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PRACTICAL NO. 01 & 2

Aim : Solving problems on strings, Sets And Binomial Coefficient.

Q1. Suppose that bank allows user to set user id for website with condition. the first, second character must be uppercase Alphabet in English alphabet. third letter may be lower or upper case alphabet or decimal digit. fourth place must be @, fifth is digit or (*) star, %, # and sixth must be a digit, how many different user id can user set.

Answer: let consider.

- 1^{st} character must be uppercase Alphabet, i.e
  1^{st} place = { upper case } = {A....Z} = 26
- 2^{nd} character must be uppercase Alphabet i.e
  2^{nd} place = {uppercase} = {A.....Z} = 26
- 3^{rd} character maybe lower or uppercase Alphabet or decimal digit. i.e
  3^{rd} place = { lower}{upper}{ decimals }
  = {a-z} {A...Z} { 0...9}
  = 26 + 26 + 10
  = 62
- fourth 4^{th} character must be a @.
  4^{th} place = { @ } = 1
- fifth character is a digit, star (*), %. #
  5^{th} place = {0-9} {*} {%} {#} = 10+1+1+1 = 13

sixth character must be a digit.

6^{th} place = {0 .... 9} = 10

The different User Id can user set are

1		2		3		4		5		6
↓		↓		↓		↓		↓		↓
26	X	26	X	62	X	1	X	13	X	10

∴ the Total Number of different User id can user settings are $\underline{26 \times 26 \times 62 \times 1 \times 13 \times 10}$.

Q 2. In a state of maharashtra license Number consist of 2 digits followed by a space and followed by 2 capital letters. the first digit cannot be a 0. How many license numbers are possible?

<u>Answer</u>- According to condition, license number consist of 2 digits followed by space followed by 2 capital letters.

Here, first digit cannot be 0. i.e digit consider from 1 to 9 and exclude 0. {1 ... 9} = 9 & second Should be {0 ... 9} = 10

And it followed by space {space} = 1 And followed by 2 capital letters, i.e 2 time [A ... Z] (26 characters of Alphabet)

X = {A, B, ... Z} = 26

Y = {A, B, ... Z} = 26

∴ Number of difference license number is string from

9 X 10 X 1 X 26 X 26.

Q 3. Suppose that a website allows users to set password with condition. the first character must be a lower case letter in the English alphabet. second and third letter may be upper or lower case alphabet or decimal digit (0-9) fourth place must be a @. fifth & sixth are lower case English letter, *, % and # and seventh place must be a digit. How many different password can be user set?

Answer:

let consider, A string of length 7.

$\therefore$ 1st place must be a lower case. i.e $\{a \dots z\} = 26$

$\therefore$ 2nd place must be a lower, upper cases and decimal digits.
i.e $\{a..z\}\{A \dots Z\}\{0 \dots 9\} = 26 + 26 + 10 = 62$

$\therefore$ 3rd place must be a lower, upper cases & decimal digits i.e $\{a..z\}$ $\{A..Z\}\{0..9\} = 26 + 26 + 10 = 62.$

$\therefore$ 4th place must be a @ i.e $\{@\} = 1$

$\therefore$ 5th place must be a lower cases, *, #, % i.e
$\{a \dots z\}\{*\}\{\#\}\{\%\} = 26+1+1+1 = 29$

$\therefore$ 6th place must be a same as fifth case i.e lower, *, #, % = 29 and 7th place must be a decimal digit i.e $\{0 \dots 9\} = 10$.

$\therefore$ The different password can be user set

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

$$26 \times 62 \times 62 \times 1 \times 29 \times 29 \times 10$$

Total Number of different password can be user sets are $26 \times 62 \times 62 \times 1 \times 29 \times 29 \times 10$

Q4. find the coefficient of x^9y^3 in $(x-7y)^{12}$

Answer:

let consider x^9y^3 in $(x-7y)^{12}$

To find the coefficient of x^9y^3 put $i=3$

By using Binomial theorem,

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i$$

$$\therefore (x-7y)^{12} = \sum_{i=0}^{12} \binom{12}{3} (x)^9 (-7y)^3 (y^3)$$

$\therefore$ Here the coefficient of x^9y^3 is $12C_3 \cdot x^9 \cdot (-7)^3 =$
$12C_3 (-7)^3$

Q5 find the coefficient of y^2x^7 in $(4x+7y)^9$

Answer:

let consider y^2x^7 in $(4x+7y)^9$

To find the coefficient of y^2x^7, put $i=7$

By using Binomial theorem.

$$(4x+7y)^9 = \sum_{i=0}^{9} \binom{9}{7} (4x)^2 (7y)^7$$

$$= \binom{9}{7} (4x)^2 (7)^7 (y)^7$$

$$= \binom{9}{7} (4)^2 (7)^7$$

$\therefore$ $9C_7 (4)^2 (7)^7$ is the coefficient of y^2x^7 in $(4x+7y)^9$..

Q 6. find the coefficient of x^5y^7 in $(3x-2y)^{12}$

Answer. let consider x^5y^7 in $(3x-2y)^{12}$ .... (Given)
To find the coefficient of x^5y^7, put $i=7$
By using Binomial theorem,

$$(3x-2y)^{12} = \sum_{i=0}^{12} \binom{12}{7} (3x)^5 (-2y)^7$$

$$= \binom{12}{7} (3)^5 (-2)^7$$

$^{12}C_7 (3)^5 (-2)^7$ is the coefficient of x^5y^7 in $(3x-2y)^{17}$.

Practical No. 03.

Aim : Solving problem Using Induction.

Q prove that sum of first n natural number $= \frac{n(n+1)}{2}$

→ let $P(n) = 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

To prove that,

Step I - for $n=1$

$LHS = n = 1$

$RHS = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$LHS = RHS$

$\therefore$ statement is true for $n=1$

Step II - Assume that statement is true for $n=k$, $k \in N$.

$\therefore 1 + 2 + 3 + \ldots\ldots + k = \frac{k(k+1)}{2}$ .... (1)

Step III - for $n = k+1$

$LHS = 1 + 2 + 3 + \ldots\ldots + k + 1$

$= 1 + 2 + 3 + \ldots\ldots + k + k+1$ .... from step II

$= \frac{k(k+1)}{2} + k+1$

$= \frac{k(k+1) + 2(k+1)}{2}$

$= \frac{k^2 + k + 2k + 2}{2} = \frac{k^2 + 3k + 2}{2}$

$$RHS = \frac{k+1(k+1+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$= \frac{k^2+2k+k+2}{2} = \frac{k^2+3k+2}{2}$$

$\therefore$ LHS = RHS.

It is true for $n=k+1$. By 1st principle of mathematical Induction, it is true for all values of n.

