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i] Gaussion elimination

$v - w = 3$

$-2u + 4v - w = 1$

$-2u + 5v - 4w = -2$

Element in 1st column ∴ performing operⁿ with 1st row

$$\begin{bmatrix} 0 & 1 & -1 \\ -2 & 4 & -1 \\ -2 & 3 & -4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

Element in 1st column ∴ performing operⁿ with 1st row

$$\begin{bmatrix} -2 & 4 & -1 \\ 0 & 1 & -1 \\ -2 & 5 & -4 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

Element in 2nd column ∴ performing operⁿ with 2nd row

$$\begin{bmatrix} -2 & 4 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

$-2 - (-2) = 0$

~~$5 - 4 =$~~

$-4 - (-1) = -4 + 1 = -3$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} -2 & 4 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

$-3 - (-1) = -3 + 1$

$-3 - 3$

∴ $-2u + 4v - w = 1$

$v - w = 3$

$-2w = -6$

∴ $w = 3$

$v - 3 = 3$

$v = 6$

$-2u + 4(6) - 3 = 1$

$2u = 24 - 3 - 1$

$u = 10$

2) Gaussian eliminaⁿ

$$x+y+z=1$$
$$x+2y+2z=1$$
$$x+2y+3z=1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad x+y+z=1$$
$$y+z=0$$
$$\underline{\underline{z=0}}$$

$\therefore \quad y+0=0$ $\qquad\qquad$ $\therefore \quad x+0+0=1$

$\therefore \underline{\underline{y=0}}$ $\qquad\qquad$ $\underline{\underline{x=1}}$

5) Gaussian elimina^n

$4y - 3z = 3$
$-x + 7y - 5z = 4$
$-x + 8y - 6z = 5$

$\cancel{0}$ $\cancel{4y}$ -

$$\begin{bmatrix} 0 & 4 & -3 \\ -1 & 7 & -5 \\ -1 & 8 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} -1 & 7 & -5 \\ 0 & 4 & -3 \\ -1 & 8 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$-1-(-1) = 0$
$-6-(-5) = -6+5$

$$\begin{bmatrix} -1 & 7 & -5 \\ 0 & 4 & -3 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$R_2 \to \frac{1}{4} R_2$

$$\begin{bmatrix} -1 & 7 & -5 \\ 0 & 1 & -3/4 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3/4 \\ 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} -1 & 7 & -5 \\ 0 & 1 & -3/4 \\ 0 & 0 & -1/4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3/4 \\ 1/4 \end{bmatrix}$$

$-1-\left(-\frac{3}{4}\right)$ $-1+\frac{3}{4}$

$\frac{-4+3}{4} = \frac{-1}{4}$

$1-\frac{3}{4}$ $\frac{4-3}{4}$

$\therefore -x + 7y - 5z = 4, \quad y - \frac{3}{4}z = \frac{3}{4}, \quad \frac{-1}{4}z = \frac{1}{4}$

$$-\frac{1}{4}z = \frac{1}{4}$$
$$z = \frac{1}{\cancel{4}} \times \frac{-\cancel{4}}{1}$$
$$z = -1$$

$$y - \frac{3(-1)}{4} = \frac{3}{4}$$
$$y = \frac{3}{4} \times \frac{4}{3}$$
$$y = 1$$

$$-x + 7(1) - 5(-1) = 4$$
$$-x + 7 + 5 = 4$$
$$x = 7 + 5 - 4$$
$$x = 8$$

$$y + \frac{3}{4} = \frac{3}{4}$$
$$y = \frac{3}{4} - \frac{3}{4}$$
$$y = \frac{0}{4}$$
$$y = 0$$

$$-x + 7(0) - 5(-1) = 4$$
$$-x + 5 = 4$$
$$x = 5 - 4$$
$$x = 1$$

Checking

$x = 8,\ y = 1,\ z = -1$

$4y - 3z = 3$

$4(1) - 3(-1)$

$4 + 3 = 7 \quad 7 \neq 3$

$x = 1,\ y = 0,\ z = -1$

$4y - 3z$

$4(0) - 3(-1)$

$+3 = 3$

$-x + 7y - 5z = 4$

$-(1) + 7(0) - 5(-1)$

$-1 + 5$

$= 4 \quad \therefore 4 = 4$

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4) linear combinaⁿ $v_1 = (-2, 1, 3), v_2 = (3, 1, -1), v_3 = (-1, -2, 1)$

