Classical Computation on a Quantum Computer

Solutions

Quick Exercises

In [1]:
```
from qiskit import Aer, execute, QuantumCircuit, QuantumRegister
from qiskit.visualization import plot_bloch_multivector,plot_histogram
```

In [2]:
```
input_bit = QuantumRegister(1,'input')
output_bit = QuantumRegister(1,'output')
garbage_bit = QuantumRegister(1,'garbage')

Uf = QuantumCircuit(input_bit,output_bit,garbage_bit)
Uf.cx(input_bit,output_bit)
# Uf.draw('mpl',justify=None)
```

Out[2]:
```
<qiskit.circuit.instructionset.InstructionSet at 0x255b734f448>
```

In [3]:
```
Vf = QuantumCircuit(input_bit,output_bit,garbage_bit)
Vf.cx(input_bit,garbage_bit)
Vf.cx(input_bit,output_bit)
# Vf.draw('mpl',justify=None)
```

Out[3]:
```
<qiskit.circuit.instructionset.InstructionSet at 0x255b734f8c8>
```

In [4]:
```
qc = Uf + Vf.inverse()
# qc.draw('mpl')
```

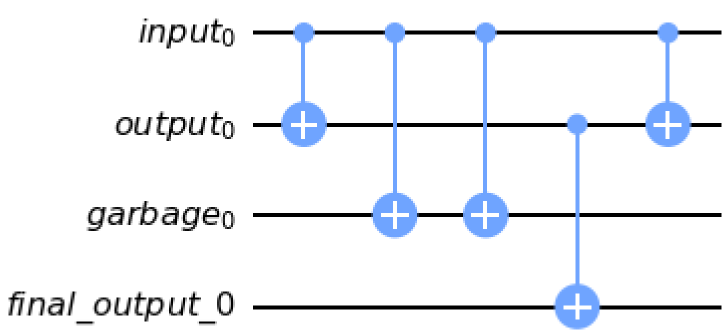
In [5]:
```
final_output_bit = QuantumRegister(1,'final_output')
```

In [6]:
```
copy = QuantumCircuit(output_bit,final_output_bit)
copy.cx(output_bit,final_output_bit)

# copy.draw('mpl',justify='None')
```

Out[6]:
```
<qiskit.circuit.instructionset.InstructionSet at 0x255b735da08>
```

In [7]:
```
(Vf.inverse() + copy + Vf).draw(output='mpl',justify='None')
```

Out[7]:

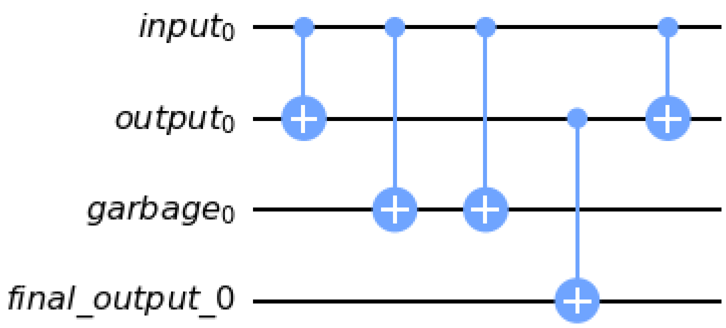


1. Show that the output is correctly written to the 'final output' register (and only to this register) when the 'output' register is initialized as $|0\rangle$.

$Solution$:
With input = 0, final_output = 0. With input = 1, final_output = 1.
We create the quantum circuit `qc` as containing the 4 registers - `input_bit` , `output_bit` , `garbage_bit` and `final_output_bit` . The circuit consists of the following operations: `Uf` , `Vf.inverse()` , `copy` , `Vf` .

