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Thursday

UNIT-I.

Q.1] Write a python program for addition subtraction of complex numbers 4+2j and 3-6j.

→

```
complex_num1 = 4+2j
complex_num2 = 3-6j

addition_result = complex_num1 + complex_num2
print ("Addition result:", addition_result)

subtraction_result = complex_num1 - complex_num2
print ("Subtraction result:", subtraction_result)

Multiplication_result = complex_num1 * complex_num2
print ("Multiplication result:", multiplication_result)
```

Q.2] Define : Galois Field, Dot Product, Convex Combinatⁿ, Span.

→ 1. Galois Field.

— It is a set of numbers that consists of a finite number of elements and has two operations, addition and multiplication, that follow specific rules. Galois fields are useful in various fields, such as cryptography, coding theory, and error correction, due to their unique mathematical properties. The size of a Galois field is represented by a prime number 'p', and it is denoted by GF(p), where p is a prime number.

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2. Dot Product

→ Dot product is defined as the sum of the products of the corresponding enteries of two sequences of numbers. The dot product of two vectors is the product of the magnitude of the two vectors and the cos of the angle between them. To recall, vectors are multiplied using two methods:

- Scalar product of vectors or dot product
- vector product of vectors or cross product.

3. Convex combination.

→ It is a linear combination of points (which can be vectors scalars, or more generally points in an affine space) where all coefficients are non-negative and sum to 1.

An expression of the form $\alpha u + \beta v$
where, $\alpha, \beta >= 0$,
$u, v \in V$ and $\alpha + \beta = 1$ is called
Convex combination of u and v.

4. Span

→ The span of a set of vectors, also called linear span, is the linear combinations of the vectors belonging to the given set. A linear span is a linear space.

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Q.3] Write a python program to find conjugate of complex number.

→
```
import numpy as np

in_complx1 = 2+4j
out_complx1 = np.conj(in_complx1)
print(" output ~~comple~~ conjugated complex number of
      2+4j :", out_complx1)

in_complx2 = 5-8j
out_complx2 = np.conj(in_complx2)
print("output conjugated complex number of
 5-8j:", out_complx2)
```

Output :-

output conjugated complex number of 2+4j : (2-4j)
output conjugated complex number of 5-8j : (5+8j)

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Q.4] Are the following vectors are linearly dependent
$v_1 = (3, 2, 7)$, $v_2 = (2, 4, 1)$ and $v_3 = (1, -2, 6)$

→ A set of vectors $\{v_1, v_2, \ldots v_p\}$ in R^n is said to be linearly independent if the vector equation
$x_1 v_1 + x_2 v_2 + \ldots + x_p v_p = 0$
has only the trival solution.

If the determinant is equal to zero, its linearly independent. Otherwise, its linearly dependent.
The determinant of vectors are as follows:-

$$\begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & -2 \\ 7 & 1 & 6 \end{vmatrix}$$

$= 3\,|(4\times6) - (1\times(-2))| - 2\,|(2\times6) - (7\times(-2))| + 1\,|(2\times1) - (7\times4)|$

$= 3(24 - (-2)) - 2(12 - (-14)) + 1(2 - 14)$

$= 3(24+2) - 2(12+14) + 1(2-14)$

$= 3(26) - 2(26) + 1(-12)$

$= 78 - 52 - 12$

$= 26 - 12$

$= 14$

∴ Determinant value not equal to zero, so vectors are linearly independent.

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Q.5] Check whether the vectors are linearly dependent $v_1 = (1, -2, 1)$, $v_2 = (2, 1, -2)$ and $v_3 = (7, -4, 1)$

→ A set of vectors $\{v_1, v_2, \ldots, v_p\}$ in R^n is said to be linearly independent if the vector eq^n
$x_1 v_1 + x_2 v_2 + \cdots + x_p v_p = 0$
has only the trivial sol^n.

If the determinant is not equal to zero, it's linearly independent. Otherwise, its linearly dependent.

