Question 1: EDA warmup (7 Points)

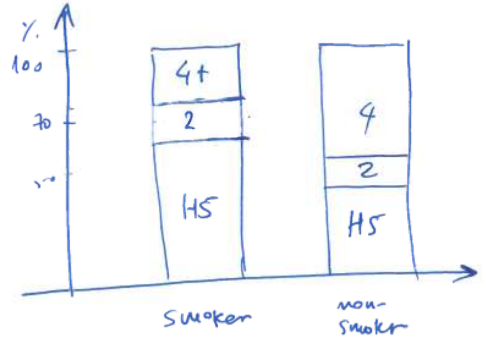
A.(5) 200 adults shopping at a supermarket were asked about the highest level of education they had completed and whether or not they smoke cigarettes. Results are summarized in the table.

	Smoker	Non-smoker	Total
High school	32	61	93
2 yr college	5	17	22
4+ yr college	13	72	85
Total	50	150	200

i.(1) What percent of the shoppers were smokers with only high school educations?
What percent of the shoppers with only high school educations were smokers?
What percent of the smokers had only high school educations?

(a) $32/200 = 16\%$
(b) $32/93 \cong 34.4\%$
(c) $32/50 = 64\%$

ii.(2) Sketch an appropriate graph/plot to for comparing education level among smokers and non-smokers. Label your graph clearly.



segmented bar chart is more appropriate here

iii.(2) Do these data suggest there is an association between smoking and education level? Give *statistical evidence* to support your conclusion. Does this indicate that students who start smoking while in high school tend to give up the habit if they complete college? Explain *shortly*.

(a) There is possible association between smoking & education level
64% of smokers had only a HS diploma
40.7% of non-smokers had only a HS-diploma
Similarly, 26% of smokers had 4+ year college
compared to 48% of non-smoker

(b) No, these data were collected at one time, about 2 different groups (smokers/non-smokers). We have no idea if smoking behavior changes over time

B.(2) An automobile service shop reported the summary statistics shown for repair bills (in €) for their customers last month.

Min	27
Q1	88
Median	132
Q3	308
Max	1442
Mean	284
SD	140

i.(1) Were any of the bills outliers? Show (using statistical evidence) how you made your decision.

Yes. $IQR = Q3 - Q1 = 308 - 88 = 220$
Upper Fence $= Q3 + 1.5 IQR = 638 << 1442$
which means that there are (at least one) outliers

ii.(1) After checking out a problem with your car the service manager gives you an estimate of "*only* €90." Is the manager right to imply that your bill will be unusually low? Explain briefly.

No. €90 is higher than Q1 (i.e. 25% of all bills)
so it's not unusually low

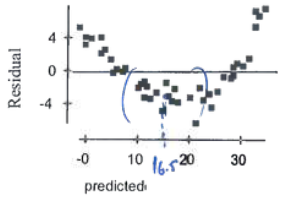
Question 2: Let's (re)discover penicillin (9 Points)

Doctors studying how the human body assimilates medication inject some patients with penicillin, and then monitor the concentration of the drug (in units/cc) in the patients' blood for seven hours. First, they tried to fit a linear model. The regression analysis and residuals plot are shown below.

```
Dependent variable is:     Concentration
No Selector
R squared = 90.8%    R squared (adjusted) = 90.6%
s =  3.472  with  43 - 2 = 41  degrees of freedom

Source        Sum of Squares    df    Mean Square    F-ratio
Regression    4900.55            1    4900.55        407
Residual      494.199           41    12.0536

Variable    Coefficient    s.e. of Coeff    t-ratio    prob
Constant    40.3266        1.295            31.1       ≤ 0.0001
Time        -5.95956       0.2956           -20.2      ≤ 0.0001
```



A.(2) Explain what the value of R squared (90.8%) means in this context and what is the difference from R squared adjusted.

90.8% in the variance of penicillin concentration can be explained by "time" variable

B.(1) Using this model, estimate what the concentration of penicillin will be after *4 hours*.

$\hat{y} = 40.3266 - 5.95956 \times \underset{\smile}{4} \cong 16.5$ units/cc

C.(2) Is that estimate likely to be accurate, too low or too high? Use the residual plot to explain.

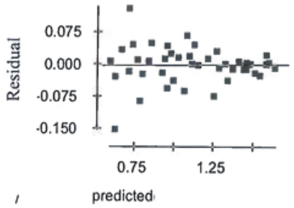
See figure: Predicted value (4 hours) is ~~in~~ an area of negative residuals meaning that the estimate will be too high

Now the researchers try a new model, using the re-expression log(*Concentration*). Examine the regression analysis and the residuals plot below.

```
Dependent variable is:     LogCnn
No Selector
R squared = 98.0%    R squared (adjusted) = 98.0%
s =  0.0451  with  43 - 2 = 41  degrees of freedom

Source        Sum of Squares    df    Mean Square    F-ratio
Regression    4.11395            1    4.11395        2022
Residual      0.083412          41    0.002034

Variable    Coefficient    s.e. of Coeff    t-ratio    prob
Constant    1.80184        0.0168           107        ≤ 0.0001
Time        -0.172672      0.0038           -45.0      ≤ 0.0001
```



D.(1) Is this model better than the previous one? Explain why you think yes or no.

Yes
(a) residuals show no pattern (random)
(b) better adj. R^2

E.(1) Using this new model, estimate the concentration of penicillin after 4 hours.

$\boxed{\log_{10} C} = 1.80184 - 0.172672 \times 4 = 1.11152$

$\Downarrow$

$C = 10^{1.11152} \cong 12.9$ units/cc

F.(2) Explain (in the context of this specific problem) what the coefficient of *"time"* means.

