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Assignment - 1

Q.1 Express the following in the standard form of complex number $(x+iy)$

① $\frac{3+2i}{2-3i}$

Solution:

$\frac{3+2i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{(3+2i)\times(2+3i)}{(2)^2-(3i)^2} = \frac{3\times2+9i+4i+6i^2}{(2)^2-(3i)^2}$

$= \frac{6+9i+4i+6i^2}{4+9}$ $\quad i^2=-1$

$= \frac{6+13i+6(-1)}{4+9}$

$= \frac{\cancel{6}+13i+\cancel{(-6)}}{13}$

$= \frac{\cancel{13}i}{\cancel{13}}$

$\boxed{z = 0-1i}$

real number = 0 imaginary = 1i

② $\frac{2-\sqrt{3}i}{1+i}$

Solution:

$\frac{(2-\sqrt{3}i)}{1+i} \times \frac{(1-i)}{(1-i)} = \frac{(2-\sqrt{3}i)\times(1-i)}{(1+i)(1-i)} = \frac{2-2i-\sqrt{3}i-\sqrt{3}i^2}{(1)^2-(i)^2}$

$= \frac{2-2i-\sqrt{3}i-\sqrt{3}i^2}{2}$

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③ $\frac{1+i}{1-i}$

Solution:

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)(1+i)}{(1)^2-(i)^2} = \frac{1+i+i+i^2}{1-(-1)}$$

$$= \frac{1+2i+i^2}{1+1}$$

$$= \frac{\cancel{1}+2i-\cancel{1}}{2} \quad \because i^2 = 1$$

$$= \cancel{2}i/\cancel{2}$$

$$Z = i$$

$$\boxed{Z = 0+1i}$$

real = 0 imaginary = 1i

Q.2 Find the complex conjugate of (change sign of imaginary)

a] $\frac{3+5i}{1+2i}$

Solution:

$$\frac{3+5i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{(3+5i)(1-2i)}{(1)^2-(2i)^2}$$

$$= \frac{3-6i+5i-10i^2}{1-4i^2}$$

$$= \frac{3-i-10(-1)}{1-4(-1)}$$

$$= \frac{3-i+10}{5}$$

$$= \frac{13-i}{5} = \frac{13}{5} + \frac{-1}{5}i$$

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b] $\frac{3+2i}{2-3i}$

Solution:

$$\frac{3+2i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{(3+2i)(2+3i)}{(2)^2-(3i)^2}$$

$$= \frac{6+9i+4i+6i^2}{4-9(i)^2}$$

$$= \frac{6+13i+6i^2}{4-9i^2}$$

$$= \frac{\cancel{6}+13i+\cancel{6(-1)}}{4-9(-1)}$$

$$= \frac{\cancel{13}i}{\cancel{13}}$$

$$= i$$

c] $\frac{2+3i}{1-i}$

Solution:

$$\frac{2+3i}{1-i} \times \frac{1+i}{1+i} = \frac{(2+3i)(1+i)}{(1)^2-(i)^2}$$

$$= \frac{2+2i+3i+3i^2}{(1)^2-(i)^2}$$

$$= \frac{2+5i+3(-1)}{1+1}$$

$$= \frac{2+5i+(-3)}{2}$$

$$= \frac{-1+5i}{2}$$

$$= \frac{-1}{2} + \frac{5}{2}i$$

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Q.3 Express the following in polar form and find their arguments.

① $z = \sqrt{3} + i$

$r = |z|$

$= \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$r = 2$

Let $\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|$

$\tan\alpha = \left(\frac{1}{\sqrt{3}}\right)$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha = \frac{\pi}{6}$

Since point $(\sqrt{3}, 1)$ lies in the first quadrant, the argument of z is given by $\theta = \alpha = \frac{\pi}{6}$

polar form $= r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

② $z = \sin\theta + i\cos\theta$

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③ $\frac{1+2i}{1-3i}$

$z = \frac{1+2i}{1-3i}$

$r = |z|$

$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$

$= \frac{(1+2i)\times(1+3i)}{(1)^2 - (3i)^2}$

$= \frac{1+3i+2i+6i^2}{1-9(-1)}$

$= \frac{-5+5i}{10}$

$= \frac{-5}{10} + \frac{5i}{10}$

$z = -\frac{1}{2} + \frac{i}{2}$

$r = |z|$

$= \sqrt{\frac{1}{4} + \frac{1}{4}}$

$r = \frac{1}{\sqrt{2}}$

Let $\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right|$

$\tan\alpha = \left|\frac{\frac{1}{2}}{-\frac{1}{2}}\right|$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

Since point $\left(-\frac{1}{2}, \frac{1}{2}\right)$ lies in the second quadrant, the argume[nt] is given by

$\theta = \pi - \alpha$

$= \pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

polar form $= r(\cos\theta + i\sin\theta)$

polar form $= \frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4} + i\sin 3\right.$

vectors

Q.4 For $u = [0,4]$ and $v = [-1,3]$, find vector $u+v$, $v-u$, $u-v$, $3v-2u$

① $u = [0,4]$ $\bar{u} = 0i + 4j$
$v = [-1,3]$ $\bar{v} = -1i + 3j$

$\bar{a} = \bar{u} + \bar{v} = -1i + 7j$
$a = [-1,7]$

② $v - u$
$u = [0,4]$ $\bar{u} = 0i + 4j$
$v = [-1,3]$ $\bar{v} = -1i + 3j$

$\bar{a} = \bar{v} - \bar{u} = -1i + (-1j)$
$a = [-1,-1]$

③ $u - v$
$u = [0,4]$ $\bar{u} = 0i + 4j$
$v = [-1,3]$ $\bar{v} = -1i + 3j$

$\bar{a} = \bar{u} - \bar{v} = +1i + 1j$
$a = [+1,1]$
$a = [1,1]$

④ $3v - 2u$
$u = [0,4]$
$v = [-1,3]$

$\bar{u} = 0i + 4j$
$\bar{v} = -1i + 3j$

$\bar{a} = 3\bar{v} - 2\bar{u}$
$= 3(-1i + 3j) - 2(0i + 4j)$
$= -3i + 9j - 8j$
$\bar{a} = -3i - 1j$
$a = [-3,-1]$

Q.5 For $u = [0,1,1]$ and $v = [1,1,1]$ over $GF(2)$ Find $v+u$ and $v+u+u$.

① $u = [0,1,1]$
$v = [1,1,1]$
so
$c = v + u$
$v = 1,1,1$
$u = 0,1,1$
$c = 1,0,0$

$1+1=0$
$1+0=1$
$0+1=1$
$0+0=0$

$v + u = [1,0,0]$

② $u = [0,1,1]$
$v = [1,1,1]$
so
$c = v + u + u$

$v = [1,1,1]$
$u = [0,1,1]$
$u = [0,1,1]$
$c = 1\ 1\ 1$

$c = v + u + u = [1,1,1]$

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Q. 6 Find a vector $x = [x_1, x_2, x_3, x_4]$ over GF(2) satisfying the following linear equations
$1100 \cdot x = 1$; $1010 \cdot x = 1$; $1111 \cdot x = 1$
Show that $x + 1111$ also satisfies the equations

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Q. 7 For each of the following pair of vector over $\mathbb{R}$, evaluate the expression $u \cdot v$:

a] $u = [1, 0]$, $v = [5, 1616]$
$u \cdot v = (1 \times 5) + (0 \times 1616) = 5$
so, $u \cdot v = 5$

b] $u = [1, 2, 3]$ $v = [3, 2, 1]$
$u \cdot v = (1 \times 3) + (2 \times 2) + (3 \times 1)$
$= 3 + 4 + 3$
$u \cdot v = 10$

c] $u = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$, $v = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$

$u \cdot v = \left[-\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}\right] + \left[\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2}\right]$

$= \left[-\frac{2}{4} + -\frac{2}{4}\right]$

$= \frac{-2-2}{4}$

$= \cancel{\frac{-4}{16}}$

$\cancel{\frac{-2}{4}}$

$u \cdot v = -1$

$-\frac{2}{4} - \frac{2}{4}$ $\quad \frac{-4}{4} = -1$

Q. 8 Calculate the magnitude of the following vector and find their unit vector using formula $\hat{u} = \frac{u}{\|u\|}$, where $\|u\|$ is the magnitude of vector given by $\|u\| = \sqrt{\sum_i u_i^2}$

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① $u = (1,1,1)$

To find the magnitude of vector $u = (1,1,1)$ we use the formula:

$$\|u\| = \sqrt{\sum_i u_i^2}$$

$$\|u\| = \sqrt{1^2+1^2+1^2} = \sqrt{1+1+1} = \sqrt{3}$$

So, the magnitude of vector u is $\sqrt{3}$

Now, to find the unit vector i, we use the formula

$$i = \frac{u}{\|u\|}$$

$$i = \frac{(1,1,1)}{\sqrt{3}}$$

So, the unit vector i is approximately.

