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Linear Algebra Using Python.
Unit - I

1) Write a Python Program for addition, subtraction, Multiplication of Complex numbers 4+2j and 3-6j

→ Program :-

```
a= 4+2j
b= 3-6j
Print (" Addition of Complex Numbers :", a+b)
Print (" Subtraction of Complex Numbers :" a-b)
Print (" Multiplication of Complex Numbers :" a*b)
```

Output :-

Addition of Complex Numbers : (7-4j)
Subtraction of Complex Numbers : (1+8j)
Multiplication of Complex Numbers : (24-18j)

2) Define :- Galois Field, Dot Product, Convex Combination, span

→

i] Galois Field :- Galois Field with two elements is denoted by GF(2). It has two elements 0 and 1.

Since it is Field there are two binary operations defined (+ & .). the behaviour of the element '0' & '1' w.r.t. (+) & (.) is given as follows.

+	0	1
0	0	1
1	1	0

.	0	1
0	0	0
1	0	1

ii] Dot Product :- The dot Product is defined as the Sum of the Products of the Correspon-ding entries of the two Sequences of Numbers

iii] Convex Combination :- An expression of the form $\alpha u + \beta v$, where $\alpha, \beta \geq 0$, $v \in V$ and $\alpha + \beta = 1$ is Called Convex Combination of u and v.

iv] Spam :- The Set of all linear Combination of Vector $V_1, V_2 \dots V_n$ is Called the Spam of the Vector. i.e.

$$\text{Spam } \{V_1, V_2, \dots V_n\} = \left\{ \sum_{i=1}^{n} K_i v_i \;\middle|\; \begin{matrix} n \in \mathbb{N} \text{ and} \\ K_i \in F \end{matrix} \right.$$

Q3. Write a python Program Program to find Conjugate of Complex Numbers.

->

```
#import Numpy
import numpy as np
a= 2+4j
a1= np. Conj(a)
Print ("Conjugate of 2+4j : ; a1)
b= 5-8j
b1 = n.p. Conj (b)
Print (" Conjugate of 5-8j : ", b1)
```

Output :-

```
Conjugate of 2+4j:  (2-4j)
Conjugate of 5-8j:  (5+8j)
```

Q4. Are the following Vectors are linear dependent
$V_1 = (3,2,7)$, $V_2 = (2,4,1)$ and $V_3 = (1,-2,6)$

→ Let's assuming there exist Constants a, b & c
Such that :

$a.V_1 + b V_2 + c.V_3 = 0$

Substituting the given Vectors

$a.(3,2,7) + b.(2,4,1) + c.(1,-2,6) = (0,0,0)$

$(3a + 2b + c,\ 2a + 4b - 2c,\ 7a + 1b + 6c) = (0,0,0)$

$(3a+2b+c) = 0$

$7a + 1b + 6c = 0$

This System has a non-trival Solution indicating that the Vectors are linearly dependent.

Q5 check whether the Vectors are linearly dependent
$V_1 = (1,-2,1)$, $V_2 = (2,1,-2)$ & $V_3 = (7,-4,1)$

→ Let's assure there exist Construct a, b & c Such that
$a.V_1 + b.V_2 + c V_3 = 0$

Substituting the gion Vectors:

$a(1,-2,1) + b(2,1,-2) + c(7,-4,1) = (0,0,0)$

Expanding this equation

$(a + 2b + 7c,\ -2a + b - 4c,\ a - 2b + c) = (0,0,0)$

$a + 2b + 7c = 0$

$a + 2b + c = 0$

This System has a non-trival Solution, ~~indeat~~ that the Vectors are linearly dependent.

Q6) Express the Polar and exponential form $1+i\sqrt{3}$

→ To express the complex number $(1+i\sqrt{3})$ in Polar & Exponential form, we 1st need to help find it's Mangnitude & argumant.

Magnitude (r) :

$r = |1+i\sqrt{3}| = \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{1+3} = 2.$

Argument (θ) =

$\theta = \text{archaz}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

Polar form

$1+i\sqrt{3} = 2 \cdot e^{i\frac{\pi}{3}}$

Exponential form :-

$1+i\sqrt{3} = 2. \left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

Q7) Find the Square root of Complex numbers $8-6i$

→ let us assign Complex number $x+iy$ to the Square root of $8-6i$ Such that both x and y are real numbers

Then we have

$x+iy = \sqrt{8-6i}$

Squaring both side, we have

$(x+iy)^2 = 8-6i$

$x^2+2ixy-y^2 = 8-6i$

$(x^2-y^2) + 2ixy = 8-6i$

$x^2-y^2 = 8$ — (i)

$2xy = -6$ — (ii)

Hence we have 2 equations (1) & (2) in two unknown x and y.

form the second equation, we have.

