demonstrate that no solution exists.

$5x - 3y + z = 7$
$2x + 3y - z = 0$
$8x + 9y - 3z = 2$

lets first row reduce this shit

$$\begin{bmatrix} 5 & -3 & 1 & 7 \\ 2 & 3 & -1 & 0 \\ 8 & 9 & -3 & 2 \end{bmatrix} -\cancel{2}4 R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 5 & -3 & 1 & 7 \\ 2 & 3 & -1 & 0 \\ 0 & -3 & 1 & 2 \end{bmatrix} \to -\frac{2}{5}R_1 + R_2 \to R_2 \begin{bmatrix} 5 & -3 & 1 & 7 \\ 0 & 21/5 & -7/5 & -14/5 \\ 0 & -3 & 1 & 2 \end{bmatrix} \; 5/7 R_2$$

$\frac{6}{5} + \frac{15}{5} = \frac{21}{5}$ $\quad \frac{-2}{5} + \frac{-5}{5} = \frac{-7}{5}$

$$\begin{bmatrix} 5 & -3 & 1 & 7 \\ 0 & 3 & -1 & -2 \\ 0 & -3 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 5 & -3 & 1 & 7 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so because of R_3, can have infinite number of solutions

From 2nd equation $3y - z = -2$ so let $y = a$
$z = 3y + 2$ then $z = 3a + 2$

so $5x = 0 + 3a - a - 7$

so $x = 1$

$$\boxed{x = 1,\ y = a,\ z = 3a + 2 \quad \text{Infinite solutions}}$$

2). Find the eigenvalues of A, and find a basis for each ~~subspace~~ eigenspace

$A = \begin{bmatrix} 0.5 & -0.6 \\ 0.75 & 1.1 \end{bmatrix}$ So first find characteristic polynomial

$\det(A - \lambda I) = \det \begin{bmatrix} 0.5-\lambda & -0.6 \\ 0.75 & 1.1-\lambda \end{bmatrix}$

$0.55 - 0.5\lambda - 1.1\lambda + \lambda^2 + 0.45 = \lambda^2 - 1.6\lambda + 1$

when determinant $= 0$ that is when there are solutions to $Ax = 0$

$\lambda^2 - 1.6\lambda + 1 = 0 \to \lambda = \frac{4}{5} + i\frac{3}{5}$ and $\frac{4}{5} - i\frac{3}{5}$

Next, find eigen vectors corresponding to eigenvalues

$-b = 1.6$ $\quad 4ac = 4$
$b^2 = 2.56$ $\quad 2a = 2$

$\lambda = \frac{1.6 \pm \sqrt{2.56 - 4}}{2} = \frac{1.6 \pm \sqrt{-1.44}}{2}$

$= 0.8 \pm 0.6i$

This question is terrible. When did we do complex eigenvectors and eigenvalues in class?

So $\lambda_1 = \frac{4}{5} + i\frac{3}{5}$ $\lambda_2 = \frac{4}{5} - i\frac{3}{5}$

λ_1 first: $\begin{bmatrix} 0.5 - (\frac{4}{5} + i\frac{3}{5}) & -0.6 \\ 0.75 & 1.1 - (\frac{4}{5} + i\frac{3}{5}) \end{bmatrix} \rightarrow$

$\begin{bmatrix} -0.3 - i\frac{3}{5} & -0.6 \\ 0.75 & 0.3 - i\frac{3}{5} \end{bmatrix}$ Row reduce now to find eigenvectors

$R_2(0.3+0.6i) \rightarrow R_2$

When did we do this in class???

$\begin{bmatrix} -0.3 - 0.6i & -0.6 \\ 0.75(0.3+0.6i) & 0.3-0.6i \cdot (0.3+0.6i) \end{bmatrix}$

no eigenvectors? $\begin{bmatrix} -0.3 - 0.6i & -0.6 \\ 0.75(0.3+0.6i) & 0.09 + 0.36 \end{bmatrix}$

$\begin{bmatrix} -0.3 - 0.6i & -0.6 \\ 0.75(0.3+0.6i) & 0.45 \end{bmatrix} \rightarrow \begin{bmatrix} -0.3 - 0.6i & -0.6 \\ 0 & 0 \end{bmatrix} v = 0$

$v = k\begin{pmatrix} 0.6 \\ -0.3 - 0.6i \end{pmatrix}$

λ_2 next: $\begin{bmatrix} 0.5 - (\frac{4}{5} - i\frac{3}{5}) & -0.6 \\ 0.75 & 1.1 - (\frac{4}{5} - i\frac{3}{5}) \end{bmatrix} \rightarrow$

$\begin{bmatrix} -0.3 + i\frac{3}{5} & -0.6 \\ 0.75 & 0.3 + i\frac{3}{5} \end{bmatrix}$ → Row reduce to find eigenvectors

$\begin{bmatrix} -0.3 + 0.6i & -0.6 \\ 0.75(0.3-0.6i) & 0.3-0.6i \cdot (0.3+0.6i) \end{bmatrix}$ Ugh??? is there no eigenvectors? So no basis

$\downarrow$

$\begin{bmatrix} -0.3 + 0.6i & -0.6 \\ 0.75(0.3-0.6i) & 0.45 \end{bmatrix}$ no basis.

$\begin{bmatrix} -0.3 + 0.6i & -0.6 \\ 0 & 0 \end{bmatrix} v = 0$

$\boxed{v_2 = k\begin{pmatrix} 0.6 \\ -0.3 + 0.6i \end{pmatrix} \quad v_1 = k\begin{pmatrix} 0.6 \\ -0.3 - 0.6i \end{pmatrix}}$

3. 3x3 matrix w eigenvalues of A matrix Eigenvalues of 0.5, 0.2 + 0.3i, and 0.2 − 0.3i is it diagonalizable?

So if eigenvalues are distinct → then eigenvectors are linearly independent, then it's diagonalizable!

$\begin{bmatrix} 1 \end{bmatrix}$ $\begin{bmatrix} 1+2i \end{bmatrix}$ $\begin{bmatrix} 1-2i \end{bmatrix}$