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

② $u = (0,11,0)$

To find the magnitude of vector $u = (0,11,0)$ we use the formula:

$$\|u\| = \sqrt{\sum_i u_i^2}$$

$$\|u\| = \sqrt{(0)^2+(11)^2+(0)^2} = \sqrt{0+121+0} = \sqrt{121} = 11$$

So, the magnitude of vector u is 11

Now, to find the unit vector i, we use the formula

$$i = \frac{u}{\|u\|}$$

$$i = \frac{(0,11,0)}{11}$$

$$i = \frac{0}{11}, \frac{11}{11}, \frac{0}{11}$$

$$i = 0, 1, 0$$

So, the unit vector i is approximately (0,1,0)

② $u = (1,-2,3)$

To find the magnitide of vector $u = (1,-2,3)$ we use formula:

$$\|u\| = \sqrt{\sum_i u_i^2}$$

$$\|u\| = \sqrt{(1)^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$$

So, the magnitude of vector u is $\sqrt{14}$

Now, to find the unit vector i, we use the formula

$$i = \frac{u}{\|u\|}$$

$$i = \frac{(1,-2,3)}{\sqrt{14}}$$

So, the unit vector i is approximately $\left(\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

③ $u = (-1,-1,4)$

To find the magnitide of vector $u = (-1,-1,4)$ we use formula

$$\|u\| = \sqrt{\sum_i u_i^2}$$

$$\|u\| = \sqrt{(-1)^2+(-1)^2+(4)^2} = \sqrt{1+1+16} = \sqrt{18}$$

So, the magnitude of vector u is $\sqrt{18}$

Now, to find the unit vector i, we use the formula

$$i = \frac{(-1,-1,4)}{\sqrt{18}}$$

So, the unit vector i is approximately $\left(\frac{-1}{\sqrt{18}}, \frac{-1}{\sqrt{18}}, \frac{4}{\sqrt{18}}\right)$

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vector spaces

Q.9 prove or give a counter example.

(a) " $\{[x,y,z] \mid x,y,z \in \mathbb{R}, x+y+z=1\}$ is a vectorspace"?

1. closure under addition : let's take two vectors $[x_1,y_1,z_1]$ and $[x_2,y_2,z_2]$ from the set. Their sum is $[x_1+x_2, y_1+y_2, z_1+z_2]$. Does this sum still satisfy $x+y+z=1$?

If we add the components, we get $(x_1+x_2)+(y_1+y_2)+(z_1+z_2) = (x_1+y_1+z_1)+(x_2+y_2+z_2) = 1+1 = 2 \neq 1$

Since closure under addition fails, the set is not a vector space.

(b) " $\{[x,y,z] : x,y,z \in \mathbb{R}$ and $x+y+z=0\}$ is a vector space"

1. closure under addition and scalar multiplication: Let's take two vectors $[x_1,y_1,z_1]$ and $[x,y_2,z_2]$. from the set.
Their sum is $[x_1+x_2, y_1+y_2, z_1+z_2]$. since $x_1+y_1+z_1=0$ & $x_2+y_2+z_2=0$
it follows that $(x_1+x_2)+(y_1+y_2)+(z_1+z_2)=0$, satisfying closure under addition. similarly, multiplying any vector in the set by a scalar still satisfies the condition $x+y+z=0$, thus closure under scalar multiplication holds.

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Alternatively, to provide a counterexample, we only need to disprove one of the vector space axioms. Lets consider the closure under scalar multiplication. if we take a vector $[1,-1,0]$ from the set and multiply it by the scalar 2, we get $[2,-2,0]$. However, $2+(-2)+0 \neq 0$, violating closure under scalar multiplication
Therefore, $\{[x,y,z] : x,y,z \in \mathbb{R}$ and $x+y+z=0$ is not a vector space.

c) " $\{[x_1,x_2,x_3,x_4,x_5] \in \mathbb{R}^5 \mid x_2=0$ and $x_3=0\}$ is a vectorspace"?

1. closure under addition ;
Let $u=[u_1,0,0,u_5]$ & $v=[v_1,0,0,v_5]$ be two vectors in v. Their sum $u+v$ is $[u_1+v_1,0,0,u_5+v_5]$ which still statisfies $x_2=0$ & $x_3=0$. so, v is closed under addition.

2. Closure under scalar multiplication :
Let k be a scalar and $u=[u_1,0,0,u_5]$ be a vector in V. then, $k \cdot u = [k \cdot u_1, 0,0, k \cdot u_5]$, which still satisfies $x_2=0$ and $x_3=0$. so, v is closed under scalar multiplication.

3. Existence of an additive identity : The zero vector in $\mathbb{R}^5$ is $[0,0,0,0]$, which satisfies $x_2=0$ and $x_3=0$. so, the zero vector is in v

4. Existence of an additive inverses : For any vector

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$u = [u_1, 0, 0, u_5]$ in r, $-u = [-u_1, 0, 0, -u_5]$ is also in r, and their sum is the zero vectors, so additive inverses exist

5. Associativity of addition: Addition in $\mathbb{R}^5$ is associative, so it holds for r as well.
6. Commutativity of addition: Addition in $\mathbb{R}^5$ is commutative so it holds for r as well
7. Distributivity of scalar multiplication over vector addition: $k\cdot(u+v) = k\cdot u + k\cdot v$, which holds for r since it holds for vectors in $\mathbb{R}^5$
8. Distributivity of scalar multiplication over scalar addition: $(k+m)\cdot u = k\cdot u + m\cdot u$, which holds for r since it holds for vectors in $\mathbb{R}^5$.
9. Scalar multiplication identity: $1\cdot u = u$ which holds for r since it holds for vector in $\mathbb{R}^5$.

Since all the properties of a vector space are satisfied, the set r forms a vector space.

Q.11 which of the following is a set of generator for $\mathbb{R}^3$?

(a) = $\{(1,1,1)\}$
(b) = $\{(1,0,0), (0,0,1)\}$
(c) $\{(1,2,1), (2,0,-1), (4,4,1)\}$
(d) $\{(1,2,1), (2,0,-1), (4,4,0)\}$

(a) $\{(1,1,1)\}$
This is not a set of generators for $\mathbb{R}^3$ because it only contains one vector, which cannot span the entire three dimensional space.

