Credit Ratings in Sovereign Bond Markets

Juyoung Yang*

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Abstract

How does credit market segmentation affect the default risk of developing countries? This paper explores the impact of market segmentation resulting from sovereign credit ratings on default risk and emphasizes its disciplinary role in shaping borrowing behavior. Empirical analysis suggests that downgrades from investment-grade to junk ratings are associated with a 30-basis-point increase in spreads. To account for this, I develop a quantitative sovereign default model that incorporates credit ratings and a segmented market structure derived from ratings. I calibrate the segmentation parameter to match the observed spread movements during downgrades, and find that the higher spread implies a 200-basis-point higher discount rate on junk bonds compared to comparable non-junk bonds. In the model, when a country accumulates debt beyond a threshold, it triggers downgrades, resulting in sovereign bonds being priced by more impatient lenders, leading to higher interest rates. Consequently, the government borrows less, reducing default risk and raising the overall bond price schedule. While the segmentation makes the country worse off in the steady-state debt level, it can benefit the government in low-debt states by mitigating overborrowing friction associated with long-maturity structures, resulting in welfare gains. An analysis of an optimal rating rule suggests that a more looser rating rule diminishes these welfare gains.

Keywords: Credit rating; Market segmentation; Sovereign bonds; Default risk

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1 Introduction

This paper examines the regulatory usage of sovereign credit ratings, particularly the constraints that institutional investors face in holding sovereign bonds with low ratings. The study investigates the consequences of these constraints on capital market segmentation and how a country's sovereign ratings affect the external credit supply it receives in an open economy. When a country is downgraded to junk status, it loses a group of investors who are subject to prudential regulations. As a result, a junk-rated country must rely on a different set of investors to raise funds compared to an investment-grade country. This market segmentation by sovereign credit ratings has a greater impact on developing countries than on developed countries. Developing countries are more likely to experience downgrades to junk during recessions or political instability, whereas advanced economies are seldom rated as junk.[1]

Along with the consequence on capital market segmentation, this paper aims to understand the role of sovereign ratings in emerging markets. While sovereign ratings have significant implications for credit access, few studies have analyzed their impacts on emerging markets and those that do often neglect the endogenous behavior of the sovereign.[2] For example, Moody's upgraded Vietnam's rating as Vietnam's National Assembly conducted prudent debt management policies (including lowering the public debt ceiling).[3] This paper addresses this gap by providing the first quantitative analysis of how developing countries respond to regulatory constraints imposed by sovereign credit ratings in segmented market structures.

Unlike the previous literature, this paper highlights the disciplinary role of sovereign ratings on countries' borrowing. Specifically, accumulating debt beyond a certain level may cause a downgrade to junk, leading to higher interest rates on the country's bonds. The country takes these consequences into account and may choose to reduce its debt level to avoid downgrades or to exit the junk territory. This disciplinary role ultimately lowers default risk and may enable the country to issue bonds at a more favorable price. The

[1] According to S&P sovereign rating, Sweden, the United Kingdom, Netherlands, and the United States have downgraded once during the entire rating history and it is from AAA (the highest rating level) to AA+ (the second highest rating level). Germany and Switzerland have been keeping their rating as AAA throughout their rating history. Singapore and Hong Kong constantly improved sovereign ratings and have kept their investment-grade status throughout their history. During the European debt crisis, only Portugal and Greece experienced downgrading to junk. Spain, Italy, and Ireland experienced multi-notch downgrades, but they never downgraded to junk.

[2] For example, Ferri et al. (1999) is motivated by Asian financial crisis episodes and conclude that sovereign ratings can represent another systemic risk to emerging markets.

[3] https://www.moodys.com/research/Moodys-upgrades-Vietnams-rating-to-Ba2-outlook-changed-to-stable--P
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segmentation by credit ratings encourages the country to keep the default risk relatively low, and the country is less vulnerable to sudden output drop. Also, this effect can reduce overborrowing inefficiency stemming from bond maturity structure and improve the country's welfare. Incorporating the endogenous response of the sovereign is critical for this result to arise.

To this goal, I build a quantitative sovereign default model with credit ratings and a segmented market structure. In my model, the sovereign borrows in long-term bonds from the international credit market to offset income shocks and to front-load its expenditure. It also chooses to default on its outstanding debt or not. A credit rating represents the country's default risk. But the rating also determines which set of creditors the government borrows from. When the country is high-rated (or investment-grade), it issues bond in a high-rating bond market and vice versa for the low-rated (or junk) bond market. Therefore, the segmented market structure generates different bond price schedules depending on the rating. I assume that those price schedules are different by the discount rate of the creditors. Those discount rate parameters controls the degree of segmentation, and are calibrated by matching statistics observed in data.

To discipline the segmentation parameter, I exploit an empirical statistic from an emerging market panel data. I use sovereign bond spreads and sovereign ratings data to capture the spread change when a country's rating changes from investment grade to junk. I find that countries experience an increase of a 30-basis-point in their yields when they are downgraded to junk. This increase is compared to those countries with similar economic fundamentals but not experiencing a downgrade to junk. This higher spread is robust to other several specifications. I find that the increase in bond spread is higher for those countries that are downgraded to junk than those that are downgraded but not cross the regulatory threshold (downgraded within investment grade or junk grade). Those empirical findings are consistent with the regulatory usage of sovereign ratings.

By replicating this statistic, calibrated the segmentation parameter represents a higher discount rate of the low-rated bond market than that of the high-rated one. This implies that a low-rated bond is charged a higher interest rate than a bond with a high rating. This discourages the government from accumulating its debt beyond the downgrade threshold. Although it prefers front-loaded expenditure, the government finds it optimal to reduce its debt level. Debt reduction as an optimal policy is apparent particularly when the economy is near a downgrade threshold. The debt issuance in a controlled manner makes the country resilient to negative output shocks, and therefore default happens less often. This lower level of default is embedded in the pricing schedule as a lower default risk compensation. This is shown by comparing the calibrated benchmark economy to the counterfactual economy without ratings. By targeting the economy to match the average of the panel data, the model simulation shows that the frequency of default, default risk, and average spread are lower in the benchmark than in the counterfactual.

Next, I study whether countries can be better off with ratings and the resulting segmentation. I find that, under the calibration, the country is worse off with ratings when at the steady-state level of debt; however, the country can be better off at low levels of debt. There are opposite forces for this welfare implication. The welfare benefit arises from the disciplinary effect: the country can borrow at a favorable price. On the other hand, borrowing less and abstaining from consumption reduces welfare. Another welfare cost arises as the country's rating is low; higher yield from impatient lenders. The calibrated sovereign is so impatient that the welfare cost quantitatively dominates the welfare benefit. However, I find that the segmentation can make the country better off in low-debt states.

Now, I investigate how the maturity of bonds affects the welfare gains associated with sovereign ratings. I compare the benchmark economy with a counterfactual one where only one-period debt is issued. My findings show that, for all levels of debt, the use of sovereign ratings results in a decrease in welfare for the country. However, ratings do have a positive effect as they act as a commitment device for the country's future borrowing behavior. This partially resolves the debt-dilution problem under long-term debts. The issue with long-term debt is that, when a new debt is issued, the country cannot commit to its future borrowing behavior, making long-term debt more expensive than one-period debt. Ratings give lenders a rational basis to anticipate that the country will not choose to borrow more in the future, especially above the junk cutoff. This anticipatory behavior of lenders leads to pay a higher price, resulting in welfare gains under long-term debts.

Conducting a counterfactual exercise using alternative segmentation rules suggests that loose rules generate negative welfare implications because they weaken the disciplinary effect and lower the welfare benefits of segmentation.

This paper is organized as follows: Section 2 goes over the literature review. Section 3 discusses the details of sovereign ratings and examples of the regulatory usage of sovereign ratings. Section 4 presents the empirical findings from emerging market data. Section 5 describes the model, and Section 6 shows the main quantitative results: the disciplinary effect of sovereign ratings, welfare implications, and the role of bond maturity. Lastly, Section 7 concludes the paper.

2 Related literature

This paper contributes to two related strands of literature. First, this paper is related to the quantitative sovereign default literature. Second, the paper is closely related to a broad set of empirical papers that analyze the impact of credit ratings on sovereigns, especially developing countries.

The sovereign default literature bases its theoretical framework developed in Eaton and Gersovitz (1981) and extended by Arellano (2008), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). The survey papers, Aguiar and Amador (2014) and Aguiar and Amador (2021), documents the sovereign default literature in detail. The Eaton-Gersovitz model is about a government that faces income risk and borrows, using a defaultable but otherwise noncontingent bond. The risk-averse government borrows to smooth consumption, but its impatient preference generates front-loading consumption. The key feature of the model is that the government cannot commit to repay its debts. This class of model is able to replicate the countercyclical movement of interest rates in emerging markets, which is documented in Aguiar and Gopinath (2007) and Neumeyer and Perri (2005).

Including long-term bond maturity allows the model to match higher debt levels and higher volatility of interest rates. Long-term bond maturity models have a computational challenge as discussed in Aguiar et al. (2020). Dvorkin et al. (2021) employs discrete choice methods to overcome the computational challenge, making the quantitative sovereign default model under long-term bonds tractable. I follow their methods and include extreme value shocks to handle the computational challenge.

There are ample empirical papers about sovereign credit ratings and their impact on sovereign bond spreads. Cantor and Packer (1996) estimate the weights of each sovereign's macroeconomic fundamentals in the determinants of sovereign credit ratings. Using cross-section data, they find that per-capital income, external debt, and inflation are crucial determinants of sovereign credit ratings. The paper shows that sovereign credit ratings summarize the country's macroeconomic fundamentals well and are highly correlated with sovereign bond spreads. Using event studies, the paper highlights that credit ratings can independently affect spreads and shows that spread movement follows the announcement of the change in the country's credit ratings in the expected direction. The paper points out that the announcement for non-investment grade, or junk grade, affects spread to a greater degree than that for investment grade bonds. Following the findings, I incorporate regulations on investing junk bonds to understand the distinction between investment grade and junk.

Hanusch et al. (2016) and Drago and Gallo (2016) are the closest papers to the empirical evidence of this paper. Both papers focus on the regulatory usage of sovereign ratings. Hanusch et al. (2016) use panel regression to estimate the impact of downgrades from investment grade to junk. They estimate the impact on short-term borrowing using Treasury Bills, whereas this paper finds evidence on long-term borrowing which is consistent with the quantitative model. Drago and Gallo (2016) use an event study approach to provide evidence of a regulatory threshold. They compare the spread response to downgrades crossing investment grade/junk classification and to downgrades without crossing it. They find that the CDS spread reacts intensely to downgrades crossing the threshold. This paper takes a similar approach, and its findings are consistent with the literature. The contribution of this paper is building a quantitative model that can generate empirical findings, which enables us to estimate the degree of investment grade/junk segmentation and to do welfare exercises.

