Representing Qubit States

Solutions

Quick Exercise (Normalisation)

1. Create a statevector that will give a 1/3 probability of measuring the state $|0\rangle$

$Solution$:

Whatever state you have constructed, first make sure that the sum of squares of the coefficients (or amplitudes) of $|0\rangle$ and $|1\rangle$ add up to 1.

Next make sure that the square of the amplitude of $|0\rangle$ is $\frac{1}{3}$.

The most straightforward state is: $\frac{1}{\sqrt{3}}|0\rangle + \frac{2}{\sqrt{3}}|1\rangle$

2. Create a different statevector that will give the same measurement probabilities.

$Solution$:

Again, the simplest statevector is one where we just switch the sign between the terms:

$$\frac{1}{\sqrt{3}}|0\rangle - \frac{2}{\sqrt{3}}|1\rangle$$

Otherwise,

$$\frac{(\sqrt{2}+i)}{3}|0\rangle \pm \frac{(2+\sqrt{2}i)}{3}|1\rangle$$

3. Verify that the probability of measuring $|1\rangle$ for these two states is 2/3 .

$Solution$:

Again, the (magnitude of coefficient of $|1\rangle$)^2 will give us the required probability.

Quick Exercise (Bloch Sphere)

Use `plot_bloch_vector()` or `plot_bloch_sphere_spherical()` to plot a qubit in the states:

In [1]:
```
from qiskit_textbook.widgets import plot_bloch_vector_spherical
from math import pi,sqrt
```

1. $|0\rangle$

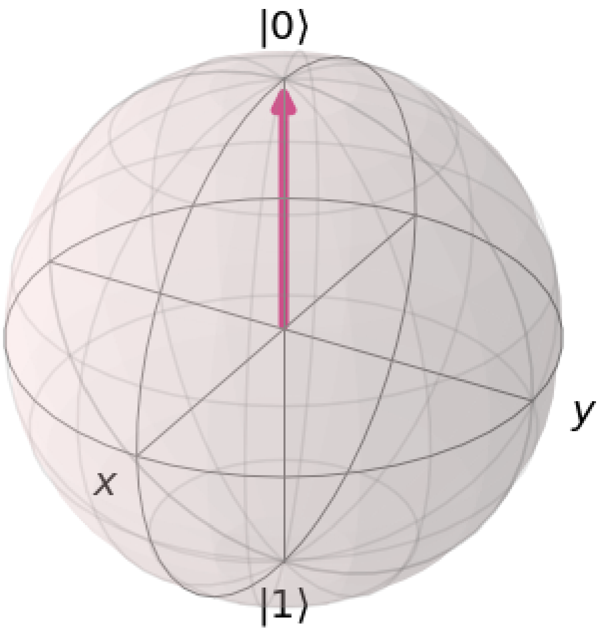
$Solution:$

We need
$\cos(\frac{\theta}{2})|0\rangle + isin(\frac{\theta}{2})|1\rangle = |0\rangle$

which implies θ must be 0, and since this eliminates the effect of ϕ, it doesn't matter what ϕ is. We let it remain 0.

In [2]:
```
coords = [0,0,1]
plot_bloch_vector_spherical(coords)
```

Out[2]:



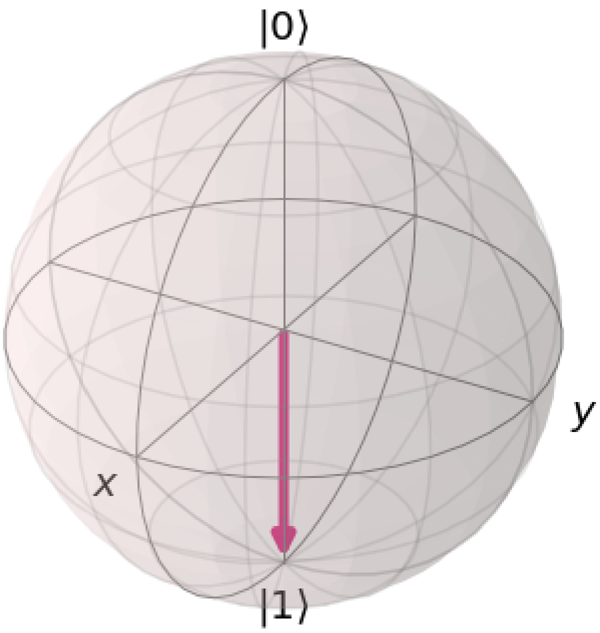
2. $|1\rangle$

We need
$\cos(\frac{\theta}{2})|0\rangle + isin(\frac{\theta}{2})|1\rangle = |1\rangle$

which implies $\dfrac{\theta}{2}$ must be $\dfrac{\pi}{2}$, and ϕ must have no effect on our state so it remains 0.