The determinant of vectors are as follows:-

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 7 \\ -4 \\ 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 7 \\ -2 & 1 & -4 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 1\,|(1\times1) - (-4\times(-2))| - 2\,|(-2\times1) - (-4\times1)| + 7\,|(-2\times(-2)) - (1\times1)|$
$= 1(1-8) - 2(-2+4) + 7(4-1)$
$= 1(-7) - 2(2) + 7(3)$
$= -7 - 4 + 21$
$= -11 + 21$
$= 10$

Determinant value not equal to zero so, vectors are linearly independent.

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Q.6] Expreu in polar and exponential form $1+i\sqrt{3}$

$\Rightarrow$ let $z = 1+i\sqrt{3}$

This is of form $a+bi$, where $a=1$, $b=i\sqrt{3}$

$r = \sqrt{a^2+b^2}$

$= \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4} = 2.$

Also, $\cos\theta = \frac{a}{r} = \frac{1}{2}$

and $\sin\theta = \frac{b}{r} = \frac{\sqrt{3}}{2}$

$\cancel{\therefore \theta = 120^\circ}$

complex number in polar form:

$1+i\sqrt{3} = 2(\cos\theta + i\sin\theta)$

And exponential form:

$1+i\sqrt{3} = 2\cdot e^{i\theta}$

However, its worth nothing that we need to be careful with the expression of the argument θ. Since $1+i\sqrt{3}$ lies in the first quadrant of the complex plane, θ is the angle between the positive real axis and the vector pointing to the complex number, which is $\pi/3$ radians or 60°. Therefore:

$\theta = \pi/3$

Thus polar form is:

$1 + i\sqrt{3} = 2(\cos(\pi/3) + i\sin(\pi/3))$

And the exponential form is:

$1 + i\sqrt{3} = 2 \cdot e^{i\pi/3}$

Q.7] Find the square root of complex number $8 - 6i$

→ let the square root be $x + iy$

$\therefore (x + iy) = \sqrt{+8 - 6i}$

Taking square on both the side.

$= x^2 - y^2 + 2xyi = 8 - 6i$

By comparing we get

$x^2 - y^2 = 8$ and $2xy = 6$

$xy = 3$

$y = \frac{3}{x}$

$$\Rightarrow x^2 - \left(\frac{3}{x}\right)^2 = -8$$

$$= x^2 - \frac{9}{x^2} = -8$$

$$= x^4 + 8x^2 - 9 = 0$$

$$= x^4 + 9x^2 - x^2 - 9 = 0$$

$$= x^2(x^2 - 1) + 9(x^2 - 1) = 0$$

$$= (x^2 + 9)(x^2 - 1) = 0$$

$$= x^2 = 9 \; ; \; x^2 = 1$$

Since x is real we get $x = \pm 1$

Taking $x = \pm 1$ we get $y = +3$

$\Rightarrow$ Square root $1 + 3i$

Taking $x = -1$ we get $y = 3$

$\Rightarrow$ Square root is $-1 + 3i$

square root is $\pm(1 - 3i)$.

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Q.8] Find the square root of complex number $-5+12i$

→ $-5+12i$

$$\sqrt{a+ib} = \pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$$

So $a=-5$, $b=12$ $\Rightarrow a^2=25$, $b^2=144$

$$\sqrt{-5+12i} = \pm\left[\sqrt{\frac{\sqrt{25+144}+(-5)}{2}} + i\sqrt{\frac{\sqrt{25+144}-(-5)}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{\sqrt{169}-5}{2}} + i\sqrt{\frac{\sqrt{169}+5}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{13-5}{2}} + i\sqrt{\frac{13+5}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{8}{2}} + i\sqrt{\frac{18}{2}}\right]$$

$$= +\left[\sqrt{4} + i\sqrt{9}\right]$$

$$\sqrt{-5+12i} = \pm\left[2+3i\right]$$

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Q.9] Find the square root of $21-20i$, where $i=\sqrt{-1}$

$\rightarrow$ $\sqrt{21-20i} = \pm\left[\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$

$\pm\left[\sqrt{\frac{\sqrt{441+400}+21}{20}} + 8\sqrt{-1}\sqrt{\frac{\sqrt{441+400}-21}{2}}\right]$

$\pm\left[\sqrt{\frac{29+21}{2}} + \sqrt{-1}\sqrt{\frac{29-21}{2}}\right]$

$= \pm\left[\sqrt{25} + \sqrt{-1}\sqrt{4}\right]$

$= \pm\ 5+\sqrt{-1}(2)$

$= \pm$

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Q.10] Express $[(3+2i)/(2+i)(1-3i)]$ in the form $x+iy$.