(b) $\{(1,0,0), (0,0,1)\}$
This is set also cannot be generators for $\mathbb{R}^3$ because it only spans the xy-plane and not the entire three dimensional space.

c) $\{(1,2,1)(2,0,-1), (4,4,1)\}$
Contain three vectors. To determine if they span $\mathbb{R}^3$, we can check if they are linealy independent. If they are, then they span $\mathbb{R}^3$

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & -1 \\ 4 & 4 & 1 \end{vmatrix} = 1\begin{bmatrix} 0 & -1 \\ 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} + 1\begin{bmatrix} 2 & 0 \\ 4 & 4 \end{bmatrix}$$
$$= 1(0+4) - 2(2+4) + 1(8-0)$$
$$= 4 - 2(6) + 8$$
$$= 4 - 12 + 8$$
$$= -8 + 8$$
$$= 0$$

d] $\{(1,2,1), (2,0,-1), (4,4,0)\}$
Contain three vectors. To determine if they span $\mathbb{R}^3$ we can check if they are linealy independent. If they are, then they span $\mathbb{R}^3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -1 \\ 4 & 4 & 0 \end{bmatrix} = 1\begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix} + 1\begin{bmatrix} 2 & 0 \\ 4 & 4 \end{bmatrix}$$

$$= 1[0+4] - 2[0+4] + 1[8-0]$$

$$= 4 - 2(4) + 8$$

$$= 4 - \cancel{8} + \cancel{8}$$

$$= 4$$

Q.B prove that the set of all $n \times m$ matrices whose entries are real is a vector space over $\mathbb{R}$ with usual matrix addition and scalar multiplecation i.e.
$(A = (a_{ij}))$ and $B = (b_{ij})$ then
$A + B := (a_{ij}) + (b_{ij}) = (a_{ij} + b_{ij})$
Also, $\alpha A = \alpha(a_{ij}) := (\alpha a_{ij})$

To prove that the set of all $n \times m$ matrices with real entries forms a vector space over $\mathbb{R}$, we need to verify the ten axioms of a vector space.

1. Closure under addition: Let A and B be two matrices in the set. Then, for any entry a_{ij} in A and b_{ij} in B. $a_{ij} + b_{ij}$ is a real number, thus A + B is also a matrix with real entries
2. Associativity of addition: This property holds true since matrix addition is associative, regardless of the entries being real or not
3. Commutativity of addition: Matrix addition is commutative, so $A + B = B + A$ for any A and B in the set.
4. Existence of additive identity: The matrix with all entries being zero serves as the additive identity, denoted as 0, since adding it to any matrix results in the same matrix
5. Existence of additive inverses: For any matrix, A, the additive inverse $-A$ exists, where each entry is the negative of the corresponding entry in A.
6. Closure under scalar multiplication:
Let α be a scalar and A be a matrix in the set. Then, for any entry a_{ij} in A, $\alpha \cdot a_{ij}$ is a real number, thus $\alpha \cdot A$ is also a matrix with real entries.
7. Distributivity of scalar multiplication with respect to vector addition: This property holds true since matrix multiplecation and addition distribute over scalar multiplication
8. Distributivity of scalar multiplication with respect to field addition: This property holds true since scalar multiplecation distribute over addition in the field of real number
9. Compatibility of scalar multiplication with field multiplication: Scalar multiplication behaves as expected over real number.
10. Identity element of scalar multiplication: Multiplying a matrix by 1 result in the same matrix.

Since all ten axioms are satisfied, the set of all $n \times m$ matrices with real entries forms a vector space over $\mathbb{R}$ with the usual matrix addition and scalar multiplication.

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Q.14 show that the set of all real symmetric matrices $S_n = \{(a_{ij}) \mid a_{ij} \in \mathbb{R},\ a_{ij} = a_{ji},\ \forall\ 1 \le i, 1 \le n\}$ is a vectorspace under usual addition of matrices and scalar multiplication.

1. closure under addition: For any two symmetric matrices A and B in S, their sum A + B is also symmetric. Let A and B be symmetric matrices in S. Then $(A+B)_{ij} = A_{ij} + B_{ij} = A_{ji} + B_{ji} = (A+B)_{ji}$, thus A + B is symmetric
2. closure under scalar multiplication: For any scalar k and any symmetric matrix A in S, the scalar multiple kA is also symmetric. Let A be a symmetric matrix in S. Then $(kA)_{ij} = kA_{ij} = kA_{ji} = (kA)_{ji}$, thus kA is symmetric.
3. Commutativity of addition: For any symmetric matrices A and B in S, A + B = B + A. This property holds due to the commutativity of addition of real number.
4. Associativity of addition: For any symmetric matrices A, B and C in S, (A+B) + C = A + (B+C). This property holds due to the associativity of addition of real numbers.
5. Identity element of addition: The zero matrix is symmetric and acts as the additive identity
6. Additive Inverse: For any symmetric matrix A in S, there exists a symmetric matrix −A in S such that A + (−A) = 0
7. Distributivity of scalar multiplication over addition: For any scalar k and symmetric matrices A and B in S, k(A + B) = kA + kB. This property holds due to the distributivity of scalar multiplication over addition for real numbers.
8. Distributivity of scalar multiplication over scalar addition: For any scalars k and l and symmetric matrix A in S, (k + l) A = kA + lA. This property holds due to the distributivity of scalar multiplication over scalar addition for real numbers.

Since all eight properties are satisfied the set of all real symmetric matrices S forms a vector space under the usual addition of matrices and scalar multiplication.

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Q.16 For each of the following problems calculate the answer over GF(2)

(a) 1 + 1 + 1 + 0
= (1+1) + (1+0)
= 0 + 1
= 1

(b) 1.1 + 0.1 + 0.0 + 1.1
= (1 xOR 1) + (0 xOR 1) + (0 xOR 0) + (1 xOR 1)
= 0 + 1 + 0 + 0
= 1

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(c) $(1+1+1+1)\cdot(1+1+1)$

$= (1\cdot1)+(1\cdot1)+(1\cdot1)+(1\cdot1)$

$= 1+1+1+1$

$= 1$

Sam

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Unit No 1

Q.1 write a python program for addition subtraction multiplication of complex numbers $4+2j$ and $3-6j$.

① $(4+2j)+(3-6j)$
$4+2j+3-6j = 7-4j$

② $(4+2j)*(3-6j)$
$3(4+2j)+(-6j)(4+2j)$
$12+6j-24j-12j^2$
$12+6j-24j-12(-1) = 12+6j-24j+12$
$24-18j$

```
Complex.py
a = 4+2j
b = 3-6j
sum = a+b
Multiplication = a*b
substraction = a-b
print(" Addition of two complex number:", sum);
print(" Multiplication of two complex number:",
        Multiplication)
print("substraction of two complex number:",
        substraction)
```

③ $(4+2j)-(3-6j)$
$4+2j-3+6j = 1+8j$

output
Addition of two complex number : 7-4j
Multiplication of two complex number : 24-18j
Substraction of two complex number : 1+8j

Q.2 Define : Galois Field, Dot product, convex combination, span.

Galois Field : Galois field with two elements is denoted by GF(2). It has two elements 0 and 1. Since it is a field there are two binary operation defined ('+' addition and '·' multiplication)

The behaviour the elements '0' and '1'

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

0+0 = 0   0·0 = 0
0+1 = 1   0·1 = 0
1+0 = 1   1·0 = 0
1+1 = 0   1·1 = 1

Dot product: the dot product is define as the sum of the products of the corresponding entries of the two sequences of number

$A^T = [A_1\ A_2\ A_3]$ $B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$

$u_1 = [u_1, u_2, \dots u_n] \in \gamma$

$v_1 = [v_1, v_2, \dots v_n] \in \gamma$

$u \cdot v_1 = u_1 v_2 + u_2 v_2 + \dots + u_n v_n$

$[A_1\ A_2\ A_3]_{3\times1} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}_{1\times3} = A_1B_1 + A_2B_2 + A_3B_3 = A \cdot B = \sum_{i=1}^{n} u_i v_i$

Convex Combination

An expression of the form $\alpha u + \beta v$ where, $\alpha, \beta \geq 0$, $v \in \gamma$ and $\alpha + \beta = 1$ is called convex combination of u and v