3 Ratings and regulations on institutional lenders

Sovereign credit ratings are the assessment by credit rating agencies, for example, Moody's and Standard and Poors', of the likelihood of a government defaulting on its debt obligations. In recent decades, the demand for sovereign ratings has risen rapidly. In the early phase of the 20$^{\text{th}}$ century, Moody's rated roughly 50 bonds of central governments. These were mainly issued by developed countries, including the United States. The number has risen to more than 130 countries as of today. This growth is because sovereign ratings facilitate countries to access international capital markets: International investors, particularly United States investors, prefer rated government securities over unrated securities of similar default risk (Cantor and Packer (1995), Luitel and Vanpée (2018)). In recent decades, developing and low-income countries, for example, African countries, have newly had their sovereign bonds assessed for credit ratings. Developing countries pay considerable attention to sovereign ratings as a means of raising external funds from international capital markets (Cantor and Packer (1995)).

Another reason developing economies care about their credit ratings is because their rating change could trigger regulatory constraints on investing their bonds. Rating-based regulations on institutional investors are intended to prevent speculative investing under prudential measures. Those regulations originated from the Volcker Rule and the Dodd-Frank law in the United States [4] (see Duffie (2012), Bernstein (2019)), and the Basel II framework spread rating-based rules widely in advanced economies as well. The restrictions are often in the form of imposing the minimum level of rating for securities investment

[4] The Volcker Rule exempts the Unites States treasuries and federal agency bonds, but it does not exempt securities issued by foreign countries.

and holdings. A well-known notion for the minimum level is the speculative grade or junk: equal to or below BB+ under S&P and Fitch or Ba1 under Moody's. Therefore, when a developing country has its sovereign rating downgraded to junk, its sovereign bonds are subject to prudential regulations, and potential investors are limited to holding junk bonds due to regulatory concerns.

South Africa in 2020 provides anecdotal evidence of developing countries' concern about being junk-rated. In November 2022, Moody's and Fitch downgraded South Africa deeper into junk territory. South Africa's Minister of Finance said, "The decision by Fitch and Moody's ... is a painful one. ... Continuous rating downgrades will translate to unaffordable debt costs, deteriorating asset values ...."[5] According to JP Morgan estimates, the downgrade of South Africa could trigger the forced selling of up to $2.4 billion worth of South Africa's dollar-denominated Eurobonds.[6] Another piece of anecdotal evidence is Greece in 2010. As its debt crisis evolved, Greece was downgraded to junk by S&P and Moody's. According to Barclays Capital, a British universal bank, this downgrade is estimated to result in Greek government bonds worth $252 billion being excluded from its bond indexes. [7] [8]

As documented in Kiff et al. (2012), BIS (2009a), credit ratings are often used by authorities to classify securities in legislation, regulations, and supervisory policies. Credit ratings affect the bond demand of institutional lenders and serve as triggers in investment decisions under regulations. These regulations are mostly motivated by prudential measures and have the goal of sheltering institutional investors' portfolios from high risks. There are nationally recognized statistical rating organizations (NRSROs) under the United States Securities and Exchange Commission (the U.S. SEC) and these regulations adopt credit ratings released by NRSROs.

Basel II is one of the prominent examples of the usage of credit ratings on regulations. Risk-based capital requirements under a standardized approach use credit ratings to map credit risks to risk weights or capital charges. Many advanced economies, for example, the European Union, Australia, Canada, Japan, and the United States, incorporate the Basel II framework into assessing the credit quality of securities in banking sectors. Also, the central banks set a limitation for acceptable collateral and margin requirements. For example, the

[5] https://www.reuters.com/article/us-safrica-ratings/painful-downgrades-will-raise-south-africas-borr

[6] https://sports.yahoo.com/news/junk-rated-debt-could-cost-south-africa-more-071113410--sector.html?guccounter=1&guce_referrer=aHR0cHM6Ly93d3cuZ29vZ2xlLmNvbS8&guce_referrer_sig=AQAAABUDy7jr94PK0m-au9_nJ8fWqfnF2DG-zWu3dBZsSsCT864flMtSS-bt14lHNZSqXCpwOn_-ThuNveGCc1ly2PSvK01khP2QzdGgjc0CAiTPbHYeLWq6kreMkhCTUdABrFm_Eh_C0v5oavsSB4LDScHXsL-xpEUMUGCVKhgEZda2

[7] https://www.wsj.com/articles/BL-MB-23516

[8] Investment decision by credit ratings is also present in internal rules of index funds. Investment-grade (rating equal to or above BBB- under S&P and Fitch or Baa3 under Moody's) bond indexes are forced to sell their junk bonds following internal guidelines. Dropping out major bond indices could trigger additional forced-sell by other investors.

European Central Bank (ECB) specifies the minimum rating level for the eligible collateral of commercial banks (ECB (2013)).

Not only banking sector is affected, but insurance sector is also affected. Mostly, insurance companies or pension funds are regulated to hold bonds above the minimum rating. In Japan, for example, estimating credit risks for insurance companies is done by calculating the solvency margin ratios, and credit ratings are used for the calculation. In the case of the United Kingdom, the Insurance Prudential Sourcebook relies on credit ratings for insurance capital requirements. In Canada, the Office of the Superintendent of Financial Institutions (OSFI) states, in its life insurer capital guideline, that "A company must consistently follow the latest ratings from a recognized, widely followed credit rating agency. Only where that rating agency does not rate a particular instrument, the rating of another recognized, widely followed credit rating agency may be used." These restrictions are present also in state legislation. For example, New York state insurance law mentions that an insurer may insure municipal obligation bonds that are not investment grade as long as at least 95 % of the insurer's aggregate liability is investment grade (see BIS (2009b)).

Non-investment grade is often subject to these regulation, and investment/non-investment is a commonly-used regulatory threshold. Investment grade is above BBB- under the Standard and Poor's and Fitch rating system or Baa3 under Moody's system. Non-investment grade which is often called a junk grade is below BB+ under the Standard and Poor's and Fitch system or Ba1 under Moody's system. Regulatory restrictions imply that it is costly to hold non-investment grade (or junk grade) securities or that the potential holders of non-investment grade sovereign debt are limited. As documented in Rigobon (2002), upgrading to investment grade means that broader types of investors, for example, pension funds, and insurance companies, can hold the sovereign bonds.

4 Empirical evidence

In this section, I present the empirical relationship between countries' credit ratings and their spreads. I investigate spreads response as a country's credit rating changes across regulatory threshold. In what follows, I use investment grade and junk grade as the regulatory threshold.

The country's default risk affects both the bond spread and the credit rating. In regression, I control for the country's default risk using its macroeconomic fundamentals. I show that the country experiencing downgrades across the regulatory threshold is associated with a statistically higher bond spread compared to the country with similar macroeconomic fundamentals that does not experience a downgrade. I use the coefficient estimate of downgrade dummy variable to calibrate the market segmentation parameter later in the calibration section. As a robust check, I compare the spread response to downgrades across the threshold to the spread response to the same degree of the downgrades but not crossing the threshold. I find a higher spread with downgrades across the threshold than downgrades not crossing the threshold.

Subsection 4.1 describes the emerging market panel data and data sources. Subsection 4.2 shows the main empirical findings.

4.1 Data description

The observation of interest is how the bond market responds to a regulatory constraints triggered by a change in the credit rating of a bond. I use bond spreads as the measure of the bond market response. In particular, I use the JP Morgan's Emerging Market Bond Index (EMBI) spread in the Global Economic Monitor (GEM) in the World Bank. These spreads are defined as the weighted averages of the bond yield spreads of US dollar-denominated external debts issued by sovereigns and sovereign entities over corresponding the United States government debt securities (see Comelli (2012)). For a country's credit rating, I use the Standard and Poor's sovereign credit ratings. [9] Under the Standard and Poor's (S&P) criteria, the regulatory threshold of interest lies between BBB- and BB+. In other words, a country with S&P ratings above BBB- is investment-graded and the one with S&P ratings below BB+ is junk-graded.

The sample period is from January 1998 to December 2019. I choose a set of countries that have experienced being both investment-graded ratings and junk-graded ratings throughout the sample period, along with data availability. Russia has a default experience during the sample period (year 1999), and I exclude the country as an outlier. The 12 countries in the sample are Azerbaijan, Brazil, Bulgaria, Colombia, Croatia, Hungary, Mexico, Namibia, Panama, Peru, the Philippines, and South Africa. Bond spreads and ratings are calculated monthly. For each country's economic fundamentals, I use real GDP growth (in percentages) and gross debt to GDP ratio (in percentages). The data is from World Economic Outlook (WEO) in the IMF, and it is at a yearly frequency.

Table 1 shows the summary statistics of bond spreads by ratings. These observations are at monthly frequency. The sample is centered around the regulatory threshold, between

[9] The Standard and Poor's is one of the major credit rating agencies authorized as an NRSRO. It is fairly documented by the literature that the ratings of different credit rating agencies rarely differ much (see Ferri et al. (1999)). El-Shagi and von Schweinitz (2018) documented empirically that the mean absolute difference to the average rating level is 0.5 notches. It means ratings across agencies are far less than one notch apart on average. Therefore, the empirical findings would not change much by using credit ratings of different agencies, for example, Moody's and Fitch.

	rating	mean	median	s.d.	min	max	freq.	obs.
Investment grade	A-	50	53	0.24	13	124	0.02	-
	BBB+	187	168	0.95	55	668	0.13	-
	BBB	183	172	0.83	43	696	0.20	-
	BBB-	263	226	1.14	45	924	0.20	-
Junk	BB+	331	312	1.46	103	937	0.16	-
	BB	357	315	1.68	97	985	0.16	-
	BB-	444	393	2.48	137	1,370	0.07	-
	B+	884	751	3.64	430	2,057	0.02	-
	B	782	803	1.94	506	1,365	0.01	-
	total	294	234	2.08	13	2,057	1.00	2,610

Table 1: Summary statistics of bond spreads by ratings (bps)

	freq.	obs.
no change	0.97	2,776
downgrade	0.01	31
upgrade	0.02	49

Table 2: Monthly rating changes

BBB- and BB+. A low level of ratings indicates the low credibility of creditors, and as expected, a lower rating is associated with a higher bond spreads. Moreover, bond spreads become more volatile for lower ratings.

Table 2 shows the monthly changes in ratings. A downgrade is defined as an event where a country's rating changes to a lower level than the rating level of the previous month. An upgrade is defined as an event where a country's rating changes to a higher level than the rating level of the previous month. In my sample, every rating changes by one notch. As shown in the table, countries' ratings are persistent, and a country's credit rating has a 97% likelihood of not changing in the next month. The literature has documented that countries' ratings tend to be persistent. (See El-Shagi and von Schweinitz (2018)).

