Computational methods and applications (AMS 147)

Homework 5 - Due Friday, March 3, 11:59 pm

Please submit to CANVAS a .zip file that includes the following Matlab functions:

`int_midpoint_rule.m`

`int_trapezoidal_rule.m`

`int_Simpson_rule.m`

`test_integration.m`

Exercise 1 Consider the following integral

$$I(f) = \int_a^b f(x)dx. \tag{1}$$

Write three Matlab functions implementing, respectively, the composite midpoint rule, the composite trapezoidal rule, and the composite Simpson rule to compute the numerical approximation of $I(f)$. Such functions should be in the form

`function [I]=int_midpoint_rule(fun,a,b,n)`	(composite midpoint rule)
`function [I]=int_trapezoidal_rule(fun,a,b,n)`	(composite trapezoidal rule)
`function [I]=int_Simpson_rule(fun,a,b,n)`	(composite Simpson rule)

Input:

`fun`: function handle representing $f(x)$

`a,b`: endpoints of the integration interval

`n`: number of evenly-spaced points in $[a, b]$ (including endpoints)

$$x_j = a + (j-1)h, \qquad h = \frac{b-a}{n-1}, \qquad j = 1, ...n.$$

to compute the numerical approximation of the integral (1).

Output:

`I`: numerical approximation of the integral (1).

Exercise 2 Use the functions you coded in Exercise 1 to compute the numerical approximation of the integral

$$I = \int_{-3}^{1} \left[\frac{1}{1+x^2} \cos\left(\frac{3}{2} e^{-x^2} \right) - \frac{x^3}{30} \right] dx. \tag{2}$$

To this end, write a Matlab script `test_integration.m` that returns the following items:

1. A plot of the integrand function in (2) (in `figure(1)`);

2. A plot of the error

   $$e(n) = |I - I_n| \qquad n = 2, 3, ..., 10000 \tag{3}$$

   where $I = 1.6851344770476$ is the reference value for the integral (2), while I_n is the numerical approximation obtained by using the integration rules you coded in Exercise 1. In particular, plot in the same `figure(2)` the error (3) versus n (use the Matlab command `loglog()`) the composite midpoint rule, the composite trapezoidal rule, and the composite Simpson rule.