5.1 Doubly-linked lists

Doubly-linked list

A ***doubly-linked list*** is a data structure for implementing a list ADT, where each node has data, a pointer to the next node, and a pointer to the previous node. The list structure typically points to the first node and the last node. The doubly-linked list's first node is called the head, and the last node the tail.

A doubly-linked list is similar to a singly-linked list, but instead of using a single pointer to the next node in the list, each node has a pointer to the next and previous nodes. Such a list is called "doubly-linked" because each node has two pointers, or "links". A doubly-linked list is a type of ***positional list***: A list where elements contain pointers to the next and/or previous elements in the list.

PARTICIPATION ACTIVITY 5.1.1: Doubly-linked list data structure.

1) Each node in a doubly-linked list contains data and _____ pointer(s).
   - one
   - two

2) Given a doubly-linked list with nodes 20, 67, 11, node 20 is the _____.
   - head
   - tail

3) Given a doubly-linked list with nodes 4, 7, 5, 1, node 7's previous pointer points to node _____.
   - 4
   - 5

4) Given a doubly-linked list with nodes 8, 12, 7, 3, node 7's next pointer points to node _____.
   - 12
   - 3

Appending a node to a doubly-linked list

Given a new node, the ***Append*** operation for a doubly-linked list inserts the new node after the list's tail node. The append algorithm behavior differs if the list is empty versus not empty:

- *Append to empty list:* If the list's head pointer is null (empty), the algorithm points the list's head and tail pointers to the new node.
- *Append to non-empty list:* If the list's head pointer is not null (not empty), the algorithm points the tail node's pointer to the new node, points the new node's previous pointer to the list's tail node, and points the list's tail pointer to the new node.

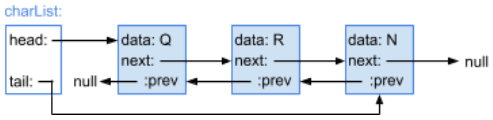
PARTICIPATION ACTIVITY 5.1.2: Doubly-linked list: Appending a node.

Animation captions:

1. Appending an item to an empty list updates the list's head and tail pointers.
2. Appending to a non-empty list adds the new node after the tail node and updates the tail pointer.
3. newNode's previous pointer is pointed to the list's tail node.
4. The list's tail pointer is then pointed to the new node.

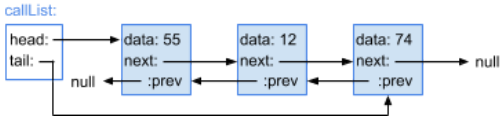
PARTICIPATION ACTIVITY 5.1.3: Doubly-linked list data structure.

1) ListAppend(charList, node F) inserts node F _____.



- after node Q
- before node N
- after node N

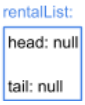
2) ListAppend(callList, node 5) executes which statement?



- `list→head = newNode`

- `list⋯→tail⋯→next = newNode`
- `newNode⋯→next = list⋯→tail`

3) Appending node K to rentalList executes which of the following statements?



- `list⋯→head = newNode`
- `list⋯→tail⋯→next = newNode`
- `newNode⋯→prev = list⋯→tail`

Prepending a node to a doubly-linked list

Given a new node, the ***Prepend*** operation of a doubly-linked list inserts the new node before the list's head node and points the head pointer to the new node.

- *Prepend to empty list:* If the list's head pointer is null (empty), the algorithm points the list's head and tail pointers to the new node.
- *Prepend to non-empty list:* If the list's head pointer is not null (not empty), the algorithm points the new node's next pointer to the list's head node, points the list head node's previous pointer to the new node, and then points the list's head pointer to the new node.

PARTICIPATION ACTIVITY 5.1.4: Doubly-linked list: Prepending a node.

Animation captions:

1. Prepending an item to an empty list points the list's head and tail pointers to new node.
2. Prepending to a non-empty list points new node's next pointer to the list's head node.
3. Prepending then points the head node's previous pointer to the new node.
4. Then the list's head pointer is pointed to the new node.

PARTICIPATION ACTIVITY 5.1.5: Prepending a node in a doubly-linked list.

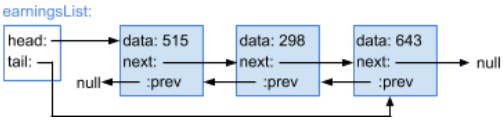
1) Prepending 29 to trainsList updates the list's head pointer to point to node _____.

- 4
- 29
- 31

2) ListPrepend(shoppingList, node Milk) updates the list's tail pointer.

- True
- False

3) ListPrepend(earningsList, node 977) executes which statement?



- `list→tail = newNode`
- `newNode→next = list→head`
- `newNode→next = list→tail`

5.2 Doubly-linked lists: Insert

Given a new node, the ***InsertAfter*** operation for a doubly-linked list inserts the new node after a provided existing list node. curNode is a pointer to an existing list node. The InsertAfter algorithm

considers three insertion scenarios:

- *Insert as first node:* If the list's head pointer is null (list is empty), the algorithm points the list's head and tail pointers to the new node.
- *Insert after list's tail node:* If the list's head pointer is not null (list is not empty) and curNode points to the list's tail node, the new node is inserted after the tail node. The algorithm points the tail node's next pointer to the new node, points the new node's previous pointer to the list's tail node, and then points the list's tail pointer to the new node.
- *Insert in middle of list:* If the list's head pointer is not null (list is not empty) and curNode does not point to the list's tail node, the algorithm updates the current, new, and successor nodes' next and previous pointers to achieve the ordering {curNode newNode sucNode}, which requires four pointer updates: point the new node's next pointer to sucNode, point the new node's previous pointer to curNode, point curNode's next pointer to the new node, and point sucNode's previous pointer to the new node.

PARTICIPATION ACTIVITY 5.2.1: Doubly-linked list: Inserting nodes.

Animation captions:

1. Inserting a first node into the list points the list's head and tail pointers to the new node.
2. Inserting after the list's tail node points the tail node's next pointer to the new node.
3. Then the new node's previous pointer is pointed to the list's tail node. Finally, the list's tail pointer is pointed to the new node.
4. Inserting in the middle of a list points sucNode to curNode's successor (curNode's next node), then points newNode's next pointer to the successor node....
5. ...then points newNode's previous pointer to curNode...
6. ...and finally points curNode's next pointer to the new node.
7. Finally, points sucNode's previous pointer to the new node. At most, four pointers are updated to insert a new node in the list.

PARTICIPATION ACTIVITY 5.2.2: Inserting nodes in a doubly-linked list.

Given weeklySalesList: 12, 30
Show the node order after the following operations:

ListInsertAfter(weeklySalesList, list tail, node 8)
ListInsertAfter(weeklySalesList, list head, node 45)
ListInsertAfter(weeklySalesList, node 45, node 76)

node 45 **node 30** **node 8** **node 76** **node 12**

	Position 0 (list's head node)
	Position 1
	Position 2
	Position 3
	Position 4 (list's tail node)

Reset

5.3 Doubly-linked lists: Remove

The ***Remove*** operation for a doubly-linked list removes a provided existing list node. curNode is a pointer to an existing list node. The algorithm first determines the node's successor (the next node) and predecessor (the previous node). The variable sucNode points to the node's successor, and the variable predNode points to the node's predecessor. The algorithm uses four separate checks to update each pointer:

- *Successor exists:* If the successor node pointer is not null (successor exists), the algorithm points the successor's previous pointer to the predecessor node.
- *Predecessor exists:* If the predecessor node pointer is not null (predecessor exists), the algorithm points the predecessor's next pointer to the successor node.
- *Removing list's head node:* If curNode points to the list's head node, the algorithm points the list's head pointer to the successor node.
- *Removing list's tail node:* If curNode points to the list's tail node, the algorithm points the list's tail pointer to the predecessor node.

When removing a node in the middle of the list, both the predecessor and successor nodes exist, and the algorithm updates the predecessor and successor nodes' pointers to achieve the ordering {predNode sucNode}. When removing the only node in a list, curNode points to both the list's head and tail nodes, and sucNode and predNode are both null. So, the algorithm points the list's head and tail pointers to null, making the list empty.

PARTICIPATION ACTIVITY 5.3.1: Doubly-linked list: Node removal.

Animation captions:

1. curNode points to the node to be removed. sucNode points to curNode's successor (curNode's next node). predNode points to curNode's predecessor (curNode's previous node).
2. sucNode's previous pointer is pointed to the node preceding curNode.
3. If curNode points to the list's head node, the list's head pointer is pointed to the successor node. With the pointers updated, curNode can be removed.
4. curNode points to node 5, which will be removed. sucNode points to node 2. predNode points node 4.
5. The predecessor node's next pointer is pointed to the successor node. The successor node's previous pointer is pointed to the predecessor node. With pointers updated, curNode can be removed.
6. curNode points to node 2, which will be removed. sucNode points to nothing (null). predNode points to node 4.
7. The predecessor node's next pointer is pointed to the successor node. If curNode points to the list's tail node, the list's tail pointer is pointed to the predecessor node. With pointers updated, curNode can be removed.

PARTICIPATION ACTIVITY 5.3.2: Deleting nodes from a doubly-linked list.

Type the list after the given operations. Type the list as: 4, 19, 3

1) numsList: 71, 29, 54

ListRemove(numsList, node 29)

numsList:

Check Show answer

2) numsList: 2, 8, 1

ListRemove(numsList, list tail)

numsList:

Check Show answer

3) numsList: 70, 82, 41, 120, 357, 66

ListRemove(numsList, node 82)

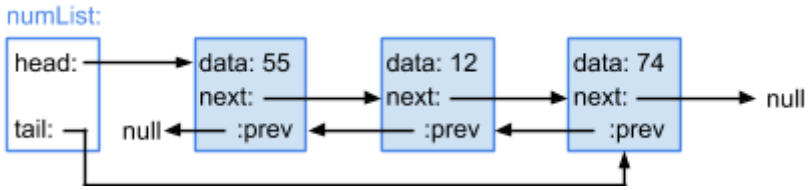
ListRemove(numsList, node 357)
ListRemove(numsList, node 66)

numsList:

Check Show answer

PARTICIPATION ACTIVITY 5.3.3: ListRemove algorithm execution: Intermediate node.

Given numList, ListRemove(numList, node 12) executes which of the following statements?



1) sucNode⇢prev = predNode

- Yes
- No

2) predNode⇢next = sucNode

- Yes
- No

3) list⇢head = sucNode

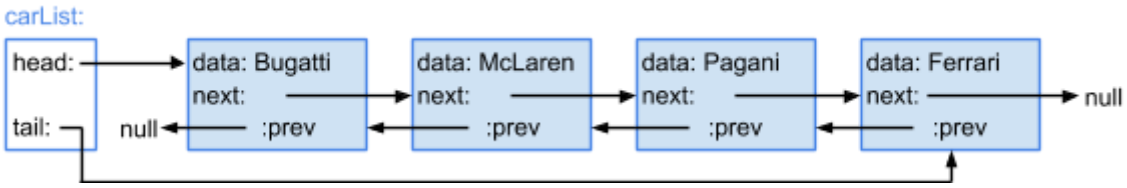
- Yes
- No

4) list⇢tail = predNode

- Yes
- No

PARTICIPATION ACTIVITY 5.3.4: ListRemove algorithm execution: List head node.

Given carList, ListRemove(carList, node Bugatti) executes which of the following statements?



1) sucNode⇢prev = predNode

- Yes
- No

2) predNode⇢next = sucNode

- Yes
- No

3) list⇢head = sucNode

- Yes
- No

4) list⇢tail = predNode

- Yes
- No

5.4 Linked list dummy nodes

Dummy nodes

A linked list implementation may use a ***dummy node*** (or ***header node***): A node with an unused data member that always resides at the head of the list and cannot be removed. Using a dummy node simplifies the algorithms for a linked list because the head and tail pointers are never null.

An empty list consists of the dummy node, which has the next pointer set to null, and the list's head and tail pointers both point to the dummy node.

PARTICIPATION ACTIVITY 5.4.1: Singly-linked lists with and without a dummy node.