4.2 The empirical results

After constructing the sample, I investigate how yields respond to a change in ratings. I focus on downgrades where regulatory restrictions are triggered (that is, when a country's rating changes from investment grade to junk).

	(1)
	spread
DowntoJunk	29.64***
	(11.47)
lag_spread	0.969***
	(0.005)
gdp	-0.312
	(0.308)
grossdebt	0.146**
	(0.057)
Observations	2528
R^2	0.981
Country FE	Y
Time FE	Y

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Regression with downgrade to junk dummy

I conduct the following panel fixed effect regression:

$$spread_{it} = \beta_0 + \beta_1 DowntoJunk_{it} + \Gamma X_{it} + \alpha_i + \delta_t + \epsilon_{it} \tag{1}$$

The variable $spread_{it}$ is country i's bond spread at time t, $DowntoJunk_{it}$ is a dummy variable which equals to one if the country i's rating is investment grade at time $t-1$ and junk at time t and equals to zero otherwise.

X_{it} is a set of control variables including the country's economic fundamentals and lagged spread. I include GDP growth and gross debt-to-GDP ratio as control variables to control a country's underlying economic fundamentals that affect both spreads and sovereign ratings. Unlike ratings and spreads in monthly frequency, economic fundamental variables are yearly. To capture the movement of the fundamentals within a year, I include the country's lagged spread (the spread of the previous month) as another control variable. I include country-fixed effect α_i and time-fixed effect δ_t in monthly frequency. The coefficient of interest is the downgrade dummy coefficient β_1.

The regression result is summarized in Table 3. The coefficient estimate of *DowntoJunk* is positive and statistically significant, implying that a country's downgrade across the regulatory threshold is associated with an increase in bond spreads. The bond spreads, on average, increases by 30 bps when the country downgrades from investment grade to junk compared to the spread of a country with similar fundamentals but that has not experienced the downgrade to junk. This finding is consistent with the literature, for example, Hanusch et al. (2016). They also find a positive and significant spread response to a country's downgrades. Unlike them, who use a short-term bond (60-day Treasury bill) spreads, I use a long-term bond spreads, which is consistent with the specification of the quantitative model in Section 5 and 6.

The estimate is robust to other specifications. I include other downgrade dummies in the regression and compare the coefficients across the dummies. $withinJunk$ and $withinInvst$ variables represent similar downgrades. The variable $withinJunk$ captures downgrades within junk: downgrade events either from BB+ to BB or from BB to BB-. Similarly, the variable $withinInvst$ denotes downgrades within investment grade: downgrade events either from BBB+ to BBB or from BBB to BBB-. Both dummies are still near investment-grade and junk threshold (between BBB- and BB+).

Formally, the regression with different downgrade dummies is as follows:

$$\begin{aligned} spread_{it} = \beta_0 + \beta_1 DowntoJunk_{it} \\ + \beta_2 withinJunk_{it} + \beta_3 withinInvst_{it} + \Gamma X_{it} + \alpha_i + \delta_t + \epsilon_{it} \end{aligned} \quad (2)$$

where $withinJunk_{it} = 1$ when a country i's rating is either BBB+ at time $t-1$ and BBB at time t or BBB at time $t-1$ and BBB- at time t. $withinInvst_{it} = 1$ when a country i's rating is either BB+ at time $t-1$ and BB at time t or BB at time $t-1$ and BB- at time t. The variables of interest are β_1, β_2 and β_3.

Table 4 depicts the regression results. Downgrades crossing the threshold, $DowntoJunk$, are associated with higher spreads than downgrades not crossing the threshold, $withinJunk$ and $withinInvst$. This is consistent with the interpretation that a downgrade from investment-grade to junk is particularly important and could be related to the regulatory concerns. In general, downgrade events are negative news about the economy, and a positive sign for the downgrade coefficient is expected. However, the magnitude of the coefficient estimates is different whether the country's rating crosses the regulatory threshold or not. Furthermore, the downgrades crossing the threshold are statistically more significant than downgrades not crossing the threshold.

This result with different downgrade dummies is consistent with the literature. Using the CDS spreads of European countries, Drago and Gallo (2016) find that crossover downgrades have a significantly greater impact than non-crossover downgrades. They conclude that downgrades leading to a country's rating category change imply intense reactions from

	(1) spread
withininvst	17.10*
	(9.572)
DowntoJunk	30.14***
	(11.46)
withinjunk	18.63*
	(10.09)
lag_spread	0.969***
	(0.005)
gdp	-0.249
	(0.309)
grossdebt	0.141**
	(0.057)
Observations	2528
R^2	0.981
Country FE	Y
Time FE	Y

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Regression with different downgrade dummies

investors due to regulatory constraints. The contribution of the part of the paper is to use a broader set of countries than just European countries to show the evidence of the regulatory aspect of sovereign ratings. Next, I use this empirical evidence to discipline a quantitative sovereign default model.

In the Appendix, I show additional tables related to the regressions and the regression result of a different specification.

5 The model

Motivated by the credit market segmentation that depends on credit ratings, I develop a dynamic small open-economy model. The model incorporates credit ratings and a global credit market segmented by these ratings. In addition, it includes a sovereign that borrows in long-term debts and strategically defaults under the timing framework proposed by Eaton and Gersovitz (1981). Including long-term debt is important as it allows the model to quantitatively match the high levels of debt observed in countries experiencing debt crises, a challenge that one-period debt models struggle with. (see, for example, Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)) Furthermore, bond maturity plays a vital role in determining the welfare implication of market segmentation. As the literature suggests (see, for example, Chatterjee and Eyigungor (2012), Hatchondo et al. (2016)), debt dilution problem is present in long-term debts, which constitutes the main friction in the model.

5.1 Model Environment

Time is discrete and infinite $t \in \{0, 1, ..., \infty\}$. The economy consists of a country, a global credit market, and the country's credit rating agency. The sovereign maximizes its expected and discounted lifetime utility. Its utility comes from expenditure each period under budget constraints. The sovereign faces a stochastic output process and has an outstanding debt. The sovereign can choose to default on its outstanding debt. If so, the country is exempted from paying all debts, but faces a cost. Under no default, the country pays the debt service and rolls over its debt by issuing new bond. The credit market is segmented by a credit rating. From which of these the sovereign raises their bond revenue depends on the credit rating the sovereign is assigned.

I model the credit rating as follows: it indicates the country's credibility and is assigned every period based on the country's default risk for next period. Not only does it indicate how likely the country is to default next period, but it determines the market where the sovereign issue new bonds. The model has multiple credit markets with a representative lender for each. I assume that the representative lender in each market is different, in particular different by discount factor. This discount factor is used for pricing the bonds. I focus on Markov equilibrium.

5.2 Timing

Following Eaton and Gersovitz (1981), the timing of the model within a period is as follows:

1. Output y realizes with inherited debt b.
2. The country chooses to default on the debt or repay.
3. Conditional on repayment, the country chooses a new debt level b'.
4. Credit rating r is assigned conditional on b'.
5. Bonds are traded in the appropriate credit market conditional on the rating. The sovereign raises bond revenue and chooses its expenditure.

5.3 The sovereign's problem

The state of the economy is output, y, and the outstanding debt level, b. Output is stochastic and exogenous. I assume it follows an AR(1) process with persistence ρ and volatility η. I estimate the process from the emerging market panel data in the empirical section.

The debt is in a long-term, and defaultable but otherwise non-contingent bond. Following Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), I consider random maturity bonds. I assume that every bond randomly matures at the rate of λ in each period. Each bond pays coupon until it matures, and when maturing, it promises to pay a principal of 1. The baseline coupon payment κ is normalized so that the price of risk-free bond equals to 1 (see Aguiar and Amador (2021)). With this specification, a unit of bond issued today is a contract that promises to pay debt service tomorrow of $\lambda + (1-\lambda)\kappa$ and of the remaining portion $(1-\lambda)$ only if the country does not default. With long-term bonds with random decay at a constant rate, the relevant variable to keep track of is the stock of the debt that the country is owed.

At the beginning of each period, the sovereign chooses to default on its debt or repay the maturing principal and coupon. If default is chosen, the sovereign is exempted from its debt obligations, but default generates two costs: 1) it generates a direct output cost, which is captured by $def(y)$, and 2) the sovereign is excluded from the global credit markets and cannot borrow for a period of time. If repayment is chosen, the sovereign pays the total debt service which is proportional to the stock of outstanding debt and raises a new revenue by issuing new bonds. Consumption takes place using the exogenous output and new bond revenue after serving the outstanding debt. The sovereign is risk-averse. The sovereign's problem is described as follows.

The sovereign, at the beginning of each period, chooses to repay or not. The value $V(y, b)$ is the value at state (y, b), $V^R(y, b)$ is the value under repayment and $V^D(y)$ is the value under default. That is:

$$V(y, b) := \max \left\{ V^R(y, b), V^D(y) \right\}$$

The function $D(y, b)$ is the default policy function where it equals to 1 when $V^R(y, b) < V^D(y)$.

Under default, the sovereign is free from its debt obligations, but suffers a direct output cost and is excluded from the credit market. It cannot borrow, but in the next period with some probability, it re-enters the credit market starting from zero debt. The parameter θ is the probability that the country re-enters the credit market under default status.

The value of default is:

$$V^D(y) = u(def(y)) + \beta \mathbb{E}_{y'|y} \left\{ \theta V(y', 0) + (1 - \theta) V^D(y') \right\}$$

Under repayment, the sovereign pays the debt service, coupon payments, and the principles of matured debt, and chooses the stock of debt tomorrow, b'. By doing so, the sovereign issues new bonds, $b' - (1 - \lambda)b$, at a price of q. As shown below, the bond price is determined by the future probability of repayment and it will depend on tomorrow's stock of debt b'. An innovation of this paper is that bond price also depends on the sovereign's credit rating. The credit rating matters because it determines in which credit market the sovereign sells its bond. When the sovereign's credit rating is high, it sells its bonds in a high-rating bond market, whereas when it has a low rating, it sells in a low-rating bond market. How the two credit markets are different is explained in the market segmentation section.

The function $u(.)$ is the sovereign's utility function. β is the sovereign's discount factor. The sovereign chooses its optimal debt issuance to smooth its consumption under the budget constraint (3).

$$V^R(y,b) = \max_{c,b'} u(c) + \beta \mathbb{E}_{y'|y}\{V(y',b')\}$$

$$s.t. \quad c + \big(\lambda + (1-\lambda)\kappa\big)b = y + q(b', R(b',y))\big(b' - (1-\lambda)b\big) \tag{3}$$

The debt issuance policy function $B(y,b)$ is determined from this optimization problem.