Example

$\alpha [3,2] + [0.5, 1]$

$= \alpha [[3.5, 3] - [0.5, 1]] + [0.5, 1]$

$= \alpha [3.5, 3] - \alpha [0.5, 1] + [0.5, 1]$

$= \alpha [3.5, 3] + (1-\alpha) [0.5, 1]$

$1 - \alpha = \beta$

put $1 - \alpha = \beta$

$\therefore \alpha [3,2] + [0.5, 1] = \alpha [3.5, 3] + \beta [0.5, 1]$

The following is true for any pair of u and v over $\mathbb{R}$. Then u to v line segment consist of the set of convex combination of u and v

Span: the set of all linear combination of vector $v_1, v_2, v_3, \dots v_n$ is called is span of the vectors that is Span$\{v_1, v_2, \dots v_n\}$ =

$$\left\{ \sum_{i=1}^{n} k_i v_i \quad n \in N \ \& \ k_i \in F \right\}$$

where the coefficients $k_1, k_2, \dots k_r$ are scalars.

n	
1	$k_1 v_1$
2	$k_1 v_1 + k_2 v_2$
3	$k_1 v_1 + k_2 v_2 + k_3 v_3$

Q.3 Write a python program to find the conjugate of complex number

The numpy.conj() function helps the user to conjugate any complex number. The conjugate of a complex number is obtained by changing the sign of its imaginary part. if the complex number is 2+5j then its conjugate is 2-5j

```
import numpy as np
Complex1 = 2+4j
Complex2 = 5-8j
out-complex1 = np.conj(complex1)
out-complex2 = np.conj(complex2)
```

print("output conjugated complex number of 2+4j:",
out-complex1)
print("output conjugated complex number of 5-8j:",
out-complex2)

output

output conjugated complex number of 2+4j : 2-4j
output conjugated complex number of 5-8j : 5+8j

Q.4 Express the following complex number in polar form and exponential form : $1+i\sqrt{3}$

$z = 1+i\sqrt{3}$

polar
$z = r(\cos\theta + i\sin\theta)$
$r = |z|$
θ = Amplitude

exponent
$z = re^{i\theta}$

$z = x+iy$
$x=1 \quad y=\sqrt{3}$

$|z| = \sqrt{x^2+y^2}$
$= \sqrt{1+3}$
$|z| = 2$
$|z| = r = 2$ —— (1)

	0°	30° ($\frac{\pi}{6}$)	45° ($\frac{\pi}{4}$)	60° ($\frac{\pi}{3}$)	90° ($\frac{\pi}{2}$)
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$\pi - \tan^{-1}\left\|\frac{y}{x}\right\|$	$\tan^{-1}\left\|\frac{y}{x}\right\|$
$-\pi - \tan^{-1}\left\|\frac{y}{x}\right\|$	$-\tan^{-1}\left\|\frac{y}{x}\right\|$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$\theta = \frac{\pi}{3}$

$z = r(\cos\theta + i\sin\theta)$
$= 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ polar form

$z = re^{i\theta}$
$z = 2e^{i\pi/3}$

Q.5 Find the square root of the following complex number $8-6i$

Let $\sqrt{8-6i} = a+bi \quad a,b \in R$
squaring on both sides we get
$8-6i = (a+bi)^2$
$8-6i = a^2+b^2i^2+2abi$
$8-6i = (a^2+b^2(-1)) + 2abi \quad [\because i^2=-1]$
$8-6i = (a^2-b^2) + 2abi$

Equating real and imaginary parts, we get
$a^2-b^2 = 8$ and $2ab = -6$
$a^2-b^2 = 8$ and $b = -\frac{3}{a}$

$\therefore a^2 - \left(-\frac{3}{a}\right)^2 = 8$

$\therefore a^2 - \frac{9}{a^2} = 8$

$\therefore \frac{a^4 - 9}{a^2} = 8$

$\therefore a^4 - 9 = 8a^2$

$\therefore a^4 - 8a^2 - 9 = 0$

$\therefore a^4 - 9a^2 + a^2 - 9 = 0$

$\therefore a^2(a^2-9) + 1(a^2-9) = 0$

$(a^2+1) = 0 \quad \& \quad (a^2-9) = 0$

$a^2 = 9 \qquad a^2 = -1$

$a^2 = \pm 3 \qquad a^2 \neq -1 \quad (a \in R)$

when $a = 3$, $b = \frac{-3}{3}$ $b = -1$

$a = -3$, $b = \frac{-3}{-3}$ $b = 1$

$\therefore \boxed{\sqrt{8-6i} = \pm(3-1i)}$

Q.6 Find the square root of complex number $-5+12i$

Let $\sqrt{-5+12i} = a+bi$ where $a, b \in R$

squaring on both sides

$-5+12i = (a+bi)^2$

$-5+12i = a^2 + b^2i^2 + 2abi$

$-5+12i = a^2 + b^2(-1) + 2abi$

$-5+12i = (a^2-b^2) + 2abi$

Equating real and imaginary parts, we get:

$a^2 - b^2 = -5 \quad \& \quad 2ab = 12$

$b = \frac{12}{2a}$

$a^2 - \left(\frac{6}{a}\right)^2 = -5$

$b = \frac{6}{a}$

$a^2 - \frac{36}{a^2} = -5$

$a^4 - 36 = -5a^2$

$a^4 + 5a^2 - 36 = 0$

$a^4 + 9a^2 - 4a^2 - 36 = 0$

$a^2(a^2+9) - 4(a^2+9) = 0$

$(a^2+9) = 0 \quad \& \quad (a^2-4) = 0$

$a^2 = -9 \qquad a^2 = 4$

$a^2 \neq -9 \quad a \in R \qquad a^2 = 4$

$\boxed{a = \pm 2}$

when $a = 2$, $b = \frac{6}{2} = 3$

$a = -2$, $b = \frac{6}{-2} = -3$

$\boxed{\therefore \sqrt{-5+12i} = \pm(2+3i)}$

Q.7 Find the square root of $21-20i$, where $i = \sqrt{-1}$

$\sqrt{21-20i} = (a+bi)$

Squaring both side

$21 - 20i = a^2 + b^2i^2 + 2abi$

$21 - 20i = a^2 + b^2i^2 + 2abi \quad [i^2 = -1]$

$21 - 20i = (a^2 - b^2) + (2abi)$

$a^2 - b^2 = 21$ & $2ab = -20$

$a^2 - \left(\frac{-10}{a}\right)^2 = 21$ $\qquad b = \frac{-\cancel{20}\ 10}{\cancel{2}a}$

$a^2 - \frac{100}{a^2} = 21$ $\qquad b = \frac{-10}{a}$

$\frac{a^4 - 100}{a^2} = 21$

$a^4 - 100 = 21a^2$

$a^4 - 21a^2 - 100 = 0$ $\qquad -100 \to -25 + 4$

$a^4 - 25a^2 + 4a^2 - 100 = 0$

$a^2(a^2-25) + 4(a^2-25) = 0$

$(a^2-25) = 0$ & $(a^2+4) = 0$

$a^2 = 25$ $\qquad a^2 \neq -4$

$a = \pm 5$ $\qquad a \in R$

when $a = 5$, $b = \frac{-10}{5} = -2$

$a = -5$, $b = \frac{-10}{-5} = 2$

$\sqrt{21 - 20i} = \pm(5 - 2i)$

Q.8 Express $[(3+2i)/(2+i)(1-3i)]$ in the form $x+iy$

$\frac{(3+2i)}{(2+i)(1-3i)} = \frac{(3+2i)}{(2+i)(1-3i)}$

$= \frac{(3+2i)}{2+(-6i)+i+(-3i^2)}$

$= \frac{(3+2i)}{2-6i+i-3i^2}$ $\quad \because i = -1$

$= \frac{(3+2i)}{2-6i+i+3}$

$= \frac{(3+2i)}{(5+5i)} \times \frac{5+5i}{5+5i}$

$= \frac{(3+2i)(5+5i)}{(5)^2-(5i)^2}$

$= \frac{15+15i+10i+10i^2}{25+25}$

$= \frac{15+15i+10i+10(-1)}{50}$

$= \frac{15+15i+10i-10}{50}$

$= \frac{5}{50} + \frac{25i}{50}$

$= \frac{5}{50} + \frac{25i}{50}$

$(x+iy) = \frac{5}{50} + i\frac{25}{50}$

Q.9 Solve the following system by backward substitution method.