$xy = -3$

$x^2y^2 = 9$

$x^2 = y^2 + 8$

$(y^2 + 8)y^2 = 9$

$(w + 8)w = 9$

$w^2 + 8w - 9 = 0$

$w(w + 9) - 1(w + 9) = 0$

$(w - 1)(w + 9) = 0$

$w = 1 ; w = -9$

Y is real number, hence it's Square Can't be negative

hence we have.

$w = 1$

$y^2 = 1$

$y = \pm 1$ — ⑥

For $y = 1$ from eq^n ⑤ we have

For $y = -1$, from eq^n ⑤ we have.

$x(-1) = -3$

$x = 3$

Hence another root is $3 - i$

Hence the Square roots of $8 - 6i$ are $-3 + i$ & $3 - i$

8] Find the Square root of Complex numbers $-5 + 12i$

→

$$\sqrt{a + ib} = \pm \left[\sqrt{\frac{a^2 + b^2 + a}{2}} + i \sqrt{\frac{a^2 + b^2 - a}{2}} \right.$$

So $a = -5$, $b = 12 \Rightarrow a^2 = 25$, $b^2 = 144$.

$$\sqrt{-5 + 12i} = \pm \left[\sqrt{\frac{25 + 144 + (5)}{2}} + i \sqrt{\frac{25 + 144 - (-5)}{2}} \right.$$

$$= \pm \sqrt{\frac{169 - 5}{2}} + i \sqrt{\frac{\sqrt{169} - 5}{2}}$$

$$= \pm \left[\sqrt{\frac{\sqrt{169}-5}{2}} + i\sqrt{\frac{\sqrt{169}+5}{2}} \right]$$

$$= \pm \left[\sqrt{\frac{13-5}{2}} + i\sqrt{\frac{13+5}{2}} \right]$$

$$= \pm \left[\sqrt{\frac{8}{2}} + i\sqrt{\frac{18}{2}} \right]$$

$$= \pm \left[\sqrt{4} + i\sqrt{9} \right]$$

$$\therefore \sqrt{5+12i} = \pm [2+3i]$$

Q9 find the Square root of $21-20i$ where $i=\sqrt{-1}$

→ Let $\sqrt{21-20i} = x+iy$

$21-20i = x^2-y^2+2ixy$

$\therefore$ On Comparing the real & imaginary Parts

$x^2-y^2=21$, $2xy=-20$

Now,

$(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$= (x^2+y^2)^2 = (21)^2+(20)^2$

$= (x^2+y^2)^2 = 441+400$

$= (x^2+y^2)^2 = 841$

$\therefore (x^2+y^2) = 29$

Solving eq^n $x^2-y^2=21$ & $x^2+y^2=29$

$x = \pm 5$, $y = \pm 2$

Since xy is negative, x & y will be of opposite signs

$\therefore \sqrt{21-20i} = \pm \sqrt{21-20i}$

$= \pm (5-2i)$

DATE

10) Express [(3+2i) / (2+i) (1-3i)] in the for 2x+iy

→ The Conjugate of (2+i) is (2-i) and

The Conjugate of (1-3i) is (1+3i)

So multiplying both the numerator and the denominator by the their respective Conjugates.

$$\frac{3+2i}{(2+i)(1-3i)} \times \frac{(2-i)(1+3i)}{(2-i)(1+3i)}$$

Expending both the numerator of the denominator

Numerator :-

$$(3+2i)(2-i)(1+3i) = (6+3i+4i-2i^2)(1+3i) = (6+i+2)(1+3i)$$

Denominator :-

$$(2+i)(2-i)(1+3i) = (4-1i+2i-i^2)(1+3i) = (4+1+2)(1+3i)$$

So after Simplification, we get.

$$= \frac{(6+i+2)(1+3i)}{(4+1+2)(1+3i)}$$

$$= \frac{(8+i)(1+3i)}{(7)(1+3i)}$$

$$= \frac{8+24i+i+3i^2}{7+21i}$$

$$= \frac{8+25i-3}{7+21i}$$

Now, divide both equal real & imaginary part of by the denominator $= \frac{5+25i}{7+21i}$

$$= \frac{35+175i-105i+525}{49+441}$$

$$= \frac{560+70i}{490}$$

Now, Simplify we get

$$\frac{(6+i+2)(1+3i)}{(4+1+2)(1+3i)}$$

$$= \frac{(8+i)(1+3i)}{(7)(1+3i)}$$

$$= \frac{8+24i+i+3i^2}{7+21i}$$

$$= \frac{8+25i-3}{7+21i}$$

Now, divide both real & imaginary part by the denominator.

