1) According to the US Bureau of Labor and Stats, the salaries of data scientists in Maryland are approximately normally distributed w̄ a mean salary of \$117K and estimated standard deviation of \$21.2k

a) a) probability that a data scientist in Maryland earns at least 100k?

- Use Z scores
- we know the population mean and standard deviation and we know they normally distributed

mean = 117k

$\sigma = 21.2k$

we want $P(X > 100k)$

Z value

$$Z = \frac{X - \mu}{\sigma} = \frac{100k - 117k}{21.2k} = \frac{-17k}{21.2k} = -0.8019$$

Use table to find $P(Z > -0.8019)$

$= 1 - P(Z \leq -0.8019) = 1 - .206 \approx \boxed{0.794 \text{ or } 79.4\%}$

b) salary range for top 5% of data scientists?

Z score for 95th percentile = 1.645

so $X = Z \times \sigma + \mu = 1.645 \times 21{,}200 + 117000$

$= \boxed{151874}$

2) A retail store is supplied with product weekly. Its weekly sales volume in thousands of units is a random variable w̄ pdf of

$$f(x) = 5(1-x)^4, \quad 0 < x < 1$$

b) ~~weekly expected sales~~? required inventory to ensure prob of being < exhau

$= \int_0^1 f(x)\,dx = \int_0^1 5(1-x)^4\,dx$

cdf

$u = 1 - t$ so $du = -dt$ when $t = 0,\ u =$

$t = x,\ u = 1 - x$

rewrite $= F(x) = \int_1^{1-x} 5u^4(-du)$

$= -5\int_1^{1-x} u^4\,du = -5\left(\frac{u^5}{5}\right)\Big|^{1-x} = -(u^5)\Big]_x^{1-x} = -(1-$

~~find x so $F(x) = 0.95$~~

~~$0.95 = -(1-x)^5 + 1$~~

~~$-0.05 = -(1-x)^5 \rightarrow 0.05 = (1-x)^5$~~

~~$x = 1 - 0.05^{1/5}$~~

~~≈ 0.4493~~

~~since 1000s of units $x \approx 449.3$ units~~

2)

a) $E[X] = \int_0^1 x \cdot f(x)\,dx = \int_0^1 x \cdot 5(1-x)^4\,dx$

$u = 1-x$ so $du = -dx$ or $-du = dx$ and when $x = 0,\ u = 1$; $x = 1,\ u = 0$

so $E[X] = \int_1^0 (1-u) \cdot 5u^4(-du)$

$= 5\int_0^1 (1-u)\,u^4\,du$ reverse signs

$5\int_0^1 (u^4 - u^5)\,du$

$= 5\left[\frac{u^5}{5} - \frac{u^6}{6}\right]_0^1$

$5\left[\frac{1}{5} - \frac{1}{6}\right] = \frac{5}{30} = \boxed{\frac{1}{6}}$

b) What is the required inventory to ensure the probability of the shelf supply being exhausted is $< 5\%$ use CDF

$F(a) = \int_0^a 5(1-x)^4\,dx$ antideriv is $\frac{(1-x)^5}{5} + C$

$= \left[\frac{5(1-x)^5}{-5}\right]_0^a = -(1-x)^5\Big]_0^a = -(1-a)^5 + 1 = 1-(1-a)^5$

so we want the 95%tile so find a so that $0.95 = 1-(1-a)^5$

$= (1-a)^5 = 0.05$

$1 - a = \sqrt[5]{0.05}$

$a = 1 - \sqrt[5]{0.05} = 0.4507$

since 1000s $\boxed{450.7}$

3) Incoming calls and likelihood function problem pg 209 textbook

$y_1 = 2.5$ min and $y_2 = 1.7$ minute

$L(\lambda) = f(y_1;\lambda) \times f(y_2;\lambda) = \lambda e^{-\lambda y_1} \times \lambda e^{-\lambda y_2}$

$= \lambda^2 e^{-\lambda(y_1 + y_2)}$

take the log because it's easier

$\ln(L(\lambda)) = \ln(\lambda^2) - \lambda(y_1 + y_2)$

$l\lambda = 2\ln(\lambda) - \lambda(y_1 + y_2)$

dx because we want to find max likelihood and set $d/dx = 0$

$d/d\lambda \; l\lambda = \frac{2}{\lambda} - (y_1 + y_2)$

$0 = \frac{2}{\lambda} - (y_1 + y_2) \rightarrow \frac{2}{\lambda} = y_1 + y_2$

$\lambda = \frac{2}{y_1 + y_2}$

$= \frac{2}{2.5 + 1.7} = \frac{2}{4.2}$

$\boxed{\approx 0.4762 = \lambda}$

4) pop have mean 30 and variance 25

Sample size 100 so it's big, what is prob that sample mean > 31?

use CLT which says we can use Z tables if the sample is big.

$SE = \frac{\sqrt{25}}{\sqrt{100}} = 0.5$ $\quad Z = \frac{31 - 30}{0.5} = 2$

so $P(\bar{x} > 31) = 1 - p(z < 2)$

$1 - 0.9772$

$\boxed{= 0.0228}$

5) joint density is $f(x,y) = k(y-x)e^{-y}$ $0 < x < y$ and $0 < y < \infty$

Find $E(Y)$ so remember $\int_0^\infty e^{-x} x^n \, dx = n!$

remember that $E(Y)$ is just $y \cdot$ marginal density function of Y over all values of Y

so integrate with respect to x to find marginal density func of Y

$$f_Y(y) = \int_0^y k(y-x)e^{-y} dx$$

$$= ke^{-y} \int_0^y (y-x) \, dx$$

$$= ke^{-y} \left(yx - \frac{x^2}{2} \right) \Big]_0^y$$

$$= ke^{-y} \left[y^2 - \frac{y^2}{2} \right] = ke^{-y} \frac{y^2}{2}$$

set this to 1 because the marginal density function of $Y = 1$

$$\int_0^\infty f_Y(y) dy = 1$$

so

$$\int_0^\infty ke^{-y} \frac{y^2}{2} dy = 1$$

so ... $\int_0^\infty e^{-y} y^n dy = n!$

$$k \frac{2!}{2} = 1$$

sub $n = 2$ into identity we get $\int_0^\infty e^{-y} y^2 dy = 2!$ because y^2 means $n = 2$

So $\int_0^\infty e^{-y} y^2 dy = 2!$

So $\int_0^\infty k e^{-y} \frac{y^2}{2} dy = k \frac{2!}{2} = k \frac{2}{2} = 1$

you can sub this top part

so $k = 1$

since $k = 1$,

$f_Y(y) = e^{-y} \frac{y^2}{2}$ now we can integrate

$$E(Y) = \int_0^\infty y \, f_Y(y) dy$$

$$= \int_0^\infty y \cdot e^{-y} \frac{y^2}{2} dy$$

$$\frac{1}{2} \int_0^\infty y^3 e^{-y} dy$$

use identity again!

$$E(Y) = \frac{1}{2} \cdot 3! = \boxed{3}$$