10) $\int_{-\infty}^{\infty} \frac{dx}{1+4x^4} = \int_{-\infty}^{\infty} \frac{dz}{1+4z^4}$ let $f_z = \frac{1}{1+4z^4}$

First find roots for use in Residue theorem because this has singularity.

when does $4z^4 = -1$?

well... really when does $z^4 = \frac{-1}{4}$? well remember $e^{\pi i} = -1$

hmmm... by Euler's formula

so

$\text{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}$

$\text{Res}_{z=z_1} \frac{1}{1+4z^4} = \frac{1}{16z^3}\Big|_{z=z_1}$ where z_1 is a root

we can do this because $p(z_0) \neq 0$ $q(z_0) = 0$ $q'(z_0) \neq 0$

Find residues now.

eventually $\int_{-\infty}^{\infty} \frac{dx}{1+4x^4} = -2\pi i(\text{res}_1 + \text{res}_2 \ldots)$. But what are the residues?