Animation captions:

1. An empty linked list without a dummy node has null head and tail pointers.
2. An empty linked list with a dummy node has the head and tail pointing to a node with null data.
3. Without the dummy node, a non-empty list's head pointer points to the first list item.
4. With a dummy node, the list's head pointer always points to the dummy node. The dummy node's next pointer points to the first list item.

PARTICIPATION ACTIVITY 5.4.2: Singly linked lists with a dummy node.

1) The head and tail pointers always point to the dummy node.
   - ○ True
   - ○ False

2) The dummy node's next pointer points to the first list item.
   - ○ True
   - ○ False

PARTICIPATION ACTIVITY 5.4.3: Condition for an empty list.

1) If myList is a singly-linked list with a dummy node, which statement is true when the list is empty?
   - ○ `myList→head == null`
   - ○ `myList→tail == null`
   - ○ `myList→head == myList→tail`

Singly-linked list implementation

When a singly-linked list with a dummy node is created, the dummy node is allocated and the list's head and tail pointers are set to point to the dummy node.

List operations such as append, prepend, insert after, and remove after are simpler to implement compared to a linked list without a dummy node, since a special case is removed from each implementation. ListAppend, ListPrepend, and ListInsertAfter do not need to check if the list's

head is null, since the list's head will always point to the dummy node. ListRemoveAfter does not need a special case to allow removal of the first list item, since the first list item is after the dummy node.

Figure 5.4.1: Singly-linked list with dummy node: append, prepend, insert after, and remove after operations.


```
ListAppend(list, newNode) {
   list→tail→next = newNode
   list→tail = newNode
}


ListPrepend(list, newNode) {
   newNode→next = list→head→next
   list→head→next = newNode
}


ListInsertAfter(list, curNode, newNode) {
   if (curNode == list→tail) { // Insert after tail
      list→tail→next = newNode
      list→tail = newNode
   }
   else {
      newNode→next = curNode→next
      curNode→next = newNode
   }
}


ListRemoveAfter(list, curNode) {
   if (curNode is not null and curNode→next is not null) {
      sucNode = curNode→next→next
      curNode→next = sucNode

      if (sucNode is null) {
         // Removed tail
         list→tail = curNode
      }
   }
}
```

PARTICIPATION ACTIVITY 5.4.4: Singly-linked list with dummy node.

Suppose dataList is a singly-linked list with a dummy node.

1) Which statement removes the first item from the list?

   - ○ `ListRemoveAfter(dataList, null)`

- `ListRemoveAfter(dataList, dataList→head)`
- `ListRemoveAfter(dataList, dataList→tail)`

2) Which is a requirement of the ListPrepend function?

- The list is empty
- The list is not empty
- newNode is not null

PARTICIPATION ACTIVITY 5.4.5: Singly-linked list with dummy node.

Suppose numbersList is a singly-linked list with items 73, 19, and 86. Item 86 is at the list's tail.

1) What is the list's contents after the following operations?

```
lastItem =
numbersList→tail
ListAppend(numbersList,
node 25)
ListInsertAfter(lastItem,
node 49)
```

- 73, 19, 86, 25, 49
- 73, 19, 86, 49, 25
- 73, 19, 25, 49, 86

2) Suppose the following statement is executed:

```
node19 =
numbersList→head→next→next
```

Which subsequent operations swap nodes 73 and 19?

- ListPrepend(numbersList, node19)
- ListInsertAfter(numbersList, numbersList→head, node19)
- ListRemoveAfter(numbersList, numbersList→head→next)

ListPrepend(numbersList, node19)

Doubly-linked list implementation

A dummy node can also be used in a doubly-linked list implementation. The dummy node in a doubly-linked list always has the prev pointer set to null. ListRemove's implementation does not allow removal of the dummy node.

Figure 5.4.2: Doubly-linked list with dummy node: append, prepend, insert after, and remove operations.

```
ListAppend(list, newNode) {
   list→tail→next = newNode
   newNode→prev = list→tail
   list→tail = newNode
}


ListPrepend(list, newNode) {
   firstNode = list→head→next

   // Set the next and prev pointers for newNode
   newNode→next = list→head→next
   newNode→prev = list→head

   // Set the dummy node's next pointer
   list→head→next = newNode

   // Set prev on former first node
   if (firstNode is not null) {
      firstNode→prev = newNode
   }
}


ListInsertAfter(list, curNode, newNode) {
   if (curNode == list→tail) { // Insert after tail
      list→tail→next = newNode
      newNode→prev = list→tail
      list→tail = newNode
   }
   else {
      sucNode = curNode→next
      newNode→next = sucNode
      newNode→prev = curNode
      curNode→next = newNode
      sucNode→prev = newNode
   }
}


ListRemove(list, curNode) {
   if (curNode == list→head) {
      // Dummy node cannot be removed
      return
   }

   sucNode = curNode→next
   predNode = curNode→prev

   if (sucNode is not null) {
      sucNode→prev = predNode
   }

   // Predecessor node is always non-null
   predNode→next = sucNode

   if (curNode == list→tail) { // Removed tail
      list→tail = predNode
   }
}
```

PARTICIPATION ACTIVITY 5.4.6: Doubly-linked list with dummy node.

1) `ListPrepend(list, newNode)` is equivalent to `ListInsertAfter(list, list→head, newNode)`.
   - ○ True
   - ○ False

2) ListRemove's implementation must not allow removal of the dummy node.
   - ○ True
   - ○ False

3) `ListInsertAfter(list, null, newNode)` will insert newNode before the list's dummy node.
   - ○ True
   - ○ False

Dummy head and tail nodes

A doubly-linked list implementation can also use 2 dummy nodes: one at the head and the other at the tail. Doing so removes additional conditionals and further simplifies the implementation of most methods.

PARTICIPATION ACTIVITY 5.4.7: Doubly-linked list append and prepend with 2 dummy nodes.

Animation captions:

1. A list with 2 dummy nodes is initialized such that the list's head and tail point to 2 distinct nodes. Data is null for both nodes.
2. Prepending inserts after the head. The list head's next pointer is never null, even when the list is empty, because of the dummy node at the tail.
3. Appending inserts before the tail, since the list's tail pointer always points to the dummy node.

Figure 5.4.3: Doubly-linked list with 2 dummy nodes: insert after and remove operations.

```
ListInsertAfter(list, curNode, newNode) {
   if (curNode == list→tail) {
      // Can't insert after dummy tail
      return
   }

   sucNode = curNode→next
   newNode→next = sucNode
   newNode→prev = curNode
   curNode→next = newNode
   sucNode→prev = newNode
}

ListRemove(list, curNode) {
   if (curNode == list→head || curNode == list→tail) {
      // Dummy nodes cannot be removed
      return
   }

   sucNode = curNode→next
   predNode = curNode→prev

   // Successor node is never null
   sucNode→prev = predNode

   // Predecessor node is never null
   predNode→next = sucNode
}
```


Removing if statements from ListInsertAfter and ListRemove

The if statement at the beginning of ListInsertAfter may be removed in favor of having a precondition that curNode cannot point to the dummy tail node. Likewise, ListRemove can remove the if statement and have a precondition that curNode cannot point to either dummy node. If such preconditions are met, neither function requires any if statements.

PARTICIPATION ACTIVITY 5.4.8: Comparing a doubly-linked list with 1 dummy node vs. 2 dummy nodes.

For each question, assume 2 list types are available: a doubly-linked list with 1 dummy node at the list's head, and a doubly-linked list with 2 dummy nodes, one at the head

and the other at the tail.

1) When list⋯→head == list⋯→tail is true in _____, the list is empty.
   - a list with 1 dummy node
   - a list with 2 dummy nodes
   - either a list with 1 dummy node or a list with 2 dummy nodes

2) list⋯→tail may be null in _____.
   - a list with 1 dummy node
   - a list with 2 dummy nodes
   - neither list type

3) list⋯→head⋯→next is always non-null in _____.
   - a list with 1 dummy node
   - a list with 2 dummy nodes
   - neither list type

5.5 Circular lists

A ***circular linked list*** is a linked list where the tail node's next pointer points to the head of the list, instead of null. A circular linked list can be used to represent repeating processes. Ex: Ocean water evaporates, forms clouds, rains down on land, and flows through rivers back into the ocean. The head of a circular linked list is often referred to as the *start* node.

A traversal through a circular linked list is similar to traversal through a standard linked list, but must terminate after reaching the head node a second time, as opposed to terminating when reaching null.

PARTICIPATION ACTIVITY 5.5.1: Circular list structure and traversal.

Animation content:

undefined

Animation captions:

1. In a circular linked list, the tail node's next pointer points to the head node.
2. In a circular doubly-linked list, the head node's previous pointer points to the tail node.
3. Instead of stopping when the "current" pointer is null, traversal through a circular list stops when current comes back to the head node.

PARTICIPATION ACTIVITY 5.5.2: Circular list concepts.

1) Only a doubly-linked list can be circular.
   - ○ True
   - ○ False

2) In a circular doubly-linked list with at least 2 nodes, where does the head node's previous pointer point to?
   - ○ List head
   - ○ List tail
   - ○ null

3) In a circular linked list with at least 2 nodes, where does the tail node's next pointer point to?
   - ○ List head
   - ○ List tail
   - ○ null

4) In a circular linked list with 1 node, the tail node's next pointer points to the tail.
   - ○ True
   - ○ False

5) The following code can be used to traverse a circular, doubly-linked list

in reverse order.

```
CircularListTraverseReverse(tail)
{
   if (tail is not null) {
      current = tail
      do {
         visit current
         current =
current→previous
      } while (current != tail)
   }
}
```

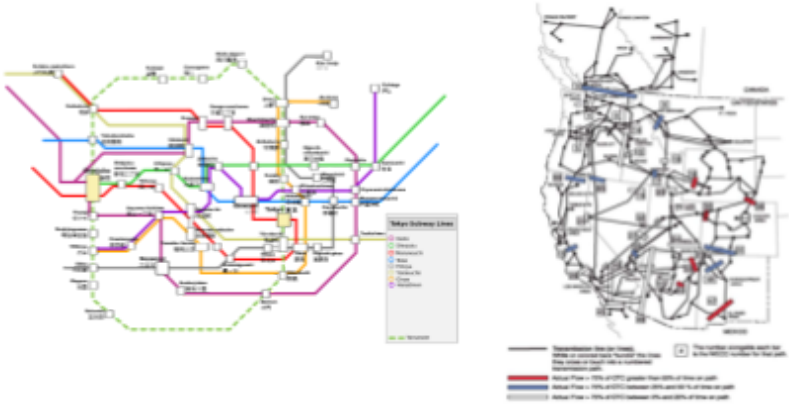

- True
- False

5.6 Graphs: Introduction

Introduction to graphs

Many items in the world are connected, such as computers on a network connected by wires, cities connected by roads, or people connected by friendship.

Figure 5.6.1: Examples of connected items: Subway map, electrical power transmission, electrical circuit.



Source: Subway map (Comicinker (Own work) / CC-BY-SA-3.0 via Wikimedia Commons), Internet map (Department of Energy / Public domain via Wikimedia Commons), Electrical circuit (zyBooks)

A ***graph*** is a data structure for representing connections among items, and consists of vertices connected by edges.

- A ***vertex*** (or node) represents an item in a graph.
- An ***edge*** represents a connection between two vertices in a graph.

PARTICIPATION ACTIVITY 5.6.1: A graph represents connections among items, like among computers, or people.

Animation captions:

1. Items in the world may have connections, like a computer network.
2. A graph's vertices represent items.
3. A graph's edges represent connections.
4. A graph can represent many different things, like friendships among people. Raj and Maya are friends, but Raj and Jen are not.

For a given graph, the number of vertices is commonly represented as V, and the number of edges as E.

PARTICIPATION ACTIVITY 5.6.2: Graph basics.

Refer to the above graphs.

