and the right one is for a given $\mu > 0$, which implies high noisy trader shock. The value 1.0 represents the government repayment, and the value 0.0 represents the government default. The default outcome satisfies (1) with $\bar{B} = 1$. The red lines represent the low prior mean economy with low θ_0, and the blue lines represent the high prior one. The solid lines represent high precision economy, and the dash lines represent low precision economy. Parameter values are assigned as Table 3.

I calculate repayment probability, defined as the average of repayment outcome in all states (θ, μ) weighted by the join probability, and the result is in the left panel of Table 4.

Note that the repayment probability with a neutral prior ($\theta_0 = 0.5$) is 0.5 which is the reference number for determining whether the government repayment is likely or not. The table shows that the low prior economy has a lower repayment probability under high information precision, whereas the high β leads the high prior economy to have a lower default probability. The reason is as follows. It is those states (θ, μ) with relatively low θ that are associated with depreciated p by a sensitive price schedule and change to default from repay. Reversely, it is those states (θ, μ) with relatively high θ that that are associated with appreciated p by a sensitive price schedule and change to repay from default. In the neutral prior or no prior case, both disadvantageous and advantageous effects from high precision are equally likely to happen, cancel each other out, and have no change in repayment probability with high precision.[7]

The low prior economy is more likely to have relatively low θ and, although those θ are good fundamentals, it is more vulnerable to being interpreted as a bad signal with a negative market noise and get changed to default with high precision. On the other hand, the high prior economy is more likely to have relatively high θ. This change can be interpreted as favorable market signals with positive market noise, and when this happens, it can get benefit from precise information since it appreciates p sufficiently so that it has higher p than θ even though θ itself is relatively high. However, note that the low prior economy has higher repayment probability than the high prior economy in both low and high information precision systems. The market has more generous good signal criteria toward the low prior economy due to the fact that the economy in general has a low level of debt. However, the market cannot distinguish the relevant fundamental and market noise so that price response and according default outcome can happen adversely to the states with low θ.

Table 4: Default risk and average bond price

Repay probability

	low β	High β
Low prior (θ_0=0.1)	0.5500	0.5438
High prior (θ_0=0.9)	0.4499	0.4561
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

Average bond price

	low β	High β
Low prior (θ_0=0.1)	0.5585	0.5501
High prior (θ_0=0.9)	0.4414	0.4498
$\theta_0 = 0.5$	0.5000	0.5000
No prior	0.5000	0.5000

Note: This table shows the repayment probability (left) and the average equilibrium bond price (right) for each specified economy. No prior economy represents the benchmark economy as in section 2. Other economies represents the economy with prior as in section 5 with a prior mean, θ_0 as specified. Parameter values are assigned as Table 2 and Table 3.

[7] I set the μ grid to be symmetric so that positive market noise can realize as equally likely as negative market noise.

Next, I calculate the average bond price, defined as the average of p in all states (θ, μ) weighted by the joint probability, and the right panel of Table 4 shows the result. The low prior economy has a higher average bond price than the high prior economy in both low precision and high precision case. This means that lenders understand the fact that θ is likely to be low in general. However, as shown in Table 4, the low prior economy has a lower average price in high precision case. Low θ state is more likely to be adversely affected by precise information, resulting in depreciation of bond price by sensitive price schedule. It is the low prior economy that is more likely to have those states realized.

6 Conclusion

This paper explores the impact of private information precision on default outcomes through bond prices. The key finding is that precise information increases the sensitivity of the price schedule to the market signal. However, this heightened sensitivity can have adverse consequences for states with low debt levels. The reason behind this is that the market struggles to distinguish between the fundamental and irrelevant market noise within the market signal. As a result, states that are fundamentally good with low debt levels are more susceptible to negative market noise compared to states with bad fundamentals. Consequently, fundamentally good states are more likely to be perceived as a pessimistic market signal, prompting the sensitive price schedule to devalue the bond price. This devaluation effect is substantial enough to push a government with low debts into default, despite the state itself being fundamentally sound and capable of repayment without the sensitive response.

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Appendices

A Multiple Equilibrium in Computation

In this section, I provide more details about how I deal with equilibrium multiplicity.

