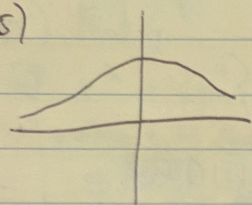
$\log_2 \ldots = 8$

$$\log_b a^y = \log_b x$$
$$y \log_b a = \log_b x$$
$$y = \frac{\log_b x}{\log_b a} \checkmark$$

$\boxed{\text{Module 6}}$

$\boxed{\text{Section 1.5}}$

5) 

not 1-to-1. while it is a function, it does not pass horizontal line test. looks like cosine function which does not pass horiz line test

6) yes, it is one-to-one. passes horiz line test

15) $f(6) = 17$ then $f^{-1}(17) = 6$

25) $y = \ln(x+3)$

inverse: $x = \ln(y+3)$

$e^x = y + 3$

$y = e^x - 3$

36) ~~Change of base~~ $\log_5 \frac{1}{125} = -3$

Check: $\log_5 1 - \log_5 125 = 0 - 3 = -3$

b) $\ln\left(\frac{1}{e^2}\right) = -2$

Check: $\ln 1 - \ln e^2 = 0 - 2 = -2$

38) a) $e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$

b) $e^{\ln(\ln e^3)} = e^{\ln(3)} = 3$

41) $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left[\ln x - \ln(x^3 + 3x + 2)^2\right] = \ln(x+2) + \frac{1}{2}\ln x - \ln(x^2 + 3x + 2)$

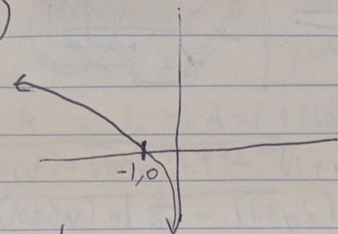
$$\cancel{\ln(x+2)} \cdot \left(\frac{\ln \sqrt{x}}{\cancel{\ln(x^2+3x+2)}} \right) = \frac{\sqrt{x}}{x+1}$$

$\ln(x+1)\cancel{(x+2)}$

42) a) $\log_5 10 = \frac{\ln 10}{\ln 5} = 1.430677$

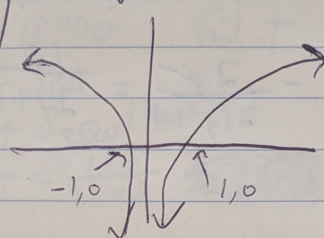
b) $\log_3 57 = \frac{\ln 57}{\ln 3} = 3.680144$

48) $y = \ln(-x)$



if x is pos then $-x$ is neg and no number that e could be raised to would get you the neg number
So domain is $(-\infty, 0)$

b) = $y = \ln|x|$



51) $e^{7-4x} = 6$ take ln of both sides

$7 - 4x = \ln 6$

$-4x = \ln 6 - 7$

$\boxed{x = \frac{\ln 6 - 7}{-4}}$

b) $\ln(3x-10) = 2$

$3x - 10 = e^2$

$3x = e^2 + 10$

$\boxed{x = \frac{e^2 + 10}{3}}$

$\boxed{\text{section } 3.6}$ 2) $f(x) = x\ln x - x$

$x\frac{1}{x} + \ln x - 1 = \boxed{\ln x}$

4) $f(x) = \ln(\sin^2 x)$

$f'(x) = \frac{1}{\sin^2 x} \cdot \frac{dy}{dx}(\sin^2 x) = \frac{1}{\sin^2 x} \cdot 2\sin x\cos x = \boxed{2\cot x}$

5) $f(x) = \ln\frac{1}{x}$

$f'(x) = \frac{1}{\frac{1}{x}} = x\frac{d}{dy}\left(\frac{1}{x}\right) = x \cdot \frac{dy}{dx}x^{-1} = x \cdot (-x^{-2}) = \frac{x}{-x^2} = \boxed{\frac{-1}{x}}$

Check: $x\frac{dy}{dx}\left(\frac{1}{x}\right) = x \cdot \frac{-1}{x^2} = \boxed{\frac{-1}{x}}$

using quotient rule

6) $y = \frac{1}{\ln x}$

$y' = -\frac{\frac{1}{x}}{\ln(x)^2} = \cancel{\frac{1}{x\ln^2 x}}$

$\boxed{\frac{-1}{x\ln^2 x}}$

Check $y = (\ln x)^{-1}$

$y' = -1 \cdot \ln x^{-2}\frac{dy}{dx}\ln x$

$= \frac{-1}{(\ln^2 x)x}$ ✓

12) $h(x) = \ln(x + \sqrt{x^2-1})$

$h'(x) = \dfrac{1}{x+\sqrt{x^2-1}} \quad \dfrac{dy}{dx}\, x+\sqrt{x^2-1} \Rightarrow 1 + \dfrac{x}{\sqrt{x^2-1}}$