2} prove that $1^2+3^2+5^2+\ldots\ldots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

$\rightarrow$ By using first mathematical Induction,

Assume, $P(n) = 1^2+3^2+5^2+\ldots\ldots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

To prove that,

Step I - for $n=1$

$LHS = (2n-1)^2 = (2(1)-1)^2 = 1$

$RHS = \frac{n(2n-1)(2n+1)}{3} = \frac{1(2(1)-1)(2(1)+1)}{3} = \frac{1(1)(3)}{3} = 1$

Statement is true for $n=1$

Step II - Assume $n=k$, for statement is true

$$1^2+3^2+5^2+\ldots+(2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

Step III - Assume that $n=k+1$,

$$LHS = [1^2+3^2+5^2+\ldots\ldots+(2k-1)^2] + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)+3(2k+1)^2}{3}$$

$$= \frac{(2k^2-k)(2k+1)+3(2k+1)^2}{3}$$

$$= \frac{4k^3 + \cancel{2k^2} - \cancel{2k^2} - k + 3(4k^2 + 4k + 1)}{3}$$

$$= \frac{4k^3 - k + 12k^2 + 12k + 3}{3}$$

$$= \frac{4k^3 + 12k^2 + 11k + 3}{3}$$

$$LHS = \frac{4k^3 + 12k^2 + 11k + 3}{3}$$

$$RHS = \frac{k+1\,(2(k+1)-1)\,(2(k+1)+1)}{3}$$

$$= \frac{k+1\,(2k+2-1)\,(2k+2+1)}{3}$$

$$= \frac{(k+1)\,(2k+1)(2k+3)}{3}$$

$$= 2k^2 + k + 2k + 1\,(2k+3)$$

$$= \frac{2k(2k^2 + k + 2k + 1) + 3(2k^2 + k + 2k + 1)}{3}$$

$$= \frac{4k^3 + 2k^2 + 4k^2 + 2k + 6k^2 + 3k + 6k + 3}{3}$$

$$= \frac{4k^3 + 6k^2 + 6k^2 + 11k + 3}{3}$$

$$= \frac{4k^3 + 12k^2 + 11k + 3}{3}$$

LHS = RHS

$\therefore$ it is true for $n = k+1$, by 1st principle of mathemathical Induction true for all n.

Q3 prove that by mathematical induction $2+5+8+\ldots+3n-1 = \frac{n(3n+1)}{2}$

Answer : let $P(n) = 2+5+8+\ldots\ldots+3n-1 = \frac{n(3n+1)}{2}$.

Step I - $n=1$
$LHS = 3(1)-1 = 2$
$RHS = \frac{n(3n+1)}{2} = \frac{1(3(1)+1)}{2} = \frac{4}{2} = 2$
LHS = RHS
$n=1$, statements are true.

Step II - Assume that $n=k$, true for all statements.
$n=k$.
$LHS = 3k-1$
$RHS = \frac{k(3k+1)}{2}$

Step III - Assume $n=k+1$

LHS =
$= 2+5+8+\ldots+3(k+1)-1$
$= \underbrace{2+5+8+\ldots\ldots+3(k-1)}_{\text{from II (k)}}+3(k+2)$
$= \frac{k(3k+1)}{2} + 3(k+2)$
$= \frac{k(3k+1)+2(3k+2)}{2}$
$= \frac{3k^2+k+6k+4}{2}$
$= LHS = \frac{3k^2+7k+4}{2}$

RHS =
$= \frac{k+1\,(3(k+1)+1)}{2}$
$= \frac{k+1\,(3(k+1)+1)}{2}$
$= \frac{k+1\,(3k+4)}{2}$
$= \frac{3k^2+3k+4k+4}{2}$
$= \frac{3k^2+7k+4}{2}$
$= RHS$

LHS = RHS

it is true for $n=k+1$, By mathematical Induction first principle all values are true for n.

21/7/23

Q. prove $\forall n \in N$, $n(n-1)(2n-1)$ is divisible by 6.

→ • Step I - for $n = 1$

$n(n-1)(2n-1)$
$1(1-1)(2(1)-1)$
$1(0)(2-1)$
$1(0)(1) = 0$

0 is divisible by 6

• Step II - Assume that statement is true for $n=k$

i.e $k(k-1)(2k-1)$ is divisible by 6

Assume, $k(k-1)(2k-1)$ is divisible by 6a

$k(k-1)(2k-1) = 6a$
$(2k-1)(k^2-k) = 6a$
$2k^3 - k - 2k^2 + k = 6a$
$2k^3 - 3k^2 + k = 6a$
$2k^3 = 6a + 3k^2 - k$ — 1

• Step3 -

Statement is true for $n = k+1$
$k+1(k+1-1)(2(k+1)-1)$ is divisible by 6

LHS = $k+1(k)(2k+2-1)$
$(k+1)(k)(2k+1)$
$(k^2+k)(2k+1)$
$2k^3 + k^2 + 2k^2 + k$
$6a + 3k^2 - \cancel{k} + k^2 + 2k^2 + \cancel{k}$

$6a + 3k^2 + k^2 + 2k^2$

$6a + 6k^2$

$6(a + k^2)$

Statement is true for $n = k+1$

By principle of mathematical induction, $n(n-1)(2n-1)$ is divisible by 6.

Q. $n^4 - 4n^2$ is divisible by 3, $n \geq 2$, prove that by mathematical induction.

→ Step I, for $n = 2$

$n^4 - 4n^2$

$(2)^4 - 4(2)^2$

$16 - 4(4)$

$16 - 16 = 0$

$\therefore$ 0 is divisible by 3.

Step II -

for $n = k$, Assume that $k^4 - 4k^2$ is divisible by 3

Assume, $k^4 - 4k^2$ is divisible by $3a$.

$k^4 - 4k^2 = 3a$

$k^4 = 3a + 4k^2$

Step III -

for $n = k+1$

$(k+1)^4 - 4(k+1)^2$ is divisible by 3.

LHS $= (k+1)^4 - 4(k+1)^2$

$= k^4 - 4k^3 + 6k^2 + 4k + 1 - 4(k^2 + 2k + 1)$

$= k^4 - 4k^3 + 6k^2 + 4k + 1 - 4k^2 - 8k - 4$

$= k^4 - 4k^3 + 2k^2 + 4k + 1 - 8k - 4$

$= k^4 - 4k^3 + 2k^2 - 4k + 1 - 4$

$= k^4 + 4k^3 + 2k^2 - 4k + 1 - 4$

$= k^4 - 4k^3 + 2k^2 - 4k - 3$.

$3a + 4k^2 - 4k^3 + 2k^2 - 4k - 3$

$3a + 4k^3 + 6k^2 - 4k - 3$

$3a + 6k^2 - 4k - 3 + 4k^3$

$= 3(a + 2k^2 - 1) + 4k^3 - 4k$

$= 3(a + 2k^2 - 1) + 4k(k^2 - 1)$

$= 3(a + 2k^2 - 1) + 4(k-1)k(k+1)$

Any 3 consequtive numbers are divisible by 3

$(k-1)k(k+1) = 3b$

$3(a + 2k^2 - 1) + 4 = 3b$

$= 3(a + 2k^2 - 1 + 4b)$ is divisible by 3

$\therefore$ statement is true for given $n = k+1$

By ~~Seco~~ first principle of mathematical Induction $n^4 - 4n^2$ is divisible by for $n \geq 2$ is proved.

Q $5^n - 4n - 1$ divisible by 16. $\forall n \geq 1$

→ Step I -
for $n = 1$

$5^1 - 4(1) - 1$

$5 - 4 - 1 = 0$ is divisible by 16.

Step II - for $n = k$, Assume that

$5^k - 4k - 1$ is divisible by 16.

Assume $5^k - 4k - 1$ is divisible by $16b$.