$1x_1 - 3x_2 - 2x_3 = 7$ — ①

$2x_2 + 4x_3 = 4$ — ②

$-10x_3 = 12$ — ③

Matrix notation

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 12 \end{bmatrix}$$

Now $-10x_3 = 12$

$x_3 = \frac{-12}{10}$

$$\boxed{x_3 = \frac{-6}{5}}$$

$x_3 = \frac{-6}{5}$ put in equation ②

$2x_2 + 4x_3 = 4$

$2x_2 + 4x_3 = 4$

$2x_2 = 4 - 4x_3$

$x_2 = \frac{4 - 4x_3}{2}$

$x_2 = \frac{4 - 4\left[-\frac{6}{5}\right]}{2}$

$= \frac{4 + 4\left(\frac{6}{5}\right)}{2}$

$= \frac{\cancel{2}\left(2 + 2\left(\frac{6}{5}\right)\right)}{\cancel{2}}$

$= 2 + \frac{2 \times 6}{5}$

$= 2 + \frac{12}{5} = \frac{10 + 12}{5} = \frac{22}{5}$

$$\boxed{x_2 = \frac{22}{5}}$$

$x_2 = \frac{22}{5}$ put in equation ①

$1x_1 - 3x_2 - 2x_3 = 7$

$x_1 - 3\left(\frac{22}{5}\right) - 2\left(\frac{-6}{5}\right) = 7$

$x_1 - \frac{66}{5} + \frac{12}{5} = 7$

$x_1 - \frac{54}{5} = 7$

$x_1 = 7 + \frac{54}{5}$

$x_1 = \frac{35 + 54}{5}$

$$\boxed{x_1 = \frac{89}{5}}$$

$x_1 = \frac{89}{5}, \; x_2 = \frac{22}{5}, \; x_3 = -\frac{6}{5}$

Q.10 Write a python program to solve system of linear equations given below.

$1x_1 - 3x_2 - 2x_3 = 7$ — ①

$2x_2 + 4x_3 = 4$ — ②

$-10x_3 = 12$ — ③

```
import numpy as np
a= np.array([[1,-3,-2],[0,2,4],[0,0,-10]])
print("Array will be :",a)
b= np.array([7,4,12])
print("Input will be array:",b)
```

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```
C = np.linalg.solve(a,b)
print("output:")
print(C)
```

Output

Array will be :
[[1 -3 -2]
 [0 2 4]
 [0 0 -10]]
Input of array will be :
[7 4 12]
Output:
[17.8 4.4 -1.2]

Q.11 Determine whether $v_1 = (2,2,2)$, $v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span vector space $\mathbb{R}^3$

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Q.12 write a python Program to rotating a complex number z = 2+3i by 180°

program:

```
import cmath
z = 2+3j
rotated-180 = z*(-1)
print(" original complex number :", z)
print(" Rotated complex number by 180 degress:",
        rotated-180)
```

Output

Original complex number : (2+3j)
Rotated complex number by 180 degrees: (-2-3j)

Q.13 write a Python Program to rotated a complex no by 90°, 180°, and 270°

program:

