

118ft 11

Linear Algebra Using Python.

Unit - I

- 1) Write a Python Program for addition, subtraction, multiplication of Complex numbers $4+2j$ and $3-6j$

→ Program :-

$$a = 4 + 2j$$

$$b = 3 - 6j$$

`Print ("Addition of Complex Numbers:", a+b)`

`Print ("Subtraction of Complex Numbers :" a-b)`

`print ("Multiplication of Complex Numbers :" a*b)`

Output:-

Addition of Complex Numbers : $(7-4j)$

Subtraction of Complex Numbers : $(1+8j)$

Multiplication of Complex Numbers : $(24-18j)$

- 2) Define :- Galois field, Dot Product, Convex Combination, span

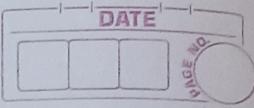
→

i) Galois field :- Galois field with two elements is denoted by $GF(2)$. It has two elements

0 and 1.

Since it is field there are two binary operations defined (+ & .). the behaviour of the element '0' & '1' w.r.t. (+) & (.) is given as follows.

		+	.
+	0	0 1	0 1
	1	1 0	0 1
		0 0	0 0



ii) Dot Product :- The dot Product is defined as the sum of the products of the corresponding entries of the two sequences of Numbers.

iii) Convex Combination:- An expression of the form $\alpha u + \beta v$, where $\alpha, \beta > 0, \alpha + \beta = 1$ is called Convex Combination of u and v .

iv) Span :- The set of all linear combination of vector v_1, v_2, \dots, v_n is called the Span of the Vector. i.e.

$$\text{Span } \{v_1, v_2, \dots, v_n\} = \left\{ \sum_{i=1}^n k_i v_i \mid k_i \in F \right\}$$

Q3. Write a python Program to find Conjugate of Complex Numbers.

→

```
#import Numpy
import numpy as np
a = 2+4j
a1 = np.conj(a)
print("Conjugate of 2+4j: ", a1)
b = 5-8j
b1 = np.conj(b)
print("Conjugate of 5-8j: ", b1)
```

Output:-

Conjugate of $2+4j$: $(2-4j)$
 Conjugate of $5-8j$: $(5+8j)$

Q4. Are the following vectors are linearly dependent

$$v_1 = (3, 2, 7), v_2 = (2, 4, 1) \text{ and } v_3 = (1, -2, 6)$$

→ Let's assume there exist constants a, b, c
such that :

$$a.v_1 + b.v_2 + c.v_3 = 0$$

Substituting the given vectors

$$a.(3, 2, 7) + b.(2, 4, 1) + c.(1, -2, 6) = (0, 0, 0)$$

$$(3a + 2b + c, 2a + 4b - 2c, 7a + 1b + 6c) = (0, 0, 0)$$

$$(3a + 2b + c) = 0$$

$$7a + 1b + 6c = 0$$

This system has a non-trivial solution indicating that the vectors are linearly dependent.

Q5. Check whether the vectors are linearly dependent

$$v_1 = (1, -2, 1), v_2 = (2, 1, -2) \text{ & } v_3 = (7, -4, 1)$$

→ Let's assume there exist constants a, b, c such that

$$a.v_1 + b.v_2 + c.v_3 = 0$$

Substituting the given vectors:

$$a(1, -2, 1) + b(2, 1, -2) + c(7, -4, 1) = (0, 0, 0)$$

Expanding this equation

$$(a + 2b + 7c, -2a + b - 4c, a - 2b + c) = (0, 0, 0)$$

$$a + 2b + 7c = 0$$

$$a - 2b + c = 0$$

This system has a non-trivial solution, instead that the vectors are linearly dependent.

(Q6) Express the Polar and exponential form
 $1+i\sqrt{3}$

→ To express the complex number $(1+i\sqrt{3})$ in Polar & Exponential form, we 1st need to find its Magnitude & Argument.