$\rightarrow$ $\dfrac{(3+2i)}{(2+i)(1-3i)}$

= multiply the numerator and denominator by the conjugate of the denominator, which is $(2-i)(1+3i)$:

$$\frac{(3+2i)}{(2+i)(1-3i)} \times \frac{(2-i)(1+3i)}{(2-i)(1+3i)}$$

Expanding the numerator and the denominator:

Numerator:
$(3+2i)(2-i)(1+3i)$

Denominator:
$(2+i)(1-3i)(2-i)(1+3i)$

After expanding, we'll have a complex number in the form $x+iy$.

let's calculate:

Numerator:

$(3+2i)(2-i)(1+3i)$
$= (3+2i)(2+6i-i-3)$
$= (3+2i)(-1+5i)$
$= -3-15i+2i-10$
$\Rightarrow -13-13i$

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Denominator:

$(2+i)(1-3i)(2-i)(1+3i)$

$=(2+i)(2+i)(1-3i)(1+3i)$

$=(4-1)(1+9 = 39$

Now, let's put it all together

$$\frac{(3+2i)}{(2+i)(1-3i)} = \frac{-13-13i}{39}$$

Thus, the expression $\frac{(3+2i)}{(2+i)(1-3i)}$. In the form $x+iy$ is $\frac{-13}{39} - \frac{13}{9} i$.

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Q.11] Solve the following system by backward substitution method.

$1x_1 - 3x_2 - 2x_3 = 7, \quad 2x_2 + 4x_3 = 4, \quad -10x_3 = 12.$

→ 1. $1x_1 - 3x_2 - 2x_3 = 7$

2. $2x_2 + 4x_3 = 4$

3. $-10x_3 = 12$

Step 1: Solve Eqⁿ 3 for x_3:

$-10x_3 = 12$

$x_3 = \frac{12}{-10}$

$x_3 = \frac{-6}{5}$

Step 2: Substitute the values of x_3 into Equation 2 and solve for x_2:

$2x_2 + 4\left(\frac{-6}{5}\right) = 4$

$2x_2 - \frac{24}{5} = 4$

$2x_2 = 4 + \frac{24}{5}$

$2x_2 = \frac{44}{5}$

$x_2 = \frac{44}{10}$

$x_2 = \frac{22}{5}.$

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Step 3: Subtitute the values of x_2 and x_3 into Equation 1 and solve for x_1:

$$1x_1 - 3\left(\frac{22}{5}\right) - 2\left(\frac{-6}{5}\right) = 7$$

$$x_1 - \frac{66}{5} + \frac{12}{5} = 7$$

$$x_1 - \frac{54}{5} = 7$$

$$x_1 = 7 + \frac{54}{5}$$

$$x_1 = \frac{35}{5} + \frac{54}{5}$$

$$x_1 = \frac{89}{5}.$$

So, the sol^n to the system of equations is

$x_1 = \frac{89}{5}$, $x_2 = \frac{22}{5}$, and $x_3 = \frac{-6}{5}$.

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Q.12] Write a python program to solve system of linear equations given below
$1x_1 - 3x_2 - 2x_3 = 7$, $2x_2 + 4x_3 = 4$, $-10x_3 = 12$

→

```
import numpy as np
coefficients = np.array([[ 1, -3 , -2 ],
                         [ 0 , 2 , 4 ],
                         [ 0 , 0 , -10 ]])

constants = np.array ([7,4,12])

solution= np.linalg.solve (coefficients, constants)

print ("solution:")
print (" x1 =", solution[0])
print ("x2 =", solution[1])
print (" x3 =", solution [2])
```

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Q.13] Determine whether $v_1 = (2,2,2)$, $v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span vector space R^3.