$\dfrac{1}{x+\sqrt{x^2-1}} \cdot \left(\dfrac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}\right) = \boxed{\dfrac{1}{\sqrt{x^2-1}}}$

13) $G(y) = \ln \dfrac{(2y+1)^5}{\sqrt{y^2+1}} = \ln(2y+1)^5 - \ln\sqrt{y^2+1}$

$= 5\ln(2y+1) - \frac{1}{2}\ln(y^2+1)$

$G'(y) = \dfrac{5}{(2y+1)} \cdot 2 - \dfrac{2y}{2(y^2+1)} = \dfrac{10}{2y+1} - \dfrac{2y}{2y^2+2}$

$2(y^2+1)$

$\boxed{\dfrac{10}{2y+1} - \dfrac{y}{y^2+1}}$

20) $H(z) = \ln\sqrt{\dfrac{a^2-z^2}{a^2+z^2}} = \dfrac{1}{2}\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)$

$\cancel{H'(z) = \dfrac{a^2+z^2}{2(a^2-z^2)} \cdot \dfrac{dH}{dz}\left(\dfrac{a^2-z^2}{a^2+z^2}\right) = \dfrac{a^2+z^2}{2(a^2-z^2)} \cdot \dfrac{-2z(a^2+z^2) - 2z(a^2-z^2)}{(a^2+z^2)^2}}$

$\frac{1}{2}\left(\ln(a^2-z^2) - \ln(a^2+z^2)\right)$

$H'(z) = \frac{1}{2}\left(\dfrac{1}{a^2-z^2} \cdot -2z - \dfrac{1}{a^2+z^2} \cdot 2z\right) = \boxed{\dfrac{-2z}{2(a^2-z^2)} - \dfrac{2z}{2a^2+z^2}}$

37) $y = (x^2+2)^2(x^4+4)^4$

$\ln y = 2\ln(x^2+2) + 4\ln(x^4+4)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2 \cdot 2x}{x^2+2} + \dfrac{4 \cdot 4x^3}{x^4+4}$

$\dfrac{dy}{dx} = \boxed{(x^2+2)^2(x^4+4)^4 \cdot \left(\dfrac{4x}{x^2+2} + \dfrac{16x^3}{x^4+4}\right)}$

43) $y = x^x$

$\ln y = x\ln x$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x}{x} + \ln x$

$\dfrac{dy}{dx} = \boxed{x^x(1+\ln x)}$

45) $y = x^{\sin x}$

$\ln y = \sin x \ln x$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\sin x}{x} + \ln x \cos x$

$\dfrac{dy}{dx} = x^{\sin x}\left(\dfrac{\sin x}{x} + \ln x \cos x\right)$

3.8) 3) a) $P(0)=100$

$P(1)=420$

$P(t)=100e^{kt}$

$P(1)=100e^{k}=420$

$e^{k}=4.2$

$k=\ln 4.2=1.4350845$

So model: $\boxed{P(t)=100e^{\ln 4.2\cdot t}}$

b) $P(3)=100e^{(\ln 4.2)(3)}\approx 7408.8$

c) $P'(t)=100\cdot e^{\ln 4.2 t}\cdot \ln 4.2$

$P'(3)=7408.8\cdot \ln 4.2$

$=p(3)\cdot k$

$=10632.254$

d) $P(t)?=10000$

$100e^{kt}=10000$ find t

$e^{kt}=100$

$\ln 4.2\, t=\ln 100$

$t=\frac{\ln 100}{\ln 4.2}\approx 3.2089$

5a) $p(0)=790$

$P(50)=980$

$790e^{kt}=$ because when $t=0$, population $=790$

$790e^{k50}=980$

$e^{50k}=\frac{980}{790}$

$50k=\ln\left(\frac{980}{790}\right)$

$k=\ln\left(\frac{98}{79\cdot 50}\right)=\ln\frac{49}{79\cdot 25}$

$\boxed{\text{model: } 790e^{\ln\frac{49}{79\cdot 25}\cdot t}}$

so year $1900=P(150)=2940$

year $1950=P(200)=3920$

It predicts a bit high?

