$$2\int_0^{\pi} (ie^{i\theta} + 1)\, d\theta$$

$$2\left(e^{i\theta} + \theta \Big|_0^{\pi}\right) = 2\left(e^{i\pi} + i\pi - e^0 - 0\right)$$

$$= 2(-2 + i\pi) = \boxed{-4 + 2\pi i}$$

8) Find Laurent Series that represents the following function in domain $0 \le |z| < \infty$

$f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ so $\sin z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$ for $|z| < \infty$

or $\sin z = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots +$

So $\sin \frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{2(2n+1)}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{4n+2}}$ for $0 < |z| < \infty$

So $\therefore z^2 \sin \frac{1}{z^2} = \sum_{n=0}^{\infty} \frac{(-1)^n z^2}{(2n+1)!\, z^{4n+2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, z^{4n}}$

9) Construct analytic function whose real part is $U(x,y) = x^3 - 3xy^2 + y$

Lets use Cauchy Riemann

$U_x = V_y$ and $U_y = -V_x$

So $U(x,y) = x^3 - 3xy^2 + y$

$\frac{\partial U}{\partial x} = 3x^2 - 3y^2$ this also V_y! so $\therefore \frac{dv}{dy} = 3x^2 - 3y^2$ too

So $V(x,y) = 3xy^2 - y^3$ + constant

So $U + iV = x^3 - 3xy^2 + y + (3xy^2 - y^3)i$

$+ C$

where C is any constant