Magnitude (r):

$$r = |1+i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3} = 2$$

Argument (θ):

$$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

Polar form

$$1+i\sqrt{3} = 2 \cdot e^{i\frac{\pi}{3}}$$

Exponential form :-

$$1+i\sqrt{3} = 2 \cdot \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)\right)$$

(Q7) Find the square root of complex numbers
 → let us assign complex number $x+iy$ to the square root of $8-6i$ such that both x and y are real numbers.
 Then we have

$$x+iy = \sqrt{8-6i}$$

Squaring both sides, we have

$$(x+iy)^2 = 8-6i$$

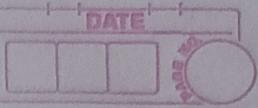
$$x^2 + 2ixy - y^2 = 8-6i$$

$$(x^2 - y^2) + 2ixy = 8-6i$$

$$x^2 - y^2 = 8 - \textcircled{1}$$

$$2xy = -6 - \textcircled{2}$$

Hence we have 2 equations $\textcircled{1}$ & $\textcircled{2}$ in two unknown x and y .



From the second equation, we have.

$$xy = -3$$

$$x^2y^2 = 9$$

$$x^2 = y^2 + 8$$

$$(y^2 + 8) y^2 = 9$$

$$(w+8)w = 9$$

$$w^2 + 8w - 9 = 0$$

$$w(w+9) - (w+9) = 0$$

$$(w-1)(w+9) = 0$$

$$w = 1 \quad ; \quad w = -9$$

y is real number, hence its square can't be negative
hence we have.

$$w = 1$$

$$y^2 = 1$$

$$y = \pm 1 \quad \text{--- ⑥}$$

For $y = 1$, from eqn ③ we have.

For $y = -1$, from eqn ③ we have.

$$x(1-1) = -3$$

$$x = 3$$

Hence another root is $3-i$

Hence the square roots of $8-6i$ are $-3+i$ & $3-i$

8] Find the square root of complex numbers $-5+12i$

$$\sqrt{a+ib} = \pm \left[\sqrt{\frac{a^2+b^2+q}{2}} + i \sqrt{\frac{a^2+b^2-q}{2}} \right]$$

$$\text{So } q = -5, b = 12 \Rightarrow a^2 = 25, b^2 = 144.$$

$$\sqrt{-5+12i} = \pm \left[\sqrt{\frac{25+144+(-5)}{2}} + i \sqrt{\frac{25+144-(-5)}{2}} \right]$$

$$= \pm \sqrt{\frac{169-5}{2}} + i \sqrt{\frac{169-5}{2}}$$

$$= \pm \left[\sqrt{\frac{\sqrt{169-5}}{2}} + i \sqrt{\frac{\sqrt{169+5}}{2}} \right]$$

$$= \pm \left[\sqrt{\frac{13-5}{2}} + i \sqrt{\frac{13+5}{2}} \right]$$

$$= \pm \left[\sqrt{\frac{8}{2}} + i \sqrt{\frac{18}{2}} \right]$$

$$= \pm [\sqrt{4} + \sqrt{9}]$$

$$= \pm \sqrt{5+12i} = \pm [2+3i]$$

Q9 Find the square root of $21-20i$ where $i = \sqrt{-1}$

$$\rightarrow \text{Let } \sqrt{21-20i} = 21+iy$$

$$21-20i = x^2 - y^2 + 2ixy$$

\therefore On Comparing the real & imaginary parts

$$x^2 - y^2 = 21, 2xy = -20$$

Now,

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= (x^2 + y^2)^2 = (21)^2 + (10)^2$$

$$= (x^2 + y^2)^2 = 441 + 100$$

$$= (x^2 + y^2)^2 = 841$$

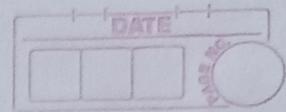
$$\therefore (x^2 + y^2) = 29$$

Solving eqn $x^2 - y^2 = 21$ & $x^2 + y^2 = 29$

$$x = \pm 5, y = \pm 2$$

Since xy is negative, x & y will be of opposite sign.