1) The computer network graph has how many vertices?
   - ○ 5
   - ○ 6

2) The computer network graph has how many edges?
   - ○ 5
   - ○ 6

3) Are Maya and Thuy friends?
   - ○ Yes
   - ○ No

4) Can a vertex have more than one edge?
   - ○ Yes
   - ○ No

No

5) Can an edge connect more than two vertices?
   - Yes
   - No

6) Given 4 vertices A, B, C, and D and at most one edge between a vertex pair, what is the maximum number of edges?
   - 6
   - 16

Adjacency and paths

In a graph:

- Two vertices are ***adjacent*** if connected by an edge.
- A ***path*** is a sequence of edges leading from a source (starting) vertex to a destination (ending) vertex. The ***path length*** is the number of edges in the path.
- The ***distance*** between two vertices is the number of edges on the shortest path between those vertices.

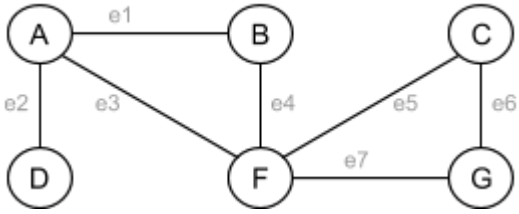
PARTICIPATION ACTIVITY 5.6.3: Graphs: adjacency, paths, and distance.

Animation captions:

1. Two vertices are adjacent if connected by an edge. PC1 and Server1 are adjacent. PC1 and Server3 are not adjacent.
2. A path is a sequence of edges from a source vertex to a destination vertex.
3. A path's length is the path's number of edges. Vertex distance is the length of the shortest path: Distance from PC1 to PC2 is 2.

PARTICIPATION ACTIVITY 5.6.4: Graph properties.

Refer to the following graph.



1) A and B are adjacent.

- True
- False

2) A and C are adjacent.

- True
- False

3) Which of the following is a path from B to G?

- e3, e7
- e1, e3, e7
- No path from B to G.

4) What is the distance from D to C?

- 3
- 4
- 5

5.7 Graph representations: Adjacency lists

Adjacency lists

Various approaches exist for representing a graph data structure. A common approach is an adjacency list. Recall that two vertices are ***adjacent*** if connected by an edge. In an ***adjacency list*** graph representation, each vertex has a list of adjacent vertices, each list item representing an edge.

PARTICIPATION ACTIVITY 5.7.1: Adjacency list graph representation.

Animation captions:

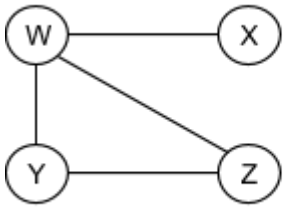
1. Each vertex has a list of adjacent vertices for edges. The edge connecting A and B appears in A's list and also in B's list.
2. Each edge appears in the lists of the edge's two vertices.

Advantages of adjacency lists

A key advantage of an adjacency list graph representation is a size of O(V + E), because each vertex appears once, and each edge appears twice. V refers to the number of vertices, E the number of edges.

However, a disadvantage is that determining whether two vertices are adjacent is O(V), because one vertex's adjacency list must be traversed looking for the other vertex, and that list could have V items. However, in most applications, a vertex is only adjacent to a small fraction of the other vertices, yielding a sparse graph. A ***sparse graph*** has far fewer edges than the maximum possible. Many graphs are sparse, like those representing a computer network, flights between cities, or friendships among people (every person isn't friends with every other person). Thus, the adjacency list graph representation is very common.

PARTICIPATION ACTIVITY 5.7.2: Adjacency lists.



Vertices	Adjacent vertices (edges)
W	X (a) Z
X	W
Y	(b)
Z	(c)

1) (a)

- Z
- Y

2) (b)

- W, X, Z
- W, X
- W, Z

3) (c)

- W
- W, X

W, Y

PARTICIPATION ACTIVITY 5.7.3: Adjacency lists: Bus routes.

The following adjacency list represents bus routes. Ex: A commuter can take the bus from Belmont to Sonoma.

Vertices	Adjacent vertices (edges)			
Belmont	Marin City	Sonoma		
Hillsborough	Livermore	Marin City	Sonoma	
Livermore	Hillsborough	Sonoma		
Marin City	Belmont	Hillsborough	Sonoma	
Sonoma	Belmont	Hillsborough	Livermore	Marin City

1) A direct bus route exists from Belmont to Sonoma or Belmont to _____.

Check Show answer

2) How many buses are needed to go from Livermore to Hillsborough?

Check Show answer

3) What is the minimum number of buses needed to go from Belmont to Hillsborough?

Check Show answer

4) Is the following a path from Livermore to Marin City? Type: Yes or No
Livermore, Hillsborough, Sonoma, Marin City

Check Show answer

5.8 Graph representations: Adjacency matrices

This section has been set as optional by your instructor.

Adjacency matrices

Various approaches exist for representing a graph data structure. One approach is an adjacency matrix. Recall that two vertices are ***adjacent*** if connected by an edge. In an ***adjacency matrix*** graph representation, each vertex is assigned to a matrix row and column, and a matrix element is 1 if the corresponding two vertices have an edge or is 0 otherwise.

PARTICIPATION ACTIVITY 5.8.1: Adjacency matrix representation.

Animation captions:

1. Each vertex is assigned to a row and column.
2. An edge connecting A and B is a 1 in A's row and B's column.
3. Similarly, that same edge is a 1 in B's row and A's column. (The matrix will thus be symmetric.)
4. Each edge similarly has two 1's in the matrix. Other matrix elements are 0's (not shown).

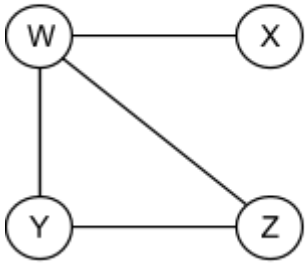
Analysis of adjacency matrices

Assuming the common implementation as a two-dimensional array whose elements are accessible in O(1), then an adjacency matrix's key benefit is O(1) determination of whether two vertices are adjacent: The corresponding element is just checked for 0 or 1.

A key drawback is $O(V^2)$ size. Ex: A graph with 1000 vertices would require a 1000 x 1000 matrix, meaning 1,000,000 elements. An adjacency matrix's large size is inefficient for a sparse graph, in which most elements would be 0's.

An adjacency matrix only represents edges among vertices; if each vertex has data, like a person's name and address, then a separate list of vertices is needed.

PARTICIPATION ACTIVITY 5.8.2: Adjacency matrix.



	W	X	Y	Z
W	0	(a)	(b)	1
X	1	0	0	(c)
Y	(d)	0	0	(e)
Z	1	0	(f)	0

1) (a)
- 0
- 1

2) (b)
- 0
- 1

3) (c)
- 0
- 1

4) (d)
- 0
- 1

5) (e)
- 0
- 1

6) (f)
- 0
- 1

PARTICIPATION ACTIVITY 5.8.3: Adjacency matrix: Power grid map.

The following adjacency matrix represents the map of a city's electrical power grid. Ex: Sector A's power grid is connected to Sector B's power grid.

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	1	1
C	1	1	0	1	1
D	1	1	1	0	0
E	0	1	1	0	0

1) How many edges does D have?

Check Show answer

2) How many edges does the graph contain?

Check Show answer

3) Assume Sector D has a power failure. Can power from Sector A be diverted directly to Sector D? Type: Yes or No

Check Show answer

4) Assume Sector E has a power failure. Can power from Sector A be diverted directly to Sector E? Type: Yes or No

Check Show answer

5) Is the following a path from Sector A to E? Type: Yes or No
AD, DB, BE

Check Show answer

Check Show answer

5.9 Graphs: Breadth-first search

This section has been set as optional by your instructor.

Graph traversal and breadth-first search

An algorithm commonly must visit every vertex in a graph in some order, known as a ***graph traversal***. A ***breadth-first search*** (BFS) is a traversal that visits a starting vertex, then all vertices of distance 1 from that vertex, then of distance 2, and so on, without revisiting a vertex.

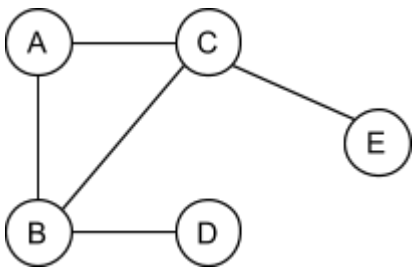
PARTICIPATION ACTIVITY 5.9.1: Breadth-first search.

Animation captions:

1. Breadth-first search starting at A visits vertices based on distance from A. B and D are distance 1.
2. E and F are distance 2 from A. Note: A path of length 3 also exists to F, but distance uses shortest path.
3. C is distance 3 from A.
4. Breadth-first search from A visits A, then vertices of distance 1, then 2, then 3. Note: Visiting order of same-distance vertices doesn't matter.

PARTICIPATION ACTIVITY 5.9.2: Breadth-first search traversal.

Perform a breadth-first search of the graph below. Assume the starting vertex is E.



1) Which vertex is visited first?

Check Show answer

2) Which vertex is visited second?

Check Show answer

3) Which vertex is visited third?

Check Show answer

4) What is C's distance?

Check Show answer

5) What is D's distance?

Check Show answer

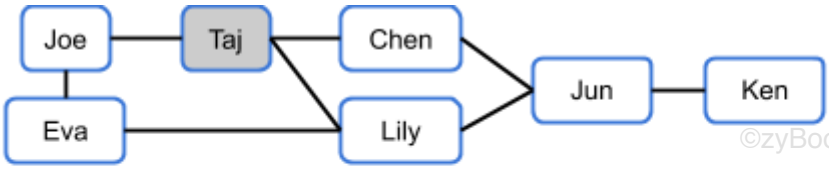
6) The BFS traversal of a graph is unique. Type: Yes or No

Check Show answer

Example: Social networking friend recommender

Example 5.9.1: Social networking friend recommender using breadth-first search.

Social networking sites like Facebook, Google+, and LinkedIn use graphs to represent "friendship" among people. For a particular user, a site may wish to recommend new friends. One approach does a breadth-first search starting from the user, recommending new friends starting at distance 2 (distance 1 people are already friends with the user).



Breadth-first traversal:	Taj	Joe	Chen	Lily	Eva	Jun	Ken
	0	1	1	1	2	2	3

Hello Taj. People you may know:

Eva

Jun

Ken

PARTICIPATION ACTIVITY 5.9.3: BFS: Friend recommender.

Refer to the friend recommender example above.

1) A distance greater than 0 indicates people are not friends with the user.
   - True
   - False

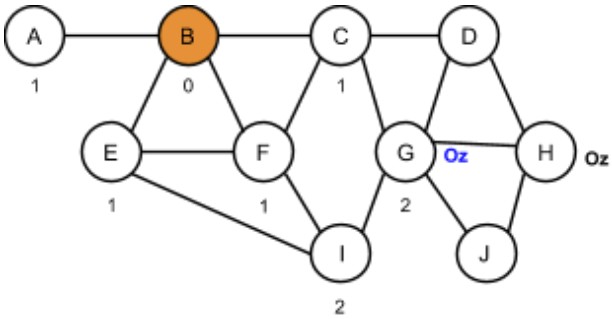
2) People with a distance of 2 are recommended before people with a distance of 3.
   - True
   - False

3) If Chen is the user, the system also recommends Eva, Jun, and Ken.
   - True
   - False

Example: Find closest item in a peer-to-peer network

Example 5.9.2: Application of BFS: Find closest item in a peer-to-peer network.

In a ***peer-to-peer network***, computers are connected via a network and may seek and download file copies (such as songs or movies) via intermediary computers or routers. For example, one computer may seek the movie "The Wizard of Oz", which may exist on 10 computers in a network consisting of 100,000 computers. Finding the closest computer (having the fewest intermediaries) yields a faster download. A BFS traversal of the network graph can find the closest computer with that movie. The BFS traversal can be set to immediately return if the item sought is found during a vertex visit. Below, visiting vertex G finds the movie; BFS terminates, and a download process can begin, involving a path length of 2 (so only 1 intermediary). Vertex H also has the movie, but is further from B so wasn't visited yet during BFS. (Note: Distances of vertices visited during the BFS from B are shown below for convenience).



Visit order:	B	A	E	F	C	I	G
	0	1	1	1	1	2	2

Oz found; terminates

PARTICIPATION ACTIVITY 5.9.4: BFS application: Peer-to-peer search.

Consider the above peer-to-peer example.