$5^k - 4k - 1 = 16b$

$5^k = 16b + 4k + 1$ — (1)

Step III -
Assume, $n = k+1$

$5^{k+1} - 4(k+1) - 1$

$5^{k+1} - 4k - 4 - 1$

$5^{k+1} - 4k - 5$

$5^k . 5^1 - 4k - 5$

$(16b + 4k + 1) . 5 - 4k - 5$

$80b + 20k + 5 - 4k - 5$

$80b + 20k - 4k$

$80b + 16k$

$\therefore 16(5b + k)$ is divisible by 16.

$\therefore$ Statement is true for $n = k+1$.
Hence proved by first principle of mathematical Induction.

Q prove that by mathematical induction

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4} \quad \forall n \geq 1$$

step I -

$\Rightarrow$ for $n=1$

$LHS = n^3 = (1)^3 = 1$

$RHS = \frac{n^2(n+1)^2}{4} = \frac{(1)^2(1+1)^2}{4} = \frac{4}{4} = 1$

$\therefore$ statements are true for $n=1$.

Step II - $n=k$

$LHS = 1^3 + 2^3 + 3^3 + \ldots\ldots + k^3$

$RHS = \frac{k^2(k+1)^2}{4}$

for $n=k$, statements are true.

step III - $n=k+1$

$LHS = 1^3 + 2^3 + 3^3 + \ldots\ldots + (k+1)^3$

$= 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3$

$= \frac{k^2(k+1)^2}{4} + (k+1)^3$

$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$

$= \frac{(k+1)^2[k^2 + 4(k+1)^3]}{4}$

$= \frac{(k+1)^2 \; k^2 + 4(k+1)(k+1)^2}{4}$

$= \frac{(k+1)^2 \; k^2 + 4(k+1)}{4}$

$$= \frac{(k+1)^2 \cdot k^2 + 4k + 4}{4}$$

$$= \frac{(k+1)^2 \cdot k^2 + 4k + 4}{4}$$

$$= \frac{(k+1)^2 (k+2)^2}{4}$$

LHS = RHS

$n = k+1$, Statement are true

Hence, proved By mathematic Induction first principle Statements are true for each steps.

Q prove that $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n+1} n^2 = \frac{(-1)^{n+1} n(n+1)}{2}$ $\forall n \in N$. By using first principle of mathematical Induction.

Answer : 1} for $n = 1$

LHS = $(-1)^{1+1} (1)^2 = 1$

RHS = $\frac{(-1)^{1+1} 1(1+1)}{2} = \frac{1(2)}{2} = \frac{2}{2} = 1$

for $n=1$, Statements are true.

2} $n = k$. Assume,

LHS = $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+1} k^2$

RHS = $\frac{(-1)^{k+1} k(k+1)}{2}$

for $n = k$, statement are true

3} Assume, $n = k+1$

LHS = $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+1} k^2$

$= 1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+1+1} (k+1)^2$

$= 1^2 - 2^2 + 3^2 - 4^2 + \dots + k^2 + (-1)^{k+2} (k+1)^2$

$$= \frac{(-1)^{k+1} k(k+1) + (-1)^{k+2} 2(k+1)^2}{2}$$

$$= \frac{(-1)^{k+1+1-1} k^{(k+1)} + (-1)^{k+2} 2(k+1)^2}{2}$$

$$= \frac{(-1)^{k+2} (-1)^1 k(k+1) + (-1)^{k+2} 2(k+1)^2}{2}$$

$$= \frac{(-1)^{k+2} (-1)^1 k(k+1) + (-1)^{k+2} 2(k+1)^2}{2}$$

$$= \frac{(-1)^{k+2} (k+1)[(-k + 2(k+1)]}{2}$$

$$= \frac{(-1)^{k+2} (k+1)[(-k + 2k + 2)]}{2}$$

$$= \frac{(-1)^{k+2} (k+1)(-k + 2k + 2)}{2}$$

$$= \frac{(-1)^{k+2} (k+1)(k+2)}{2}$$

LHS = RHS

$\therefore$ Statement is true for $n = k+1$

$\therefore$ By First principle of Mathematical Induction statement are true.

Q prove that $1+2+2^2+2^3+\ldots 2^n = 2^{n+1}-1 \; \forall n \geq 0$

$\Longrightarrow$ step I -

$n = 0$

LHS = $n = 0$, $2^n = 2^0 = 1$

RHS = $2^{0+1}-1 = 2-1 = 1$

LHS = RHS statement are true for $n=0$

Step II - Assume that $n=k$

LHS = $1+2+2^2+2^3+\ldots 2^k$

RHS = ~~1+2~~ $2^{k+1}-1$

Statements are true for $n=k$

Step III - Assume that $n=k+1$

RHS = $2^{k+1+1}-1 = 2^{k+2}-1$

LHS = $1+2+2^2+2^3+\ldots+2^{k+1}$

$= 1+2+2^2+2^3+\ldots+2^k+2^{k+1}$

$= 2^{k+1}-1+2^{k+1}$

$= 2^1(2^{k+1})-1$

$= 2^{k+1+1}-1$

$= 2^{k+2}-1$

LHS = RHS

for $n=k+1$, Statement are true,

Hence proved By First principle of Mathem[atical] Induction.

* prove that $n(n^2-1)$ is divisible by 3, $n \in N$

⇒ step I : $n=1$

$n(n^2-1) = 1((1)^2-1) = 1(1-1) = 1(0) = 0$

∴ $n=1$ is true for statement.

0 is divisible by 3

step II - $n=k$, Assume that statement is true.

i.e $k(k^2-1)$ is divisible by 3

Assume $k(k^2-1)$ is divisible by 3a

$k(k^2-1) = 3a$

$k^3 - k = 3a$

$k^3 = 3a + k \quad — (1)$

Step 3 -

Statement is true for $n=k+1$

i.e $(k+1)[(k+1)^2-1]$ is divisible by 3

$LHS = k+1[(k+1)^2-1]$.

$= k+1[(k^2+2k+1-1]$

$= k+1[k^2+2k]$

$= k^3+2k^2+k^2+2k$

$= k^3+3k^2+2k$

$= 3a+k+3k^2+2k$

$= 3a+3k^2+3k$ ∴ statement is true

$= 3(a+k^2+k)$ for $n=k+1$.

∴ By principle of mathematical Induction $n(n^2-1)$ is divisible by 3.

* prove that $2^{2n}-1$ is divisible by 3. $n \in N$

→ Step I

for $n=1$

$2^{2(1)}-1 = 2^2-1 = 4-1 = 3$

3 is divisible by 3

Step II - Assume that statement is divisible by 3.

$n = k$ i.e $2^{2k}-1$

Assume, $2^{2k}-1$ is divisible by 3a

$$2^{2k}-1 = 3a$$

$$2^{2k} = 3a+1$$

Step III -

Assume for $n = k+1$

$$2^{2}(k \quad = 2^{2(k+1)}-1$$

$$= 2^{2k+2}-1$$

$$= 2^{2k}\cdot 2^2 - 1$$

$$= 2^{2k}\cdot 4 - 1$$

$$= (3a+1)4 - 1$$

$$= 12a+4-1$$

$$= 12a+3$$

$$= 3(4b+1)$$

By principle of mathematical equation, statement is true for $n = k+1$

$2^{2n}-1$ is divisible by 3. Hence proved prove that $n^4+n^3-2n^2$ is always even for any n belongs to natural number $\forall n \in N$.

