

5. Evaluate  $\int_C (x+2) ds$  where  $C$  is  $r(t) = t\mathbf{i} + \frac{4}{3}t^{\frac{3}{2}}\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$

$$r(t) = \left[ t, \frac{4}{3}t^{\frac{3}{2}}, \frac{1}{2}t^2 \right]$$

$$r'(t) = \left[ 1, 2t^{\frac{1}{2}}, t \right]$$

$$|r'(t)| = \sqrt{1^2 + 2^2 + t^2} = \sqrt{t^2 + 4 + 1} = \sqrt{t^2 + 5}$$

Find vector field as function of  $r(t) = F(r(t)) = t + 2$

Setup integral

$$\int_C f(x,y) ds = \int_0^2 F(r(t)) \cdot |r'(t)| dt$$

$$= \int_0^2 [t+2, 0, 0] \cdot [1, 2t^{\frac{1}{2}}, t] dt$$

$$= \int_0^2 t + 2 dt = \left[ \frac{1}{2}t^2 + 2t \right]_0^2 = 2$$

6) Find square roots of  $1 - \sqrt{3}i$  expressed in rect coordinates

rect form =  $a + bi$

polar form =  $r \cos \theta + i r \sin \theta$

where  $r = \sqrt{a^2 + b^2}$  and  $\tan^{-1} \frac{b}{a} = \theta$

De Moivre  $z^n = (r^n (\cos n\theta + i \sin n\theta))$

So in polar form  $z = 1 - \sqrt{3}i$

$$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

$$\tan \theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \Rightarrow \theta = -60^\circ$$

$$\text{let } z^{\frac{1}{2}} = 2^{\frac{1}{2}} e^{i \frac{-\pi}{6}}$$

$$\text{so } 1 - \sqrt{3}i = \sqrt{2} (\cos -\frac{\pi}{6} + i \sin -\frac{\pi}{6}) = \sqrt{2} \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right)$$

$$= \frac{1}{\sqrt{2}} (\sqrt{3} - i) \text{ so } z \text{ is } \pm \frac{1}{\sqrt{2}} (\sqrt{3} - i) \text{ so } \pm \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$$