Corrections To Approximation Algorithms Paper

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1 Paper Rules

There are *Three Basic Rules* to keep in mind when writing a paper.

1. Do not start a sentence with a symbol.
2. Avoid run together mathematical expressions.
3. Watch out for misplaced symbols.

2 Equation in the Title

One of the most common of our violated rules can be found in the title of the paper. It is a violation to start a paper with an equation. The title of this paper violates rules 1 and 2, which can be seen below in red while a corrected version will follow in blue.

$O\left(\sqrt{\log(n)}\right)$ **Approximation Algorithms for Min UnCut, Min 2CNF Deletion, and Directed Cut Problems**

We give $O\left(\sqrt{\log(n)}\right)$ **Approximation Algorithms for Min UnCut, Min 2CNF Deletion, and Directed Cut Problems**

Going one step further.

Improvement on Approximation Algorithms for Min UnCut, Min 2CNF Deletion, and Directed Cut Problems

3 Abstract

A **Abstract** is a *self-contained* summary of the article used to grab the attention and interest of the reader. Typical things to avoid are Formulas, References, unsubstantiated statements and claims, and in some journals use of "We". Violation will be in red font while a corrected version will follow in blue.

We give $O(\sqrt{\log(n)})$-approximation algorithms for the Min UnCut, Min 2CNF Deletion, Directed Balanced Separator, and Directed Sparsest Cut problems. The previously best-known algorithms give an $O(\log(n))$-approximation for Min UnCut [9], Directed Balanced Separator [17], Directed Sparsest Cut [17], and an $O(\log(n)\log(n)\log(n))$-approximation for Min 2CNF Deletion [14].We also show that the integrality gap of an SDP relaxation.

An improvement on previously best-known approximation algorithms for several combinatorial problems, namely: Min UnCut, Directed Balanced Separator, Directed Sparsest Cut and Min 2CNF Deletion. New methods obtained using semidefinite relaxation on directed graphs with a simpler algorithm that deals with Min UnCut.

4 Definition 1

Definition 1 (Min CSP(F)). Consider boolean variables $b_1, ..., b_n$ and a set of constraints C from F. The goal is to find an assignment that minimizes the number of unsatisfied constraints.

Corrections to the above definition.

- A Minimum Constraint Satisfaction Problem or Min CSP may have just been defined it is somewhat confusing in the definition.
- A set of constraints C from a set F.
- Ldots used for boolean variable range.

Definition 1 (Min CSP(F)) Consider a set C of constraints from a set F with boolean variable $b_1, \ldots, b_n$. We want to show an assignment that minimizes the number of unsatisfied constraints.

- A Minimum Constraint Satisfaction Problem or Min CSP may have just been defined it is somewhat confusing in the definition.
- A set of constraints C from a set F.
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5 Three More Violations

5.1 How Not to State a Theorem 3.1

Theorem 3.1. There is a randomized polynomial-time algorithm for finding an $O(\sqrt{\log(n)})$ approximation for the Min 2CNF Deletion problem.

- The statement of this theorem is not self-contained.
- Statement is vague and unclear.
- Min 2CNF Deletion problem is defined earlier.

5.2 How Not to State a Theorem 2.1

Theorem 2.1. There is a randomized polynomial-time algorithm for finding an $O(\sqrt{\log(n)})$ approximation for the Min UnCUT problem.

- The statement of this theorem is not self-contained.
- Statement is vague and unclear.
- Min UnCUT Deletion problem is defined earlier.

5.3 "Any" Violations

In red I have identified 3 of the 4 "Any" violations.

The previously best-known approximation ratio for Min UnCut is $O(\log n)$ [9], and the best previously known approximation for Min 2CNF Deletion is $O(\log n \log\log n)$[14]. Both problems are known to be Max SNP-hard [18]. The best-known lower bound for Min 2CNF Deletion is $8\sqrt{5} - 15 \approx 2.88854$ [7]. Moreover, if the Unique Games Conjecture holds true, then Min 2CNF Deletion cannot be approximated within any constant factor [13].

LEMMA 5.1. There is a polynomial-time algorithm for the following task. Given any feasible SDP solution with $\beta = \sum_{(i,j)\in E} d(i,j)$, and a vertex k such that the ball of squared-radius $1/(8n^2)$ around v_k contains at least $n/2$ vectors (other than v), the algorithm finds a cut $(S, \bar{S})$ with directed expansion at most $O(\beta n)$.

THEOREM 6.2 Given r, $d = 4k$, $k \geq r \geq 2$, there exist a positive constant $\epsilon = \epsilon(r)$ so that in any set of more than $(2 - \epsilon(r))^d$ (± 1)-vectors there are r pairwise orthogonal vectors.

Corrected "Any" violations will be shown below in blue.

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LEMMA 5.1. There is a polynomial-time algorithm for the following task. Given an arbitrarily feasible SDP solution with $\beta = \sum_{(i,j)\in E} d(i,j)$, and a vertex k such that the ball of squared-radius $1/(8n^2)$ around v_k contains at least $n/2$ vectors (other than v), the algorithm finds a cut $(S, \bar{S})$ with directed expansion at most $O(\beta n)$.

THEOREM 6.2 Given r, $d = 4k$, $k \geq r \geq 2$, there exist a positive constant $\epsilon = \epsilon(r)$ so that in every set of more than $(2 - \epsilon(r))^d$ (± 1)-vectors there are r pairwise orthogonal vectors.