$$\therefore \sqrt{21-20i} = \pm \sqrt{21-20i}; \\ = \pm \sqrt{29} \quad \pm (5-2i)$$



- 10) Express $\frac{(3+2i)}{(2+i)(1-3i)}$ in the form $a+bi$
- The conjugate of $(2+i)$ is $(2-i)$ and
The conjugate of $(1-3i)$ is $(1+3i)$
So multiplying both the numerator and the denominator by their respective conjugates.

$$\frac{3+2i}{(2+i)(1-3i)} \times \frac{(2-i)(1+3i)}{(2-i)(1+3i)}$$

Expanding both the numerator & the denominator

Numerator :-

$$(3+2i)(2-i)(1+3i) = (6+3i+4i-2i^2)(1+3i) = \\ (6+i+2)(1+3i)$$

Denominator :-

$$(2+i)(2i)(1+3i) = (4-1+2i-i^2)(1+3i) = (4+1+2)(1+3i)$$

So after Simplification, we get

$$= \frac{(6+i+2)(1+3i)}{(4+1+2)(1+3i)}$$

$$= \frac{(8+i)(1+3i)}{(7)(1+3i)}$$

$$= \frac{8+24i+i-3i^2}{7+21i}$$

$$= \frac{8+25i-3}{7+21i}$$

Now, divide both equal real & imaginary part of by the denominator

$$\frac{8+25i}{7+21i}$$

$$= \frac{35+175i-105i+525}{49+441}$$

$$= \frac{560+70i}{490}$$

Now, Simplify, we get

$$\begin{aligned}
 & \frac{(6+i+2)(1+3i)}{(4+i+2)(1+3i)} \\
 &= \frac{(8+i)(1+3i)}{(7)(1+3i)} \\
 &= \frac{8 + 24i + i + 3i^2}{7 + 21i} \\
 &= \frac{8 + 25i - 3}{7 + 21i}
 \end{aligned}$$

Now, divide both real & imaginary part by the denominator.

$$= \frac{5 + 25i}{7 + 21i}$$

Now, multiply by the conjugate of the denominator

$$= \frac{(5 + 25i)(7 - 21i)}{-7^2 - 21^2}$$

$$= \frac{35 + 175i - 105i + 525}{49 + 49i}$$

$$= \frac{560 + 70i}{490}$$

Now, simplify

$$= \frac{112 + 14i}{98}$$

$$= \frac{56}{49} + \frac{7}{49}i$$

$$= \frac{8}{7} + \frac{1}{7}i$$

i) Write a Python Program to solve system of linear equations given below.

$$1u_1 - 3u_2 - 2u_3 = 7$$

$$2u_2 + 4u_3 = 4$$

$$-10u_3 = 12$$

→ import numpy as np

a = np.array ([[1,-3,-2], [0,2,4], [0,0,-10]])

b = np.array ([7,4,12])

c = np.linalg.solve(a,b)

print(c)

Output:-

[17.8 4.4 -1.2]

ii) Determine whether $v_1 = (2, 2, 2)$, $v_2 = (0, 1, 3)$ & $v_3 = (0, 1, 1)$ Span Vector Space \mathbb{R}^3

$$\begin{bmatrix} 2 & 6 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

We can observe that the third column is linear combination of first two columns so it doesn't add any new information to the span of the set of vectors we can discard it, thus.

$$\begin{bmatrix} 2 & 0 \\ 2 & 0 \\ 2 & 3 \end{bmatrix}$$

We can see that the rank of this matrix is 2 which is 2 which is less than the dimension so the vectors $(v_1), (v_2)$ & (v_3) do not span.