→ step I - for $n=1$

$n^4+n^3-2n^2$

$(1)^4+(1)^3-2(1)^2$

$1+1-2(1) = 2-2 = 0$

0 is divisible by 2

Step II - for $n=k$

$k^4+k^3-2k^2$ is divisible by 2

Assume that, $k^4+k^3-2k^2$ is divisible by 2a.

$k^4+k^3-2k^2 = 2a$

$k^4 = 2a-k^3+2k^2$

statement is true for $n=k$

step III - for $n=k+1$

$LHS = (k+1)^4+(k+1)^3-2(k+1)^2$

$(k+1)^2[(k+1)^2+(k+1)-2]$

$k^2+2k+1\ [(k^2+2k+1)+(k+1)-2]$

$(k^2+2k+1)[k^2+3k]$

$k^4+3k^3+2k^3+6k^2+k^2+3k$

$k^4+5k^3+7k^2+3k$

$2a-k^3+2k^2+5k^3+7k^2+3k$

$2a-k^3+5k^3+9k^2+3k$

$2a+4k^3+9k^2+3k$

$= 2(a+2k^3) + 3k(3k+1)$

Any 2 consecutive numbers are divisible by 2.

$3k(3k+1) = 2b$

$2(a+2k^3) + 2b$

$= 2(a + 2k + b)$ is divisible by 2.

$\therefore$ Statement are true for $n = k+1$.

PRACTICAL NO. 4

Aim: Solving problems based on Second Induction

Q1. If $u_1 = 3$, $u_2 = 5$, $u_n = 3u_{n-1} - 3u_{n-2}$ for $n \geq 3$, then using Induction show that $u_n = 2^n + 1$, $\forall n \geq 3$.

→ Step I :

for $n = 3$

LHS = $u_n = 3u_{n-1} - 2u_{n-2}$

$u_n = 3u_{3-1} - 2u_{3-2}$

$u_n = 3(5) - 2(3)$

$u_n = 15 - 6$

$u_n = 9$

RHS = $u_n = 2^n + 1$

$= 2^3 + 1$

$= 8 + 1$

$= 9$

$\therefore$ LHS = RHS

$\therefore$ Statement is true for $P(n) = 3$

• Step II :

Assume statement is true for $3 < k < m$.

$u_{m-1} = 2^{m-1} + 1$ is true.

Similarly, $u_{m-2} = 2^{m-2} + 1$ is also true.

• Step III -

To prove statement for $n=m$.

i.e To prove

$$U_m = 2^m + 1$$

$LHS = U_m = 3U_{m-1} - 2U_{m-2}$ — ①

put U_{m-1} & U_{m-2} values in equation ①

$$\begin{aligned} U_m &= 3(2^{m-1}+1) - 2(2^{m-2}+1) \\ &= 3\cdot 2^{m-1} + 3 - 2\cdot 2^{m-2} - 2 \\ &= 2\cdot 2^{m-1} + 1 \\ &= 2^1\cdot 2^{m-1} + 1 \\ &= 2^{1+m-1} + 1 \\ &= 2^m + 1 \end{aligned}$$

$LHS = RHS$

statement is true for $n=m$

∴ By 2nd Mathematical Induction, Statement is true for all $n \geq 3$.

Q

Let a_n be the recursive relation defined by.
$a_n = 2a_{n-1} + a_{n-2}$ $n \geq 2$ with initial condition
$a_0 = 1$, $a_1 = 2$
prove that $a_n \leq (5/2)^n$

→ Step I -

for $n = 2$

LHS = $a_n = 2a_{2-1} + a_{2-2}$
$= 2a_1 + a_0$
$= 2(2) + 1$
$= 4 + 1$
$= 5$

RHS = $a_n \leq (5/2)^n$
$= 5 \leq (5/2)^2$
$= 5 \leq 6.25$

LHS = RHS

P(2) is true for statement $a_n = 2a_{n-1} + a_{n-2}$

Step II -
Assume, Statement is true for $2 < k < n$

$a_{m-1} \leq (5/2)^{m-1}$ is true.
similarly, $a_{m-2} \leq (5/2)^{m-2}$ is true

Step III -
To prove statement for $n = m$

i.e To prove,
$a_m \leq (5/2)^m$

Q

Let a_n be the recursive relation defined by.
$a_n = 2a_{n-1} + a_{n-2}$ $n \geq 2$ with initial condition
$a_0 = 1$, $a_1 = 2$
prove that $a_n \leq (5/2)^n$

→ Step I -

for $n = 2$

LHS $= a_n = 2a_{2-1} + a_{2-2}$
$= 2a_1 + a_0$
$= 2(2) + 1$
$= 4 + 1$
$= 5$

RHS $= a_n \leq (5/2)^n$
$= 5 \leq (5/2)^2$
$= 5 \leq 6.25$

LHS = RHS

$P(2)$ is true for statement $a_n = 2a_{n-1} + a_{n-2}$

Step II -
Assume, Statement is true for $2 < k < m$

$a_{m-1} \leq (5/2)^{m-1}$ is true.
similarly, $a_{m-2} \leq (5/2)^{m-2}$ is true

Step III -
To prove Statement for $n = m$

i.e To prove,
$a_m \leq (5/2)^m$

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Page No.

LHS =

$$a_m = 2a_{m-1} + a_{m-2} \quad -① $$

put a_{m-1} & a_{m-2} value in eqn ①

$$a_m = 2a_{m-1} + a_{m-2} \leq 2(5/2)^{m-1} + (5/2)^{m-2}$$

$$\leq 2(5/2)^{m-1} + \left(\frac{5}{2}\right)^{m-1-1}$$

$$\leq 2(5/2)^{m-1} + (5/2)^{m-1}(5/2)^{-1}$$

$$\leq (5/2)^{m-1}\left[2 + \frac{2}{5}\right]$$

$$\leq (5/2)^{m-1}(12/5)$$

$$\leq (5/2)^{m-1}(5/2) \leq (5/2)^m$$

i.e $a_m \leq \left(\frac{5}{2}\right)^m$

∴ statement is true for $n = m$

∴ By second principle of mathematical Induction statement is true for all $n \geq 2$

Q3. Let a_n be recursive Relation, Defined by $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$, with initial condition. $a_0 = 1$, $a_1 = 1$, $a_2 = 1$

(I) Obtain first few (7) terms of Recursive Relation
(II) prove that all a_n are odd
(III) prove that $a_n \leq 2^{n-1}$, $\forall n \geq 1$

→ (i) first few 7 terms of Recursive Relation

$a_0 = 1$, $a_1 = 1$, $a_2 = 1$

$\therefore a_3 = a_{3-1} + a_{3-2} + a_{3-3}$
$= a_2 + a_1 + a_0$
$= 1 + 1 + 1$
$a_3 = 3$

$\therefore a_4 = a_{4-1} + a_{4-2} + a_{4-3}$
$a_4 = a_3 + a_2 + a_1$
$a_4 = 3 + 1 + 1$
$a_4 = 5$

$a_5 = a_{5-1} + a_{5-2} + a_{5-3}$
$= a_4 + a_3 + a_2$
$= 5 + 3 + 1$
$a_5 = 9$

$a_6 = a_{6-1} + a_{6-2} + a_{6-3}$
$= a_5 + a_4 + a_3$
$a_6 = 9 + 5 + 13 = 17$

$$a_7 = a_{7-1} + a_{7-2} + a_{7-3}$$
$$= a_6 + a_5 + a_4$$
$$= 17 + 9 + 5$$
$$= 31$$