1) In some BFS traversals starting from vertex B, vertex H may be visited before vertex G.
   - ○ True
   - ○ False

2) If vertex J sought the movie Oz, the download might occur from either G or H.
   - ○ True
   - ○ False

3) If vertex E sought the movie Oz, the download might occur from either G or H.

- ○ True
- ○ False

Breadth-first search algorithm

An algorithm for breadth-first search pushes the starting vertex to a queue. While the queue is not empty, the algorithm pops a vertex from the queue and visits the popped vertex, pushes that vertex's adjacent vertices (if not already discovered), and repeats.

PARTICIPATION ACTIVITY 5.9.5: BFS algorithm.

Animation captions:

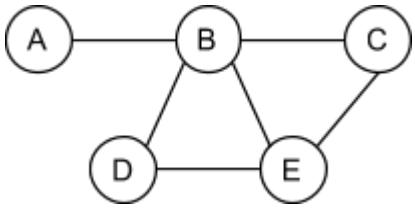
1. BFS pushes start vertex (in this case A) to frontierQueue, and adds to discoveredSet.
2. Pop vertex from frontierQueue, and visits popped vertex.
3. Pushes adjacent undiscovered vertices to frontierQueue, and adds to discoveredSet.
4. Vertex A's visit is complete. Proceed to next vertex in the frontierQueue (D). Then next (B).
5. Visit E. Then, even though B and F are adjacent to E, they are already in the discoveredSet so are not again added to frontierQueue or discoveredSet.
6. Continue until frontierQueue is empty. Note that discoveredSet above shows the visit order. Note that each vertex's distance from start is shown on the graph.

When the BFS algorithm first encounters a vertex, that vertex is said to have been ***discovered***. In the BFS algorithm, the vertices in the queue are called the ***frontier***, being vertices thus far discovered but not yet visited. Because each vertex is visited at most once, an already-discovered vertex is not pushed to the queue again.

A "visit" may mean to print the vertex, append the vertex to a list, compare vertex data to a value and return the vertex if found, etc.

PARTICIPATION ACTIVITY 5.9.6: BFS algorithm.

BFS is run on the following graph. Assume C is the starting vertex.



1) Which vertices are in the frontierQueue before the first iteration of the while loop?

- A
- C
- A, B, C, D, E

2) Which vertices are in discoveredSet after the first iteration of the while loop?

- C, B, E
- B, E
- C

3) In the second iteration, currentV = B. Which vertices are in discoveredSet after the second iteration of the while loop?

- C, B, E, A
- B, E, A, D
- C, B, E, A, D

4) In the second iteration, currentV = B. Which vertices are in frontierQueue after the second iteration of the while loop?

- C, B, E, A, D
- E, A, D
- frontierQueue is empty

5) BFS terminates after the second iteration, because all vertices are in the discoveredSet.

- True
- False

5.10 Graphs: Depth-first search

This section has been set as optional by your instructor.

Graph traversal and depth-first search

An algorithm commonly must visit every vertex in a graph in some order, known as a ***graph traversal***. A ***depth-first search*** (DFS) is a traversal that visits a starting vertex, then visits every vertex along each path starting from that vertex to the path's end before backtracking.

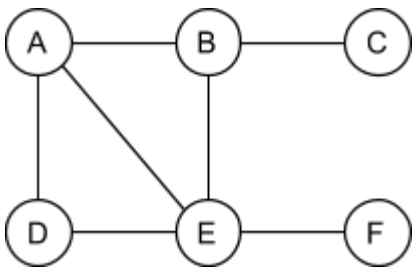
PARTICIPATION ACTIVITY 5.10.1: Depth-first search.

Animation captions:

1. Depth-first search starting at A descends along a path to the path's end before backtracking.
2. Reach path's end: Backtrack to F, visit F's other adjacent vertex (E). B already visited, backtrack again.
3. Backtracked all the way to A. Visit A's other adjacent vertex (D). No other adjacent vertices: Done.

PARTICIPATION ACTIVITY 5.10.2: Depth-first search traversal.

Perform a depth-first search of the graph below. Assume the starting vertex is E.



1) Which vertex is visited first?

Check Show answer

2) Assume DFS traverses the following verticies: E, A, B. Which vertex is visited next?

Check Show answer

3) Assume DFS traverses the following vertices: E, A, B, C. Which vertex is visited next?

Check Show answer

4) Is the following a valid DFS traversal? Type: Yes or No
E, D, F, A, B, C

Check Show answer

5) Is the following a valid DFS traversal? Type: Yes or No
E, D, A, B, C, F

Check Show answer

6) The DFS traversal of a graph is unique. Type: Yes or No

Check Show answer

Depth-first search algorithm

An algorithm for depth-first search pushes the starting vertex to a stack. While the stack is not empty, the algorithm pops the vertex from the top of the stack. If the vertex has not already been visited, the algorithm visits the vertex and pushes the adjacent vertices to the stack.

PARTICIPATION ACTIVITY 5.10.3: Depth-first search traversal with starting vertex A.

Animation content:

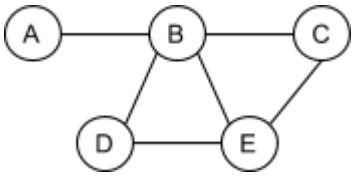
undefined

Animation captions:

1. A depth-first search at vertex A first pushes vertex A onto the stack. The first loop iteration then pops vertex A off the stack and assigns currentV with that vertex.
2. Vertex A is visited and added to the visited set. Each vertex adjacent to A is pushed onto the stack.
3. Vertex B is popped off the stack and processed as the next currentV.
4. Vertices F and E are processed similarly.
5. Vertices F and B are in the visited set and are skipped after being popped off the stack.
6. Vertex C is popped off the stack and visited. All remaining vertices except D are in the visited set.
7. Vertex D is the last vertex visited.

PARTICIPATION ACTIVITY 5.10.4: DFS algorithm.

DFS is run on the following graph. Assume C is the starting vertex.



1) Which vertices are in the stack before the first iteration of the while loop?

- A
- C
- A, B, C, D, E

2) Which vertices are in the stack after the first iteration of the while loop?

- B, E, C
- B, E
- B

3) Which vertices are in visitedSet after the first iteration of the while loop?

- B, E, C
- C
- B

4) In the second iteration, currentV = B. Which vertices are in the stack after the second iteration of the while loop?

- C
- A, C, D, E
- A, C, D, E, E

5) Which vertices are in visitedSet after the second iteration of the while loop?

- C, B
- C
- B

6) DFS terminates once all vertices are added to visitedSet.

- True
- False

Recursive DFS algorithm

A recursive DFS can be implemented using the program stack instead of an explicit stack. The recursive DFS algorithm is first called with the starting vertex. If the vertex has not already been visited, the recursive algorithm visits the vertex and performs a recursive DFS call for each adjacent vertex.

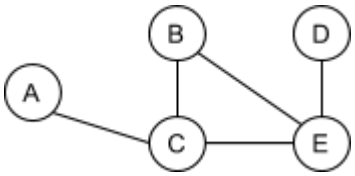
Figure 5.10.1: Recursive depth-first search.

```
RecursiveDFS(currentV) {
   if ( currentV is not in visitedSet ) {
      Add currentV to visitedSet
      "Visit" currentV
      for each vertex adjV adjacent to currentV
         RecursiveDFS(adjV)
   }
```

```
}
```

PARTICIPATION ACTIVITY 5.10.5: Recursive DFS algorithm.

The recursive DFS algorithm is run on the following graph. Assume D is the starting vertex.



1) The recursive DFS algorithm uses a queue to determine which vertices to visit.
   - True
   - False

2) DFS begins with the function call RecursiveDFS(D).
   - True
   - False

3) If B is not yet visited, RecursiveDFS(B) will make subsequent calls to RecursiveDFS(C) and RecursiveDFS(E).
   - True
   - False

5.11 Directed graphs

Directed graphs

A ***directed graph***, or ***digraph***, consists of vertices connected by directed edges. A ***directed edge*** is a connection between a starting vertex and a terminating vertex. In a directed graph, a vertex Y is ***adjacent*** to a vertex X, if there is an edge from X to Y.

Many graphs are directed, like those representing links between web pages, maps for navigation, or college course prerequisites.

Figure 5.11.1: Directed graph examples: Process flow diagram, airline routes, and college course prerequisites.

Source: Fluid catalyic cracker (Mbeychok / Public Domain via Wikimedia Commons), Florida airline routes 1974 (Geez-oz (Own work) / CC-BY-SA-3.0 via Wikimedia Commons), college course prerequisites (zyBooks)

PARTICIPATION ACTIVITY 5.11.1: A directed graph represents connections among items, like links between web pages, or airline routes.

Animation captions:

1. Items in the world may have directed connections, like links on a website.
2. A directed graph's vertices represent items.
3. Vertices are connected by directed edges.
4. A directed edge represent a connection from a starting vertex to a terminating vertex; the terminating vertex is adjacent to the starting vertex. B is adjacent to A, but A is not adjacent to B.
5. A directed graph can represent many things, like airline connections between cities. A flight is available from Los Angeles to Tucson, but not Tucson to Los Angeles.

PARTICIPATION ACTIVITY 5.11.2: Directed graph basics.

Refer to the above graphs.

1) E is a _____ in the directed graph.

- vertex
- directed edge

2) A directed edge connects vertices A and D. D is the _____ vertex.
   - starting
   - terminating

3) The airline routes graph is a digraph.
   - True
   - False

4) Tucson is adjacent to _____.
   - San Francisco
   - Los Angeles
   - Dallas

Paths and cycles

In a directed graph:

- A ***path*** is a sequence of directed edges leading from a source (starting) vertex to a destination (ending) vertex.
- A ***cycle*** is path that starts and ends at the same vertex. A directed graph is ***cyclic*** if the graph contains a cycle, and ***acyclic*** if the graph does not contain a cycle.

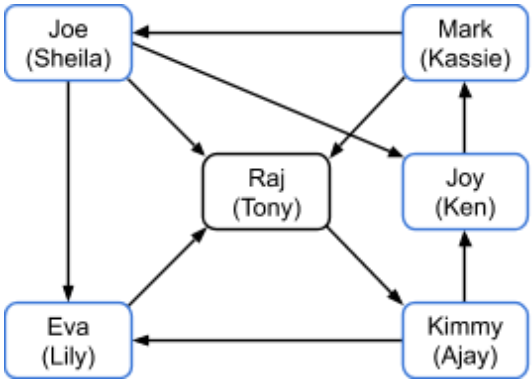
PARTICIPATION ACTIVITY 5.11.3: Directed graph: Paths and cycles.

Animation captions:

1. A path is a sequence of directed edges leading from a source (starting) vertex to a destination (ending) vertex.
2. A cycle is a path that starts and ends at the same vertex. A graph can have more than one cycle.
3. A cyclic graph contains a cycle. An acyclic graph contains no cycles.

Example 5.11.1: Cycles in directed graphs: Kidney transplants.

A patient needing a kidney transplant may have a family member willing to donate a kidney but is incompatible. That family member is willing to donate a kidney to someone else, as long as their family member also receives a kidney donation. Suppose Gregory needs a kidney. Gregory's wife, Eleanor, is willing to donate a kidney but is not compatible with Gregory. However, Eleanor is compatible with another patient Joanna, and Joanna's husband Darrell is compatible with Gregory. So, Eleanor donates a kidney to Joanna, and Darrell donates a kidney to Gregory, which is an example of a 2-way kidney transplant. In 2015, a 9-way kidney transplant involving 18 patients was performed within 36 hours (Source: SF Gate). Multiple-patient kidney transplants can be represented as cycles within a directed graph.



3-way transplant: Eva (Lily), Raj (Tony), Kimmy (Ajay)

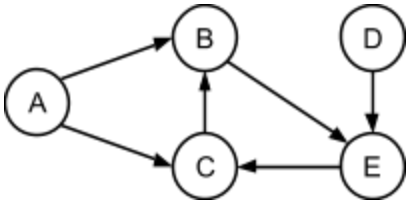
4-way transplant: Raj (Tony), Kimmy (Ajay), Joy (Ken), Mark (Kassie)

In this graph, vertices represent patients, and edges represent compatibility between a patient's family member (shown in parentheses) and another patient. An N-way kidney transplant is represented as a cycle with N edges. Due the complexity of coordinating multiple simultaneous surgeries, hospitals and doctors typically try to find the shortest possible cycle.