Q13 Solve the following system by backward substitution method.

$$\rightarrow \begin{aligned} x_1 - 3x_2 - 2x_3 &= 7 & \text{(i)} \\ 2x_2 + 4x_3 &= 4 & \text{(ii)} \\ -10x_3 &= 12 & \text{(iii)} \end{aligned}$$

$$-10x_3 = 12$$

$$x_3 = \frac{12}{-10}$$

$$x_3 = -1.2$$

Put the value of x_3 in eqn (ii)

$$2x_2 + 4x_3 = 4$$

$$2x_2 + 4(-1.2) = 4$$

$$2x_2 + (-4.8) = 4$$

$$2x_2 = 8.8$$

$$x_2 = 4.4$$

Put the value of x_2, x_3 in eqn (i)

$$x_1 - 3x_2 - 2x_3 = 7$$

$$x_1 - 3(4.4) - 2(-1.2) = 7$$

$$x_1 - 13.2 + 2.4 = 7$$

$$x_1 - 10.8 = 7$$

$$x_1 = 17.8$$

$$\therefore x_1 = 17.8$$

$$x_2 = 4.4$$

$$x_3 = -1.2$$

Q14 Show that Vectors $v_1 = (1, 0, 1)$, $v_2 = (2, 1, 4)$ and $v_3 = (1, 1, 3)$ do not Span Vector Space.

→ In this $v_1 = (1, 0, 1)$, $v_2 = (2, 1, 4)$, $v_3 = (1, 1, 3)$ let's denote an arbitrary vector in the space as (x, y, z) . Now, we'll try to find coefficients a, b, c , such that.

$$a \cdot (1, 0, 1) + b \cdot (2, 1, 4) + c \cdot (1, 1, 3) = (x, y, z)$$

$$a + 2b + c = x \quad \text{--- i}$$

$$b + c = y \quad \text{--- ii}$$

$$a + 4b + 3c = z \quad \text{--- iii}$$

$$b + c = y$$

$$b = (y - c)$$

Put the value of b in eqn (i) & (iii)

$$a + 2b + c = x \quad \text{in eqn i}$$

$$a + 2(y - c) + c = x$$

$$a + 2y - 2c + c = x$$

$$a = x - 2y + c$$

in eqn (iii)

$$a + 4b + 3c = z$$

$$a + 4(y - c) + 3c = z$$

$$a + 4y - 4c + 3c = z$$

$$a = z - 4y + c$$

Equating the expressions for a :

$$x - 2y + c = z - 4y + c$$

$$x - z = 2y - 4y$$

$$x - z = -2y$$

$$\frac{x - z}{-2} = y$$

This implies that y is dependent on x and z . Since y is dependent on other variables, it means that the vectors cannot span the entire space, as there are not unique solution. That's why they not span vector space.

5] write a Python Program For rotating a Complex number $z = 2+3i$ by 180° .

→ Code:-

```
def rotate_complex_number(z):
    rotated_z = z * (-1)
    return rotated_z
```

$z = 2+3j$

```
rotated_z = rotate_complex_number(z)
print("Original Complex number:", z)
print("Complex number rotated by 180°:", rotated_z)
```

Output:-

Original Complex number : $(2+3j)$

Rotated Complex number : $(-2-3j)$

6] Write a Python Program to rotate a Complex no. by 90 degree, 180° and 270° .

→ Code:-

```
def rotate_90_deg(z):
```

```
    return -z.imag + 1j * z.real
```

```
def rotate_180_deg(z):
```

```
    return -z.real - z.imag * 1j
```

```
def rotate_270_deg(z):
```

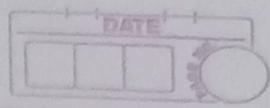
```
    return z.imag - 1j * z.real
```

Complex-number = $3+4j$

rotated_90 = rotate_90_deg(complex.number)

rotated_180 = rotate_180_deg(Complex.number)

rotated_270 = rotate_270_deg(Complex.number)



```

Print ("Original Complex number : ", Complex_number)
Print ("Rotated by 90 degrees : ", rotated_90)
Print ("Rotated by 180 degrees : ", rotated_180)
Print ("Rotated by 270 degrees : ", rotated_270)