```
import cmath
z = 2+3j
rotated-90 = z*(1j)
rotated-180 = z*(-1)
rotated-270 = z*(-1j)
print ("original complex number :", z)
print ("Rotated complex number by 90 degrees:",
        rotated-90)
print ("Rotated complex number by 180 degrees :",
        rotated-180)
print ("Rotated complex number by 270 degrees:",
        rotated-270).
```

$R \pm (kR)$

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Output

Original complex number : (2+3j)
Rotated complex number by 90 degrees : (-3+2j)
Rotated complex number by 180 degrees : (-2-3j)
Rotated complex number by 270 degrees : (3-2j)

Q.14 Express the following as a linear combination of $r_1 = (-2,1,3)$, $r_2 = (3,1,-1)$ and $r_3 = (-1,-2,1)$ with $w = (6,-2,5)$

Solution ⇒

$(6,-2,5) = x(-2,1,3) + y(3,1,-1) + z(-1,-2,1)$

$(6,-2,5) = (-2x, x, 3x) + (3y, y, -y) + (-z, -2z, z)$

$6 = -2x + 3y - z$

$-2 = x + y - 2z$

$5 = 3x - y + z$

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & 1 & -2 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$

$$\begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & -5 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ 5 \end{bmatrix}$$

$R_3 \to 2R_3 + 3R_2$

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$$\begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & -5 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -20 \end{bmatrix}$$

$R_3 \longrightarrow R_3 + R_2$

$$\begin{bmatrix} -2 & 3 & -1 \\ 0 & -1 & -5 \\ 0 & 0 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -30 \end{bmatrix}$$

$-6z = -30$

$z = \frac{-30}{-6}$

$\boxed{z = 5}$

$-y - 5z = -10$

$-y - 5(5) = -10$

$-y = -10 + 25$

$\boxed{y = -15}$

$-2x + 3y - z = 6$

$-2x + 3(-15) - 5 = 6$

$-2x - 45 - 5 = 6$

$-2x - 50 = 6$

$-2x = 6 + 50$

$x = \frac{-56}{2}$

$\boxed{x = -28}$

$(6, -2, 5) = -28(-2, 1, 3) - 15(3, 1, -1) + 5(-1, -2, 1)$

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Q.1 Are the following vectors are linearly dependent
$r_1 = (3,2,7)$, $r_2 = (2,4,1)$ and $r_3 = (1,-2,6)$

Solution:
lets take vector $r_1 = (3,2,7)$, $r_2 = (2,4,1)$, $r_3 = (1,-2,6)$
Linear combination of vector,

$c_1 r_1 + c_2 r_2 + c_3 r_3 = 0$

$c_1(3,2,7) + c_2(2,4,1) + c_3(1,-2,6) = 0$

$c_1 3 + c_2 2 + c_3 = 0$

$c_1 2$

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Q.2 Are the following vectors are linearly dependent
$r_1 = (3,2,7)$, $r_2 = (2,4,1)$ and $r_3 = (1,-2,6)$

set of vectors $r_1 = (3,2,7)$, $r_2 = (2,4,1)$, $r_3 = (1,-2,6)$
linear combination of vector is

$c_1(3,2,7) + c_2(2,4,1) + c_3(1,-2,6) = 0$

$3c_1 + 2c_2 + c_3 = 0$

$2c_1 + 4c_2 + (-2)c_3 = 0$

$7c_1 + c_2 + 6c_3 = 0$

$3x + 2y + z = 0$ —— ①

$2x + 4y - 2z = 0$ —— ②

$7x + y + 6z = 0$ —— ③

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

lets take determinant of above matrix

$$3\begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ 7 & 6 \end{bmatrix} + 1\begin{bmatrix} 2 & 4 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{bmatrix} = 3\begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ 7 & 6 \end{bmatrix} + 1\begin{bmatrix} 2 & 4 \\ 7 & 1 \end{bmatrix}$$

$= 3((4\times6) - (-2\times1)) - 2(12 - (-2\times7)) + 1(2 - 28)$

$= 3(24+2) - 2(12+14) + 1(-26)$

$= 3(26) - 2(26) + 1(-26)$

$= 78 - 78$

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determinant of the matrix is 0

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{bmatrix} = 0$$

A set of vectors $\{v_1, v_2, v_3 \dots v_n\}$ is set to be linearly dependant if $c_1v_1 + c_2v_2 + \dots + c_nv_n = 0$ are prove.

In above example determinat of the matrix are zero so $v_1 = \{3, 2, 7)$, $v_2 = (2, 4, 1)$ and $v_3 = (1, -2, 6)$ are linearly dependant

Q.3 Check whether the vectors are linearly dependent $v_1 = (1, -2, 1)$, $v_2 = (2, 1, -2)$, and $v_3 = (7, -4, 1)$

Solution:

Lote vector $v_1 = (1, -2, 1)$ $v_2 = (2, 1, -2)$, $v_3 = (7, -4, 1)$

the linear combination of the vectors

$$c_1v_1 + c_2v_2 + c_3v_3 = 0$$

$$x(1, -2, 1) + y(2, 1, -2) + z(7, -4, 1) = 0$$

$$x + 2y + 7z = 0 \quad \text{--- (1)}$$

$$-2x + y + -4z = 0 \quad \text{--- (2)}$$

$$x - 2y + 1z = 0 \quad \text{--- (3)}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 1 & -4 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

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$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 5 & -10 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 5 & -10 \\ 0 & -4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

R.

Second method.

$$\begin{vmatrix} 1 & 2 & 7 \\ -2 & 1 & -4 \\ 1 & -2 & 1 \end{vmatrix} = 1\begin{vmatrix} 5 & -10 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & -10 \\ 1 & 1 \end{vmatrix} + 7\begin{vmatrix} 0 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= 1[5-20] - 2[0+10] + 7[0-5]$$

$$= -15 - 20 - 35$$

$$= -70$$

determinant of the matrix $= -70$

A set of vector $\{v_1, v_2, v_3 \dots v_n\}$ is set to be linearly dependant if $c_1v_1 + c_2 + v_2 + \dots + c_nv_n = 0$ here determinant of the vectors non-zero it is linearly independant.

$\therefore$ In above example determinat of the matrix of vector are non-zero so $v_1 = (1, -2, 1)$, $v_2 = (2, 1, -2)$ and $v_3 = (7, -4, 1)$ are linearly independant

Q.13 Determine whether $v_1 = (2,2,2)$, $v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span vector space R_3

Solution:

Lets take vector $v_1 = (2,2,2)$, $v_2 = (0,0,3)$, $v_3 = (0,1,1)$

Linear combination of the vector is the

$$c_1v_1 + c_2v_2 + c_3v_3 = 0$$

$\therefore$ here c_1, c_2, c_3 are scalar of vector

$$x(2,2,2) + y(0,0,3) + z(0,1,1) = 0$$
$$2x + 0 + 0 = 0$$
$$2x + z = 0$$
$$2x + 3y + z = 0$$

$2x = 0$ —— ①

$2x + z = 0$ —— ②

$2x + 3y + z = 0$ —— ③

First method

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$z = 0$

$2x = 0 \quad x = 0$

$3y + z = 0$

$y = 0$

$x = 0,\ y = 0,\ z = 0$

here scalar $c_1 = 0$, $c_2 = 0$, $c_3 = 0$ so vectors are linearly independent $\therefore$ & mainly they are ~~not~~ span $\mathbb{R}^3$

Second method

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix}$$

$$= 2(0-3) - 0(2-2) + 0(6-0)$$

$$= -6$$

$\therefore$ Here Determinant of matrix non-zero so vector are linearly independent, they are span $\mathbb{R}^3$

Q.14 Show that vector $v_1 = (1,0,1)$, $v_2 = (2,1,4)$ and $v_3 = (1,1,3)$ do not span vector space.

solution:

lets take vector $v_1 = (1,0,1)$, $v_2 = (2,1,4)$, $v_3 = (1,1,3)$ linear combination of vector.

$c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1(1,0,1) + c_2(2,1,4) + c_3(1,1,3) = 0$

$1c_1 + 2c_2 + c_3 = 0$

$c_2 + c_3 = 0$

$c_1 + 4c_2 + 3c_3 = 0$

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 4 \end{vmatrix}$$

$= 1[3-4] - 2[0-1] + 1[0-1]$

$= 1(-1) - 2(-1) + 1(-1)$

$= -1 + 2 - 1$

$= 1 - 1$

$= 0$

here determinant of vector matrix is zero the vectors are linearly dependant & mainly they are Not span vector space.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \text{dependant}$$

Q.15 Express the following as a linear combination of $v_1 = (-2,1,3)$, $v_2 = (3,1,-1)$ and $v_3 = (-1,-2,1)$ with $w = (6,-2,5)$

Given:

$v_1 = (-2,1,3)$, $v_2 = (3,1,-1)$, $v_3 = (-1,-2,1)$

$w = (6,-2,5)$

a vector v is said to be linear combination of vector $v_1, v_2, v_3, \dots v_n$ if v can be expressed as

$v = k_1v_1 + k_2v_2 + \dots + k_nv_n$

Here, we can represent it as,

$w = av_1 + bv_2 + cv_3$

where, a, b, c are the scalars we get equation as

$-2a + 3b - c = 6$ — (I)

$a + b - 2c = -2$ — (II)

$3a - b - c = 5$ — (III)

we can rearrange the equation to express a, b, c in terms of one variable.

Now, let's rearrange eqn (II)

$-2 = a + b - 2c$

$a = -b + 2c - 2$ — (V)

Now substituting this in eqn (I)

$-2a + 3b - c = 6$

$-2(-b + 2c - 2) + 3b - c = 6$

$2b - 4c + 4 + 3b - c = 6$

$5b - 5c = 2$

$b - c = 2/5$

$b = c + 2/5$ — (IV)

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Now we substitute the value of a and b in eqn (III)

$3a - b - c = 5$

$3(-b + 2c - 2) - b - c = 5$

$-3b + 6c - 6 - b - c = 5$

$-4b + 7c = 5 + 6$

$-4b + 7c = 11$

$-4\left(c + \frac{2}{5}\right) + 7c = 11$

$-4c - \frac{8}{5} + 7c = 11$

$3c - \frac{8}{5} = 11$

$3c = 11 + \frac{8}{5}$

$3c = \frac{55 + 8}{5}$

$3c = \frac{63}{5}$

$c = \frac{63}{3 \times 5}$

$c = \frac{63}{15}$

$\boxed{c = \frac{21}{5}}$

Now put $c = 21/5$ in equn. IV

$b = c + 2/5$

$b = \frac{21}{5} + \frac{2}{5}$

$b = 23/5$

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Substitute value of a and b in eqn V

$a = -b + 2c - 2$

$a = -\left(\frac{23}{5}\right) + 2\left(\frac{21}{5}\right) - 2$

$= -\frac{23}{5} + \frac{42}{5} - 2$

$= \frac{-23 + 42 - 10}{5}$

$= \frac{-33 + 42}{5}$

$a = \frac{9}{5}$

Hence the ω can be expressed as.

$\omega = ax_1 + bx_2 + cx_3$

$\omega = \frac{9}{5}x_1 + \frac{23}{5}x_2 + \frac{21}{5}x_3$

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Unit. No. 2

Q.1 Define: Indentity matrix, Symmetric Matrix, Null Space, Inner Product, Outer product, Forest Spanning Subgraph, Spanning subgraph. Cycle, path, Basis, Row rank of Matrix Column Rank of matrix

① Indentity matrix :

An identity matrix is a square matrix in which all the elements of principal diagonals are ~~Tzre~~ one, and all other elements are zeros.

It is denoted by the notation (I)

Example :

I_n (or) $I = [a_{ij}]_{n \times n}$, where

$$[a_{ij}] = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

Example :

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② Symmetric Matrix

A symmetric matrix is a matrix that is equal to its transpose.

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 6 & 5 & 9 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 6 & 5 & 9 \end{bmatrix}$$

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③ Null Space

the null space of matrix a is set of all those vector γ such that $A * \gamma = 0$ and the set is denoted by null Null(A).

④ Inner Product

$[u = [1, 4, 0], \; v = [2, 2, 1]]$

let u and v are D vector then $u^t \cdot v$ is known as inner product of two vector and it is denoted by Two $\langle u, v \rangle$

Inner product $= u^t \cdot v$

$$= \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$$

Example

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 0 & 3 \end{bmatrix}_{3 \times 2}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2 \times 1}$$

$$AB = \begin{bmatrix} (1)(2) + (1)(1) \\ (2)(2) + (4)(1) \\ (0)(2) + (3)(1) \end{bmatrix} = \begin{bmatrix} 2 + 1 \\ 4 + 4 \\ 0 + 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 3 \end{bmatrix}$$

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④ outerproduct

outerproduct of two vectors let u and v are D-vector then $u \cdot v^t$ is known as outer product of two vectors.

$u \cdot [1 \quad 4 \quad 0]$

$v \cdot [2 \quad 2 \quad 1]$

outer product : $u \cdot v^t$

$$= [1 \quad 4 \quad 0] \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

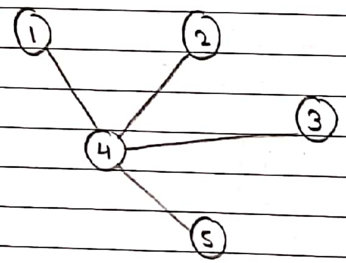
$= 1\times 2 + 4\times 2 + 0\times 1$

$= 2+8+0$

$= 10$

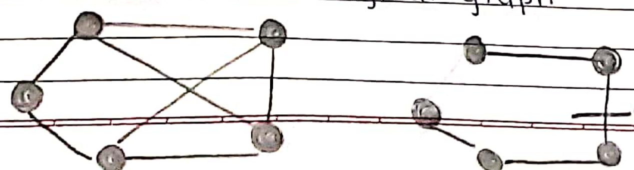
⑤ Forest

Forest. A forest is an undirected graph in which any two vertices are connected by at most one path.



⑥ Spanning subgraph

A spanning subgraph is a subgraph that contains all the vertices of the original graph.



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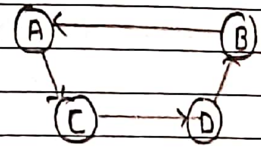
⑦ Cycle

In graph theory, a cycle in a graph is a non-empty trail in which only the first and last vertices are equal.



⑧ Basis

Let v be a vector space, the set $B = \{v_1, v_2, v_3, \dots v_n\}$ is the basis of vector space v [span the set of all linear combinations of vectors in s]

⑨ Row rank of Matrix

The row rank of a matrix is the maximum number of rows, thought of as vectors, which are linearly independent

⑩ Column rank of Matrix

the column rank is the maximum number of column which are linearly independent.

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Q.2 Find the co-ordinate representation of $v = [1,3,5,3]$ in terms of $a_1 = [1,1,0,0]$, $a_2 = [0,1,1,0]$, $a_3 = [0,0,1,1]$

$$v = \alpha_1[1,1,0,0] + \alpha_2[0,1,1,0] + \alpha_3[0,0,1,1]$$

where $\alpha_1, \alpha_2, \alpha_3$ are elements of $GF(2)$

$$v = \alpha_1, \alpha_1+\alpha_2, \alpha_2+\alpha_3, \alpha_3$$

$$[1,3,5,3] = \alpha_1, \alpha_1+\alpha_2, \alpha_2+\alpha_3, \alpha_3$$

We can rewrite this equation as a system of linear equations:

$\alpha_1 = 1$

$\alpha_1+\alpha_2 = 3$

$\alpha_2+\alpha_3 = 5$

$\alpha_3 = 3$

Solving this system, we find:

$\boxed{\alpha_1 = 1}$, $\boxed{\alpha_3 = 3}$, $\alpha_2 + 3 = 5$

$\alpha_2 = 5-3$

$\boxed{\alpha_2 = 2}$

So, the coordinate representation of v in terms of the given vectors is $[1 \; 2 \; 3]$ over $GF(2)$

Q.3 Find the co-ordinate representation of $v = [0,0,0,1]$ in term of the vector $[1,1,0,1]$, $[0,1,0,1]$ and $[1,1,0,0]$ in $GF(2)$

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$$v = \alpha_1[1,1,0,1] + \alpha_2[0,1,0,1] + \alpha_3[1,1,0,0]$$

$$v = \alpha_1+\alpha_3, \alpha_1+\alpha_2+\alpha_3, \alpha_1+\alpha_2$$

For First component:

$0 = \alpha_1 + 0 + \alpha_3$

$\alpha_1 = \alpha_3$

For second component:

$0 = \alpha_1+\alpha_2+\alpha_3$

$\alpha_2 = \alpha_1+\alpha_3$

For Fourth component

$1 = \alpha_1+\alpha_2+0$

$1 = \alpha_1 + (\alpha_1+\alpha_3)$

$1 = \alpha_1+\alpha_1+\alpha_1$

$1 = \alpha_1$

so, $\alpha_1 = 1$, and therefore $\alpha_3 = \alpha_1 = 1$

$\alpha_2 = \alpha_1+\alpha_3$

$= 1+1$

$\boxed{\alpha_2 = 0}$

therefore, the coordinate representation of v in terms of the given vectors is $(1,0,1)$ in $GF(2)$

Q.4 write a program in python to multiply two matrices using nested loops