5.4 Credit rating

In each period, the sovereign's credit rating, r, is assigned. The credit rating takes two values: $r \in \{h, l\}$. [10] Investment-grade bonds are high rating bonds ($r = h$), and junk bonds are low rating bonds ($r = l$). I assume that there is no incomplete or imperfect information in the model. [11]

As in the real world where credit ratings reflect the credibility of the country's debt obligations, in the model, the credit rating reflects the sovereign's default probability. If the sovereign is likely to default in the next period, a low rating is assigned to the bonds, and vice versa. I assume that the assignment of a rating follows an exogenous rule, which is characterized by a rating rule parameter, $\bar{p}$. This parameter serves as a threshold between high and low ratings: if the sovereign's default probability next period is higher than $\bar{p}$, the country's bonds are assigned low rating and vice versa. It is important to remember that a credit rating is assigned after the sovereign's debt issuance choice. Therefore, the sovereign takes into account that its debt choice affects how its bonds are rated.

The credit rating is assigned as follows. For each state s, given sovereign's debt choice b',

$$R(b'(s), s) = l \quad if \quad \mathbb{E}_{s'|s}\big(D(s', b'(s))\big) > \bar{p} \tag{4}$$

$$R(b'(s), s) = h \quad if \quad \mathbb{E}_{s'|s}\big(D(s', b'(s))\big) \leq \bar{p} \tag{5}$$

The function $R(b'(s), s)$ is the rating policy function under the rule $\bar{p}$.

5.5 Global credit markets

There are two credit markets; one for high rating bonds and one for low rating bonds. Each market is perfectly competitive and is composed of a continuum of lenders. Lenders

[10] Credit rating agencies, like Moody's and S&P, issue sovereign credit ratings with the highest AAA+ and the lowest D. There are 20 different levels of credit ratings.

[11] There are literature studies on sovereign credit ratings under the global game framework. See Carlson and Hale (2006), Holden et al. (2012), Holden et al. (2018)

in each market are homogeneous, and therefore, each market has a representative lender. Both representative lenders are risk-neutral and have deep pockets. I assume that the credit markets are segmented: the representative lender of the high rating bonds markets purchases and holds the high rating bonds and they cannot purchase and hold the low rating bonds, and vice versa.[12]

Each lender maximizes the expected profit from holding the sovereign's risky bonds. In each period, the representative lender purchases bonds at the market price. The payoff of the bonds is conditional on the country's repayment next period and consists of debt services and the market value of the remaining bonds next period.[13] In equilibrium, the bond price schedules of high and low rating bonds satisfy a zero expected profit for each lender.

The main assumption is that the two lenders are different in terms of their discount factors: $\frac{1}{R_h}$ for the high rating bond market lender and $\frac{1}{R_l}$ for the low rating bond market lender. I normalize R_h to a risk-free rate.

It is reasonable to assume that $R_l > R_h$, which means the lender in the low-rated bond market is more impatient than the high-rated bond market lender. Because hedge funds usually seek high-return investment through short-selling and speculative investment practices, their outside investment options are higher than those of commercial banks and pension funds. Therefore, sovereign bonds that are junk-rated need to compensate hedge funds to find investing in junk bonds is as profitable as investing in high-yield outside options. In addition, investing in junk bonds involves regulatory costs that financial institutions have to bear, for example, higher capital requirements and not being counted as eligible collateral by central banks. Pricing of junk bonds incorporates the compensation for those costs. It also captures the different risk tolerance between traditional financial institutions and hedge funds.

The pricing equations for each market are as follows. For each state, and given the country's choice of b', the pricing schedule for high rating bonds is

[12] I do not allow lenders to choose which rated bonds to hold but take it as exogenous. This enables the model to be solved in a tractable way, especially the pricing schedules.

[13] When the country is either upgraded or downgraded next period, the current bondholder can no longer hold the bonds and has to sell the holding bonds to the secondary market. When the country keeps its rating, the current bondholder can sell the bonds or resell them to the secondary market. I assume both primary and secondary markets are perfectly competitive, and the secondary market has no liquidity friction. Therefore, primary and secondary markets share the same market price schedule, and I can recursively express bond prices with a single price schedule.

$$q(b', s, R(b', s) = h) = \frac{1}{R_h} \mathbb{E}_{s'|s} \Big[(1 - D(b', s')) \big(\lambda + (1 - \lambda)\kappa + (1 - \lambda) q(B(b', s'), s', R(B(b', s'), s')) \big) \Big]$$

The pricing schedule for low rating bonds is

$$q(b', s, R(b', s) = l) = \frac{1}{R_l} \mathbb{E}_{s'|s} \Big[(1 - D(b', s')) \big(\lambda + (1 - \lambda)\kappa + (1 - \lambda) q(B(b', s'), s', R(B(b', s'), s')) \big) \Big]$$

It is important to notice that both pricing schedules are the same except for the discount factor. Regarding the pricing of the bonds, not only does its expected payoff next period matter, but also the bonds' rating, thus to whom the bonds are sold. Furthermore, the bond price incorporates the expected value of the remaining bonds next period, which depends on the credit rating next period. Assuming that the discount factor of the representative lender of low rating bonds market is low, the more likely it is that the sovereign's bonds will be low rating tomorrow, the lower the value of the remaining bonds next period. This expectation is embedded in the pricing schedules. The bonds could be priced relatively low even though it is high rating, if there is a high chance of downgrade next period.

Another important feature of the model is the endogenous response of the sovereign. This response is embedded in the pricing equations above. The equilibrium pricing schedules depend on the sovereign's policy functions. The sovereign internalizes how its borrowing decision affects its credit rating and therefore the pricing schedule that it faces. The impatient sovereign has a temptation to front-load consumption, which is a main driver of high borrowing. Long bond maturity is another factor of high borrowing. The opposite tensions of high borrowing are present in the model. It is not only the fact that the bond is devalued by the default risk but also the fact that borrowing over a certain threshold (so that the sovereign downgrades to low rating) triggers another bond devaluation by the low discount factor of the representative lender of low rating bonds market. The sovereign internalizes this devaluation.

5.6 Markov equilibrium

I define a Markov equilibrium of the economy as follows. Given the exogenous rating rule $\bar{p}$, a Markov equilibrium consists of the sovereign's value functions $V(b,s)$, $V^D(y)$, $V^R(b,s)$, the sovereign's policy functions $B(b,s), D(b,s)$, rating policy function $R(b',s)$ and bond pricing schedule $q(b',s)$ such that

1. Given the rating policy function and the bond pricing schedule, the sovereign's value functions and policy functions satisfy the sovereign's bellman equations and maximization problems.

2. Given the sovereign's policy functions, the rating policy function is consistent with the rating rule $\bar{p}$.

3. Given rating policy functions and the sovereign's policy functions, the bond pricing schedule satisfies zero expected discounted profit conditions.

5.7 Bond maturity and efficiency

The literature on quantitative sovereign defaults suggests that the maturity of bonds plays a crucial role in the sovereign's borrowing behavior (see Hatchondo and Martinez (2009); Chatterjee and Eyigungor (2012); Hatchondo et al. (2016)). The literature finds that debt dilution happens under long-term debt, and it creates overborrowing inefficiency. Debt dilution means the reduction in the value of outstanding debt caused by new debt issuance. This happens because sovereigns cannot commit, at the time of borrowing, the borrowing behavior of future sovereigns. Long-term bond allows the sovereign to postpone the costs of current borrowing to the future sovereign. Given today's borrowing, the future sovereign has incentives to borrow high and the current sovereign is limited to constrain it. Rational lenders anticipate that additional borrowing by the future sovereign will increase default risk and reduce the market value of bonds issued by current sovereign. The bonds issued in current period is low-priced, and the sovereign has to issue more debts in order to raise a certain level revenue.

Chatterjee and Eyigungor (2012) quantifies debt dilution inefficiency by comparing equilibrium with long-term and with one-period debt. They find that sovereign's welfare is highest for one-period debt and declines monotonically as maturity parameter increases. Hatchondo et al. (2016) compare long-term bond equilibrium with dilution effect and without dilution effect, and find that the one with dilution has higher debt level and more frequent defaults. These findings suggest that long-duration maturity induces the sovereign to borrow more due to debt dilution problem, and it is welfare-reducing.

parameters	description	value	source
σ	risk aversion	2	literature
ρ	output persistence	0.844	the panel data
η	output volatility	0.034	the panel data
R_h	high rating market	1.027	risk-free rate
λ	bond structure	0.1	10-years maturity
$\bar{p}$	rating rule	0.0064	Moody's rating rule

Table 5: Parameters calibrated outside of the model

Aguiar and Amador (2019) find that borrowing is constraint efficient in one-period-bond Eaton and Gersovitz (1981) model by using dual-contracting approach. Constraint efficiency here means Pareto-optimal allocation between lenders and sovereign in the presence of market incompleteness and lack of commitment on default decision next period. Bond maturity plays a vital role in determining the welfare implications of market segmentation in this paper. I will explain the details in Section 7.6.

6 Calibration

The model is calibrated to a yearly frequency. The parameters of the economy are chosen either from outside of the model or by matching moments. The model targets the average across countries in the panel data. The details of calibration strategy are in the Appendix.

The flow utility function is assumed to be a the constant relative risk aversion (CRRA) utility on consumption:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

The default cost is in units of output. I follow Arellano (2008) and Chatterjee and Eyigungor (2012), and it is non-linear in output.

$$def(y) = max\{0, d_0 + d_1 y\}$$

Table 5 shows the list of parameters that I have calibrated outside of the model. The country's risk aversion σ is set to the standard value in the literature. The stochastic output is assumed to follow an AR(1) process and is estimated from the emerging market panel data. For each country, I linearly detrend real GDP growth and estimate an AR(1) process from the cyclical components. I take the sample mean on estimated output process weighted by data observations.[14] The estimated value for persistence and volatility on output are $\rho = 0.84$ and $\eta = 0.03$ for each. Consistent with the previous literature, as in Neumeyer and Perri (2005) and Aguiar and Gopinath (2007), the estimated parameter features the emerging market's volatile output.

I normalize the discount rate of the high-rated bond market to be the international risk-free rate. I take a long-term government bond for Germany as the benchmark safe asset. That is, the risk-free rate is estimated as the average of 10-year German bond yield from FRED in 1998-2019 (the same time period of the panel data). The bond maturity parameter λ is set to resemble the 10-year maturity bond.

The rating rule parameter $\bar{p}$ is taken from *idealized cumulative expected default rates* [15] released by Moody's (2018), one of the major credit rating agencies. These rates suggest the benchmark expected default rates where a rated-counterparty will fail to perform its debt obligation (Moody's (2022)). The rates are available for each rating level. Since the model is in a yearly frequency, the rates in the 1-year horizon are used for the estimation. Since $\bar{p}$ captures the threshold of credit ratings under regulatory purposes, I take the mean of expected default rates of the Baa3 rating and the Ba1 rating. The estimated value of $\bar{p}$ is 0.64%. It means that in the model country is low-rated if it is expected to default at a higher rate than 0.64% next period.