Step II -

Assume statement is true for $1<k<m$

i.e statement is true for $(m-3)(m-2)(m-1)$

$$\therefore \quad a_{m-1} \leq 2^{(m-1)-1} \leq 2^{m-2}$$
$$a_{m-1} \leq 2^{(m-1)-2} \leq 2^{m-3}$$
$$a_{m-1} \leq 2^{(m-1)-3} \leq 2^{m-4}$$

Step III -

To prove for $n=m$ i.e $a_m \leq 2^{m-1}$

$$LHS = a_m = a_{m-1} + a_{m-2} + a_{m-3}$$
$$a_m \leq 2^{(m-2)} + 2^{(m-3)} + 2^{(m-4)}$$
$$a_m \leq 2^{m-2} + 2^{(m-3)} \cdot 2^1 \cdot 2^{-1} + 2^{m-4} \cdot 2^2 \cdot 2^{-2}$$
$$a_m \leq 2^{m-2}\, 2^{-1} + 2^{m-2} \cdot 2^{-2} + 2^{m-2}$$
$$a_m \leq 2^{m-2}\left(\frac{1}{2} + \frac{1}{1} + \frac{1}{4}\right)$$
$$a_m \leq 2^{m-2}\left(\frac{4+2+1}{4}\right)$$
$$a_m \leq 2^{m-2}\left(\frac{7}{4}\right)$$
$$a_m \leq 2^{m-2}\,(1.7)$$
$$a_m \leq 2^{m-2}\,(2)$$
$$a_m \leq 2^{m-2} \cdot (2^1)$$
$$a_m \leq 2^{m-2+1}$$
$$a_m \leq 2^{m-1}$$

Statement is true for $n=m$.

By 2nd principle of Mathematical Induction. Statement is true $\forall n \geq 1$

II. prove that all a_n's are odd

Step I -

for $n=3$

$a_3 = 3$ is odd

$\therefore$ statement is true for $n=3$

Step II - Assume statement is true for $3<k<m$.

a_{m-1} is odd

similarly, a_{m-2} is odd

a_{m-3} is odd.

Step III - Statement is true for $n=m$ ie a_m is odd

$a_m = a_{m-1} + a_{m-2} + a_{m-3}$

But sum of 3 odd numbers is always odd.

$\therefore$ a_m is odd

$\therefore$ Statement is true for $n=m$.

By 2nd principle of mathematical Induction, for all ~~a_n's odd~~ a_n's are odd

Q 4. prove that integer n can be expressed as product of positive primes, $n \geq 2$.

→ step I -

for $n=2$

'2' itself a prime number

statement is true for $n=2$

Step II -

Assume statement for $2 < k < m$.

i.e if $k < m$, it can be expressed as Product of primes.

step III -

To prove statement for $n=m$

if m is a prime number that result is clear.

suppose m is not a prime.

$\therefore$ m is composite

$\therefore$ m is product of 2 integers

Say, $m = r \cdot s$ ...(1)

where $r < m$, $s < m$ & $r, s \in Z$

Here, $r < m$ and $s < m$

from step II, r, s can be expressed as product of primes

says, $r = r_1 \cdot r_2 \cdot \ldots \cdot r_i$ where $r_1 \ldots r_i$, $s_1 \ldots s_j$ are primes

$s = s_1 \cdot s_2 \cdot \ldots \cdot s_j$

with these equation (i) becomes,

$m = (r_1 \cdot r_2 \ldots r_i) \cdot (s_1 \cdot s_2 \ldots s_j) = (r_1 \cdot r_2 \ldots r_i) \cdot (s_1, s_2 \ldots s_j)$

= product of primes

$\therefore$ Statement is true for $n = m$

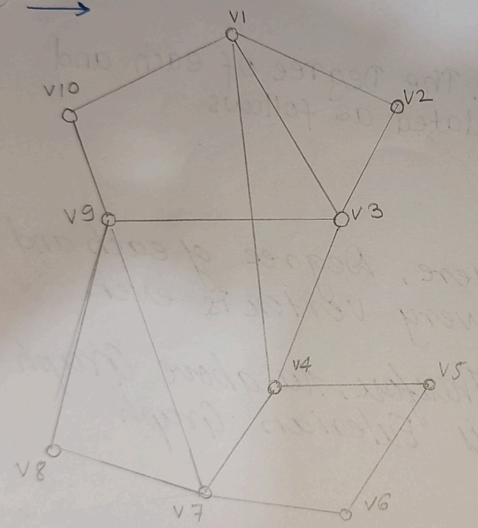
By Second Principal of mathematical Induction is true

PRACTICAL No. 5 & 6

Aim: Solving problems on Eulerian & Hamittovian Graph.

Q1. To check whether the following graph is Eulerian or Hamittorian.

→



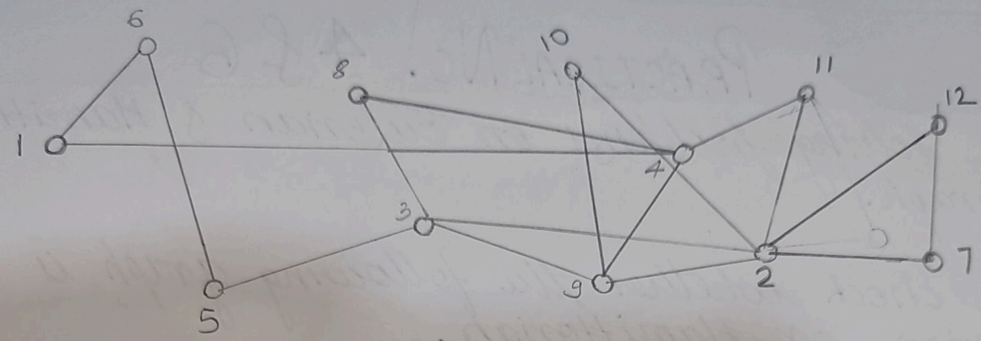
For, the given graph The Degree of each and every vertex is calculated as follows

D(V1) = 4
D(V2) = 2
D(V3) = 4
D(V4) = 4
D(V5) = 2
D(V6) = 2
D(V7) = 4
D(V8) = 2
D(V9) = 4
D(V10) = 2

Here, Degree of each and every vertex is even.

∴ The above graph is Eulerian Graph.

Q find an eulerian circuit in the graph G or explain why one does not exist.



