Time Allowed: 120 minutes **Name:**____________________
Total Weight: Make up points **Due by 11:59pm 17 October 2020**

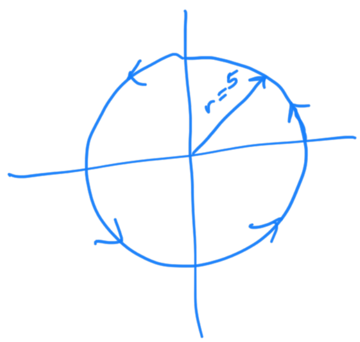
Multivariable and Complex Analysis Quiz IV Makeup

SHOW ALL WORK TO RECEIVE EXTRA CREDIT

1. Evaluate $\int_C (x^2 - y^2)\, ds$ where C is given by

$$x = 5\cos t, \quad y = 5\sin t, \quad 0 \le t \le 2\pi$$

Hint: Draw a picture!



$x^2 = 25\cos^2 t$ $\quad x^2 - y^2 = 25(\cos^2 t - \sin^2 t)$

$y^2 = 25\sin^2 t$ $\quad = 25\cos 2t$

$ds = \sqrt{25(\cos^2 t + \sin^2 t)}$

$= \sqrt{25(1)} = 5$

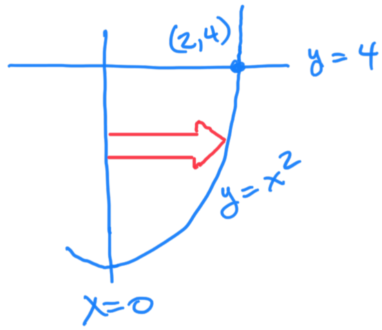
$\int_0^{2\pi} 25\cos 2t\,(5)\, dt$

$= \int_0^{2\pi} 125 \cos 2t\, dt$

$= \frac{125}{2} \sin 2t \Big|_0^{2\pi} = \emptyset$

!!

2. Evaluate $\displaystyle\int\int_R xe^{y^2}\, dA$, where R is the region in the first quadrant bounded by the graphs of $y = x^2$, $x = 0$, and $y = 4$. *Hint: Draw a picture and evaluate your integral from left to right.*



$$\iint_R xe^{y^2}\,dA = \int_0^4 \int_0^{\sqrt{y}} xe^{y^2}\,dx\,dy$$

$$= \int_0^4 \frac{x^2}{2} e^{y^2} \Big|_0^{\sqrt{y}}\, dy$$

$$= \int_0^4 \frac{1}{2} y e^{y^2}\, dy$$

$$= \frac{1}{4} e^{y^2} \Big|_0^4 = \frac{1}{4}\left(e^{16} - 1\right)$$

$$\approx \underline{\underline{2.221 \times 10^6}}$$

$$\ddot\smile$$

3. Evaluate $\int_C (x^5 + 3y)\, dx + (2x - e^{y^3})\, dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$. *Hint: Change to polar coordinates after applying Green's Theorem.*

$$F_1 = (x^5 + 3y) \qquad \frac{\partial F_1}{\partial y} = 3$$

$$F_2 = (2x - e^{y^3}) \qquad \frac{\partial F_2}{\partial x} = 2$$

This gives...

$$\iint_R (2-3)\, dA = -\iint_R dA$$

$$= -\int_0^{2\pi} \int_0^2 r\, dr\, d\theta$$

$$= \underline{\underline{-4\pi}}$$

☺

4. While subject to the force

$$\mathbf{F}(x,y) = (-16y + \sin x^2)\mathbf{i} + (4e^{y^2} + 3x^2)\mathbf{j}$$

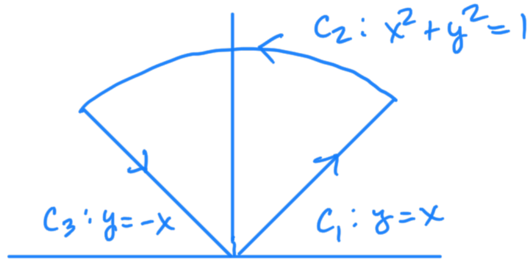
acting on a particle traveling counterclockwise once around the simple closed curve C where

$$C_1 : y = x$$

$$C_2 : x^2 + y^2 = 1$$

$$C_3 : y = -x$$

Use Green's Theorem to find the work done by $\mathbf{F}$. *Hint: Draw a ... you know what I am going to say...*



$$W = \int_C \vec{F} \cdot d\vec{r} = \int_C (-16y + \sin x^2)\,dx + (4e^{y} + 3x^2)\,dy$$

By Green's Theorem ...

$$W = \iint_R (6x + 16)\,dA$$ Look like a job for polar coordinates.

R is the pie slice $0 \le r \le 1$ & $\pi/4 \le \theta \le 3\pi/4$

$$W = \int_{\pi/4}^{3\pi/4} \int_0^1 (6r\cos\theta + 16)\,r\,dr\,d\theta$$

$$= \int_{\pi/4}^{3\pi/4} \left(2r^3\cos\theta + 8r^2\right)\Big|_0^1 \,d\theta$$

$$= \int_{\pi/4}^{3\pi/4} (2\cos\theta + 8)\,d\theta = 4\pi$$