$$= \frac{5+25i}{7+21i}$$

Now, Multiply by the Conjugat of the denominato

$$= \frac{(5+25i)(7-21i)}{7^2+21^2}$$

$$= \frac{35+175i-105i+525}{49+441}$$

$$= \frac{560+70i}{490}$$

Now, simplify

$$= \frac{112+14i}{98}$$

$$= \frac{56}{49} + \frac{7}{49}i$$

$$= \frac{8}{7} + \frac{1}{7}i$$

1) Write a Python Program to solve System of linear equations given below.

$1x_1 - 3x_2 - 2x_3 = 7$

$2x_2 + 4x_3 = 4$

$-10x_3 = 12$

→

```
import numpy as np
a= np.array ([[1,-3,-2], [0,2,4], [0,0,-10]])
b= np.array ([7,4,12])
c= np.linalg.solve (a,b)
print (c)
```

output :-

> [17.8 4.4 -1.2]

2) Determine whether $V_1 = (2,2,2)$, $V_2 = (0,0,3)$ & $V_3 = (0,1,1)$ Span Vector Space R^3

→ $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

We can observe that the third Column is linear Combination of first two Columns so it doesn't add any ~~two~~ new information to the span of the set of Vector we can discard it, thus.

$\begin{bmatrix} 2 & 0 \\ 2 & 0 \\ 2 & 3 \end{bmatrix}$

We can see that the rank of this matrix is 2. which is 2 . which is less than the dimension So the Vector (v1) (v2) & (v3) do not Span.

Q13 Solve the following System by backward Substitution method.

→ $1x_1 - 3x_2 - 2x_3 = 7$ — (i)

$2x_2 + 4x_3 = 4$ — (ii)

$-10x_3 = 12$ — (iii)

$-10x_3 = 12$

$x_3 = \frac{12}{-10}$

$\boxed{x_3 = -1.2}$

Put the value of x_3 in eq^n (ii)

$2x_2 + 4x_3 = 4$

$2x_2 + 4 \times -1.2 = 4$

$2x_2 + (-4.8) = 4$

$2x_2 = 8.8$

$\boxed{x_2 = 4.4}$

Put the Value of x_2, x_3 in eq^n (i)

$1x_1 - 3x_2 - 2x_3 = 7$

$1x_1 - 3 \times 4.4 - 2 \times (-1.2) = 7$

$1x_1 - 13.2 + 2.4 = 7$

$x_1 - \cancel{10.8} - 10.8 = 7$

$\boxed{x_1 = 17.8}$

∴ $x_1 = +17.8$

$x_2 = +4.4$

$x_3 = -1.2$

Q.14 Show that Vectors $v_1 = (1,0,1)$, $v_2 = (2,1,4)$ and $v_3 = (1,1,3)$ do not Span Vector Space.

→ In this $v_1 = (1,0,1)$, $v_2 = (2,1,4)$, $v_3 = (1,1,3)$
let's denote an arbitary vector in the space as (x,y,z). Now, we'll try to find Coefficients a,b,c, such that.

$$a.(1,0,1) + b.(2,1,4) + c.(1,1,3) = (x,y,z)$$

$$a + 2b + c = x \quad -i$$

$$b + c = y \quad -ii$$

$$a + 4b + 3c = z \quad -iii$$

$$b + c = y$$

$$b = (y - c)$$

Put the Value of b in eq^n (i) & (iii)

$$a + 2b + c = x \quad \text{in (i)}$$

$$a + 2(y-c) + c = x$$

$$a + 2y - 2c + c = x$$

$$a = x - 2y + c$$

in eq^n (iii)

$$a + 4b + 3c = z$$

$$a + 4(y-c) + 3c = z$$

$$a + 4y - 4c + 3c = z$$

$$a = z - 4y + c$$

Equating the expressions for a:

$$x - 2y + c = z - 4y + c$$

$$x - z = 2y - 4y$$

$$x - z = -2y$$

$$\frac{x-z}{-2} = y$$

This ~~8~~ implies that y is dependent on x and ~~4~~ z. Since y is dependent on other variables, it means that the Vectors Cannot Span the entire Space, as there are not unique Solution. That's why they not Span vector space.

15] Write a Python Program for rotating a Complex number z = 2+3i by 180°.