$w = (6, -2, 5)$

Express w in linear combinaⁿ form.

i.e. $w = av_1 + bv_2 + cv_3$

for that we have to find (a, b & 3)

$a(-2, 1, 3) + b(3, 1, -1) + c(-1, -2, +1) = (6, -2, 5)$

$-2a + 3b - c = 6$ — (1)

$+a + b - 2c = -2$ — (2)

$3a - b + c = 5$ — (3)

solving using ~~echelon form~~ Gussian eliminaⁿ

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & 1 & -2 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ -2 & 3 & -1 & 6 \\ 3 & -1 & 1 & 5 \end{array}\right]$$

$R_2 \to R_2 + 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 5 & -5 & 2 \\ 3 & -1 & 1 & 5 \end{array}\right]$$

$-1 + 2(-2)$

$-1 - 4$

$6 + 2(-2)$

$6 - 4$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 5 & -5 & 2 \\ 0 & -4 & 7 & 11 \end{array}\right]$$

$R_2 \to \frac{1}{5} R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 1 & -1 & 2/5 \\ 0 & -4 & 7 & 11 \end{array}\right]$$

$1 - 3(-2)$

$-1 - 3$ $\quad 1 + 6$

$1 - (-2)3$

$5 - 3(-2)$

$5 + 6$

$5 -$

$R_3 \to R_3 + 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 1 & -1 & 2/5 \\ 0 & 0 & 3 & \cancel{19}\,63/5 \end{array}\right] \to$$

$11 + 4\left(\frac{2}{5}\right)$ $\quad 11 + \frac{8}{5}$

$7 + 4(-1)$

$7 - 4$

$11 + 4(2)$

$7 + 4(-1) \quad 7 - 4 = 3$

$5 - 3(-2) \quad 5 + 6$

$\frac{55 + 8}{5} \quad 11 + 4\left(\frac{2}{5}\right) = 11 + \frac{8}{5}$

$\therefore \quad a + b - 2c = -2$

$b - c = \frac{2}{5}$ $\qquad 3c = \frac{63}{5}$

$a + b - 2c = -2$

$b - c = \frac{2}{5}$

$3c = \frac{63}{5}$

$\therefore \quad c = \frac{\cancel{63}^{21}}{5} \times \frac{1}{\cancel{3}}$

$\boxed{c = \frac{21}{5}}$

$b - \frac{21}{5} = \frac{2}{5}$

$b = \frac{2}{5} + \frac{21}{5} \quad \boxed{b = \frac{23}{5}}$

$a + \frac{23}{5} - 2\left(\frac{21}{5}\right) = -2$

$a = -2 - \frac{23}{5} + \frac{42}{5}$

$a = -10 - 23 + 42 / 5$

$\boxed{a = \frac{9}{5}}$

Checking

$a = \frac{9}{5}, \; b = \frac{23}{5}, \; c = \frac{21}{5}$

$-2a + 3b - c = 6$

$-2\left(\frac{9}{5}\right) + 3\left(\frac{23}{5}\right) - \frac{21}{5}$

$\frac{30}{5} = 6 = 6$

$a + b - 2c = -2$

$\frac{9}{5} + \frac{23}{5} - 2\left(\frac{21}{5}\right)$

$\frac{-10}{5} = -2$

5) find eigen value & eigen vector of
10) $(8,-8,-2),(4,-3,-2)$ & $(3,-4,1)$

$\Rightarrow$ step 1 : $(A-\lambda I)x=0$

$$\begin{bmatrix}8 & -8 & -2\\4 & -3 & -2\\3 & -4 & 1\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

step 2: $|A-\lambda I|=0$

$$\begin{vmatrix}8-\lambda & -8 & -2\\4 & -3-\lambda & -2\\3 & -4 & 1-\lambda\end{vmatrix}=0$$

step 3: $\lambda^3-\begin{bmatrix}\text{Sum of diagonal}\\\text{element}\end{bmatrix}\lambda^2+\begin{bmatrix}\text{Sum of diagonal}\\\text{minors}\end{bmatrix}\lambda-|A|=0$