```
#Multiplication of two matrix
x = [[2,4,6],
     [8,3,2],
     [1,4,5]]
print("Matrix x is:")
For i in x :
    print(i)

Y=[[1,2,3,4],[5,6,7,8],[9,4,3,2]]
print("Matrix Y is :")
For i in Y :
    print(i)

# Two matrix multiplication.
# 3x4
M=[[0,0,0,0],[0,0,0,0],[0,0,0,0]]
print(M)
Foriinrange(len(x)):
    For j in range(len(Y[0])):
        For k in range(len(Y)):
            M[i][j] = x[i][k] * Y[k][j]

For r in M:
    print(r)
```

$$x = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 3 & 2 \\ 1 & 4 & 5 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 4 & 3 & 2 \end{bmatrix}$$

3x3   3x4

Output:

```
[[76 64 52 52]
 [41 42 51 60]
 [66 46 46 46]
```

Q.5 write python code to print diagonal matrix with diagonal elements [1,2,3,4]

```
def print-diagonal-matrix(Diagonal-elements):
    n= len(Diagonalelements)
    For i in range(n):
        For j in range(n):
            if i==j:
                print(diagonal-elements[i],
                      end=" ")
            else
                print(0, end=" ")
        print()

Diagonal-elements = [1,2,3,4]
print-diagonal-matrix(diagonal-elements)
```

Output

```
1 0 0 0
0 1 0 0
0 0 3 0
0 0 0 4
```

Q.6 Find the null space of matrix A = [[1 5 6]
[2 6 8]
[3 4 7]]

$$A = \begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 3 & 4 & 7 \end{bmatrix}$$

mathematical representation matrix Null space

$Null(A): A \cdot v = 0$

$$\begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 3 & 4 & 7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 5 & 6 \\ 0 & -4 & -4 \\ 3 & 4 & 7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 5 & 6 \\ 0 & -4 & -4 \\ 0 & -11 & -11 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2/-4$

$$\begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & -11 & -11 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3/-11$

$$\begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_2 + v_3 = 0$

$\boxed{v_2 = -v_3}$

$v_2 = -v_3$ put in $v_1 + 5v_2 + 6v_3 = 0$

$v_1 + 5(-v_3) + 6v_3 = 0$

$v_1 - 5v_3 + 6v_3 = 0$

$v_1 + v_3 = 0$

$\boxed{v_1 = -v_3}$

$v = [v_1 \ v_2 \ v_3]$

$= [-v_3, -v_3 \ v_3]$

$= v_3[-1 \ -1 \ 1]$

$\therefore Null(A) = \{v \mid v_3[-1 \ -1 \ 1]\}$

Q.7 write dot product defination of matrix-vector multiplication with an example.

dot product defination : multiplication involves taking the dot product of each row of the matrix with the vector.

Mathematical representation :

Given a A of size $m \times n$ and a vector x of size $n \times 1$, the matrix-vector multiplication is defined as :

$$A \cdot x = \underset{3\times 3}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$$

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$$A \cdot x = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$$

Example

