Time Allowed: 120 minutes **Name:**_____
Total Weight: 100 points

Introduction to Probability and Statistics Midterm Exam

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (20) According to the U.S. Bureau of Labor and Statistics, the salaries of data scientists in Maryland are approximately normally distributed with a mean annual salary of \$117K and estimated standard deviation of \$21.2K.

   (a) What is the probability a data scientist in Maryland earns at least a six-figure salary?

   (b) What is the salary range for the top 5% of data scientists in Maryland?

2. (20) A retail store is supplied with product on a weekly basis. It's weekly sales volume in thousands of units is a random variable with probability density given by,

$$f(x) = 5(1-x)^4, \quad 0 < x < 1$$

   (a) What are the expected weekly sales?

   (b) What is the required inventory to ensure the probability of the shelf supply being exhausted is less than 5%. *Hint: use the CDF*

3. (20) If the average rate at which telephone calls come into the Student Company switchboard is too great, the number of potential customers put on hold will be unacceptably high. According to a joint analysis conducted by OneKTone Analytics Inc and Arrow Analytics Inc, Student Company's current office staff should be adequate provided the rate of incoming calls is no more than 0.50 per minute. Suppose that two intervals between the next three calls received are $y_1 = 2.5$ minutes, and $y_2 = 1.7$ minutes. Based on those two observations, What is the maximum likelihood estimate, λ, for the true incoming call rate given the data follow an Exponential distribution, namely

$$f(y; \lambda) = \lambda e^{-\lambda y}$$

Hint: Start by find the Likelihood function, $L(y)$.

4. (20) Suppose a population has a mean of 30 and a variance of 25. If a sample size of 100 is drawn from the population, what is the probability that the sample mean will be larger than 31? *Hint: Think in terms of the central limit theorem.*

5. (20) The joint density of random variables, X and Y, is given by

$$f(x,y) = k(y-x)e^{-y} \quad -y < x < y, \;\; 0 < y < \infty$$

Find $E(Y)$.
Hint: A very useful identity is

$$\int_0^\infty e^{-x}x^n dx = n!$$