PARTICIPATION ACTIVITY 5.11.4: Directed graphs: Cyclic and acyclic.

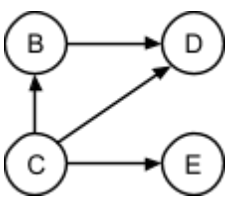
Determine if each of the following graphs is cyclic or acyclic.

1)



- Cyclic
- Acyclic

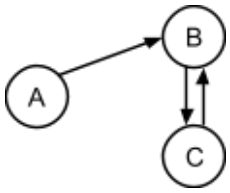
2)



Cyclic

Acyclic

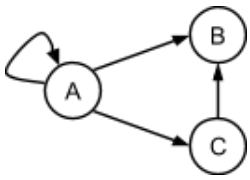
3)



Cyclic

Acyclic

4)



Cyclic

Acyclic

5.12 Weighted graphs

Weighted graphs

A ***weighted graph*** associates a weight with each edge. A graph edge's ***weight***, or ***cost***, represents some numerical value between vertex items, such as flight cost between airports, connection speed between computers, or travel time between cities. A weighted graph may be directed or undirected.

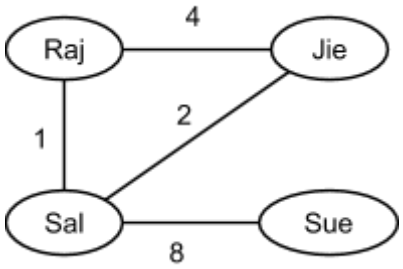
PARTICIPATION ACTIVITY 5.12.1: Weighted graphs associate weight with each edge.

Animation captions:

1. A weighted graph associates a numerical weight, or cost, with each edge. Ex: Edge weights may indicate connection speed (Mbps) between computers.
2. Weighted graphs can be directed. Ex: Edge weights may indicate travel time (hours) between cities; travel times may vary by direction.

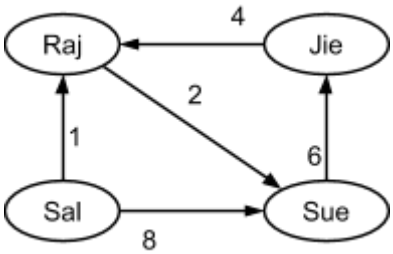
PARTICIPATION ACTIVITY 5.12.2: Weighted graphs.

1) This graph's weights indicate the number of times coworkers have collaborated on projects. How many times have Raj and Jie teamed up?



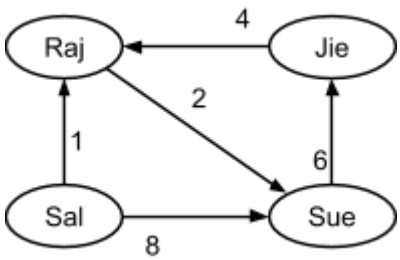
- 1
- 4

2) This graph indicates the number of times a worker has nominated a coworker for an award. How many times has Sal nominated Sue?



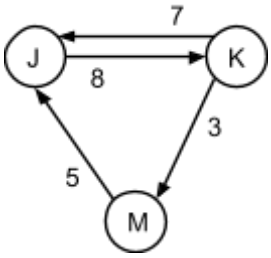
- 0
- 8

3) This graph indicates the number of times a worker has nominated a coworker for an award. How many times has Raj nominated Jie?



- 4
- 1
- 0

4) The weight of the edge from K to J is _____.



- 7
- 8

Path length in weighted graphs

In a weighted graph, the ***path length*** is the sum of the edge weights in the path.

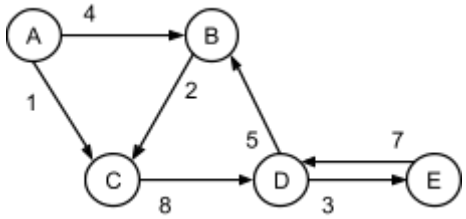
PARTICIPATION ACTIVITY 5.12.3: Path length is the sum of edge weights.

Animation captions:

1. A path is a sequence of edges from a source vertex to a destination vertex.
2. The path length is the sum of the edge weights in the path.
3. The shortest path is the path yielding the lowest sum of edge weights. Ex: The shortest path from Paris to Marseille is 6.

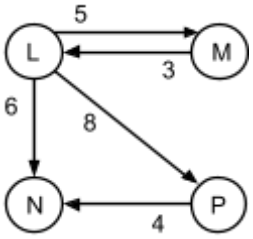
PARTICIPATION ACTIVITY 5.12.4: Path length and shortest path.

1) Given a path A, C, D, the path length is _____.



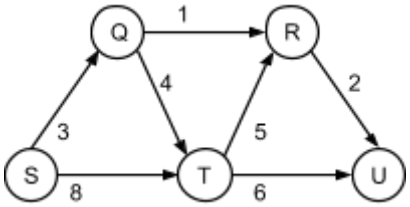
Check Show answer

2) The shortest path from M to N has a length of _____.



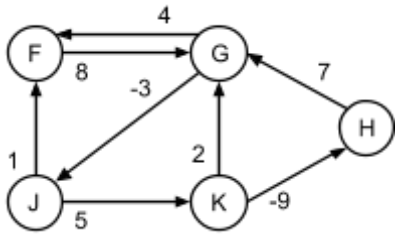
Check Show answer

3) The shortest path from S to U has a length of _____.



Check Show answer

4) Given a path H, G, J, F, the path length is _____.

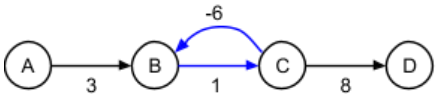


Check Show answer

Negative edge weight cycles

The ***cycle length*** is the sum of the edge weights in a cycle. A ***negative edge weight cycle*** has a cycle length less than 0. A shortest path does not exist in a graph with a negative edge weight cycle, because each loop around the negative edge weight cycle further decreases the cycle length, so no minimum exists.

Figure 5.12.1: A shortest path from A to D does not exist, because the cycle B, C can be repeatedly taken to further reduce the cycle length.



Path 1: A, B, C, B, C, D — Length = 3 + 1 + -6 + 1 + 8 = 7

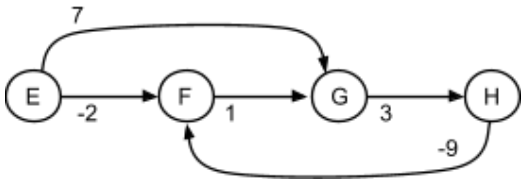
Path 2: A, B, C, B, C, B, C, D — Length = 3 + 1 + -6 + 1 + -6 + 1 + 8 = 2

Path 3: A, B, C, B, C, B, C, B, C, D — Length = 3 + 1 + -6 + 1 + -6 + 1 + -6 + 1 + 8 = -3

and so on...

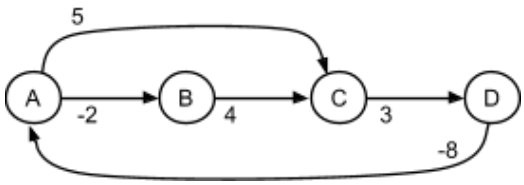
PARTICIPATION ACTIVITY 5.12.5: Negative edge weight cycles.

1) The cycle length for F, G, H, F is _____.



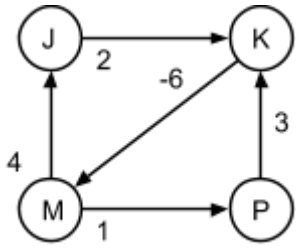
Check Show answer

2) Is A, C, D, A a negative edge weight cycle? Type Yes or No.



Check Show answer

3) The graph contains _____ negative edge weight cycles.



Check Show answer

5.13 Applications of graphs

Geographic maps and navigation

Graphs are often used to represent a geographic map, which can contain information about places and travel routes. Ex: Vertices in a graph can represent airports, with edges representing available flights. Edge weights in such graphs often represent the length of a travel route, either in total distance or expected time taken to navigate the route. Ex: A map service with access to real-time traffic information can assign travel times to road segments.

PARTICIPATION ACTIVITY 5.13.1: Driving directions use graphs and shortest path algorithms to determine the best route from start to destination.

Animation content:

undefined

Animation captions:

1. A road map can be represented by a graph. Each intersection of roads is a vertex. Destinations like the beach or a house are also vertices.
2. Roads between vertices are edges. A map service with realtime traffic information can assign travel times as edge weights.
3. A shortest path finding algorithm can be used to find the shortest path starting at the house and ending at the beach.

PARTICIPATION ACTIVITY 5.13.2: Using graphs for road navigation.

1) The longer a street is, the more vertices will be needed to represent that street.
   - True
   - False

2) Using the physical distance between vertices as edge weights will often suffice in contexts where the fastest route needs to be found.
   - True
   - False

3) Navigation software would have no need to place a vertex on a road in a location where the road does not intersect any other roads.
   - True
   - False

4) If navigation software uses GPS to automatically determine the start location for a route, the vertex closest to the GPS coordinates can be used as the starting vertex.
   - True
   - False

PARTICIPATION ACTIVITY 5.13.3: Using graphs for flight navigation.

Suppose a graph is used to represent airline flights. Vertices represent airports and edge weights represent flight durations.

1) The weight of an edge connecting two airport vertices may change based on _____.
   - flight delays
   - weather conditions
   - flight cost

2) Edges in the graph could potentially be added or removed during a single day's worth of flights.
   - True
   - False

Product recommendations

A graph can be used to represent relationships between products. Vertices in the graph corresponding to a customer's purchased products have adjacent vertices representing products that can be recommended to the customer.

PARTICIPATION ACTIVITY 5.13.4: A graph of product relationships can be used to produce recommendations based on purchase history.

Animation content:

undefined

Animation captions:

1. An online shopping service can represent relationships between products being sold using a graph.
2. Relationships may be based on the products alone. A game console requires a TV and games, so the game console vertex connects to TV and game products.
3. Vertices representing common kitchen products, such as a blender, dishwashing soap, kitchen towels, and oven mitts, are also connected.
4. Connections might also represent common purchases that occur by chance. Maybe several customers who purchase a Blu-ray player also purchase a blender.
5. A customer's purchases can be linked to the product vertices in the graph.
6. Adjacent vertices can then be used to provide a list of recommendations to the customer.

PARTICIPATION ACTIVITY 5.13.5: Product recommendations.

1) If a customer buys only a Blu-ray player, which product is not likely to be recommended?
   - Television 1 or 2
   - Blender

- Blender
- Game console

2) Which single purchase would produce the largest number of recommendations for the customer?

- Tablet computer
- Blender
- Game console

3) If "secondary recommendations" included all products adjacent to recommended products, which products would be secondary recommendations after buying oven mitts?

- Blender, kitchen towels, and muffin pan
- Dishwashing soap, Blu-ray player, and muffin mix
- Game console and tablet computer case

Social and professional networks

A graph may use a vertex to represent a person. An edge in such a graph represents a relationship between 2 people. In a graph representing a social network, an edge commonly represents friendship. In a graph representing a professional network, an edge commonly represents business conducted between 2 people.

PARTICIPATION ACTIVITY 5.13.6: Professional networks represented by graphs help people establish business connections.

Animation content:

`undefined`

Animation captions:

1. In a graph representing a professional network, vertices represent people and edges represent a business connection.

2. Tuyet conducts business with Wilford, adding a new edge in the graph. Similarly, Manuel conducts business with Keira.
3. The graph can help professionals find new connections. Shayla connects with Octavio through a mutual contact, Manuel.

PARTICIPATION ACTIVITY 5.13.7: Professional network.

Refer to the animation's graph representing the professional network.

1) Who has conducted business with Eusebio?
   - Giovanna
   - Manuel
   - Keira

2) If Octavio wishes to connect with Wilford, who is the best person to introduce the 2 to each other?
   - Shayla
   - Manuel
   - Rita

3) What is the length of the shortest path between Salvatore and Reva?
   - 5
   - 6
   - 7

5.14 Algorithm: Dijkstra's shortest path

This section has been set as optional by your instructor.