Every 1 hour passes ⇒ the logarithm of concentration is reduced by −0.172672

1. vi - A : $(x_1,x_2) \Leftrightarrow (ax_1,x_2)$: shrinks in the x_1-direction
FT: $\mathcal{F}(f(ax_1,x_2)) = \frac{1}{a}\mathcal{F}(f(\xi_1/a,\xi_2))$: stretched in ξ_1 direction and the whole figure stretch

ii - F

2. $f(x_1,ax_2)$ similar with 1.

3. v - C : shift x_1 by a to the left
FT: only a phase change ⇔ same

4. iv - D : A: rotation by $\pi/6$ ⇒ similar with FT

5. i - E : similar with 4.

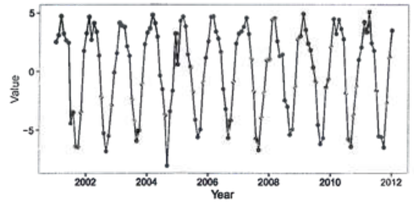
6. iii - B : well, only one choice left ☺
$\mathcal{F}(f(x_1,x_2)\cdot \mathrm{sinc}(ax_1)\cdot \mathrm{sinc}(ax_2)) = \mathcal{F}(f(f_1,f_2)\cdot \Pi_a(f_1)\Pi_a(f_2))$
FT: cutts-off the FT by a function of a
spatial domain: low-pass

Question 5: Time is of the essence (6 points)

A. (2) Why is stationarity a desirable property for a time series process?

With a stationary process, a longer time series $(n \gg)$ gives more info about the statistical properties (means, autocorrelation etc) and they don't change

B. (2) Below is a plot of a monthly time series. The investigator who collected it expects there to be an annual seasonal pattern, but she is more interested in the long-term trend.



Suggest a method the investigator could use to model the seasonality, along with shortly motivating why.

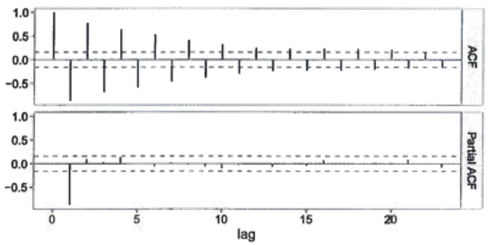
More than one correct answer:

Examples:

(a) calculate monthly means and model deviations from the mean

(b) model the residuals of regression against the months

C. (2) Given the autocorrelation plot and partial autocorrelation plot below, what is your estimate for which kind of ARIMA model the researcher should attempt?



ARIMA(1,0,0) :

Question 6: Small Dimensionality Reduction (5 points)

We have a small database of ratings of 5 movies by 6 users (negatives are allowed, in the case that a user dislikes a movie much). They are represented by a 6 × 5 matrix X, where each row corresponds to a user and each column corresponds to a movie.

$$\begin{array}{cc} & \text{movies} \\ \text{users} & X = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -3 & -3 & -3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

A.(1) What is the sample user mean, i.e. how is the average user rating movies?

obviously [0, 0, 0, 0, 0]

B.(2) If SVD (Singular Value Decomposition) is applied, how many singular values you expect to have? Explain why. What would change if two of the zeros were replaced by ones?

2 singular values (due to the structure of X)

If zeros are removed, that accounts for noise meaning that I expect more than 2 (3 or 4)

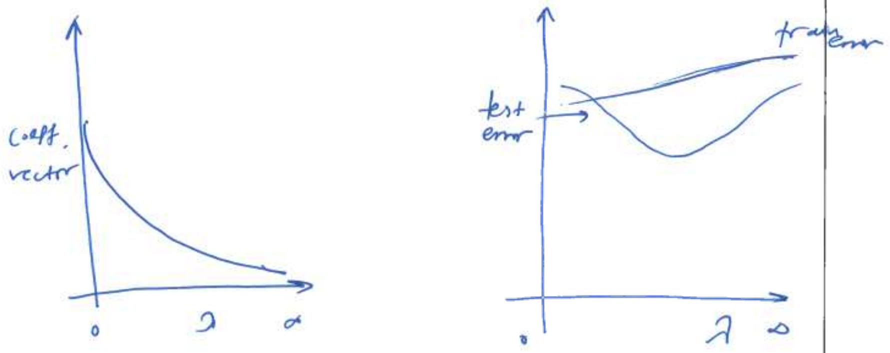
C.(2) A student very eager to apply a recommendation system on the above data, decided to do a per-user normalization (i.e. find the average rating of each user and subtract it from the values). Explain why the student decided to do this pre-processing step and what might be the problem in the dataset above.

- Remove rating bias for each user

- Problem because everything will become 0

Question 7: Regularization magic (6 points)

A.(3) Suppose you are doing linear regression using an L2 regularization penalty. Sketch the average cross-entropy training and testing error you would expect to see when you run the algorithm on a dataset, as well as the norm of the coefficient vector, as a function of the regularization parameter λ (make one graph for the error curves and another for the coefficient vector norm). No explanation is necessary.



B.(3) Suppose you used cross-validation to find a good value for λ, on a dataset of 500 examples with 100 features. But, you have now acquired more data, so instead of the original 500 examples, now you have 50000. Would the same value of λ work best, or would you expect a higher or lower value of λ to work better? Justify your answer.

I expect a lower value of λ

λ: gives a bias-variance trade-off

data volume ↑, bias ↓, no need for high λ

Bonus (Meta)-Question (1 point)

Find a mistake (typo, syntax or other) in this exam or the course slides.

Extra answer sheet.