7) Evaluate $\int_C f(z)\,dz$ where C is the semicircle $z=2e^{i\theta}$

$0\le\theta\le\pi$ $\quad \int \frac{z+2}{z}\,dz \quad$ not analytic at $z=0$

$\int_C f(z)\,dz = \int_0^\pi f\{z(\theta)\}\, z'\theta \, d\theta = \int_0^\pi \left(\frac{2e^{i\theta}+2}{2e^{i\theta}}\right)(2ie^{i\theta})\,d\theta$

$\int_0^\pi 2(ie^{i\theta}+i)\,d\theta$

$2\int_0^\pi (ie^{i\theta}+1)\,d\theta$

$2\left(e^{i\theta}+\theta\Big|_0^\pi\right) = 2\left(e^{i\pi}+i\pi-e^0-0\right)$

$= 2(-2+i\pi) = \boxed{-4+2\pi i}$

8) Find Laurent Series that represents the following function in domain $0\le|z|\le\infty$

$f(z) = z^2\sin\left(\frac{1}{z^2}\right) \quad$ So $\quad \sin z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$ for $|z|<\infty$

or $\sin z = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots +$

So $\sin\frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{2(2n+1)}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{4n+2}}$ for $0<|z|<\infty$

So $\therefore z^2\sin\frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{(-1)^n z^2}{(2n+1)!\, z^{4n+2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{4n}}$