6.1 Calibrated parameters

The rest of the parameter values are jointly calibrated by matching moments. A key parameter is the discount rate of the low rating market R_l which governs the degree of market segmentation. I calibrate the segmentation parameter to target the downgrade coefficient in the analysis of Section 4. To be specific, I run the same regression using the model simulation as in Section 4. In the model simulations, the downgrade dummy variable captures downgrades from $r = h$ to $r = l$. I use the same controls for this regression as in the data: output level y, debt-to-output ratio b/y, and the last period spread. I do not control for country-fixed effect and time-fixed effect in the model simulation because it is a single-country model and because there is no exogenous time-varying risk-free rates. [16]

[14] The dataset is not balanced, and some counties, for example Azerbaijan and Namibia, have fewer observations on spreads.

[15] The word idealized may imply a potential discrepancy between the expected default rate by the time the credit rating agency assigns a rating to the entity and the default rate afterwards. The default risk estimate using historical ratings path could be different from the initial design of each rating category.

[16] Including time-fixed effect in the data is to control time-varying risk-free rates and risk premium. Another purpose is to improve on the frequency issue of macro-fundamental variables. The spread movement within a year could potentially result from the fundamental change within a year. This is hard to capture using yearly data. Month-year fixed effect can control this potential fundamental change. In the model, the current

parameter	description	value	moments	target	model
R_l	low rating market	1.047	downgrade coefficient	29.64	29.98
β	country's impatience	0.918	mean b/y	47.1 %	25.5 %
$d0$	default cost	-0.217	mean spread	294 bps	293 bps
$d1$	default cost	0.254	s.d. spread	1.80	1.37

Table 6: Parameters calibrated inside of the model

Following the literature, the country's discount factor β and default cost parameters d_0, d_1 are mapped to the mean debt-to-output ratio, mean spread level, and the standard deviation of the spread. The data moments are calculated as sample averages across countries in the panel data. In the end, I calibrate the segmentation parameter R_l, country's impatience β, and default cost d_0, d_1 jointly to match the four moments: the downgrade coefficient, the mean debt-to-output ratio, mean spread level, and the standard deviation of the spread.

Table 6 show the calibrated values for four parameters and the targeted moments. The model has a limitation in matching the mean debt-to-output ratio. It is because high enough R_l relative to β is necessary for generating the downgrade coefficient, but that R_l discourages the sovereign from accumulating the debt in the model simulation. On the other hand, low enough β relative to R_l weakens the segmentation effect, and the downgrade coefficient is not generated under the model simulation. Including the variance of taste shock in the calibration can potentially improve on debt-to-output ratio matching. [17]

7 Quantitative results

I now describe some features of the Markov equilibrium in the calibrated model. To understand the role of sovereign ratings, I compare the benchmark model to a counterfactual economy with no ratings and no market segmentation. I demonstrate the disciplinary role of ratings.

7.1 The equilibrium bond pricing schedule

Figure 1 depicts the equilibrium bond spread schedule as a function of the debt choice b'. The schedule is evaluated at the mean y level, and the b' level in the horizontal axis is relative

spread perfectly captures the fundamental change between last period and current period.

[17] Following Dvorkin et al. (2021), I employ extreme value shock to tackle computational issues solving a sovereign default model with long-term bond. Taste shock-related parameters are directly taken from the literature for now, but potentially I can include those parameters in the joint calibration. For the literature to include taste shocks in the calibration, see Arce (2021).

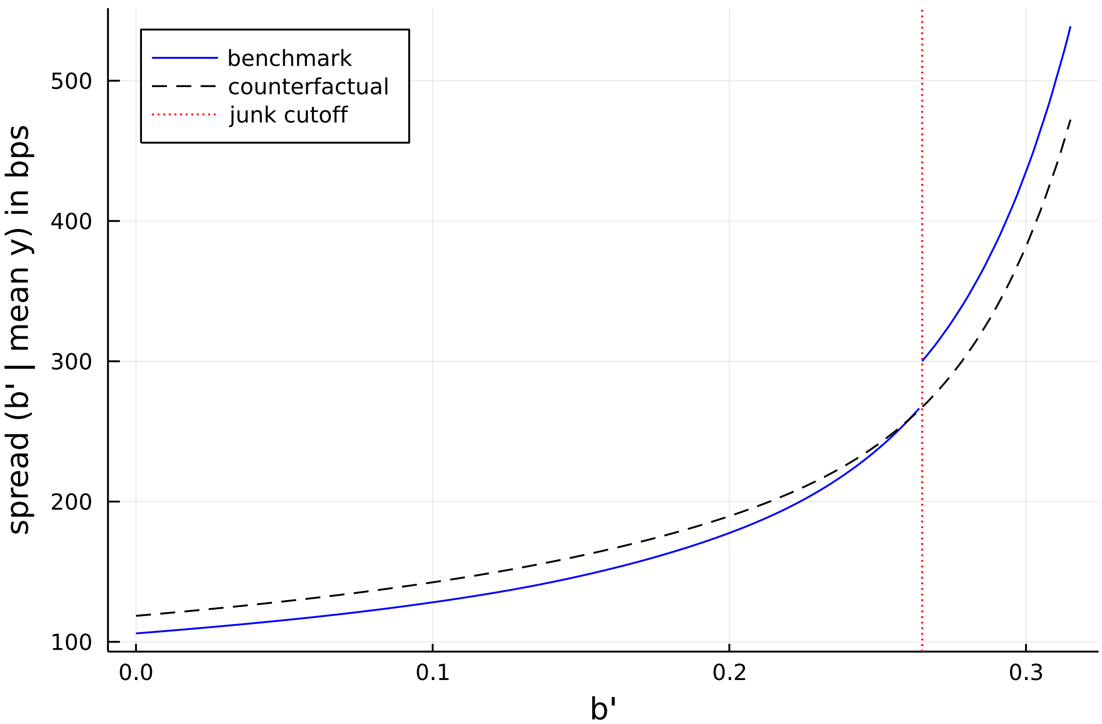

Figure 1: The equilibrium bond pricing schedule

to the mean y level. The junk cutoff indicates that the country's rating is high with a lower b' level than the cutoff, and a higher b' level than the cutoff produces the low rating for the country. As usual, the spread schedule is upward-sloping: the higher b' level increases the equilibrium spread. A high level of debt choice today means a high level of outstanding debt tomorrow, raising the sovereign's incentive to default tomorrow. That higher default risk requires a higher interest rate (or a lower bond price). The upward-sloping pricing schedule is present in both the benchmark and the counterfactual.

Unlike the counterfactual, the first distinctive feature of the benchmark is a discontinuity. This discontinuity arises from the segmentation by credit ratings. As the debt choice b' increases above the cutoff, the country's default probability increases above the rating rule threshold, $\bar{p}$, and the country's rating changes from high to low. As the country's rating changes from high to low, the lender who prices the bond switches from the patient lenders (the lenders in the high-rated bond market) to the impatient lenders (the lenders in the low-rated bond market). This switch means a discrete change in the discount rate of the equilibrium pricing equation. This discrete increase in interest rate is the driving force of the disciplinary effect on sovereign's borrowing. As shown in the counterfactual, the default risk compensation grows smoothly as the default risk increases around the threshold. Therefore, the discrete increase in the spread is not from default risk compensation but from the switch to a different credit market.

Another feature of the pricing schedule is the presence on anticipation effect. Note that the bond in the model is a long-duration bond. Thus, the bond price reflects all the future credit ratings and default probabilities. If the country downgrades to a low rating tomorrow, the market value of its bonds tomorrow will be low. Even though the sovereign's rating is high today and patient lenders price the bond with a low yield, the bond price could be noticeably low if there is a high probability of a downgrade tomorrow. That is, the expectation of future downgrades forces the devaluation of bonds in the current period. This devaluation is visualized when b is lower than the threshold but substantially high enough: the spread increases faster as b' rises closer to the threshold. This more rapid growth in the spread (or faster devaluation of the bonds) is not because the default risk evolves faster under the benchmark [18] but because downgrades in the next period are more likely to be anticipated.

Combining the first and the second features tells us how the discontinuity is determined in the model. The discontinuity is higher as the R_l parameter increases. On the other hand, the greater the anticipation effect is, the lower the discontinuity is. The country's borrowing incentives govern the anticipation effect. In particular, they govern how likely it is that a downgrade will happen next period and how likely it is for the downgrade to revert if that happened. The country's impatience and default cost parameters are relevant for the borrowing incentives. The downgrade coefficient captures this discontinuity under the simulation. To identify R_l in the model, I need to consider the interaction of this discontinuity with other parameters, which provides a rationale for the joint calibration.

The last feature to highlight is the lower spread in the benchmark under a low enough level of b'. As long as b' is low and downgrades are less likely to happen tomorrow, the sovereign can borrow cheaply. This cheap borrowing arises from the disciplined borrowing and lower probability of default, a result that is explained in detail in the following subsections.

7.2 The optimal policy of the sovereign

Figure 2 shows the optimal policy of the sovereign in the benchmark calibration. The borrowing policy is shown as a function of the outstanding debt level b evaluated at the mean y level. The level of the horizontal axis is normalized by the mean y level. The function is weakly increasing in b, which means the sovereign finds it optimal to choose a high b' when it inherits a high level of b. It is because the sovereign rolls over the inherited debt and, conditional on repayment, consumption which the sovereign gains flow utility

[18] The growth rate of default probability with b' around the cutoff under the benchmark is almost equivalent to the one under the counterfactual.