Sum of diagonal $=8+(-3)+1=\underline{6}$

Sum of diagonal minors $=$

$$\begin{vmatrix}-3 & -2\\-4 & 1\end{vmatrix}+\begin{vmatrix}8 & -2\\3 & 1\end{vmatrix}+\begin{vmatrix}8 & -8\\4 & -3\end{vmatrix}=$$

$=|-3-(8)|+|8-(-6)|+|-24-(-32)|$

$=-11+14+8$

$=\underline{\underline{11}}$

$$|A|=8\begin{vmatrix}-3 & -2\\-4 & 1\end{vmatrix}-(-8)\begin{vmatrix}4 & -2\\3 & 1\end{vmatrix}+(-2)\begin{vmatrix}4 & -3\\3 & -4\end{vmatrix}$$

$=8(-3-(8))+8(4-(-6))-2(-16-(-9))$

$=8(-11)+8(10)-2(-7)=-88+80+14$

$=\underline{6}$

putting the value in step 3 eq^n

we have to take this value by our own

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

Here we get $\lambda = 1$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda = 2$, $\lambda = 3$

6, -2, 3

$\therefore$ Eigen values are $\lambda = (1, 2, 3)$

for finding eigen vectors, we have to put these values in step 2 matrix

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix}$$

puting $\lambda = 1$

$$\therefore \begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Crammer's Rule.

$7x - 8y - 2z = 0$ —— (1)

$4x - 4y - 2z = 0$ —— (2)

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x}{16-8} = \frac{-y}{-14+8} = \frac{z}{-28+32}$$

$$\frac{x}{8} = \frac{+y}{+6} = \frac{z}{4}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$\therefore$ 1st vector $= \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

putting $\lambda = 2$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$6x - 8y - 2z = 0$

$4x - 5y - 2z = 0$

Crammer's rule:

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -5 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 6 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix}}$$

$$\frac{x}{6} = \frac{+y}{+4} = \frac{z}{2}$$

$\therefore$ 2nd vector = $\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

putting $\lambda = 3$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$5x - 8y - 2z = 0$

$4x - 6y - 2z = 0$

Crammer's rule.

$$\frac{x}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\frac{x}{4} = \frac{+y}{+2} = \frac{z}{2}$$

$\therefore$ 3rd vector = $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$

7] Eigen values: $(3, -1, 1)\ (-1, 3, 1)\ (1, -1, 3)$

Step 1: $(A - \lambda I)x = 0$

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 2: $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 3-\lambda & 1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

Step 3: $\lambda^3 - \lambda^2(\text{sum of diagonal}) + \lambda(\text{sum of dia minors}) - |A| = 0$

Sum of diagonal $= 3 + 3 + 3 = \underline{9}$

Sum of diagonal minors $=$

$$= \begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix}$$

$= 10 + 8 + 8$

$= \underline{\underline{26}}$

$$|A| = 3\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix}$$

$= 3(10) + 1(-4) + 1(-2)$

$= \underline{\underline{24}}$

$\therefore\ \cancel{\lambda^3 - \lambda\ 9\lambda^2 + 26\lambda -}$

$\underline{\lambda^3 - 9\lambda^2 + 26\lambda - 24 = 0}$

Finding eigen value using synthetic division

$$\begin{array}{c|cccc} 1 & 1 & -9 & 26 & -24 \\ & & 1 & -8 & 18 \\ \hline & 1 & -8 & 18 & -6 \neq 0 \end{array} \qquad \begin{array}{c|cccc} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

For "2" we get the result as 0

$\therefore$ 1st eigen value is 2.

$\lambda^2 - 7\lambda + 12 = 0$ $\qquad$ $+12 =$

$(\lambda - 4)(\lambda - 3) = 0$

$\therefore \lambda = 4, \lambda = 3$

$\therefore$ Eigen values are $\lambda = 2, 3, 4$

putting $\lambda = 2$ in matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x - y + z = 0$

$-x + y + z = 0$

Cramer's rule

$$\frac{x}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}}$$

$$\frac{x}{-2} = \frac{-y}{2} = \frac{z}{0}$$

putting $\lambda = 3$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x - y + z = 0$

$-x + 0y + z = 0$

Cramer's rule

$$\frac{x}{\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}}$$

$$\frac{x}{-1} = \frac{-y}{1} = \frac{z}{-1}$$

putting $\lambda = 4$.