8) $M(0)=50$ mg

find $M(t)$ $\quad 50e^{kt}$

$M(28)=25$ so $50e^{k28}=25$

$e^{k28}=\frac{1}{2}$

$28k=\ln 0.5$

$k=\frac{\ln 0.5}{28}=\boxed{\frac{-\ln 2}{28}}$

a) $\boxed{\text{model: } 50e^{\frac{\ln 0.5}{28}t}}$

b) $M(40): 50e^{\frac{\ln 0.5}{28}\cdot 40}=35.7142$

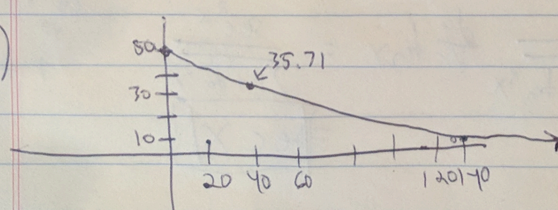
c) $50e^{\frac{\ln 0.5}{28}t}=2$

$e^{\frac{\ln 0.5 t}{28}}=\frac{1}{25}$

$\frac{\ln 0.5 t}{28}=\ln\frac{1}{25} \Rightarrow t=\frac{\ln\frac{1}{25}\cdot 28}{\ln 0.5}$

$\boxed{t=130}$

d)



20) Interest $= 8\%$ $\quad A_0\left(1+\frac{r}{n}\right)^{nt}$

base $= \$1000$

i) $1000(1+0.08)^3 = 1259.71$ annual compound

ii) $1000\left(1+\frac{0.08}{4}\right)^{12} = 1268.24$ quarterly

iii) $1000\left(1+\frac{0.08}{12}\right)^{36} = 1270.24$ monthly

iv) $1000\left(1+\frac{0.08}{52}\right)^{156} = 1271.01$ weekly

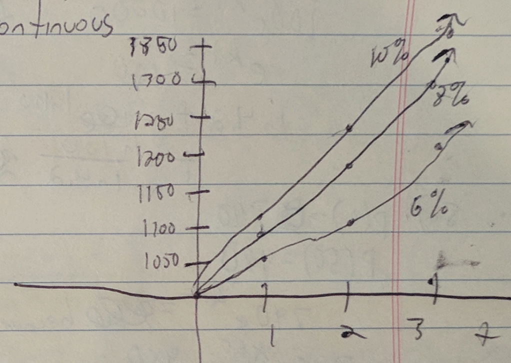
v) $1000\left(1+\frac{0.08}{365}\right)^{1095} = 1271.22$ daily

vi) $1000\left(1+\frac{0.08}{8760}\right)^{8760\cdot 3} = 1271.25$ hourly

vii) $1000(e)^{0.08\cdot 3} = 1271.25$ continuous

b)

	6%	8%	10%
0	1000	1000	1000
1	1061.83	1083.29	1105.17
2	1127.50	1173.51	1221.40
3	1197.21	1271.25	1349.86



22a) $e^{0.06\cdot t} = 2$

$0.06t = \ln 2$

$\boxed{t = \frac{\ln 2}{0.06}} = \boxed{11.55}$ units of time

Section 3.11

2) $\cosh 0 = \frac{e^0 + e^0}{2} = 1 \qquad \sinh 0 = \frac{e^0 - e^0}{2} = 0$

a) $\tanh 0 = 0$ $\qquad$ b) $\tanh(1)$

$= \frac{e - e^{-1}}{e + e^{-1}} = \frac{e^2 - 1}{e^2 + 1} \approx 0.76159$

15) $\sinh 2x = 2\sinh x \cosh x$

$= 2 \cdot \frac{e^x - e^{-x}}{2} \cdot \frac{e^x + e^{-x}}{2} = \boxed{\frac{e^{2x} - e^{-2x}}{2} = \sinh 2x}$

37) $y = e^{\cosh 3x}$

$y' = e^{\cosh 3x} \cdot \sinh 3x \cdot 3$

40) $y = \sinh^{-1}(\tan x)$

$y' = \frac{1}{\sqrt{1+\tan^2 x}} \frac{dy}{dx}\tan x = \frac{\sec^2 x}{\sqrt{1+\tan^2 x}}$

$= \boxed{\sec x}$

$\frac{\cos^2 x}{s^2} + \frac{\sin^2 x}{\cos^2} = \frac{1}{\cos^2}$

$1 + \tan^2 x = \sec^2 x$

41) $y = \cosh^{-1}\sqrt{x}$

$$y' = \frac{1}{\sqrt{x-1}}\,\frac{d}{dx}\sqrt{x} = \boxed{\frac{1}{2\sqrt{x-1}\,\sqrt{x}}}$$