```

Output :-

Original Complex number :- $(3+4j)$
 Rotated by 90 degrees: $(-4+3j)$
 Rotated by 180 degrees: $(-3-4j)$
 Rotated by 270 degrees: $(4-3j)$

- i) Which of the following is a set of generators of \mathbb{R}^3
- $\{(4,0,0), (0,0,2)\}$
 - $\{(1,0,0), (0,1,0), (0,0,1)\}$
- i) $\{(4,0,0), (0,0,2)\}$ - This set of generators of \mathbb{R}^3 given by $\{(4,0,0), (0,0,2)\}$ consist of two vectors: $(4,0,0)$ and $(0,0,2)$. Since there are only two vectors in the set they cannot generate \mathbb{R}^3 because they do not span the entire three-dimensional space. So they not a set of generators of \mathbb{R}^3 .
- ii) $\{(1,0,0), (0,1,0), (0,0,1)\}$

The set of vectors $\{(1,0,0), (0,1,0), (0,0,1)\}$ forms a basis for \mathbb{R}^3 because it consists of three linearly independent vectors that span the entire three-dimensional space.

Therefore, this set serves as a set of generators for \mathbb{R}^3 .

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

18] Express the following as a linear combination of v_1 , v_2 , v_3 and w .
 $v_1 = (-2, 1, 3)$, $v_2 = (3, 1, -1)$ and $v_3 = (-1, -2, 1)$ with $w = (6, -2, 5)$
 \rightarrow To express vector $w = (6, -2, 5)$ as linear combination of vectors $v_1 = (-2, 1, 3)$, $v_2 = (3, 1, -1)$ and $v_3 = (-1, -2, 1)$, we need to find scalars a, b, c such that
 $w = av_1 + bv_2 + cv_3$

$$6 = -2a + 3b - c$$

$$-2 = a + b - 2c$$

$$5 = 3a - b + c$$

From the first equation $c = -2a + 3b - 6$

Substitute c in terms of a and b into the 2nd eqn

$$-2 = a + b - 2c$$

$$-2 = a + b + 4a - 6b + 12$$

$$-2 = 5a - 5b + 12$$

$$5a - 5b = -12 - 2$$

$$5a - 5b = -14$$

$$a - b = \frac{-14}{5} \quad \text{--- i}$$

Similarly, substitute c in terms of a and b into the third equation:

$$5 = 3a - b + (-2a + 3b - 6)$$

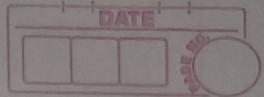
$$5 = 3a - b - 2a + 3b - 6$$

$$5 = a + 2b - 6$$

$$a + 2b = 11 \quad \text{--- ii}$$

Multiply eqn (i) by 2.

$$2a - 2b = \frac{-28}{5} \quad \text{--- iii}$$



Add this to eqn (2) and eqn (3)

$$a + 2b = 11$$

$$2a - 2b = -\frac{28}{5}$$

$$3a = 11 - \frac{28}{5}$$

$$3a = \frac{27}{5}$$

$$\boxed{a = \frac{9}{5}}$$

Put the value of $a = \frac{9}{5}$ in eqn (1)

$$a - b = -\frac{14}{5}$$

$$\frac{9}{5} - b = -\frac{14}{5}$$

$$-b = -\frac{14}{5} - \frac{9}{5}$$

$$-b = -\frac{23}{5}$$

$$\boxed{b = \frac{23}{5}}$$

Put the value of a and b in $s = 3a - bt + c$

$$s = 3 \times \frac{9}{5} - \frac{23}{5} + c$$

$$s = \frac{27}{5} - \frac{23}{5} + c$$

$$s = \frac{4}{5} + c$$

$$\frac{25}{5} - \frac{4}{5} = c$$

$$\therefore c = \frac{21}{5}$$

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