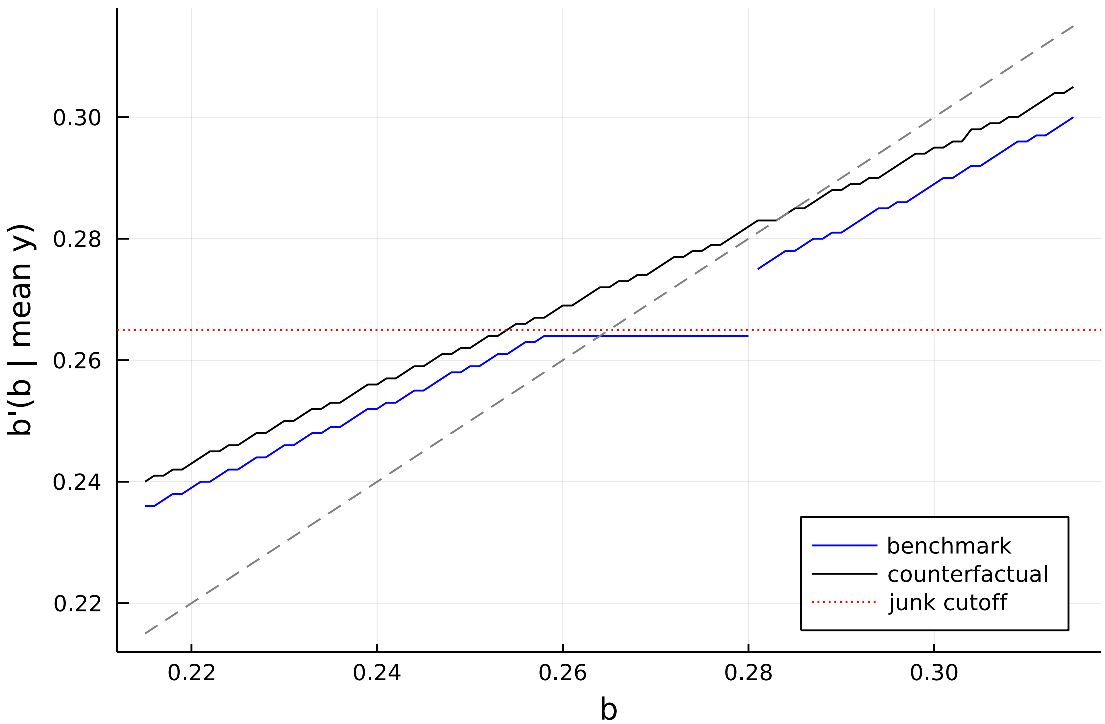

Figure 2: Sovereign's borrowing policy

from reduces with a higher outstanding debt. The junk cutoff denotes the level of debt such that at the given y if the government chooses b' above it, its rating is low. Therefore, the government faces today the impatient lenders of the low-rated bond market.

The distinguishing feature under the benchmark is that the sovereign stops borrowing near the cutoff. To be more precise, the country reduces the debt stock to a level right below the cutoff. That is, the sovereign internalizes the consequence of borrowing over the cutoff. In the simulation, the sovereign finds it optimal to keep its high rating and avoids the high yield from the impatient lenders. However, reducing borrowing means less consumption and less flow utility today. With a high enough outstanding debt level, the country has no choice but to cross the threshold and as a result, its bonds are low rated. But still, the sovereign chooses less debt than it would have chosen in the counterfactual; the country exits the low-rating territory by gradually reducing debt until above, but nearby the threshold.

Not only when the state is near the cutoff, but less borrowing is also present in the entire debt state b, including when the country's rating is high and is far from the cutoff. [19] This less borrowing can be interpreted as a precautionary behavior of risk-averse agents in

[19]The entire debt state refers to a set of debt state $[0, \bar{b}]$ where the default probability reaches to 1 under $\bar{b}$ given y as the mean level. In this region, the optimal borrowing in the benchmark is strictly lower than that in the counterfactual.

	benchmark	counterfactual	% change
mean default risk	1.83 %	2.60 %	- 30 %
junk freq.	37 %	51 %	- 27 %
annual default freq.	1.7 %	2.5 %	- 32 %
mean debt/y	25.5 %	26.9 %	- 5 %
mean spread	293 bps	313 bps	- 6 %
s.d. spread	1.37	1.45	- 6 %

Table 7: Model simulation results

the incomplete market (Aiyagari (1994)). A sufficient negative output shock could lead to downgrades, and the country pays the high yield as a consequence. Due to an incomplete market structure, the government cannot hedge from this output shock and the following rating downgrades. Instead, the risk-averse government chooses to borrow less out of the precautionary motive, even when the state is far away from the cutoff. By doing so, the probability of downgrade, which comes from the sufficient negative output shock, is low. This motive becomes more apparent as the probability of downgrades rises: as the state is closer to the cutoff, the government chooses to borrow much less. Less borrowing in the entire debt region lowers the default probability, and the pricing schedule embraces it. Therefore, the sovereign can borrow at a lower spread under a high rating than it would have had in the counterfactual, as shown in Figure 1.

7.3 Simulation

Table 7 compares the model simulation result to that of the counterfactual economy. To make this comparison, I feed the same time series of stochastic variables, the output series and the re-entry shock upon default. After discarding the first 100 periods of each simulation, I imposed the same initial debt level of the benchmark simulation on the counterfactual simulation. Each simulation is over 1,000 periods, and the listed moments are calculated as the sample average over 100 simulations. Percentage change denotes the percentage change of the moment under the benchmark relative to the counterfactual. The periods when the country is in default are not included in calculating the moments.

As documented before, a lower mean debt level relative to output in the benchmark is expected. Regarding the average spread, disciplined borrowing lowers the incentive to default at a given output level and could result in a lower spread, especially when the country's rating is high. On the other hand, the presence of impatient lenders could contribute to a higher spread on average when low ratings happen frequently. Under the calibration simulation, the country's rating is high as often as 63%, and the average spread of the benchmark economy is lower than without ratings.

Another finding is that the country's default risk is lower with the segmentation by ratings, and fewer defaults happen on the equilibrium path. This result is consistent with a lower spread volatility and mean spread. Spread surges are observed with high default risk, which contributes to high spread volatility. Frequent downgrades could also contribute to higher spread volatility, as the spread surges discretely with the switching to impatient lenders. Despite the small magnitude of the reduction in spread volatility compared to the significant reduction in default risk, the first force quantitatively dominates the second force, resulting in lower spread volatility under the benchmark.

The fact that the reduction in default risk is relatively significant compared to the decrease in mean debt level highlights the crucial role played by the segmentation by ratings. This segmentation restricts the country from remaining in a positive default risk state, thereby preventing the default risk from evolving further. This is supported by the optimal borrowing policy, which reveals that the debt reduction is most pronounced near the cutoff or low-rating territory, where the default risk is strictly positive, and the economy is vulnerable to adverse output shocks.

7.4 Debt dynamics in recession

Figure 3 illustrates that the segmentation keeps the debt stock low, which prevents default from happening. To assess the impact of the segmentation, I simulate an output process that reflects a recession in both the benchmark and counterfactual economies. The process begins with a high level of output, gradually declines, and experience a significant drop at time $t = 6$. The two economies start with the same initial debt level. Given this output process and initial debt level, I plot the equilibrium debt level, default probability, and spreads over time. The unit of time is a year.

In the counterfactual economy, the sovereign defaults at $t = 6$, whereas in the benchmark economy, it manages to survive the recession without defaulting. The likelihood of default increases as the sovereign holds a higher level of debt and as the output level drops. During a recession, defaults could happen in equilibrium when the sovereign is sufficiently indebted.At $t = 6$, when the output level drops significantly, the counterfactual economy had a debt-to-GDP ratio of up to 29%, making repayment costly for the government. In contrast, the benchmark economy had a debt-to-GDP ratio of only 26%, which is attainable because the government starts reducing the debt stock from $t = 2$. As the output level falls from $t = 2$, the default probability increases, bringing the economy closer to the junk cutoff.

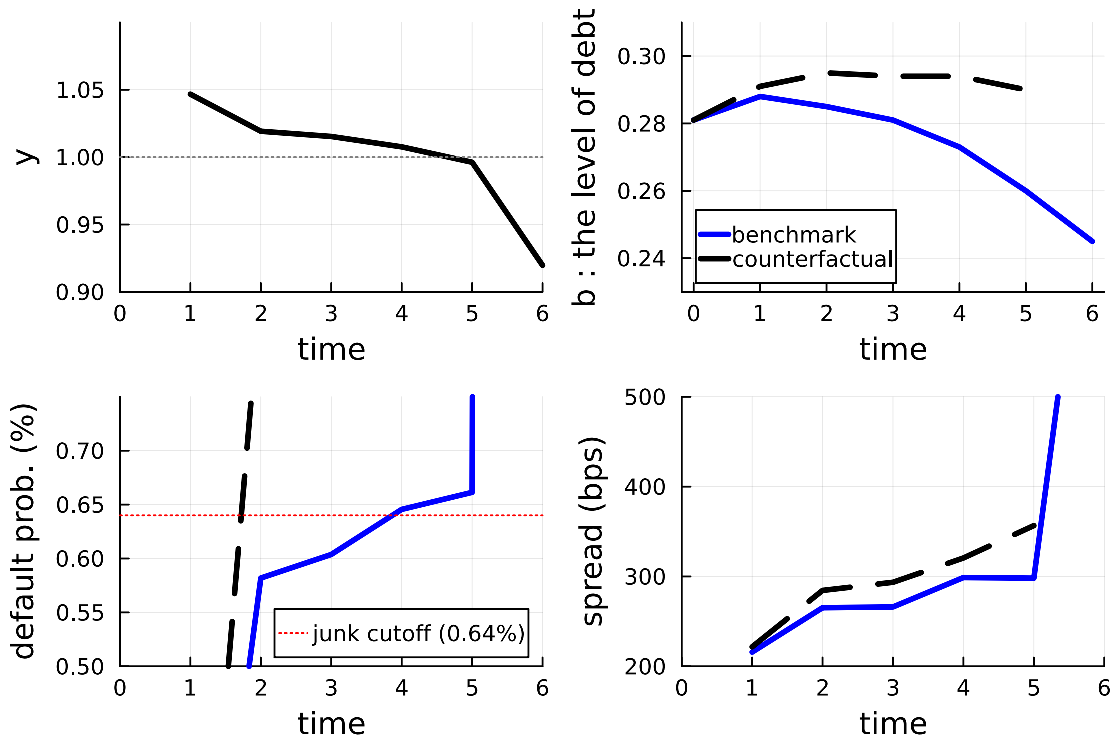

Figure 3: Debt, default probability, and the spreads dynamics during recession

By $t = 4$, the country is rated as junk in the benchmark. The segmentation encourages the government to reduce its debt stock, keeping the default risk relatively low and the economy less vulnerable to sudden output drops. At $t = 5$, the default probability under the benchmark is approximately 0.66%, while under the counterfactual, it is as high as 3%.

The right bottom panel of Figure 3 displays the bond spreads of the two economies. In general, the spreads in the counterfactual economy are higher than those in the benchmark economy. Even though the discount rate in the counterfactual economy is equal to or lower than that in the benchmark economy, the sufficiently high default probability in the counterfactual offsets the high discount rate of the junk-rated bond market in the benchmark. One interesting observation is that due to the anticipation effect, the movement of spreads upon downgrade is relatively smooth. From $t = 3$ to $t = 4$, the government is downgraded to junk. Although the bond at $t = 4$ is discounted more by 200 bps, the spreads increase by approximately 30 bps. This is because the spreads at $t = 3$ already include the possibility of a downgrade next period and incorporate the difference in the discount rate before the downgrade.