$-x - y + z = 0$

$-x - y + z = 0$

$$\frac{x}{\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix}}$$

$$\frac{x}{0} = \frac{-y}{0} = \frac{z}{0}$$

$$\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

putting $\lambda = 4$

$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-x - y + z = 0 \quad - (1)$

$-x - y + z = 0 \quad - (2)$

$x - y - z = 0 \quad - (3)$

Using (2) & (3)

$$\frac{x}{\begin{vmatrix} -1 & -1 \\ 1 & -1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -1 & -1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x}{2} = \frac{-y}{0} = \frac{z}{2}$$

Q.8 Program to convert 2x2 matrix to row echelon form

```
import numpy as np

def row_echelon_form(matrix):
    matrix = np.array(matrix, dtype=float)
    if matrix.shape != (2,2)
        print("Input matrix should be a 2x2 matrix.")
        return none.

    ratio = matrix[1,0]/matrix[0,0]
    matrix[1] -= ratio * matrix[0]
    return matrix

matrix = [[2,4],[1,3]]
result = row_echelon_form(matrix)
if result is not none:
    print("Row echelon form of the matrix")
    print(result)
```

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III] Eigen value & vectors $A = ([12 \; -51] \; [2 \; -11])$

$\Rightarrow$ step 1: $(A - \lambda I)x = 0$

$$\begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

step 2: $|A - \lambda I| = 0$

$$\begin{vmatrix} 12-\lambda & -51 \\ 2 & -11-\lambda \end{vmatrix} = 0$$

$$(12-\lambda)(-11-\lambda) - (-51 \times 2) = 0$$

$$(-132 - 12\lambda + 11\lambda + \lambda^2) - (-102) = 0$$

$$-132 - \lambda + \lambda^2 + 102 = 0$$

$$\lambda^2 - \lambda^2 - 30 = 0$$

$$(\lambda + 5)(\lambda - 6) = 0$$

$$\therefore \lambda = -5, \; \lambda = 6$$

Eigen value $= -5, 6$

putting $\lambda = -5$

$$\begin{bmatrix} 17 & -51 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$17x_1 - 51x_2 = 0$$

$$2x_2 - 6x_2 = 0$$

$$\frac{-x_1}{-6}, \; \frac{x_2}{2}$$

$$\therefore \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

putting $\lambda = 6$

$$\begin{bmatrix} 6 & -51 \\ 2 & -17 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$6x_1 - 51x_2 = 0$$

$$2x_1 - 17x_2 = 0$$

$\cancel{6x_1 - 51x_2}$

$6x_1 -$

$$\frac{-x_1}{-17} = \frac{x_2}{2} \quad \begin{bmatrix} 17 \\ 2 \end{bmatrix}$$

13] Echelon form

$$\begin{bmatrix} 1 & \cdots & & \\ 0 & 1 & \cdots & \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As this is in the 1st col ∴ we have to perform operⁿ with 1st row

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ 4 & 3 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 4 & 3 & 2 \end{bmatrix}$$

this is also in 1st col ∴ operⁿ with 1st row

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 3 & -2 \end{bmatrix}$$

second col ∴ perform operⁿ with 2nd row

$R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

second col ∴ perform operⁿ with 2nd row

Upper triangular but not in echelon form Coz every pivot non-zero element should be 1, but we have here -1

$R_2 \to -1 R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

echelon form

(iv) Python program for prime factors

```
def prime_factor(n):
    factor = []
    divisor = 2

    while n > 1:
        while n % divisor == 0:
            factor.append(divisor)
            n //= divisor

    return factor

num = int(input("Enter positive number:"))
print(prime_factor(num))
```

Output:

- Enter positive number: 24
  [2, 2, 2, 3]

15) Python program for project u on v

```
import numpy as np

def orthogonal_projection(u,v)
    scalar_projection= np.dot(u,v)/np.dot(v,v)
    projection= scalar_projection * v

    return projection

u= np.array([1,2,3])
v= np.array([4,5,6])
~~poting~~
print("~~Projec~~ Orthogonal project" of u on v",
                orthogonal_projection(u,v))
```

Output:

Orthogonal projection of u on v: [1.66 2.07 2.49]