In [3]:
```
coords = [pi,0,1]
plot_bloch_vector_spherical(coords)
```

Out[3]:

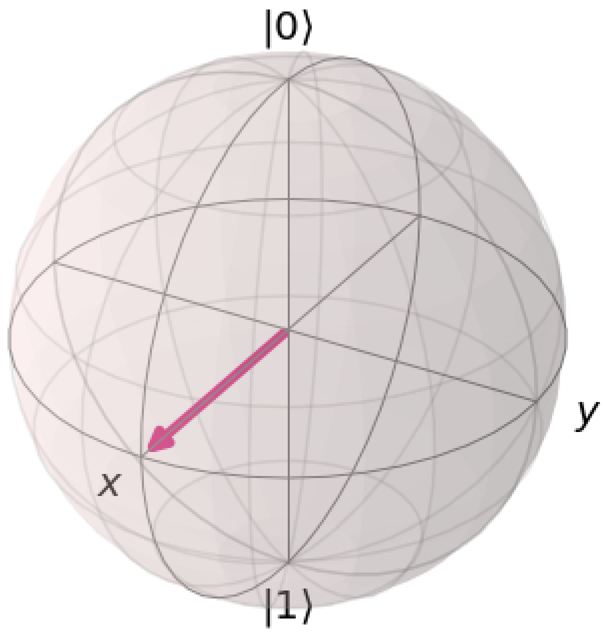


3. $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$cos(\frac{\theta}{2})= \frac{1}{\sqrt{2}}$ and $sin(\frac{\theta}{2})= \frac{1}{\sqrt{2}}$

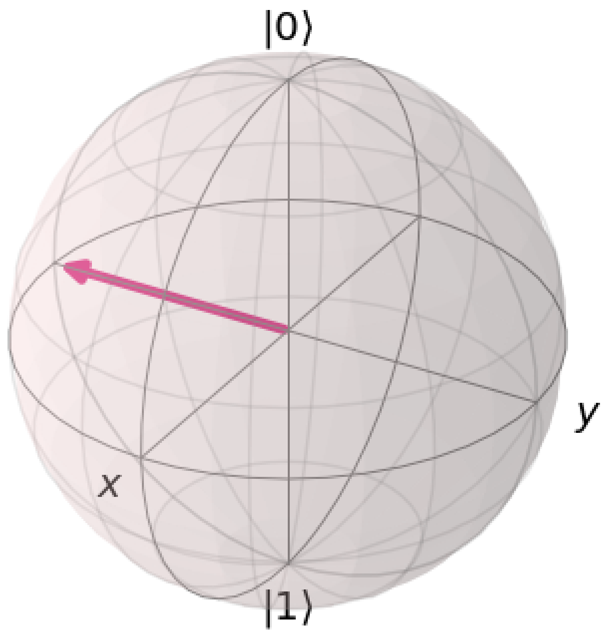
We need ϕ to not affect, hence $\phi = 0$

In [4]:
```
coords = [pi/2,0,1]
plot_bloch_vector_spherical(coords)
```
Out[4]:



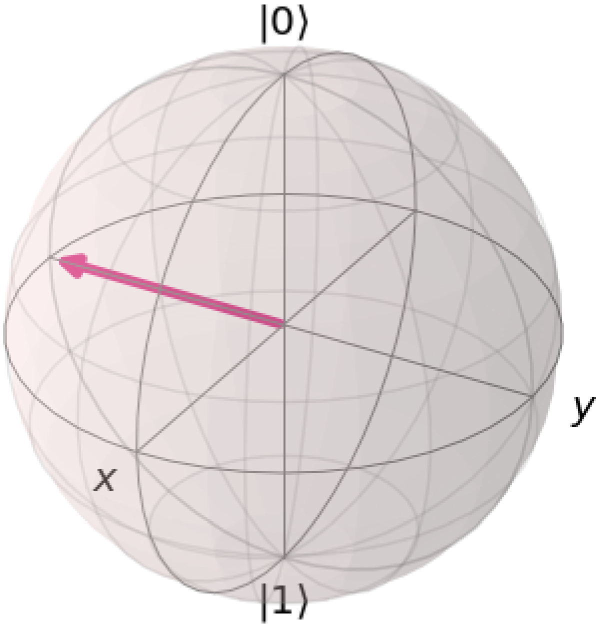
4. $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

In [5]:
```
coords = [pi/2,-pi/2,1]
plot_bloch_vector_spherical(coords)
```
Out[5]:



5. $\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}$

In [6]:
```
coords = [pi/2,-pi/2,1]
plot_bloch_vector_spherical(coords)
```
Out[6]:



In [7]:
```
import qiskit
qiskit.__qiskit_version__
```
Out[7]:
```
{'qiskit-terra': '0.16.1',
 'qiskit-aer': '0.7.1',
 'qiskit-ignis': '0.5.1',
 'qiskit-ibmq-provider': '0.11.1',
 'qiskit-aqua': '0.8.1',
 'qiskit': '0.23.1'}
```
In [ ]:

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