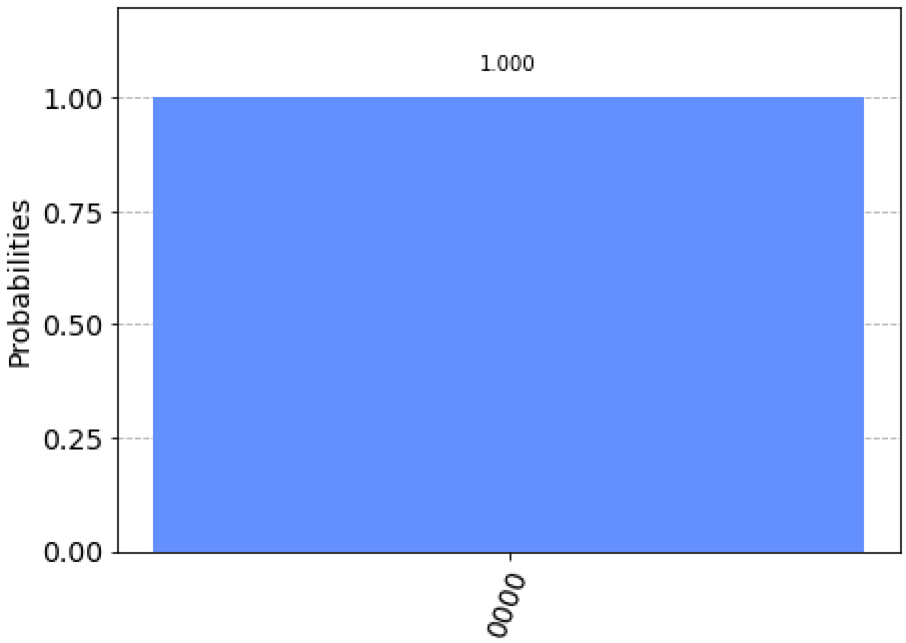
In [8]:
```
qc0 = QuantumCircuit(input_bit,output_bit,garbage_bit,final_output_bit)
qc0 = qc0 + Vf.inverse() + copy + Vf

qc0.draw('mpl',justify='none')
```

Out[8]:



In [9]:
```
qasm_sim = Aer.get_backend('qasm_simulator')
qc0.measure_all()
result = execute(qc0,backend=qasm_sim).result()
counts = result.get_counts()
plot_histogram(counts)
```

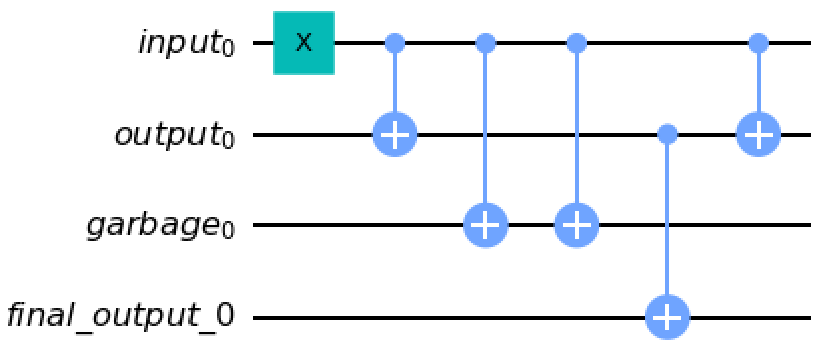
Out[9]:



The circuit gives the desired output for the input 0. Next we check for input 1.

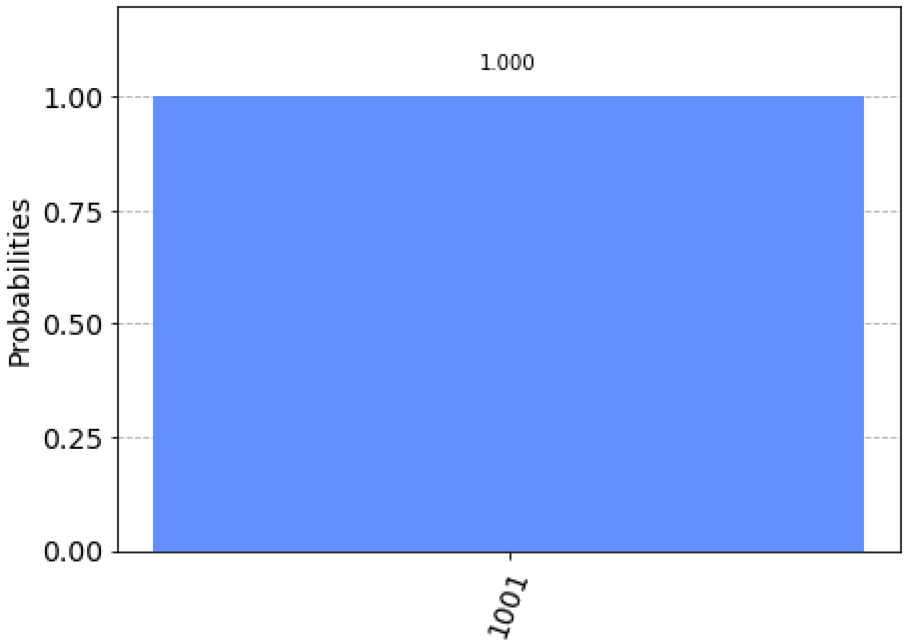
In [10]:
```
qc1 = QuantumCircuit(input_bit,output_bit,garbage_bit,final_output_bit)
qc1.x(0)
qc1 = qc1 + Vf.inverse() + copy + Vf

qc1.draw('mpl',justify='none')
```

Out[10]:



In [11]:
```
qc1.measure_all()
result = execute(qc1,backend=qasm_sim).result()
counts = result.get_counts()
plot_histogram(counts)
```

Out[11]:



The output and garbage bits are 0, and the final_output bit contains the required output $f(x)$.