This paper is focused on comparative statics in unique price equilibrium. To ensure the uniqueness of the equilibrium, I restrict parameter sets so that the precision is low enough. Numerically, I define that a unique price is attained at a given parameter β, σ, and α if fixed points with different initial guesses of p are equivalent within a certain tolerance level for all $x \in R$. Specifically, for a given value β, σ, and α, I set $0.5, 0.0$, and 1.0 as different initial guesses and compute a fixed point for each initial guess. As long as all fixed points are equivalent within tolerance level $1e-6$ for x grid $\in [0.0, 1.0]$, I denote that β, σ, and α guarantees a unique price.

In this model, it turns out that β, σ, and α guarantees a unique price as long as $\beta + \beta\sigma\alpha$ < 2.439. I continue numerical exercise with parameter sets that satisfy the condition for guaranteeing a unique price.

B Computational Algorithm

For the numerical exercise, I set a grid for θ in the interval $[0.0, 1.0]$, as these values are relevant for the dependency of default outcome on $p \in [0.0, 1.0]$. For the μ grid, I symmetrically set it around 0. Given that the mean of μ is 0, $\mu > 0$ is interpreted as a high noisy demand shock relative to the average, and $\mu < 0$ as a low noisy demand shock relative to the average. In essence, $\mu > 0$ corresponds to a favorable market noise, making the market signal x more likely to be interpreted as favorable for a given θ. The interpretation of x as favorable or unfavorable critically determines whether high precision has an adverse impact on the default outcome.

The distribution of (θ, μ) is constructed conditionally within the truncated intervals.

C Additional Figures

In this section, I present additional figures that provide further insights into the equilibrium dynamics.

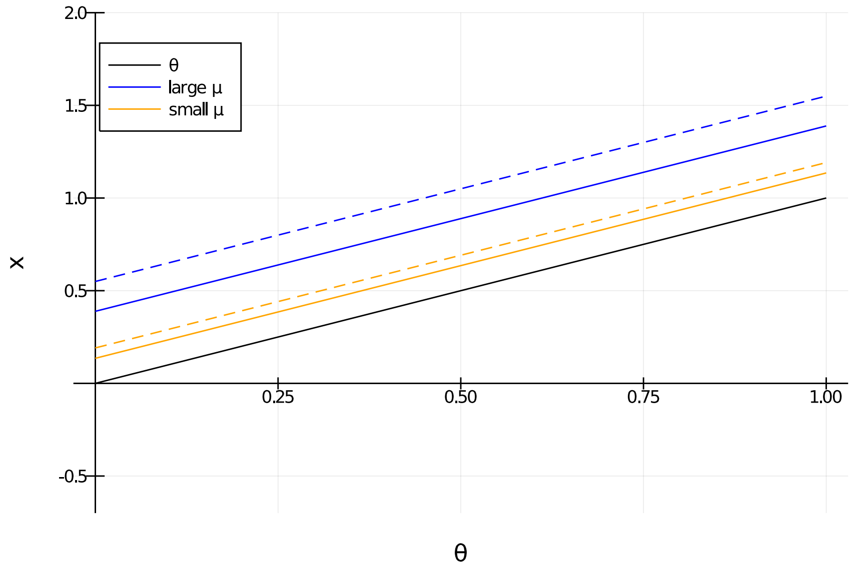

Figure C.1: Magnitude effect of μ on x

C.1 The magnitude of noisy trader shocks

Along with examining the effect of different sign of noisy trader shock, μ, I analyze how different magnitudes of μ impact market signal, x. Intuitively, for a given debt level, θ, when there is higher demand from noisy trader, the market signal is likely to underestimate θ since the bond price is largely driven by μ shock relative to the fundamental.