Solution -

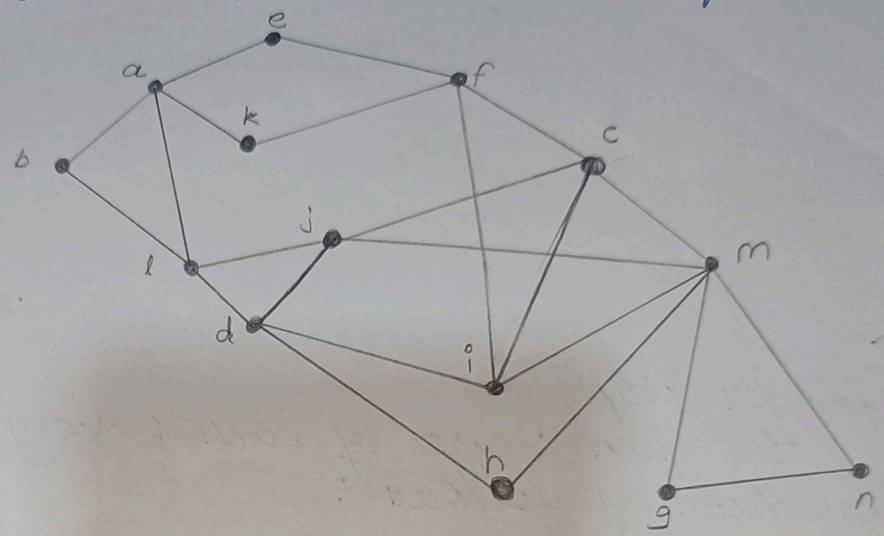
For the given graph, The Degree of each and every vertex is calculated as follows

D(1) = 2
D(2) = 6
D(3) = 4
D(4) = 4
D(5) = 2
D(6) = 2
D(7) = 2
D(8) = 2
D(9) = 4
D(10) = 2
D(11) = 2
D(12) = 2

Here, Degree of each and every vertex is even

∴ Therefore, the above Graph is Eulerian Graph

Q find an Eulerian circuit in the Graph G



for given graph

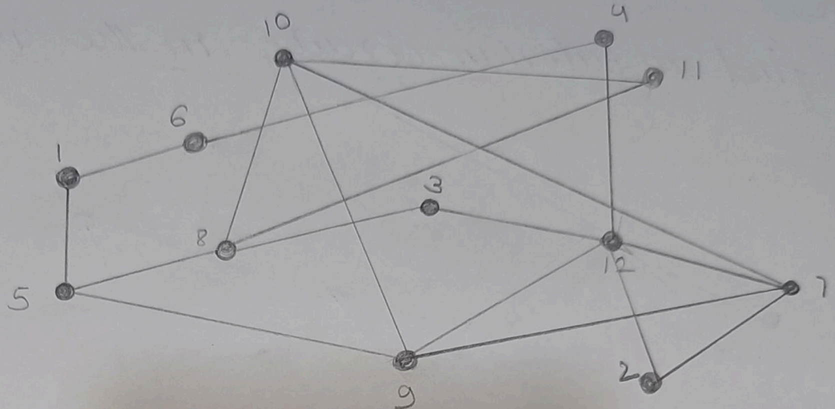
The Degree of the each & every vertex is calculated as follows:

D(a) = 4
D(b) = 2
D(c) = 4
D(d) = 4
D(e) = 2
D(f) = 4
D(g) = 2
D(h) = 2
D(i) = 4
D(j) = 4
D(k) = 2
D(L) = 4
D(m) = 6
D(n) = 2

Degree of Each and Every Vertex is even

∴ Therefore, the above graph is Eulerian graph.

Q find an eulerian circuit in the graph G.



→

for the Graph
the given Degree of each & every vertex is calculated as follows:

D(1) = 2
D(2) = 2
D(3) = 2
D(4) = 2
D(5) = 3
D(6) = 2
D(7) = 4
D(8) = 4
D(9) = 4
D(10) = 4
D(11) = 2
D(12) = ~~4~~ 5

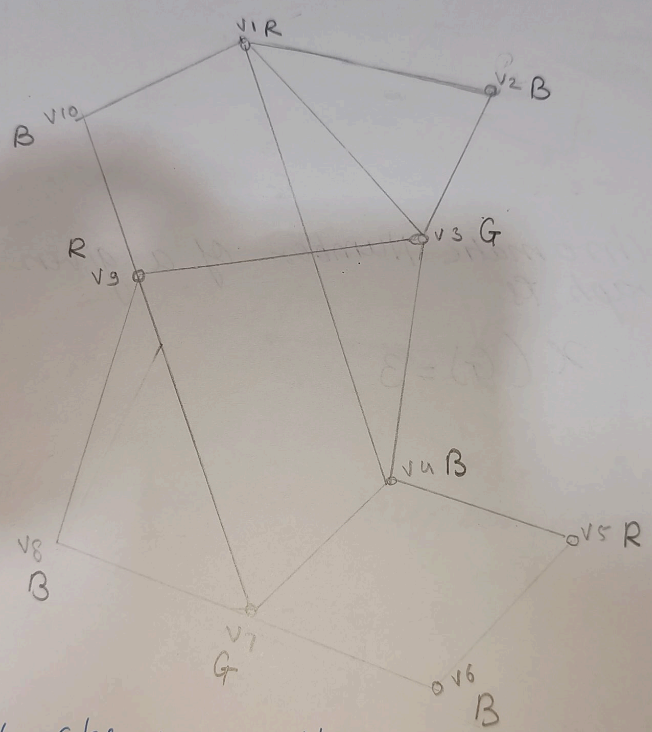
Here, Each & every vertex is not a even
∴ the above graph is Hamiltonian graph.

To make it as Eulerian graph Add the edge from 5 to 12

Practical No. 07

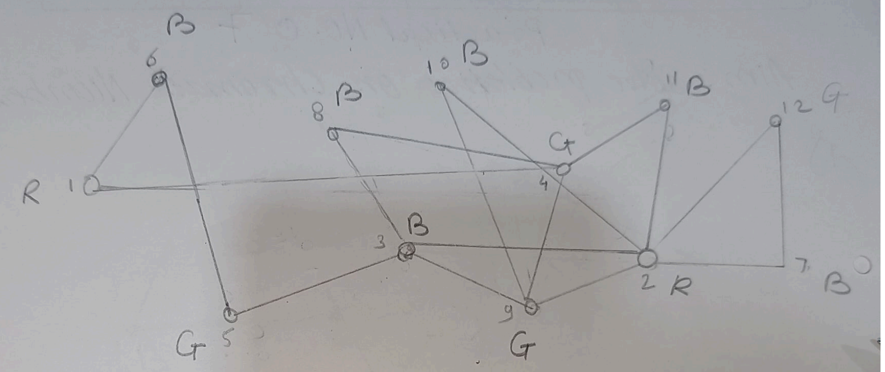
Aim: Solve problems on Chromatic Number

1)