→ To determine, whether the vector $v_1 = (2,2,2)$ $v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span the vector space R^3, we need to check if any vector in R^3 can be expressed as a linear combⁿ of v_1, v_2 and v_3

A vector (x,y,z) can be expressed as a linear combination of v_1, v_2 and v_3 if the following system of equations has a solution:

$$\begin{cases} 2a + 0b + 0c = x \\ 2a + 0b + 1c = y \\ 2a + 3b + 1c = z \end{cases}$$

where a, b and c are scalars.

let's solve this system of equations using Gaussian elimination:

We start by representing the augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & x \\ 2 & 0 & 1 & y \\ 2 & 3 & 1 & z \end{array}\right]$$

perform Gaussian elimination to determine if the system is consistent.

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$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & x \\ 2 & 0 & 1 & y \\ 2 & 3 & 1 & z \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 0 & x \\ 0 & 0 & 1 & y-x \\ 0 & 3 & 1 & z-x \end{array}\right]$$

Here the second row can never be 0 unless $y = x$. Similarly, the third row can never be 0 unless $z = x$, which makes the system inconsistent. So, the vectors v_1, v_2 and v_3 do not span $\mathbb{R}^3$.

Q.14] Show that vectors $v_1 = (1,0,1)$, $v_2 = (2,1,4)$ and $v_3 = (1,1,3)$ do not span vector space.

→ To show that vectors v_1, v_2 and v_3 do not span the vector space, we need to demonstrate that there exist vectors in the space that cannot be expressed as a linear combination of these 3 vectors.

matrix:-

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 3 \end{bmatrix}$$

Calculate the determinant of matrix A. if it equals zero, the vectors are linearly dependent:

$$\det(A) = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 4 \end{vmatrix}$$

$$= 1(3-4) - 2(0-1) + 1(0-1)$$
$$= 1(-1) - 2(-1) + 1(-1)$$
$$= -1 + 2 - 1$$
$$= -2 + 2$$
$$= 0$$

$\det(A) = 0$.

Since the determinant of A is zero, the vectors v_1, v_2 and v_3 are linearly dependent, which means they do not span the vector space $\mathbb{R}^3$.

Q.15 Write a python Program for rotating a complex number $z = 2+3i$ by $180°$

→
```
import cmath
z = 2 +3j
rotated_z = z*cmath.exp(1j* cmath.pi)
print ('original z:", z)
print ("Rotated z by 180 degrees :", rotated_z)
```

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16 Write a python program for rotating a complex number ~~z = 2 + 3~~ by 90°, 180° and 270°.

→ Rotating by 90 degrees:

H(z) = i*z

```
import numpy as np
import matplotlib.pyplot as plt
S = np.array([2+2j, 3+2j, 1.75+1j, 2+1j, 2.25+1j,
   2.75+1j, 3+1j, 3.25+1j])
L = np.array([z*1j for z in S])
print(L)
X = L.real
X
Y = L.imag
Y
plt.scatter(X, Y, label = "Complex Numbers", color = 'b',
 s = 25 marker = 'o')
plt.xlabel('X')
plt.ylabel('Y')
plt.show.
```

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→ Rotating by 180°

```
import numpy as np
import matplotlib.pyplot as plt.
S=np.array([2+2j, 3+2j, 1.75+1j, 2+1j, 2.25+1j, 2.5+1j,
        2.75+1j, 3+1j, 3.25+1j])
 L = np.array([z*-1 for z in s])
print(L)
 X = L.real
  X
 Y = L.imag
 Y
plt.scatter(x, y, label="Complex Number",
      color='b', s=25, marker='o')
plt.xlabel('x')
plt.ylabel('y')
plt.show().
```

→ Rotation by 270 degree

```
import numpy as np
import matplotlib.pyplot as plt
S=np.array([2+2j, 3+2j, 1.75+1j, 2+1j, 2.25+1j,
     2.50+1j, 2.75+1j, 3+1j, 3.25+1j])
L=np. array([z*(2.7182**(5*3.14*1j/4)) for z in s])
pint(L)
x = L.real
x
y=L.imag
y
plt.scatter(x, y, label="Complex Number",
 color='b', s=25, marker='o')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```