Figure C.1 visualizes the market signal, x, under different magnitudes of μ, assuming all positive values. x is plotted as a function of the debt state, θ. The blue lines correspond to larger μ values compared to the yellow lines, and both μ values are positive shocks. In short, $\mu_{blue} > \mu_{yellow} > 0$. The dashed lines represent the market signal in a low information precision economy (low β case), while the solid lines represent the market signal in a high β economy. In both cases of large and small μ, the market signal reveals the fundamental debt state more precisely as the private signals become more precise. This is shown by the solid lines, for both large and small μ, being closer to the black line representing θ compared to the dashed line, indicating a smaller deviation of x from θ. As the magnitude of the μ shock increases, the deviation of x from θ becomes larger, and the effect of high β becomes more powerful in reducing this deviation. The figure illustrates that the reduction between the solid line and the dashed line is larger in the case of a large μ compared to the case of a small μ.

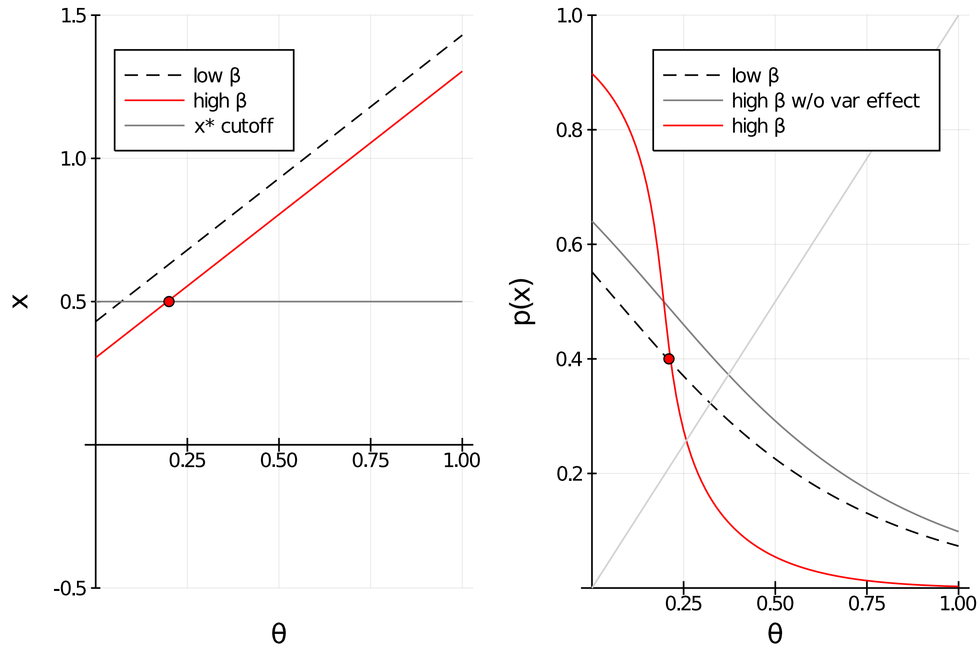

Figure C.2: x and $p(x)$ in negative μ case

C.2 Negative noisy trader shocks

I will now provide an explanation of what happens under negative trader shocks, in contrast to the previous explanation of positive noisy trader shocks as described in subsection 4.2.

Here, negative noisy trader shocks, $\mu < 0$, imply relatively low residual demands for bonds, and these shocks are irrelevant to the fundamental. A low residual demand results in a low price for a given debt state, θ, which makes the market signal appear unfavorable to the marginal lender. In other words, $x > \theta$ for all θ, as illustrated in the left panel for both the low and high β cases.

Whether precise information has a negative impact on debt financing depends on the market signal relative to the market signal cutoff, x^*. The marginal lender with precise information generally responds sensitively to market signals and devalues the bond prices under unfavorable market signals relative to x^*, or when $x > x^*$. In the parameterized economy described in subsection 4.2, $x^* = 0.5$. The sensitive pricing induced from information precision has negative consequences for states with $\theta > 0.235$. These are the states where, for a given $\mu < 0$, $x > 0.5$ and the marginal lender interprets the fundamental as unfavorable. As a result, the government in those states sells bonds at lower prices and collects lower revenue compared to what would have been achieved with low β.

It is interesting to note that without this sensitive response, higher precision of private signals could have been beneficial because the market signal would be a more accurate estimate of the fundamental, offsetting the underestimating effect of negative noisy trader shocks. This is shown by the gray line in the right panel of Figure C.2. The gray line represents a counterfactual bond price where higher precision results in a more accurate estimate of θ through market clearing condition. However, the marginal lender does not respond as sensitively in this counterfactual economy. In this scenario, the bond price, p, is always higher with precise information. It is the marginal lender's sensitive response that generates the adverse impact, especially in those θ states where the corresponding x is higher than x^* and is inferred as unfavorable.