Dijkstra's shortest path algorithm

Finding the shortest path between vertices in a graph has many applications. Ex: Finding the shortest driving route between two intersections can be solved by finding the shortest path in a directed graph where vertices are intersections and edge weights are distances. If edge weights instead are expected travel times (possibly based on real-time traffic data), finding the shortest path will provide the fastest driving route.

Dijkstra's shortest path algorithm, created by Edsger Dijkstra, determines the shortest path from a start vertex to each vertex in a graph. For each vertex, Dijkstra's algorithm determines the vertex's distance and predecessor pointer. A vertex's ***distance*** is the shortest path distance from the start vertex. A vertex's ***predecessor pointer*** points to the previous vertex along the shortest path from the start vertex.

Dijkstra's algorithm initializes all vertices' distances to infinity (∞), initializes all vertices' predecessors to 0, and pushes all vertices to a queue of unvisited vertices. The algorithm then assigns the start vertex's distance with 0. While the queue is not empty, the algorithm pops the vertex with the shortest distance from the queue. For each adjacent vertex, the algorithm computes the distance of the path from the start vertex to the current vertex and continuing on to the adjacent vertex. If that path's distance is shorter than the adjacent vertex's current distance, a shorter path has been found. The adjacent vertex's current distance is updated to the distance of the newly found shorter path's distance, and vertex's predecessor pointer is pointed to the current vertex.

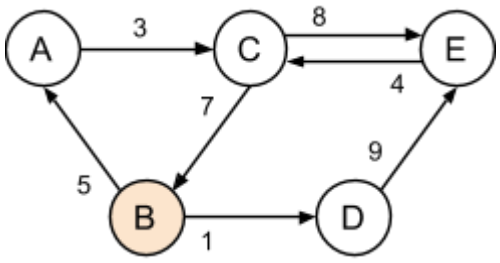
PARTICIPATION ACTIVITY 5.14.1: Dijkstra's algorithm finds the shortest path from a start vertex to each vertex in a graph.

Animation captions:

1. Dijsktra's shortest path algorithm initializes each vertex's distance to Infinity, initializes each vertex's predecessor to 0, and pushes each vertex into the unvisitedQueue.
2. The start vertex's distance is 0. The algorithm visits the start vertex first.
3. For each adjacent vertex, if a shorter path from the start vertex to the adjacent vertex is found, the vertex's distance and predecessor pointer are updated.
4. B has the shortest path distance, and is popped from the queue. The path through B to C is not shorter, so no update is done. The path through B to D is shorter, so D's distance and predecessor pointer are updated.
5. D is then popped from the queue. The path through D to C is shorter, so C's distance and predecessor pointer are updated.
6. C is then popped from the queue. The path through C to D is not shorter, so no update is made.
7. The algorithm terminates when all vertices are visited. Each vertex's distance is the shortest path distance from the start vertex. The vertex's predecessor pointer points to the previous vertex in the shortest path.

PARTICIPATION ACTIVITY 5.14.2: Dijkstra's shortest path traversal.

Perform Dijkstra's shortest path algorithm on the graph below with starting vertex B.



1) Which vertex is visited first?

Check Show answer

2) A's distance after the algorithm's first while loop iteration is _____.
Type inf for infinity.

Check Show answer

3) D's distance after the first while loop iteration is _____.
Type inf for infinity.

Check Show answer

4) C's distance after the first iteration of the while loop is _____.
Type inf for infinity.

Check Show answer

5) Which vertex is visited second?

Check Show answer

6) E's distance after the second while

loop iteration is _____.

Check Show answer

Finding shortest path from start vertex to destination vertex

After running Dijkstra's algorithm, the shortest path from the start vertex to a destination vertex can be determined using the vertices' predecessor pointers. If the destination vertex's predecessor pointer is not 0, the shortest path is traversed in reverse by following the predecessor pointers until the start vertex is reached. If the destination vertex's predecessor pointer is 0, then a path from the start vertex to the destination vertex does not exist.

PARTICIPATION ACTIVITY 5.14.3: Determining the shortest path from Dijkstra's algorithm.

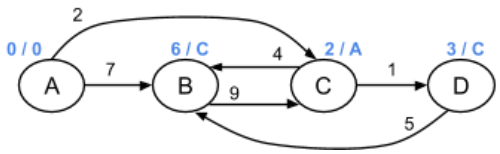
Animation captions:

1. The vertex's predecessor pointer points to the previous vertex in the shortest path.
2. Starting with the destination vertex, the predecessor pointer is followed until the start vertex is reached.
3. The vertex's distance is the shortest path distance from the start vertex.

PARTICIPATION ACTIVITY 5.14.4: Shortest path based on vertex predecessor.

Type path as: A, B, C
If path does not exist, type: None

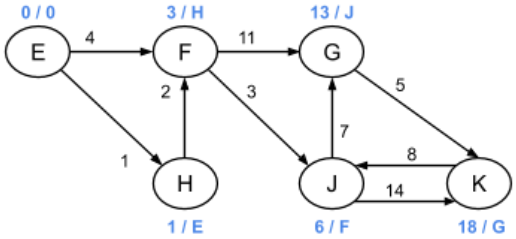
1) After executing DijkstraShortestPath(A), what is the shortest path from A to B?



Check Show answer

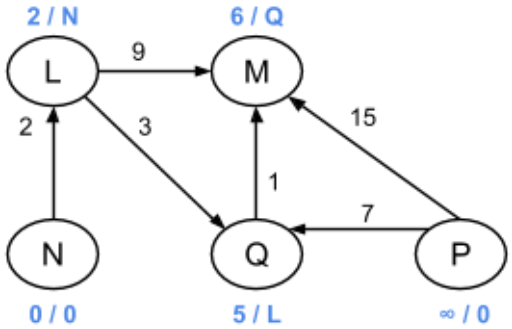
2) After executing

DijkstraShortestPath(E), what is the shortest path from E to G?



Check Show answer

3) After executing DijkstraShortestPath(N), what is the shortest path from N to P?



Check Show answer

Algorithm efficiency

If the unvisited vertex queue is implemented using a list, the runtime for Dijkstra's shortest path algorithm is $O(V^2)$. The outer loop executes V times to visit all vertices. In each outer loop execution, popping the vertex from the queue requires searching all vertices in the list, which has a runtime of O(V). For each vertex, the algorithm follows the subset of edges to adjacent vertices; following a total of E edges across all loop executions. Given $E < V^2$, the runtime is $O(V*V + E) = O(V^2 + E) = O(V^2)$. Implementing the unvisited vertex queue using a standard binary heap reduces the runtime to O((E + V) log V), and using a Fibonacci heap data structure (not discussed in this material) reduces the runtime to O(E + V log V).

Negative edge weights

Dijkstra's shortest path algorithm can be used for unweighted graphs (using a uniform edge weight of 1) and weighted graphs with non-negative edges weights. For a directed graph with negative edge weights, Dijkstra's algorithm may not find the shortest path for some vertices, so the algorithm should not be used if a negative edge weight exists.

PARTICIPATION ACTIVITY 5.14.5: Dijkstra's algorithm may not find the shortest path for a graph with negative edge weights.

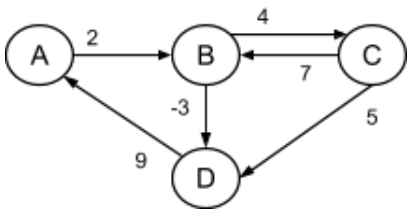
Animation captions:

1. A is the start vertex. Adjacent vertices resulting in a shorter path are updated.
2. Path through B to D results in a shorter path. Vertex D is updated.
3. D has no adjacent vertices.
4. A path through C to B results in a shorter path. Vertex B is updated.
5. Vertex B has already been visited, and will not be visited again. So, D's distance and predecessor are not for the shortest path from A to D.

PARTICIPATION ACTIVITY 5.14.6: Dijkstra's shortest path algorithm: Supported graph types.

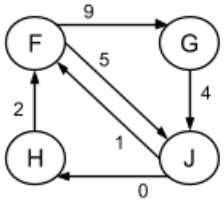
Indicate if Dijkstra's algorithm will find the shortest path for the following graphs.

1)

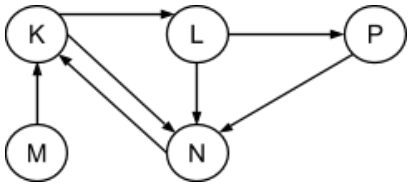


- Yes
- No

2)



- Yes
- No

3) 

- Yes
- No

5.15 Algorithm: Bellman-Ford's shortest path

This section has been set as optional by your instructor.

Bellman-Ford shortest path algorithm

The ***Bellman-Ford shortest path algorithm***, created by Richard Bellman and Lester Ford, Jr., determines the shortest path from a start vertex to each vertex in a graph. For each vertex, the Bellman-Ford algorithm determines the vertex's distance and predecessor pointer. A vertex's ***distance*** is the shortest path distance from the start vertex. A vertex's ***predecessor pointer*** points to the previous vertex along the shortest path from the start vertex.

The Bellman-Ford algorithm initializes all vertices' current distances to infinity (∞) and predecessors to 0, and assigns the start vertex with a distance of 0. The algorithm performs V-1 main iterations, visiting all vertices in the graph during each iteration. Each time a vertex is visited, the algorithm follows all edges to adjacent vertices. For each adjacent vertex, the algorithm computes the distance of the path from the start vertex to the current vertex and continuing on to the adjacent vertex. If that path's distance is shorter than the adjacent vertex's current distance, a shorter path has been found. The adjacent vertex's current distance is updated to the newly found shorter path's distance, and the vertex's predecessor pointer is pointed to the current vertex.

The Bellman-Ford algorithm does not require a specific order for visiting vertices during each main iteration. So after each iteration, a vertex's current distance and predecessor may not yet be the shortest path from the start vertex. The shortest path may propagate to only one vertex each iteration, requiring V-1 iterations to propagate from the start vertex to all other vertices.

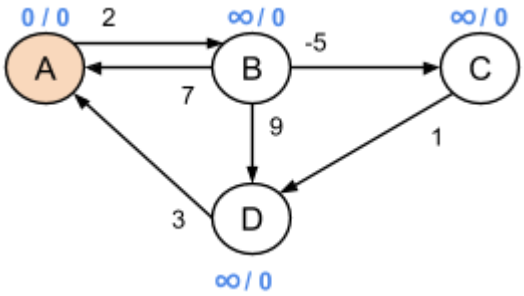
PARTICIPATION ACTIVITY 5.15.1: The Bellman-Ford algorithm finds the shortest path from a source vertex to all other vertices in a graph.

Animation captions:

1. The Bellman-Ford algorithm initializes each vertex's distance to Infinity and each vertex's predecessor to 0, and assigns the start vertex's distance with 0.
2. For each vertex in the graph, if a shorter path from the currentV to the adjacent vertex is found, the adjacent vertex's distance and predecessor pointer are updated.
3. The path through B to D results in a shorter path to D than is currently known. So vertex D is updated.
4. The path through C to B results in a shorter path to B than is currently known. So vertex B is updated.
5. D has no adjacent vertices. After each iteration, a vertex's distance and predecessor may not yet be the shortest path from the start vertex.
6. In each main iteration, the algorithm visits all vertices. This time, the path through B to D results in an even shorter path. So vertex D is updated again.
7. During the third main iteration, no shorter paths are found, so no vertices are updated. V-1 iterations may be required to propagate the shortest path from the start vertex to all other vertices.
8. When done, each vertex's distance is the shortest path distance from the start vertex, and the vertex's predecessor pointer points to the previous vertex in the shortest path.

PARTICIPATION ACTIVITY 5.15.2: Bellman-Ford shortest path traversal.

The start vertex is A. In each main iteration, vertices in the graph are visited in the following order: A, B, C, D.



1) What are B's values after the first iteration?

- 2 / A
- ∞ / 0

2) What are C's values after the first iteration?