→ The Chromatic Number of a Graph is
$\chi(G) = 3$

2}



→ The Chromatic Number of a given graph is

$\chi(G) = 3$

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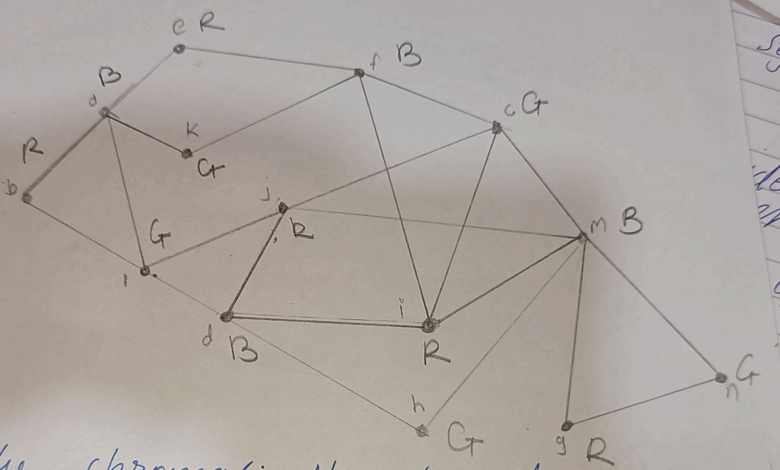
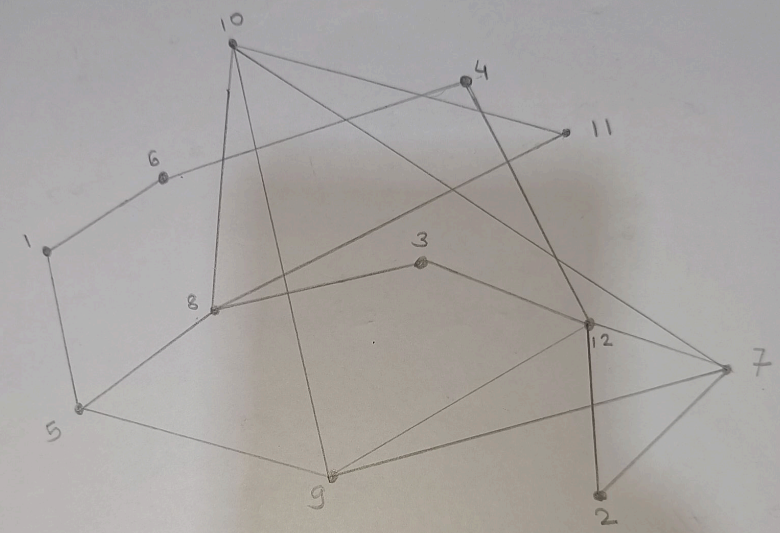
3)



→ the chromatic Number of an given graph is

$\chi(G) = 3$

43



The chromatic Number of a given graph is

$$\chi(G) = 4$$

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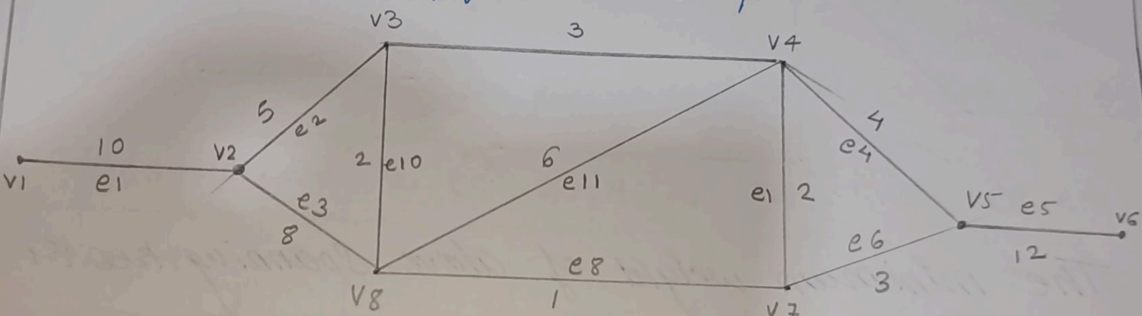
Practical No. 08

Aim: Solving the problems on kruskal's Algorithm.

1) Using kruskal's algorithm find shortest spanning tree of following graph.



Now consider, the weights in increasing order sequence i.e.

				×			×	×		
Edges	e8	e10	e7	e6	e3	e4	e2	e11	e3	e1	e5
weights	1	2	2	3	3	4	5	6	8	10	12

consider, minimum weight 1 i.e edge e8, Next trace the edge e10, i.e weight 2.

Next consider minimum weight 2 i.e e7

Now next weight is 3 i.e e3 which form a circuit or loop

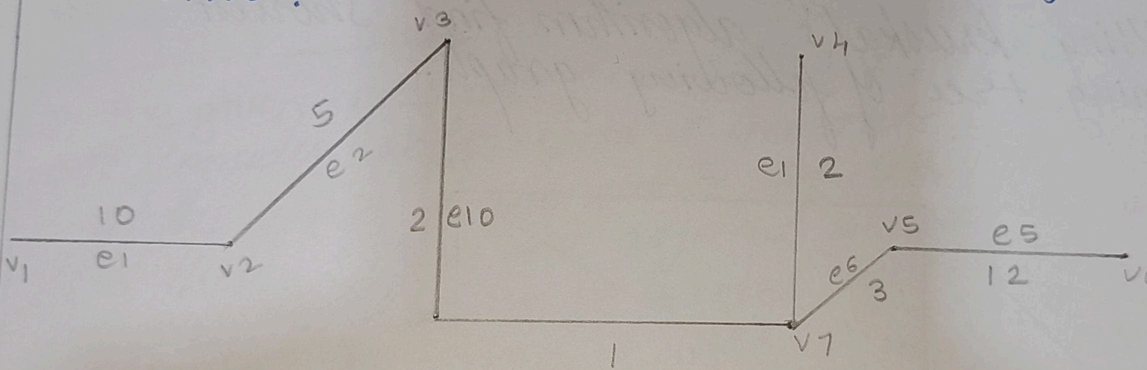
∴ avoid it (Discarded)

Using Krushkals Algorithm
the following Graph.

And consider the edge e6 of weight 3.
Now, next minimum weight is 4 which is again Discarded

After that, consider the edge e2 of weight 5
Next, After that discarded the edges e9 & e11 having corresponding weight 8 & 6 and consider the edge e1 & e5 with corresponding weights 10 & 12.

Now, the minimum Spanning tree for given graph as above:



The minimum weight of above Spanning tree is

$\therefore\ 10+5+2+1+2+3+12$
$= 35$

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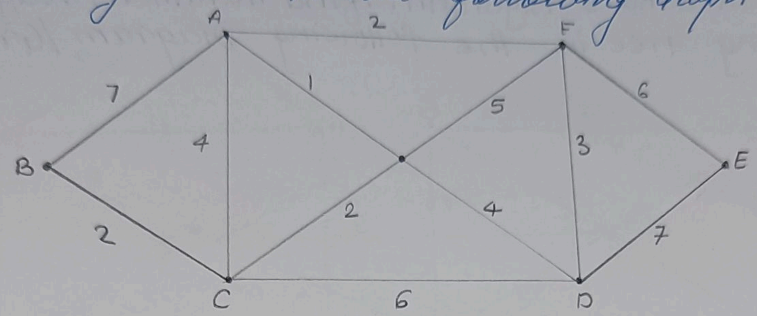
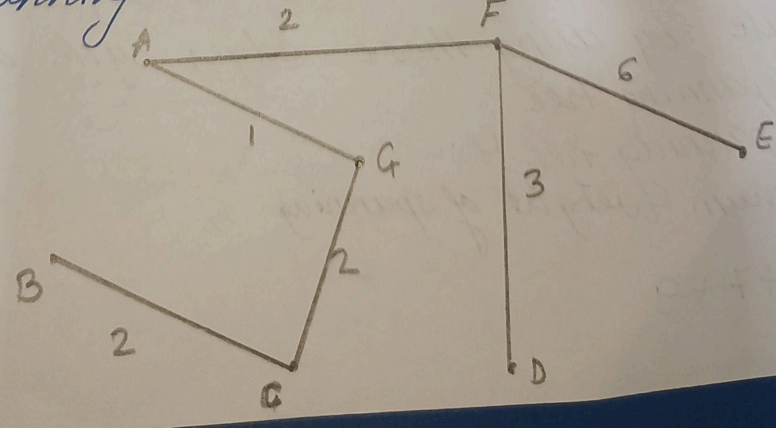
Q. Using krushkal's Algorithm find maximum weighted spanning tree in the following Graph.