Default outcome The devaluation coming from the sensitive price schedule affect bond revenue and give a consequence to default outcome. When the residual demand from noisy trader is relatively low, the debt state should be sufficiently low in order for the marginal lender to infer it as favorable. In those debt states that are not as low sufficiently, a fundamental-irrelevant shock can influence default outcome. The marginal lender more confidently believes that the government's debt level is unfavorable and devalues the bond price sufficiently. This result in the government failing to raise enough bond revenue, ending up with default.

The right panel of Figure 5 illustrates that $\theta \in [0.265, 0.375]$ is the state where the government defaults in high β economy, whereas the default would not have occurred in the case of low β. The reason for this is that the market cannot fully separate the default-relevant fundamental, θ, from x, and there exists a θ where the marginal lender interprets it as an unfavorable signal with sufficient market noise, even though the debt level is relatively low. In those states, insolvency risk is relatively low, but rollover risk is high. Default occurs due to the failure of rollover.

C.3 Price schedule in low prior economy

Figure C.3 plots the price schedule of the low and the high prior economy, respectively. The red plots represent the low prior economy, while the blue plots represents the high prior economy. The dash lines correspond to the low β economy, and the solid lines corresponds to the high β economy. In the low prior economy, θ tends to realize as low on average, and the equilibrium bond price is always higher compared to the case of the high prior economy, regardless of the precision level.

The influence of different priors, θ_0, on the bond price schedule is manifested through the market signal cutoff, x^*. The plot shows that the high prior economy has a lower cutoff, indicating that the market has a more stringent criteria for interpreting the market signal, x.

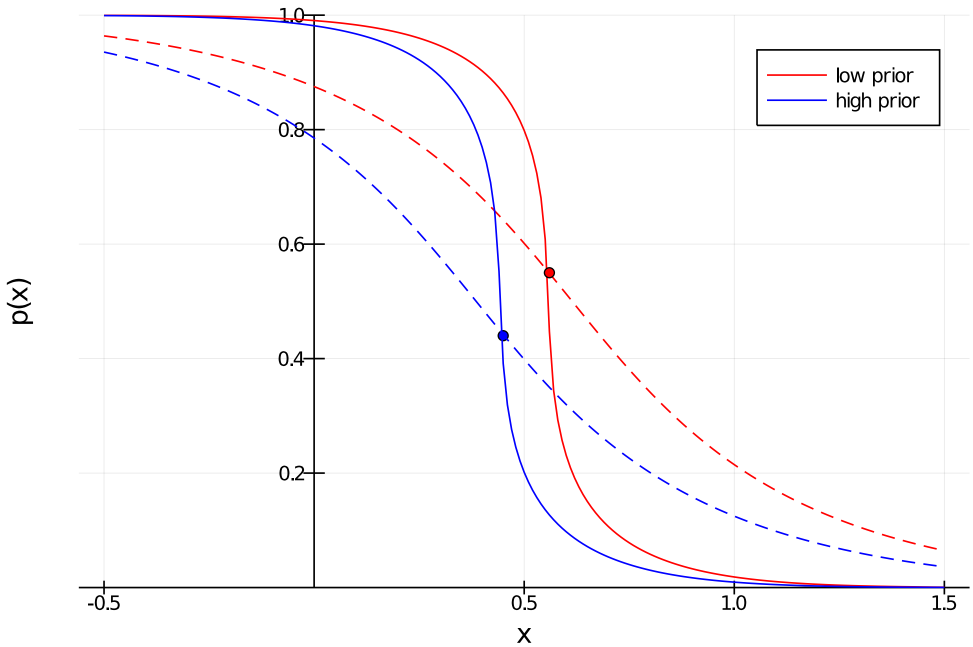

Figure C.3: Price schedule with low and high θ_0

The same value of x can be interpreted as an unfavorable signal in the high prior economy, whereas it is considered a favorable signal in the low prior economy.