```
import numpy as np

A = np.array([[1, 2, 3],
              [4, 5, 6],
              [7, 8, 9]])
x = np.array([1, 2, 3])
result = np.dot(A, x)
print("Result of matrix-vector multiplication:")
print(result)
```

Output:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times1+2\times2+3\times3 \\ 4\times1+5\times2+6\times3 \\ 7\times1+8\times2+9\times3 \end{bmatrix} = \begin{bmatrix} 1+4+9 \\ 4+10+18 \\ 7+16+27 \end{bmatrix} = \begin{bmatrix} 14 \\ 32 \\ 50 \end{bmatrix}$$

Result of matrix-vector multiplication:
[14 32 50]

Q.8 Writ dot product definition of vector-matrix multiplication with an example

dot product definition of vector-matrix multiplication:
dot product involves taking dot product of the vector with each colum of matrix

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mathematical representation
Given vector x size $1 \times m$ and matrix will M size will be $n \times m$

$$x \cdot A = \underset{3\times1}{[x_1 \; x_2 \; x_3]}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$x \cdot A = [x_1 a_{11} + x_2 a_{12} + x_3 a_{13} \; \ldots \; x_1 a_{31} + x_2 a_{32} + x_3 a_{33}]_{3\times3}$$

Example

```
import numpy as np

x = np.array([1, 2, 3])
A = np.array([[1, 2, 3],
              [4, 5, 6],
              [7, 8, 9]])

result = np.dot(x, A)
print("Result of vector-matrix multiplication:")
print(result)
```

Output

$$\underset{3\times1}{[1, 2, 3]}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = [1\times1+2\times4+3\times7 \quad 1\times2+2\times5+3\times8 \quad 1\times3+2\times6+3\times9]$$

$$= [1+8+21 \quad 2+10+24 \quad 3+12+27]$$

$$= \underset{3\times3}{[30 \quad 36 \quad 42]}$$

Result of vector-matrix multiplication:
[30 36 42]

Q.9 write a python code to check whether a given matrix M = [[1,9,5],[3,2,4],[5,4,1]] is a symmetrix matrix

```
def is-symmetric(matrix):
        rows = len(matrix)
        cols = len(matrix[0])

        if rows != cols:
            return False

        for i in range(rows):
            for j in range(cols):
                if matrix[i][j] != matrix[j][i]:
                    return False
        return True.

M = [[1, 3, 5],
     [3, 2, 4],
     [5, 4, 1]]
if is-symmetric(M):
        print("Matrix M is symmetric.")
else :
        print("Matrix M is not symmetric.")
```

Q-10 Find the dimension of the vector space spanned by vectors (1,1,-2,0,1), (1,2,0,-4,1), (0,1,3,-3,2), (2,3,0,-2,0) and also find the basis

$v_1 = (1,1,-2,0,-1)$ $v_3 = (0,1,3,-3,2)$
$v_2 = (1,2,0,-4,1)$ $v_4 = (2,3,0,-2,0)$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}$$
$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 3 \\ -2 & 0 & 3 & 0 \\ 0 & -4 & -3 & -2 \\ -1 & 1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$.

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ -2 & 0 & 3 & 0 \\ 0 & -4 & -3 & -2 \\ -1 & 1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_1$
$R_5 \to R_5 + R_1$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & -4 & -3 & -2 \\ 0 & 2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_5 \to R_5 - R_3$
$R_4 \to R_4 + 2R_3$
$R_3 \to R_3 - 2R_2$
$R_5 \to R_5 + R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 9 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$R_4 \to R_4 - 3R_3$

Pivot element

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Rank of matrix = no of pivot element

It is define as the dimention of span of s where s = set of v) it is denoted by rank(s)

$\therefore$ basis = $\{v_1, v_2, v_3\}$

$\therefore$ dimensions of vector space = 3 = Rank

11. Check whether the set of function are linearly independent?

$2 - x + 4x^2,\ 3 + 6x + 2x^2,\ 2 + 10x - 4x^2$

$2 - x + 4x^2,$

$3 + 6x + 2x^2$

$2 + 10x - 4x^2$

1. $2a + 3b + 2c = 0$ (coefficients of constant terms)
2. $-a + 6b + 10c = 0$ (coefficients of linear terms)
3. $4a + 2b - 4c = 0$ (coefficients of quadratic terms)

$$\begin{bmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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Find determinal of matrix

$$\begin{vmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{vmatrix} = 2\begin{vmatrix} 6 & 10 \\ 2 & -4 \end{vmatrix} - 3\begin{vmatrix} -1 & 10 \\ 4 & -4 \end{vmatrix} + 2\begin{vmatrix} -1 & 6 \\ 4 & 2 \end{vmatrix}$$

$= 2(-24-20) - 3(4-40) + 2(-2-4)$

$= 2(-44) + 3(36) + 2(-6)$

$= -88 + 108 - 12$

$= 8$

determinat of matrix is non-zero above function are linearly ~~dependent~~ independent

Q.12 Show that vector $\{(1,2,1), (2,1,0), (1,-1,2)\}$ of $\mathbb{R}^3$ form a basic of $\mathbb{R}^3$

To show that the vector $\{(1,2,1), (2,1,0), (1,-1,2)\}$ form a basis of $\mathbb{R}^3$, we need to demonstrate two things:

(1) They span $\mathbb{R}^3$

(2) They are linearly independent

To check linear independence, we set up the equation:

$a(1,2,1) + b(2,1,0) + c(1,-1,2) = (0,0,0)$

This gives us the system of equation

$a + 2b + c = 0$

$2a + b - 1c = 0$

$a + 2c = 0$

$a + 2c = 0$

$\boxed{a = -2c}$

$2a + b - c = 0$

$-4c + b - c = 0$

$b - 5c = 0$

$\boxed{b = 5c}$

$a + 2b + c = 0$

$-4c + 2(5c) + c = 0$

$-4c + 10c + c = 0$

$6c + c = 0$

$7c = 0$

$\boxed{c = 0}$

$a = 0,\ b = 0,\ c = 0$, the vector are linearly independent, mainly it is Span $\mathbb{R}^3$ above both condition are meet so it is Span vectors form a basis of $\mathbb{R}^3$

Q.13 prove that, for any vector $v \in V$; there is exactly one representation of v in terms of the basis vectors. If $a_1, a_2, \dots a_n$ be a basis for a vectorspace V.

prove

Late $v \in V$ $B = \{a_1, a_2, \dots a_n\}$

$v = \alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_n a_n$ — ①

Assume, those is another representation

$v = \beta_1 a_1 + \beta_2 a_2 + \dots + \beta_n a_n$ — ②

$v - v = \beta_1 a_1 - \alpha_1 a_1 + \beta_2 a_2 - \alpha_2 a_2 + \dots + \beta_n a_n - \alpha_n a_n$

$0 = a_1(\beta_1 - \alpha_1) + a_2(\beta_2 - \alpha_2) + \dots a_n(\beta_n - \alpha_n)$

By comparing

$0 = \beta_1 - \alpha_1$ $\quad$ $0 = \beta_2 - \alpha_2$ $\quad$ $0 = \beta_n - \alpha_n$

$\beta_1 = \alpha_1$ $\quad$ $\beta_2 = \alpha_2$ $\quad$ $\beta_n = \alpha_n$.

Thus there exactly one unique representation of $v \in V$ in terms of Basis.

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deter $= 0 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ dependant

dete $\neq 0 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ independant

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ i, j, k must

given set of vector set of generators of $\mathbb{R}^3$ if it satisfies following creterity

1. set must of have at least 3 vector
2. It have i, j, k (represent dimention
3. Set must be linearly independant.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{dependent}$$