- -5 / B
- -3 / B

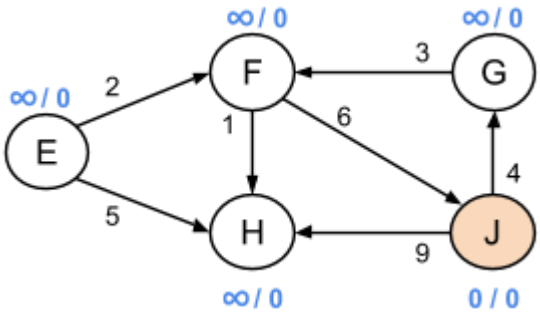
- ∞ / 0

3) What are D's values after the first iteration?

- 3 / A
- -2 / C
- ∞ / 0

PARTICIPATION ACTIVITY 5.15.3: Bellman-Ford: Distance and predecessor values.

The start vertex is J. Vertices in the graph are processed in the following order: E, F, G, H, J.



1) How many iterations are executed?

- 5
- 4
- 6

2) What are F's values after the first iteration?

- ∞ / 0
- 7 / G

3) What are G's values after the first iteration?

- 4 / J
- ∞ / 0

4) What are E's values after the final iteration?

- 9 / F
- ∞ / 0

Algorithm Efficiency

The runtime for the Bellman-Ford shortest path algorithm is O(VE). The outer loop (the main iterations) executes V-1 times. In each outer loop execution, the algorithm visits each vertex and follows the subset of edges to adjacent vertices, following a total of E edges across all loop executions.

Checking for negative edge weight cycles

The Bellman-Ford algorithm supports graphs with negative edge weights. However, if a negative edge weight cycle exists, a shortest path does not exist. After visiting all vertices V-1 times, the algorithm checks for negative edge weight cycles. If a negative edge weight cycle does not exist, the algorithm returns true (shortest path exists), otherwise returns false.

PARTICIPATION ACTIVITY 5.15.4: Bellman-Ford: Checking for negative edge weight cycles.

Animation captions:

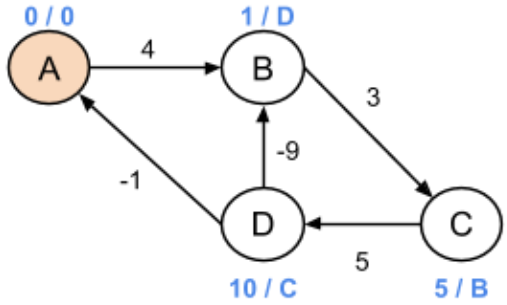
1. After visiting all vertices V-1 times, the Bellman-Ford algorithm checks for negative edge weight cycles.
2. For each vertex in the graph, adjacent vertices are checked for a shorter path. No such path exists through A to B.
3. But, a shorter path is still found through B to C, so a negative edge weight cycle exists.
4. The algorithm returns false, indicating a shortest path does not exist.

Figure 5.15.1: Bellman-Ford shortest path algorithm.

```
BellmanFord(startV) {
   for each vertex currentV in graph {
      currentV⋯➝distance = Infinity
      currentV⋯➝predV = 0
   }

   // startV has a distance of 0 from itself
   startV⋯➝distance = 0

   for i = 1 to number of vertices - 1 { // Main iterations
      for each vertex currentV in graph {
         for each vertex adjV adjacent to currentV {
            edgeWeight = weight of edge from currentV to adjV
            alternativePathDistance = currentV⋯➝distance + edgeWeight

            // If shorter path from startV to adjV is found,
            // update adjV's distance and predecessor
            if (alternativePathDistance < adjV⋯➝distance) {
               adjV⋯➝distance = alternativePathDistance
               adjV⋯➝predV = currentV
            }
         }
      }
   }

   // Check for a negative edge weight cycle
   for each vertex currentV in graph {
      for each vertex adjV adjacent to currentV {
         edgeWeight = weight of edge from currentV to adjV
         alternativePathDistance = currentV⋯➝distance + edgeWeight

         // If shorter path from startV to adjV is still found,
         // a negative edge weight cycle exists
         if (alternativePathDistance < adjV⋯➝distance) {
            return false
         }
      }
   }

   return true
}
```

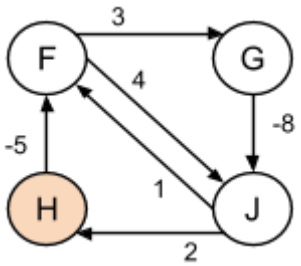

PARTICIPATION ACTIVITY 5.15.5: Bellman-Ford algorithm: Checking for negative edge weight cycles.

1) Given the following result from the Bellman-Ford algorithm for a start vertex of A, a negative edge weight cycle is found when checking adjacent vertices of vertex _____.



- B
- C
- D

2) What does the Bellman-Ford algorithm return for the following graph with a start vertex of H?



- True
- False

5.16 All pairs shortest path

This section has been set as optional by your instructor.

Overview and shortest paths matrix

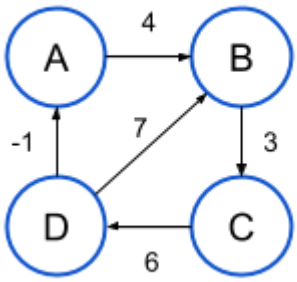
An ***all pairs shortest path*** algorithm determines the shortest path between all possible pairs of vertices in a graph. For a graph with vertices V, a |V| x |V| matrix represents the shortest path lengths between all vertex pairs in the graph. Each row corresponds to a start vertex, and each column in the matrix corresponds to a terminating vertex for each path. Ex: The matrix entry at row F and column T represents the shortest path length from vertex F to vertex T.

PARTICIPATION ACTIVITY 5.16.1: Shortest path lengths matrix.

Animation captions:

1. For a graph with 4 vertices, the all-pairs-shortest-path matrix has 4 rows and 4 columns. An entry exists for every possible vertex pair.
2. An entry at row F and column T represents the shortest path length from vertex F to vertex T.
3. The shortest path from vertex A to vertex D has length 9.
4. The shortest path from B to A has length 2.
5. Only total path lengths are stored in the matrix, not the actual sequence of edges for the corresponding path.

PARTICIPATION ACTIVITY 5.16.2: Shortest path lengths matrix.



	A	B	C	D
A	0	4	7	?
B	8	0	3	9
C	5	9	?	6
D	-1	?	6	0

1) What is the shortest path length from A to D?

Check Show answer

2) What is the shortest path length from C to C?

Check Show answer

3) What is the shortest path length from D to B?

Check Show answer

PARTICIPATION ACTIVITY 5.16.3: Shortest path lengths matrix.

1) If a graph has 11 vertices and 7 edges, how many entries are in the all-pairs-shortest-path matrix?
   - 11
   - 7
   - 77
   - 121

2) An entry at row R and column C in the matrix represents the shortest path length from vertex C to vertex R.
   - True
   - False

3) If a graph contains a negative edge weight, the matrix of shortest path lengths will contain at least 1 negative value.
   - True
   - False

4) For a matrix entry representing path length from A to B, when no such path exists, a special-case value must be used to indicate that no path exists.
   - True
   - False

Floyd-Warshall algorithm

The ***Floyd-Warshall all-pairs shortest path algorithm*** generates a |V| x |V| matrix of values representing the shortest path lengths between all vertex pairs in a graph. Graphs with cycles and negative edge weights are supported, but the graph must not have any negative cycles. A

negative cycle is a cycle with edge weights that sum to a negative value. Because a negative cycle could be traversed repeatedly, lowering the path length each time, determining a shortest path between 2 vertices in a negative cycle is not possible.

The Floyd-Warshall algorithm initializes the shortest path lengths matrix in 3 steps.

1. Every entry is assigned with infinity.
2. Each entry representing the path from a vertex to itself is assigned with 0.
3. For each edge from X to Y in the graph, the matrix entry for the path from X to Y is initialized with the edge's weight.

The algorithm then iterates through every vertex in the graph. For each vertex X, the shortest path lengths for all vertex pairs are recomputed by considering vertex X as an intermediate vertex. For each matrix entry representing A to B, existing matrix entries are used to compute the length of the path from A through X to B. If this path length is less than the current shortest path length, then the corresponding matrix entry is updated.

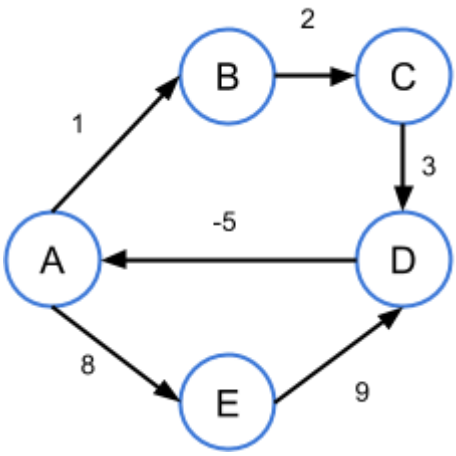
PARTICIPATION ACTIVITY 5.16.4: Floyd-Warshall algorithm.

Animation captions:

1. All entries in the shortest path lengths matrix are first initialized with ∞. Vertex-to-same-vertex values are then initialized with 0. For each edge, the corresponding entry for from X to Y is initialized with the edge's weight.
2. The k = 0 iteration corresponds to vertex A. Each vertex pair X, Y is analyzed to see if the path from X through A to Y yields a shorter path. Shorter paths from C to B and from D to B are found.
3. During the k = 1 iteration, 4 shorter paths that pass through vertex B are found from path from A to C, from A to D, from C to D, and from D to C.
4. During the k = 2 iteration, 1 shorter path that passes through vertex C is found from the path from B to A.
5. During the k=3 iteration, no entries are updated. The shortest path lengths matrix is complete.

PARTICIPATION ACTIVITY 5.16.5: Floyd-Warshall algorithm.

Consider executing the Floyd-Warshall algorithm on the graph below.



1) How many rows and columns are in the shortest path length matrix?

- 5 rows, 5 columns
- 5 rows, 6 columns
- 6 rows, 5 columns
- 6 rows, 6 columns

2) After matrix initialization, but before the k-loop starts, how many entries are set to non-infinite values?

- 5
- 11
- 14
- 25

3) After the algorithm completes, how many entries in the matrix are negative values?

- 0
- 3
- 5
- 7

4) After the algorithm completes, what is the shortest path length from vertex B to vertex E?

- -4
- 5
- 8

- Infinity (no path)

Path reconstruction

Although only shortest path lengths are computed by the Floyd-Warshall algorithm, the matrix can be used to reconstruct the path sequence. Given the shortest path length from a start vertex to an end vertex is L. An edge from vertex X to the ending vertex exists such that the shortest path length from the starting vertex to X, plus the edge weight, equals L. Each such edge is found, and the path is reconstructed in reverse order.

PARTICIPATION ACTIVITY 5.16.6: Path reconstruction.

Animation captions:

1. The matrix built by the Floyd-Warshall algorithm indicates that the shortest path from A to C is of length 3.
2. The path from A to C is determined by backtracking from C.
3. Since A is the path's starting point, shortest distances from A are relevant at each vertex.
4. Traversing backwards along the only incoming edge to C, the expected path length at B is 3 - 5 = -2.
5. The computation at the edge from A to B doesn't hold, so the edge is not part of the path.
6. The computation on the D to B edge holds.
7. The computation on the A to D edge holds, and brings the path back to the starting vertex. The final path is A to D to B to C.

Figure 5.16.1: FloydWarshallReconstructPath algorithm.

```
FloydWarshallReconstructPath(graph, startVertex, endVertex, distMatrix) {
   path = new, empty path

   // Backtrack from the ending vertex
   currentV = endVertex
   while (currentV != startVertex) {
      incomingEdges = all edges in the graph incoming to current vertex
      for each edge currentE in incomingEdges {
         expected = distMatrix[startVertex][currentV] - currentE→weight
         actual = distMatrix[startVertex][currentE→fromVertex]
         if (expected == actual) {
            currentV = currentE→fromVertex
            Prepend currentE to path
            break
         }
      }
   }

   return path
}
```

PARTICIPATION ACTIVITY 5.16.7: Path reconstruction.