Now, consider, the weights in increasing order.

Edges	AG	AF	CG	BC	DF	AC	GD	FG	CD	EF	ED	AB
						×	×	×	×		×	×
Weights	1	2	2	2	3	4	4	5	6	6	7	7

from the given graph edges like AG, CG, AF, BC, FD, FE are consider or accepted to Generate Spanning tree of the given graph while AC, GD, FD, CD, AB, ED these edges are rejected because of by using this edges can be formed cycles in the created in the spanning tree
the spanning tree is -

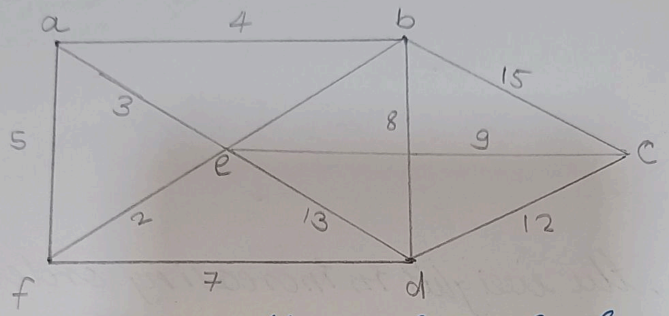


the minimum weight of Spanning tree is

$\therefore 2 + 2 + 1 + 2 + 3 + 6$

$= 16$

Q3. Using kruskal's Algorithm, find minimum weighted spanning tree in the following Diagram /Graph.



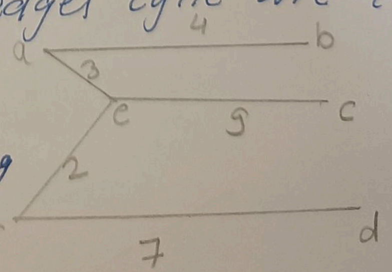
Now consider, the weights in increasing order sequence i.e.

Edges	ef	ae	ab	af	eb	fd	bd	ec	cd	ed	bc
Weights	2	3	4	5	6	7	8	9	12	13	15

from the given graph Edges like ef, ae, ab, fd, ec bc are consider or accepted to generate Spanning tree of the given graph

While af, eb, bd, cd, ed these edges are rejected because by using these edges cycle are created in spanning tree

The Spanning tree is :-



The minimum weight of spanning tree is

$2 + 3 + 4 + 7 + 9$

$= 25$

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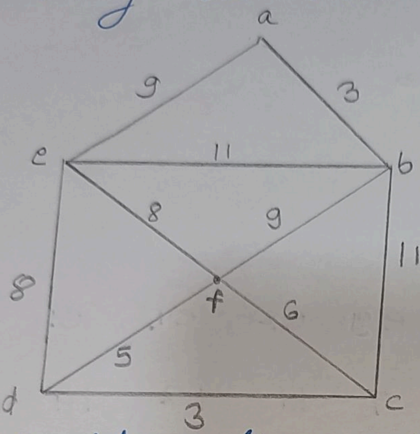
Q4 Using krustal's Algorithm, find minimum weighted Spanning tree in the following Diagram / Graph.



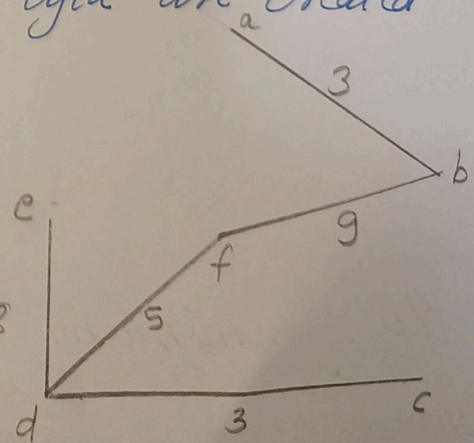
Now consider, the weights in increasing order sequence

Edges	ab	dc	fd	fc	ed	ef	bf	ae	be	bc
weight	3	3	5	6	8	8	9	9	11	11
				x		x		x	x	x

From the given graph edges like ab, dc, fd, ed, bf, are consider or accepted to generate spanning tree.

While The edges fc, ef, ae, be, bc are rejected because by using these edges cycle are created in the spanning tree.

The spanning tree is -

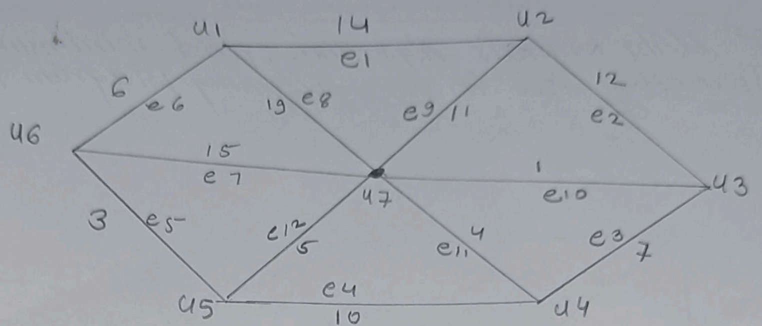


The minimum weight of Spanning tree is

3 + 3 + 5 + 8 + 9

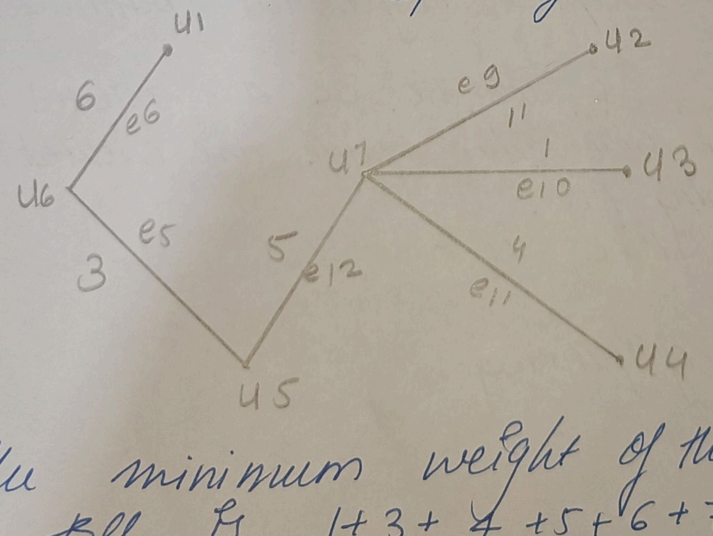
= 28

Q Using Krushkal's Algorithm, find out the minimum weighted Spanning tree in the following Diagram/ Graph:



edges	e10	e5	e11	e12	e6	e3	e9	e2	e1	e7	e8
weight	1	3	4	5	6	7	11	12	14	15	19
						x		x	x	x	x

from the given graph edges like e10, e5, e11, e12, e6 e3, e9 are consider or accepted to generate the spanning tree of given Graph
And other Remaining edges are ~~formin~~ in e3, e2, e1, e7 e8 are forming an cycle in the Graph are Rejected.
the minimum Spanning tree is



the minimum weight of the given Spanning tree is 1+3+4+5+6+7+11+
= 30