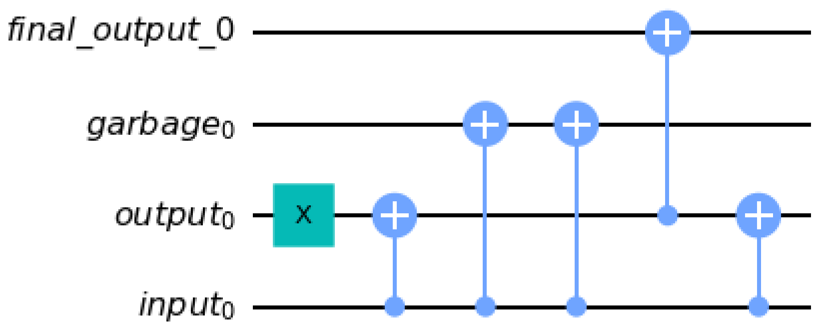
2. Determine what happens when the 'output' register is initialized as $|1\rangle$.

Solution :

In [12]:

```
qc = QuantumCircuit(input_bit,output_bit,garbage_bit,final_output_bit)
qc.x(1)
qc = qc + Vf.inverse() + copy + Vf
qc = qc.reverse_bits()

qc.draw('mpl',justify='none')
```

Out[12]:



In the circuit above, if we track the changes that occur along the circuit, we can see that if the output qubit is initialized as $|1\rangle$, the final_output bit will contain the opposite of the desired output.

When $x = 0$,
$|0, 1, 0, 0\rangle \rightarrow |0, 1, 0, 0\rangle \rightarrow |0, 1, 0, 0\rangle \rightarrow |0, 1, 0, 0\rangle \rightarrow |0, 1, 0, 1\rangle \rightarrow |0, 1, 0, 1\rangle$

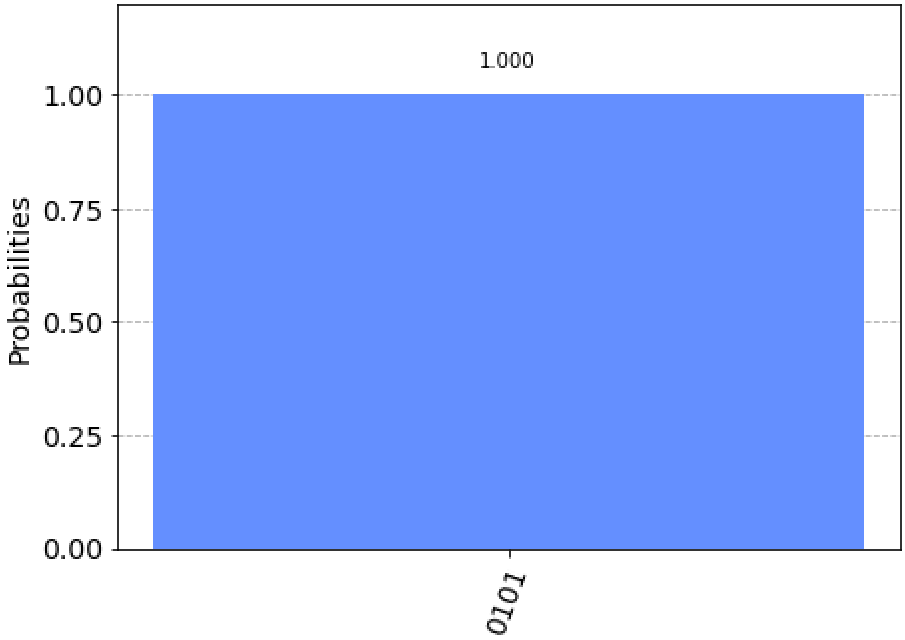
When $x = 1$,
$|1, 1, 0, 0\rangle \rightarrow |1, 0, 0, 0\rangle \rightarrow |1, 0, 1, 0\rangle \rightarrow |1, 0, 0, 0\rangle \rightarrow |1, 0, 0, 0\rangle \rightarrow |1, 1, 0, 0\rangle$