1) Path reconstruction from vertex X to vertex Y is only possible if the matrix entry is non-infinite.
   - True
   - False

2) A path with positive length may include edges with negative weights.
   - True
   - False

3) More than 1 possible path sequence from vertex X to vertex Y may exists, even though the matrix will only store 1 path length from X to Y.
   - True
   - False

4) Path reconstruction is not possible if the graph has a cycle.
   - True
   - False

Algorithm efficiency

The Floyd-Warshall algorithm builds a |V| x |V| matrix and therefore has a space complexity of $O(|V|^2)$. The matrix is constructed with a runtime complexity of $O(|V|^3)$.

5.17 Set abstract data type

This section has been set as optional by your instructor.

Set abstract data type

A ***set*** is a collection of distinct elements. A set ***add*** operation adds an element to the set, provided an equal element doesn't already exist in the set. A set is an unordered collection. Ex: The set with integers 3, 7, and 9 is equivalent to the set with integers 9, 3 and 7.

PARTICIPATION ACTIVITY 5.17.1: Set abstract data type.

Animation content:

`undefined`

Animation captions:

1. Adding 67, 91, and 14 produces a set with 3 elements.
2. Because 91 already exists in the set, adding 91 any number of additional times has no effect.
3. Set 2 is built by inserting the same numbers in a different order.
4. Because order does not matter in a set, the 2 sets are equivalent.

PARTICIPATION ACTIVITY 5.17.2: Set abstract data type.

1) Which of the following is not a valid set?

- { 78, 32, 46, 57, 82 }
- { 34, 8, 92 }
- { 78, 28, 91, 28, 15 }

2) How many elements are in a set that is built by adding the element 28 6 times, then the element 54 9 times?

- 1
- 2

- 15

3) Which 2 sets are equivalent?

- { 56, 19, 71 } and { 19, 65, 71, 56 }
- { 88, 54, 81 } and { 81, 88, 54 }
- { 39, 56, 14, 11 } and { 14, 56, 93, 11 }

Element keys and removal

Set elements may be primitive data values, such as numbers or strings, or objects with numerous data members. When storing objects, set implementations commonly distinguish elements based on an element's ***key value***: A primitive data value that serves as a unique identifier for the element. Ex: An object for a student at a university may store information such as name, phone number, and ID number. No two students will have the same ID number, so the ID number can be used as the student object's key.

Sets are commonly implemented to use keys for all element types. When storing objects, the set retrieves an object's key via an external function or predetermined knowledge of which object property is the key value. When storing primitive data values, each primitive data value's key is itself.

Given a key, a set ***remove*** operation removes the element with the specified key from the set.

PARTICIPATION ACTIVITY 5.17.3: Element keys and removal.

Animation content:

`undefined`

Animation captions:

1. Different students at the same university may have the same name or phone number, but each student has a unique ID number.
2. A set for the course roster uses the student ID as the key value, since the exact same student cannot enroll twice in the same course.
3. The call to remove Student C provides only the student ID.

PARTICIPATION ACTIVITY 5.17.4: Element keys and removal.

Refer to the example in the animation above.

1) If the student objects contained a field for GPA, then GPA could be used as the key value instead of student ID.
   - True
   - False

2) `SetRemove(courseRosterSet, "Student D")` would remove Student D from the set.
   - True
   - False

3) SetRemove will not operate properly on an empty set.
   - True
   - False

Searching and subsets

Given a key, a set ***search*** operation returns the set element with the specified key, or null if no such element exists. The search operation can be used to implement a subset test. A set X is a ***subset*** of set Y only if every element of X is also an element of Y.

PARTICIPATION ACTIVITY 5.17.5: SetIsSubset algorithm.

Animation content:

`undefined`

Animation captions:

1. To test if set2 is a subset of set1, each element of set 2 is searched for in set1. Elements 19, 22, and 26 are found in set1.
2. Element 34 is in set2 but not set1, so set2 is not a subset of set1.
3. The first element in set3, 88, is not in set1, so set3 is not a subset of set1.
4. All elements of set4 are in set1, so set4 is a subset of set1.
5. No other set is a subset of another.
6. But each set is always a subset of itself.

PARTICIPATION ACTIVITY 5.17.6: Searching and subsets.

1) Every set is a subset of itself.
   - True
   - False

2) For X to be a subset of Y, the number of elements in Y must be greater than or equal to the number of elements in X.
   - True
   - False

3) The loop in SetIsSubset always performs N iterations, where N is the number of elements in subsetCandidate.
   - True
   - False

5.18 Set operations

This section has been set as optional by your instructor.

Union, intersection, and difference

The ***union*** of sets X and Y, denoted as X ∪ Y, is a set that contains every element from X, every element from Y, and no additional elements. Ex: { 54, 19, 75 } ∪ { 75, 12 } = { 12, 19, 54, 75 }.

The ***intersection*** of sets X and Y, denoted as X ∩ Y, is a set that contains every element that is in both X and Y, and no additional elements. Ex: { 54, 19, 75 } ∩ { 75, 12 } = { 75 }.

The ***difference*** of sets X and Y, denoted as X \ Y, is a set that contains every element that is in X but not in Y, and no additional elements. Ex: { 54, 19, 75 } \ { 75, 12 } = { 54, 19 }.

The union and intersection operations are commutative, so X ∪ Y = Y ∪ X and X ∩ Y = Y ∩ X. The difference operation is not commutative.

PARTICIPATION ACTIVITY 5.18.1: Set union, intersection, and difference.

Animation content:

undefined

Animation captions:

1. The union operation begins by adding all elements from set1.
2. Each element from set2 is added. Adding elements 82 and 93 has no effect, since 82 and 93 already exist in the result set.
3. The intersection operation iterates through each element in set1. Each element that is also in set2 is added to the result.
4. The difference of set1 and set2, denoted set1 \ set2, iterates through all elements in set1. Only elements 61 and 76 are added to the result, since these elements are not in set2.
5. Set difference is not commutative. SetDifference(set2, set1) produces a result containing only 23 and 46, since those elements are in set2 but not in set1.

PARTICIPATION ACTIVITY 5.18.2: Union, intersection, and difference.

1) How many elements are in the set { 83, 5 } ∪ { 9, 77, 83 }?
   - ○ 2
   - ○ 4
   - ○ 5

2) How many elements are in the set { 83, 5 } ∩ { 9, 77, 83 }?
   - ○ 1
   - ○ 2
   - ○ 3

3) { 83, 5 } \ { 9, 77, 83 } = ?
   - ○ { 83 }
   - ○ { 5 }
   - ○ { 83, 5 }

4) { 9, 77, 83 } \ { 83, 5 } = ?

- { 9, 77 }
- { 9, 77, 83 }
- { 5 }

5) Which set operation is not commutative?

- Union
- Intersection
- Difference

6) When X and Y do not have any elements in common, which is always true?

- X ∪ Y = X ∩ Y
- X ∩ Y = X \ Y
- X \ Y = X

7) Which is true for any set X?

- X ∪ X = X ∩ X
- X ∪ X = X \ X
- X \ X = X ∩ X

Filter and map

A ***filter*** operation on set X produces a subset containing only elements from X that satisfy a particular condition. The condition for filtering is commonly represented by a ***filter predicate***: A function that takes an element as an argument and returns a Boolean value indicating whether or not that element will be in the filtered subset.

A ***map*** operation on set X produces a new set by applying some function F to each element. Ex: If X = {18, 44, 38, 6} and F is a function that divides a value by 2, then SetMap(X, F) = { 9, 22, 19, 3 }.

PARTICIPATION ACTIVITY 5.18.3: SetFilter and SetMap algorithms.

Animation content:

undefined

Animation captions:

1. SetFilter is called with the EvenPredicate function passed as the second argument.
2. SetFilter calls EvenPredicate for each element. EvenPredicate returns true for each even element, and false for each odd element.
3. Every element for which the predicate returned true is added to the result, producing the set of even numbers from set1.
4. SetFilter(set1, Above90Predicate) produces the set with all elements from set1 that are greater than 90.
5. SetMap is called with the OnesDigit function passed as the first argument. Like SetFilter, SetMap calls the function for each element.
6. The returned value from each OnesDigit call is added to the result set, producing the set of distinct ones digit values.
7. SetMap(set1, StringifyElement) produces a set of strings from a set of numbers.

PARTICIPATION ACTIVITY 5.18.4: Using SetFilter with a set of strings.

Suppose stringSet = { "zyBooks", "Computer science", "Data structures", "set", "filter", "map" }. Filter predicates are defined below. Match each SetFilter call to the resulting set.

```
StartsWithCapital(string) {
   if (string starts with capital letter)
      return true
   else
      return false
}

Has6OrFewerCharacters(string) {
   if (length of string <= 6)
      return true
   else
      return false
}

EndsInS(string) {
   if (string ends in "S" or "s")
      return true
   else
      return false
}
```


SetFilter(stringSet, EndsInS)

SetFilter(stringSet, StartsWithCapital)

SetFilter(stringSet, Has6OrFewerCharacters)

	{ "zyBooks", "Data structures" }
	{ "Computer science", "Data structures" }
	{ "set", "filter", "map" }

Reset

PARTICIPATION ACTIVITY 5.18.5: Using SetMap with a set of numbers.

Suppose numbersSet = { 6.5, 4.2, 7.3, 9.0, 8.7 }. Map functions are defined below. Match each SetMap call to the resulting set.

```
MultiplyBy10(number) {
   return number * 10.0
}

Floor(number) {
   return floor(number)
}

Round(number) {
   return round(number)
}
```

SetMap(numbersSet, MultiplyBy10) **SetMap(numbersSet, Floor)**

SetMap(numbersSet, Round)

	{ 65.0, 42.0, 73.0, 90.0, 87.0 }
	{ 7.0, 4.0, 7.0, 9.0, 9.0 }
	{ 6.0, 4.0, 7.0, 9.0, 8.0 }

Reset

PARTICIPATION ACTIVITY 5.18.6: SetFilter and SetMap algorithm concepts.

1) A filter predicate must return true for elements that are to be added to the resulting set, and false for elements that are not to be added.
   - True
   - False

2) Calling SetFilter on set X always produces a set with the same number of elements as X.
   - True
   - False

3) Calling SetMap on set X always produces a set with the same number of elements as X.
   - True
   - False

4) Both SetFilter and SetMap will call the function passed as the second argument for every element in the set.
   - True
   - False

5.19 Static and dynamic set operations

This section has been set as optional by your instructor.

A ***dynamic set*** is a set that can change after being constructed. A ***static set*** is a set that doesn't change after being constructed. A collection of elements is commonly provided during construction of a static set, each of which is added to the set. Ex: A static set constructed from the list of integers (19, 67, 77, 67, 59, 19) would be { 19, 67, 77, 59 }.

Static sets support most set operations by returning a new set representing the operation's result. The table below summarizes the common operations for static and dynamic sets.

Table 5.19.1: Static and dynamic set operations.

Operation	Dynamic set support?	Static set support?
Construction from a collection of values	Yes	Yes
Count number of elements	Yes	Yes
Search	Yes	Yes
Add element	Yes	No
Remove element	Yes	No
Union (returns new set)	Yes	Yes
Intersection (returns new set)	Yes	Yes
Difference (returns new set)	Yes	Yes
Filter (returns new set)	Yes	Yes
Map (returns new set)	Yes	Yes

PARTICIPATION ACTIVITY 5.19.1: Static and dynamic set operations.

1) Static sets do not support union or intersection, since these operations require changing the set.
   - ○ True
   - ○ False

2) A static set constructed from the list of integers (20, 12, 87, 12) would be { 20, 12, 87, 12 }.
   - ○ True
   - ○ False

3) Suppose a dynamic set has N elements. Adding any element X and then removing element X will always result in the set still having N elements.

- ○ True
- ○ False

PARTICIPATION ACTIVITY 5.19.2: Choosing static or dynamic sets for real-world datasets.

For each real-world dataset, select whether a program should use a static or dynamic set.

1) Periodic table of elements

- ○ Static
- ○ Dynamic

2) Collection of names of all countries on the planet

- ○ Static
- ○ Dynamic

3) List of contacts for a user

- ○ Static
- ○ Dynamic