Lesson 8

1) $u = (1,-1,2)$ and $v = (2,0,-1)$

a) $u - 2v = (1,-1,2) - 2(2,0,-1) = (1,-1,2) - (4,0,-2) = \boxed{(-3,-1,4)}$

b) $2(u+v) = 2(3,-1,1) = \boxed{(6,-2,2)}$

2) Say $u = (2,1,-1)$ and $v = (1,3,2)$ solve $2w + u = -v$ for w

$2w + (2,1,-1) = (-1,-3,-2)$

$2w = (-1,-3,-2) - (2,1,-1)$

$2w = (-3,-4,-1)$

$\boxed{w = (-1.5,-2,-0.5)}$

3) $3(-1,0,1,2) - (2,3,0,1) + 2w = 0$

$(-3,0,3,6) - (2,3,0,1) + 2w = 0$

$(-5,-3,3,5) + 2w = 0$ $\quad 2w = (5,3,-3,-5)$

$\boxed{w = (2.5, 1.5, -1.5, -2.5)}$

4) $(3,-2,-1) = a(1,0,-1) + b(2,1,2) + c(1,3,1)$

$$\begin{aligned} a + 2b + c &= 3 \\ b + 3c &= -2 \\ -a + 2b + c &= -1 \end{aligned} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ -1 & 2 & 1 & -1 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 4 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & -10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$\boxed{v = 2u_1 + u_2 - u_3}$

$c = -1$
$b = 1$
$a = 2$

5) ~~closed~~ under multiplication does not hold. ex:
$-1 \cdot (1,2) \rightarrow (-1,-2)$ which is not in V

no additive inverse either ex: $(1,2)$ there is no vector in V that when added to $(1,2)$ gives you $(0,0)$

6) Say matrix is $A=\begin{bmatrix}2 & 1\\ 4 & 5\end{bmatrix}$ $B=\begin{bmatrix}3 & 1\\ 4 & 5\end{bmatrix}$ $A+B=\begin{bmatrix}5 & 2\\ 8 & 10\end{bmatrix}$ is not in the form $\begin{bmatrix}a & 1\\ b & c\end{bmatrix}$ even though A and B are in that form so not closed under addition

- not closed under multiplication say $A=\begin{bmatrix}3 & 1\\ 3 & 3\end{bmatrix}$ and $c=2$ then:

$cA=\begin{bmatrix}6 & 2\\ 6 & 6\end{bmatrix}$ which is not in the form $\begin{bmatrix}a & 1\\ b & c\end{bmatrix}$ so not closed under multiplication.

- no additive identity no 0 vector, vector $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ is not in format $\begin{bmatrix}a & 1\\ b & c\end{bmatrix}$ even though

$\begin{bmatrix}2 & 1\\ 3 & 3\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}2 & 1\\ 3 & 3\end{bmatrix}$

- no additive inverse. say $A=\begin{bmatrix}3 & 1\\ 3 & 3\end{bmatrix}$. no vector B so that $A+B=0$

if $B=\begin{bmatrix}-3 & -1\\ -3 & -3\end{bmatrix}$, that is not in form $\begin{bmatrix}a & 1\\ b & c\end{bmatrix}$.

7) $\begin{bmatrix}a & 0\\ b & c\end{bmatrix}$ say $A=\begin{bmatrix}3 & 0\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & 3\end{bmatrix}$ $A+B=\begin{bmatrix}4 & 0\\ 4 & 4\end{bmatrix}$ which is $\begin{bmatrix}a & 0\\ b & c\end{bmatrix}$

so closed under addition

say $c=2$ $cA=\begin{bmatrix}6 & 0\\ 2 & 1\end{bmatrix}$ which is in form $\begin{bmatrix}a & 0\\ b & c\end{bmatrix}$ so closed under multiplication

8) - closed under addition? ~~no~~ no because

$A=(1,1,2)$ $B=(3,4,2)$ $A+B=(4,5,4)$ which is not in the form of $(a,b,2)$ so not closed under addition.

- Say c is 2 so $cA=(2,2,4)$ which is not in form $(a,b,2)$ so not closed under multiplication