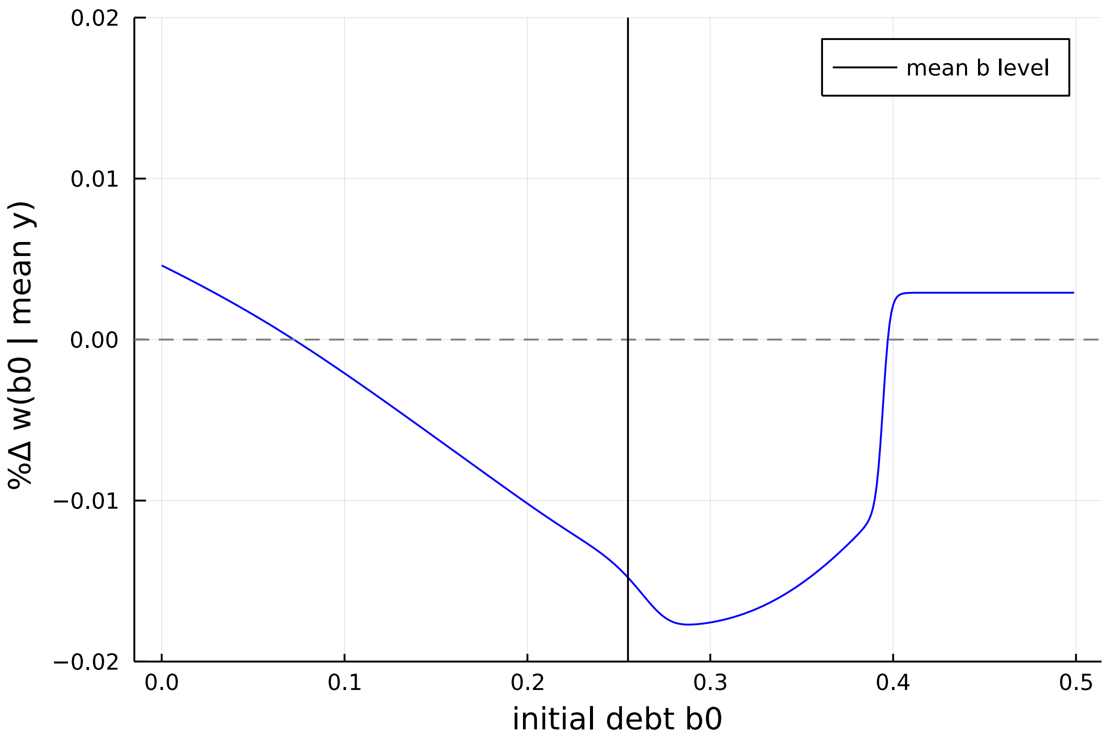

Figure 4: The welfare under the segmentation

7.5 The welfare implications of ratings

In this section, I discuss how the ratings and resulting segmentation affects the sovereign's welfare. Figure 4 shows the percentage change in welfare under the benchmark compared to the counterfactual, the economy without segmentation. The welfare is in consumption equivalent units. I can define welfare in a following way:

$$1 + \Lambda(b, s) = \frac{V_{NS}(b, s)}{V_S(b, s)}^{\frac{1}{1-\sigma}}$$

where $V_{NS}(b, s)$ is the value of the sovereign at state (b, s) in benchmark economy with segmentation, and $V_S(b, s)$ is the value of sovereign at state (b, s) in counterfactual economy without segmentation. A positive number means the segmentation generates welfare gain for the sovereign. I plot the welfare for different initial debt levels b_0 and for a given mean y level.

The difference in welfare is a decreasing function of debt for low levels of the initial debt. It is because the welfare benefit from the segmentation decreases with the high debt level, whereas the welfare cost rises with the high debt level. Those opposite forces help to understand the welfare implications of segmentation. The disciplining behavior of ratings lowers the country's default risk and allows the government to borrow at a better price, which is a potential source of welfare benefit. This benefit is maximized when the debt state is far from the junk cutoff at a given y level. As the debt state is closer to the cutoff, the anticipated bond devaluation counteracts the disciplinary effect, and the welfare benefit is reduced. Although the sovereign currently maintains a high rating, there is a decent chance that the country may be downgraded to junk next period when the debt state is close enough to the cutoff.

The primary source of welfare cost is low consumption from reducing debt issuance. In this model, the country's welfare is derived from the flow utility from consumption. As the debt state is closer to the cutoff at a given y level, the government actively shrinks the bond issuance, and the welfare cost from low consumption amplifies. Not only is reducing debt stock painful to the country, but the high yield from the impatient lenders is also a crucial source of welfare cost. Adverse output shocks trigger downgrades to low ratings, and the sovereign pays high yields to impatient lenders in addition to a high default risk compensation. This is not ideal from the perspective of risk sharing: the sovereign may want to hedge from low-income shock.

Figure 4 shows how each opposite force aggregates quantitatively. First, the segmentation delivers a welfare loss to the sovereign in the stationary debt level. The percentage change in welfare under the mean debt level is negative under the calibration. The calibrated sovereign is so impatient that it accumulates debt sufficiently close to the cutoff. The anticipated devaluation counterbalances the welfare benefit from the segmentation, at the same time the country suffers from controlled borrowing as it is near the cutoff. Moreover, the government is exposed to a decent chance of downgrades next period (coming from low-income shocks). In the end, the welfare cost quantitatively dominates the welfare benefit, and the welfare loss, in the long run, is as much as around -0.015% under the calibration.

However, the country gains welfare from the segmentation at low debt level. As long as the country is far from the cutoff, downgrades rarely occur in the near term, and the government does not need to reduce the debt issuance aggressively. The benefit of borrowing at a better price is maximized without anticipated depreciation. Quantitatively, the country's welfare is enhanced by the segmentation by roughly 0.005%. Also, the welfare gain happens under the default region. The debt region over the level of 0.4 visually shows it. In this model, the country in default status returns to the credit market with zero debt, where the welfare benefit is maximized.

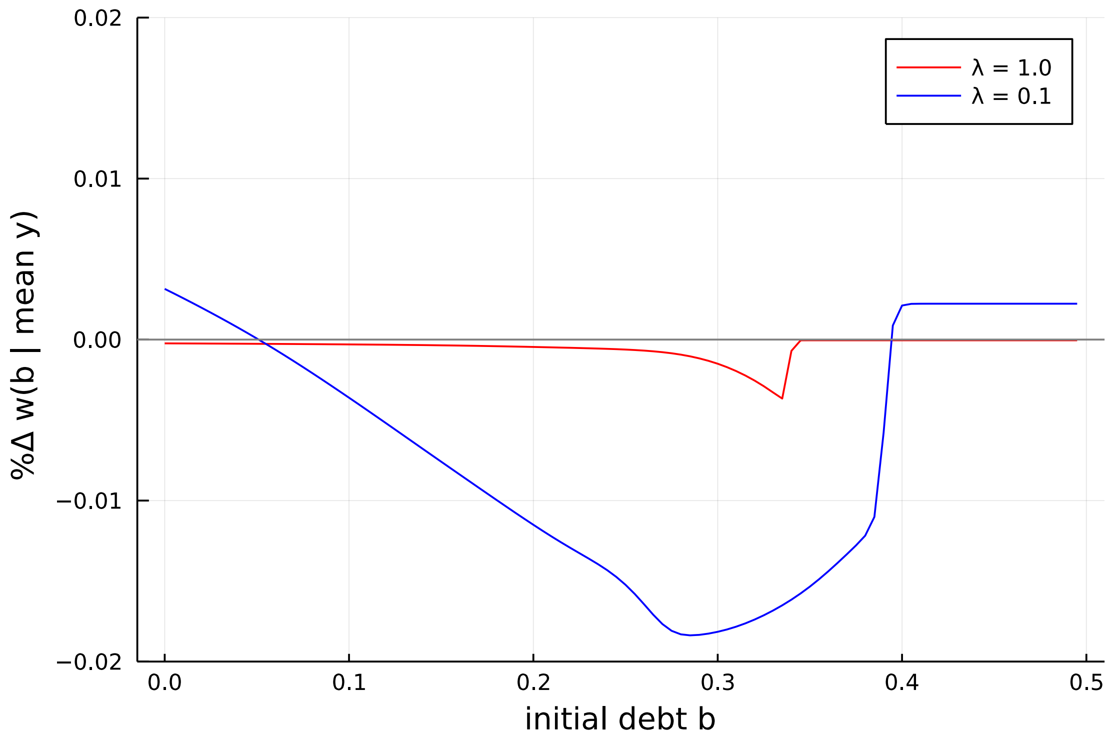

Figure 5: The welfare under the segmentation with different bond maturity

7.6 The role of bond maturity

Here, I study how the welfare results change with different bond maturity. Figure 5 outlines the sovereign's welfare as in Figure 4, but I vary bond maturity parameter, λ. The blue line is the benchmark where it captures 10-years maturity whereas the red line is with $\lambda = 1.0$, which is 1-year maturity. Under one-period bond, the debt service for debt stock b is $(\lambda + (1-\lambda)\kappa)b = b$. It means that the total principle of outstanding debt should be paid in order to raise new bond revenue. This is different from the benchmark where the sovereign can raise new revenue as long as at least 10% of outstanding debt is repaid. I keep all other parameters, other than κ, the same in the counterfactual. [20]

With $\lambda = 1.0$, the sovereign's welfare under the segmentation is negative across all debt levels, suggesting that the sovereign is always worse off with the segmentation under this calibration. The main friction under one-period debt is that sovereigns cannot commit, at the time the debt is issued, on their next period default decision. Adding the segmentation cannot resolve commitment issue on not defaulting next period under Eaton and Gersovitz (1981) timing. Given the current debt level, the probability of default in the next period

[20] In the benchmark, coupon parameter κ is normalized so that the bond price with zero risk is equal to 1. Under one-period bond case, every bond matures next period, and there is no notion for coupon. Under one-period bond case, I set $\kappa = 0$

is contingent on the exogenously drawn output level, which follows a stochastic process. As the debt level increases, the sovereign's credit rating approaches the junk cutoff, and the associated welfare cost rises. At sufficiently high levels of debt, the sovereign chooses to default. In this case, the sovereign's value is independent of the debt level, and this is represented as a horizontal line for debt-to-GDP ratios higher than around 32%.

The finding that the sovereign can benefit from segmentation under long-term debts suggests that segmentation can partially address the issue of lack of commitment regarding future borrowing. The segmentation works as a commitment device by influencing market prices for future borrowing. When new debt is issued, lenders anticipate that the future sovereign is limited to borrow more, especially borrow beyond the junk cutoff. The future value of outstanding debt will not be diluted as much, and the lenders are willing to pay a higher price for current debt. This partially resolves the welfare-reducing debt-dilution problem, and the sovereign can benefit under sufficiently low levels of debt. This result is consistent with the existing literature. As an alternative solution to the debt-dilution problem, Chatterjee and Eyigungor (2012) suggests fixing the future value of outstanding debt at its value at issue. The segmentation can be viewed as a lower bound of the future value of outstanding debt.

