Computational methods and applications (AMS 147)

Homework 4 - Due Thursday, February 23, 11:59 pm

Please submit to CANVAS a .zip file that includes the following Matlab functions:

`poly_least_squares.m`
`test_least_squares_finance.m`
`test_least_squares_interp.m`

Exercise 1 Write a Matlab function `poly_least_squares.m` that implements the least squares method we discussed in class to approximate a data set in terms of a polynomial model of degree M. The function should be in the form

```
function [a,err] = poly_least_squares(x,y,M)
```

Input:
`x`: vector of nodes `x=[x(1) ... x(N)]`
`y`: vector of data points `y=[y(1) ... y(N)]` corresponding to `[x(1) ... x(N)]`
`M`: degree of the polynomial model

$$\psi(x) = \texttt{a(1)} + \texttt{a(2)}x + \texttt{a(3)}x^2 + \cdots + \texttt{a(M+1)}x^M \tag{1}$$

Output:
`a`: vector of coefficients representing the polynomial (1)
`err`: Error between the model and the data in the 2-norm

$$\texttt{err} = \sum_{i=1}^{N} \left[y_i - \psi(x_i)\right]^2 . \tag{2}$$

Exercise 2 Use the function you coded in Exercise 1 to determine the least squares polynomial approximant of the attached financial data set `Dow_Jones_2012_2017.dat` (normalized closing price of the Dow Jones index from 2-14-2012 to 2-13-2017). To this end, write a Matlab function `test_least_squares_finance.m` that plots in the same figure the data points $\{x_i, y_i\}_{i=1,\ldots,n}$ (in blue) and the least-squares polynomial model (1) for $M = 1, 2, 4, 8$. Assume that the domain of the polynomial is the same as domain of the data points, i.e., plot the polynomial model in the interval $[0, 1]$.

Exercise 3 Show that the least squares polynomial approximant can be an interpolant. To this end, apply the function you coded in Exercise 1 to the following data set

$$x_j = -1 + 2\frac{j}{15}, \qquad y_j = \frac{1}{2 + \sin\left(20 x_j^2\right)}, \qquad j = 0, ..., 15. \tag{3}$$

Set `M`=15 in (1), and write a Matlab function `test_least_squares_interp.m` that returns a plot the data points (3) (blue circles) and a plot of the polynomial model (1) (red line) in the same figure.