Let's verify by simulating the circuits:

When $x = 0$:

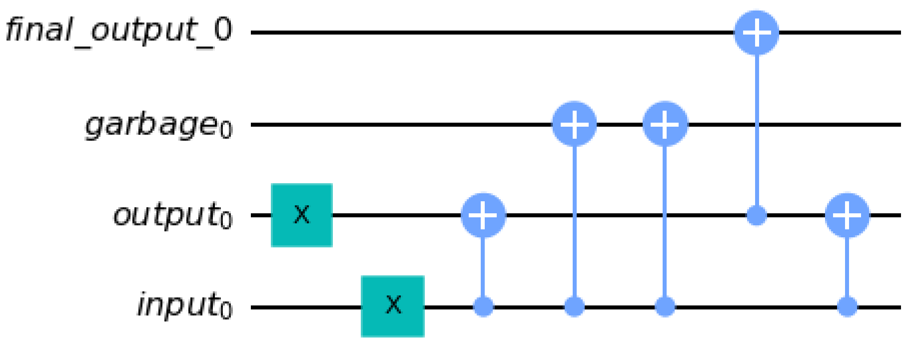
In [13]:
```
qc.measure_all()
result = execute(qc,backend=qasm_sim).result()
counts = result.get_counts()
plot_histogram(counts)
```

Out[13]:



In [14]:
```
qc = QuantumCircuit(input_bit,output_bit,garbage_bit,final_output_bit)
qc.x([0,1])
qc = qc + Vf.inverse() + copy + Vf
qc = qc.reverse_bits()

qc.draw('mpl',justify='none')
```

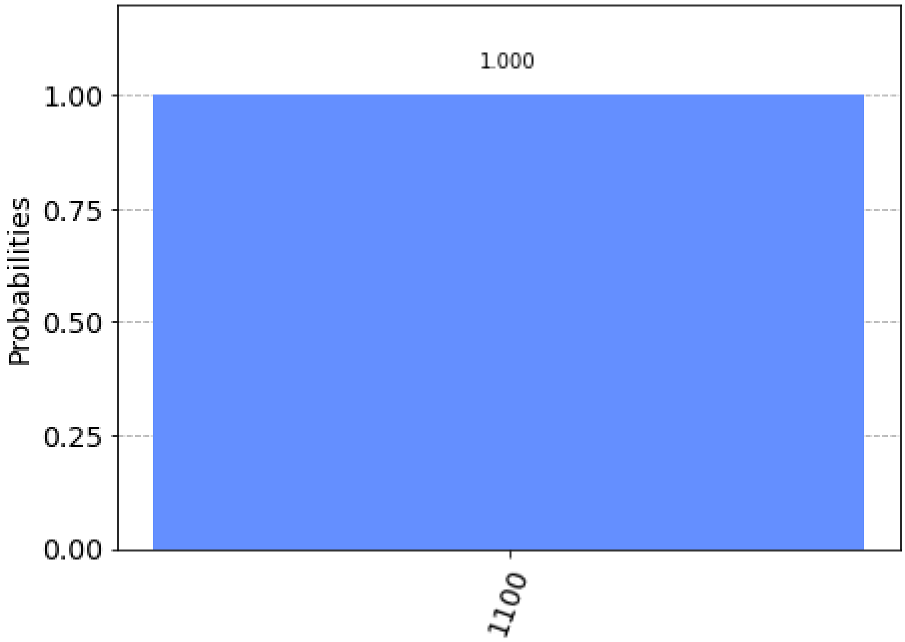
Out[14]:



In [15]:

```
qc.measure_all()
result = execute(qc,backend=qasm_sim).result()
counts = result.get_counts()
plot_histogram(counts)
```

Out[15]:



The results agree with the explanation. When the 'output' is initialized as $|1\rangle$, the answer obtained in final_output is the opposite of the correct output.

In [16]:

```
import qiskit
qiskit.__qiskit_version__
```

Out[16]:

```
{'qiskit-terra': '0.16.1',
 'qiskit-aer': '0.7.1',
 'qiskit-ignis': '0.5.1',
 'qiskit-ibmq-provider': '0.11.1',
 'qiskit-aqua': '0.8.1',
 'qiskit': '0.23.1'}
```