Computational methods and applications (AMS 147)

Homework 6 - Due Friday, March 17, 11:59 pm

Please submit to CANVAS a .zip file that includes the following Matlab functions:

`tridiag_solver.m`

`AB3.m`

`solve_ODE_system.m`

Exercise 1 Write a Matlab function that implements the Thomas algorithm to solve tridiagonal linear systems of equations in the form

$$\begin{bmatrix} a_1 & c_1 & 0 & \cdots & 0 \\ e_1 & a_2 & c_2 & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & & a_{n-1} & c_{n-1} \\ 0 & \cdots & 0 & e_{n-1} & a_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} \tag{1}$$

The function should be in the form

```
function x = tridiag_solver(e,a,c,b)
```

Input:

`e`$=[e_1\ e_2\ \cdots\ e_{n-1}]$ `a`$=[a_1\ a_2\ \cdots\ a_n]$ `c`$=[c_1\ c_2\ \cdots\ c_{n-1}]$ `b`$=[b_1\ b_2\ \cdots\ b_n]^T$

Output:

`x`$=[x_1\ x_2\ \cdots\ x_n]^T$

Exercise 2 Consider the system of nonlinear ordinary differential equations

$$\begin{cases} \dfrac{dy(t)}{dt} = f(y(t),t) \\ y(0) = y_0 \end{cases} \tag{2}$$

where $f : \mathbb{R}^n \times [0,T] \to \mathbb{R}^n$, $y : [0,T] \to \mathbb{R}^n$. Write a Matlab function that solves the system (2) by using the third-order Adams-Bashforth scheme. To start-up the scheme (first two steps) you can use the Euler-forward or the Heun method. The function should be in the form

```
function [y,t] = AB3(fun,y0,T,DT,IOSTEP)
```

Input:

`fun`: function handle representing $f(y,t)$

`y0`: column vector representing the initial condition

`T`: period of integration

`DT`: time step

`IOSTEP`: Input/output step. The solution is saved in the output matrix `y` every `IOSTEP` steps.

Output:

`y`: $n \times S$ matrix collecting the time snapshots of the solution to (2). Note that the total number of snapshots S (including the initial condition) is `floor(T/(IOSTEP*DT))+1`.

`t`: vector collecting the time instants at which the solution is saved in the output matrix `y`.

Exercise 3 Consider the following nonlinear dynamical system

$$\begin{cases} \dfrac{dy_1(t)}{dt} = -y_1(t) + y_2(t)y_3(t) \\ \dfrac{dy_2(t)}{dt} = -y_2(t) + (y_3(t) - 2)y_1(t) \\ \dfrac{dy_3(t)}{dt} = 1 - y_1(t)y_2(t) \end{cases} \tag{3}$$

It is known that the solution to (3) is chaotic in time and it settles on a strange attractor. By using the function `AB3.m` you coded in Exercise 2, compute the numerical solution to (3). To this end, set `NSTEPS`=1000000, `IOSTEP`=50, `DT`= 1e-3, `y0`=[1 2 3]^T, and write a function

```
function solve_ODE_system()
```

that returns the following items:

1. The plots of the trajectories $y_1(t)$, $y_2(t)$ and $y_3(t)$ versus time in `figure(1)`.
2. A three-dimensional plot of the curve $(y_1(t), y_2(t), y_3(t))$ in `figure(2)` (use the command `plot3()`). Such curve provides a visualization of the strange attractor.