→ Code:-

```
def rotate_Complex_number(z):
    rotated_z = z*(-1)
    return rotated_z

z = 2+3j
rotated_z = rotate_Complex_number(z)
print("original Complex number:", z)
Print("Complex number rotated by 180°:", rotated_z)
```

Output:-

```
original Complex number: (2+3j)
Rotated Complex number: (-2-3j)
```

16] Write a Python Program to rotate a Complex no. by 90 degree, 180° and 270°.

→ Code:-

```
def rotate_90_deg(z):
    return -z.imag + 1j * z.real
def rotate_180_deg(z):
    return -z.real - z.imag * 1j
def rotate_270_deg(z):
    return z.imag - 1j * z.real

Complex_number = 3 + 4j
rotated_90 = rotate_90_deg(complex_number)
rotated_180 = rotate_180_deg(Complex_number)
rotated_270 = rotate_270_deg(Complex_number)
```

```
Print ("original Complex number :", Complex_number)
Print (" Rotated by 90 degrees: ", rotated_90)
Print (" Rotated by 180 degrees :", rotated _180)
Print (" Rotated by 270 degrees :" rotated_270)
```

Output :-

```
original Complex number :- (3+4j)
Rotated by 90 degrees: (-4+3j)
Rotated by 180 degrees: (-3-4j)
Rotated by 270 degrees: (4-3j)
```

17] Which of the Following is a set of generators of R^3

i] $\{(4,0,0), (0,0,2)\}$ ii] $\{(1,0,0), (0,1,0), (0,0,1)\}$

→ i] $\{(4,0,0)\ (0,0,2)\}$ - This set of generators of R^3 given by $\{(4,0,0), (0,0,2)\}$ Consist of two Vectors: $(4,0,0)$ and $(0,0,2)$ Since there are only two vectors in the set they cannot generate R^3 because they do not span the entire three-dimensional Space. So they not a set of generators of R^3

ii] $\{(1,0,0), (0,1,0), (0,0,1)\}$

The Set of vectors $\{(1,0,0), (0,1,0), (0,0,1)\}$ forms a basis for R^3 because it Consists of three linearly independent Vectors that Span the entire three-dimensional Space. Therefore, this Set Serves as a set of generators for R^3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18] Express the following as a linear Combination of $v_1 =$ $(-2,1,3)$, $v_2 = (3,1,-1)$ and $v_3 = (-1,-2,1)$ with $w = (6,-2,5$

→ To express vector $w = (6,-2,5)$ as linear Combination of Vectors $v_1 = (-2,1,3)$, $v_2 = (3,1,-1)$ and $v_3 = (-1,-2,1)$, we need to find Scalars a, b, c Such that

$$w = av_1 + bv_2 + cv_3$$

$$6 = -2a + 3b - c$$
$$-2 = a + b - 2c$$
$$5 = 3a - b + c$$

From the first equation $c = -2a + 3b - 6$

Substitute c in terms of a anb into the 2nd eqn

$$-2 = a + b - 2c$$
$$-2 = a + b + 4a - 6b + 12$$
$$-2 = 5a - 5b + 12$$
$$5a - 5b = -12 - 2$$
$$5a - 5b = -14$$
$$a - b = \frac{-14}{5} \quad - (i)$$

Similarly, substitute c in terms of a and b into the third equation:

$$5 = 3a - b + (-2a + 3b - 6)$$
$$5 = 3a - b - 2a + 3b - 6$$
$$5 = a + 2b - 6$$
$$a + 2b = 11 \quad - (ii)$$

Multiply eqn (1) by 2.

$$2a - 2b = \frac{-28}{5} \quad - (iii)$$

add this to eq^n (2) and eq^n(3)

$$a + \cancel{2b} = 11$$
$$2a - \cancel{2b} = \frac{-28}{5}$$

$$3a = 11 - \frac{28}{5}$$

$$3a = \frac{27}{5}$$

$$\boxed{a = \frac{9}{5}}$$

Put the value of $a = \frac{9}{5}$ in eq^n(1)

$$a - b = \frac{-14}{5}$$

$$\frac{9}{5} - b = \frac{-14}{5}$$

$$-b = \frac{-14}{5} - \frac{9}{5}$$

$$-b = -\frac{23}{5}$$

$$\boxed{b = \frac{23}{5}}$$

Put the value of a and b in $5 = 3a - b + c$

$$5 = 3 \times \frac{9}{5} - \frac{23}{5} + c$$

$$5 = \frac{27}{5} - \frac{23}{5} + c$$

$$5 = \frac{4}{5} + c$$

$$\frac{25}{5} - \frac{4}{5} = c \qquad \boxed{\therefore c = \frac{21}{5}}$$