7.7 Different segmentation rules and welfare implications

In this exercise, I vary the rating rule parameter, $\bar{p}$, and analyze how it affects the sovereign's welfare. The benchmark rule segments the credit market between BBB-/Baa3 and BB+/Ba1, which distinguishes between investment-grade and junk. Figure 6 shows the percentage change in welfare with different segmentation rules normalized by welfare under the benchmark. I map the $\bar{p}$ values to the rating level according to Moody's rating rule, which I use in calibration. The assumption is that each segmented market has the same discount rate as in the benchmark. The welfare change is evaluated under the mean output level y with two different initial debt levels: starting the economy with zero or the mean debt level in the benchmark.

The result suggests that the current rule is reasonable, whereas the optimal segmentation rule is a bit looser than that. Under the calibration, between BB-/Ba3 and B+/B1 is the optimal rule, and the country is better off even at the mean debt level. The sovereign is significantly impatient in the calibration, and looser rule can reduce the welfare cost. However, if the rule is loosened too much, it reduces welfare because it weakens disciplinary motives. On the other hand, tight segmentation rules aggravate the country's welfare.

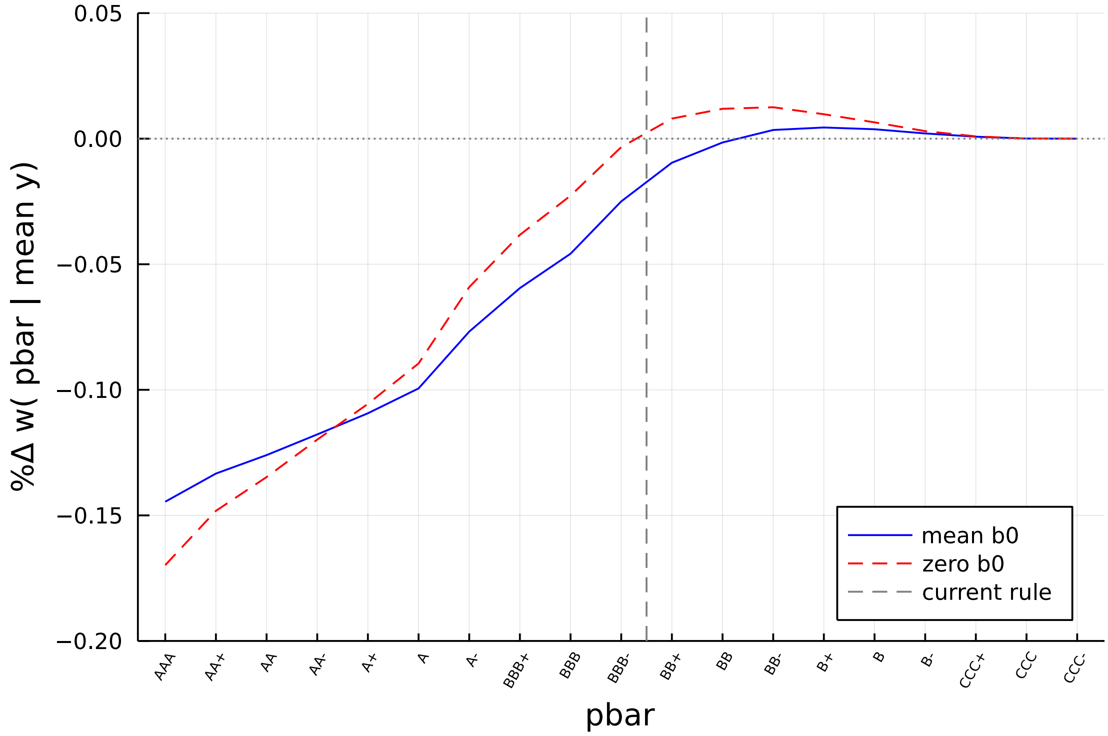

Figure 6: Change in welfare under the counterfactual segmentation

8 Conclusion

From the regulatory usage of sovereign credit ratings, this paper highlights the disciplinary role of ratings on governments' overborrowing. I build a model of a country's borrowing, default, credit rating, and the consequent credit market segmentation. I incorporate the endogenous response of the government. This delivers a different implication of sovereign ratings in the international credit market, which the previous literature has neglected. I calibrate the segmentation parameter using the spread response to countries' downgrade to junk observed in the data. The consequence of downgrades to junk gives sovereigns incentives to manage their credit ratings and discourages them from borrowing over the threshold. Under the calibration, downgrades to junk imply that the impatient lenders of junk bond markets charge a high yield to the sovereign. This consequence of downgrades and the country's ability to manage its ratings are the driving forces of disciplined borrowing behavior.

This disciplined behavior lowers the country's default risk and allows it to borrow at a better price, which is a potential source of welfare benefit. The welfare cost of ratings and implied segmentation is from the anticipated devaluation of bonds before downgrades and controlled borrowing when the country is near the threshold. Under the calibration, the cost dominates the benefit, and the segmentation by sovereign ratings results in welfare loss to the impatient country in the long run. However, the government gains from the segmentation during the transition, especially when the debt stock is low.

This welfare analysis suggests the importance of moderate segmentation in the international capital market. The finding suggests a rationale for the current segmentation rule. The paper proposes that loose segmentation policies weaken disciplinary motives and deliver negative welfare implications to developing countries. Imposing an adequate punishment as a form of market segmentation could alleviate commitment issues and give developing countries better credit access.

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Appendices

A Additional tables for empirical evidence

Table A1: additional table for the regression (1)

	(1)	(2)	(3)	(4)	(5)
	spread	spread	spread	spread	spread
DowntoJunk	141.4*	33.58*	37.72**	37.42**	29.64***
	(78.73)	(18.02)	(16.85)	(16.90)	(11.47)
lag_spread		0.973***	0.976***	0.973***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.879***	1.132***	-0.312
			(0.325)	(0.379)	(0.308)
grossdebt			0.0627	0.124	0.146**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.001	0.948	0.949	0.949	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note:* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is montly frequency across 12 countries.

Table A2: additional table for the regression (2)

	(1) spread	(2) spread	(3) spread	(4) spread	(5) spread
withininvst	174.7***	18.71	19.57	20.00	17.10*
	(65.84)	(15.07)	(14.06)	(14.10)	(9.572)
DowntoJunk	142.2*	33.84*	38.46**	38.12**	30.14***
	(78.65)	(17.99)	(16.82)	(16.86)	(11.46)
withinjunk	26.52	47.11***	50.75***	51.30***	18.63*
	(69.39)	(15.87)	(14.82)	(14.85)	(10.09)
lag_spread		0.973***	0.976***	0.974***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.977***	1.247***	-0.249
			(0.325)	(0.379)	(0.309)
grossdebt			0.0583	0.120	0.141**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.004	0.948	0.950	0.950	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note:* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is montly frequency across 12 countries.

Table A3: regression result with a different specification

	(1) spread	(2) spread	(3) spread	(4) spread	(5) spread
DowntoBBB	29.91	74.07***	77.26***	78.10***	17.74
	(104.0)	(23.78)	(22.15)	(22.20)	(15.13)
DowntoBBB-	23.81	25.55	29.52	29.85	19.38
	(93.06)	(21.27)	(19.85)	(19.89)	(13.52)
DowntoJunk	142.2*	33.83*	38.43**	38.07**	30.26***
	(78.68)	(17.99)	(16.81)	(16.85)	(11.47)
DowntoBB	174.9**	24.40	25.86	26.47	24.20**
	(78.68)	(18.00)	(16.78)	(16.82)	(11.43)
DowntoBB-	174.1	5.412	4.734	4.642	0.513
	(120.1)	(27.46)	(25.57)	(25.64)	(17.45)
lag_spread		0.973***	0.976***	0.974***	0.969***
		(0.004)	(0.005)	(0.005)	(0.005)
gdp			0.974***	1.240***	-0.245
			(0.325)	(0.379)	(0.309)
grossdebt			0.0605	0.123	0.142**
			(0.052)	(0.079)	(0.057)
Observations	2610	2597	2528	2528	2528
R^2	0.004	0.948	0.950	0.950	0.981
Country FE	N	N	N	Y	Y
Time FE	N	N	N	N	Y

Note:* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. In all specification, the dependent variable is EMBI+ sovereign bond spread. DowntoJunk is a dummy variable which equals to 1 when a country's S&P sovereign rating is above or equal to BBB- in period $t-1$, and below or equal to BB+ in period t. DowntoBBB is a dummy variable which equals to 1 when a country's rating is BBB+ at period $t-1$ and BBB at period t. Other dummy variables, DowntoBBB-, DowntoBB, and DowntoBB-, are constructed similarly (BBB in period $t-1$ and BBB- in period t, BB+ in period $t-1$ and BB in period t, and BB in period $t-1$ and BB- in period t, respectively). Lag spread is a 1-month lag variable. gdp is real gdp growth rate, and grossdebt is public debt to gdp ratio. The data is montly frequency across 12 countries.

B Numerical Algorithm

The algorithm iterates on value functions, $V(b,s), V^D(b,s)$ and price function, $q(b,s)$, until convergence. Using Tauchen's method, I discretize stochastic output process, y, as a Markov chain. I also discretize endogenous state variable, b into finite grid. I use 151-grid points for y, and 301-grid points for b.

The challenge of computing long-term debts has been documented in the literature. Following Dvorkin et al. (2021), I use Extreme Value shock to resolve this issue. I assume there is taste shock, additive utility shocks, associated with each possible debt level choice and default choice. For parameters governing the distribution of extreme value shock, I take the value from Dvorkin et al. (2021).

First, I set initial guesses for $V(b,s), V^D(b,s)$, and $q(b,s)$. I update $V^D(b,s)$ using initial guess $V(b,s)$ and $V^D(b,s)$. Then, for each (b,s), I use discrete-search method to calculate the optimal debt. That is, given initial guess $V(b,s)$ and $q(b,s)$, I look for a b that gives the highest value under repayment than any other b in the grid can give. By doing so, I can define debt policy function, $B(b,s)$. Given initial $V(b,s), q(b,s)$ and defined $B(b,s)$, I can define value under repayment, $V^R(b,s)$. With updated $V^D(b,s)$ and $V^R(b,s)$, I can update $V(b,s)$ and define default policy function, $D(b,s)$. Given $D(b,s)$, I can calculate next period default probability for each state (b,y). Given a rating parameter, I can define rating function, $R(b,s)$ following the rating rule as specified. Given $R(b,s)$, $D(b,s)$, and the initial $q(b,s)$, I can update price function. Rating function determines the discount rate, and initial price function is used for calculating the future value of bonds.

I check if the updated $V(b,s), V^D(b,s), q(b,s)$ are close enough to the initial $V(b,s), V^D(b,s)$, $q(b,s)$. If it close enough within a tolerance level, I stop the iteration. Otherwise, I use the updated $V(b,s), V^D(b,s), q(b,s)$ as initial guesses and re-do the iteration.