$$2\int_0^6 36-x^2-x^2+36$$

$$2\int_0^6 72-2x^2 \to 2\left[72x-\frac{2}{3}x^3\right]_0^6$$

$$\to 2[432-144] = 288\cdot 2 = \boxed{576}$$ ✓

Module 12

6.2 Section 6.2

3) $y=\sqrt{x-1}$, $y=0$, $x=5$, about x axis

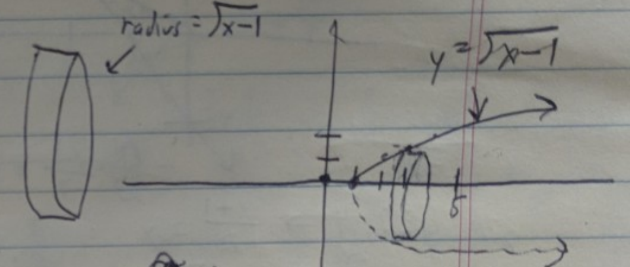
Area cross section $=\pi(\sqrt{x-1})^2$

$=\pi(x-1)$

$=\pi x-\pi$

so... $\int_1^5 \pi x-\pi\,dx = \pi\int_1^5 \pi x-\pi = \pi\left[\frac{x^2}{2}-x\right]_1^5 = \pi\left[\frac{25}{2}-5-\frac{1}{2}+1\right]$

$=\pi[8] = \boxed{8\pi}$



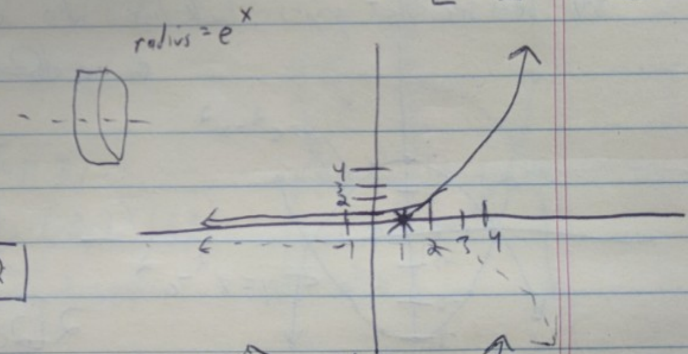
4) $y=e^x$, $y=0$, $x=-1$, $x=1$, x axis

Area cross section $=\pi(e^x)^2$

$=\pi e^{2x}$

$$\pi\int_{-1}^1 e^{2x} = \pi\left[\frac{e^{2x}}{2}\right]_{-1}^1$$

$$=\left[\frac{e^2}{2}-\frac{e^{-2}}{2}\right]\pi = \boxed{\pi \sinh 2}$$



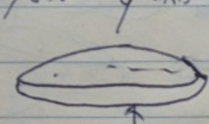
5) $x=2\sqrt{y}$, $x=0$, $y=9$, about y axis

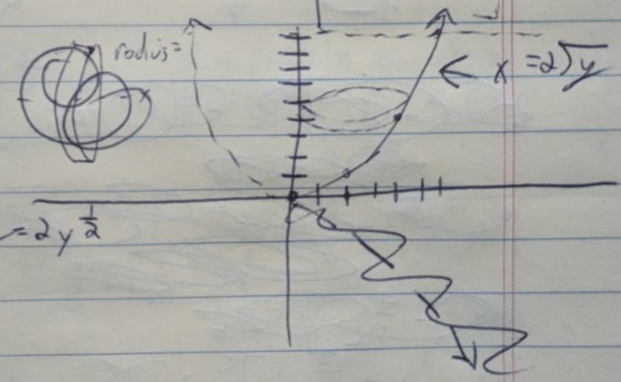
Area cross section $=\pi(2y^{\frac{1}{2}})^2$

$=\pi 4y$

$$\pi\int_0^9 4y\,dy = \pi\left[4\frac{y^2}{2}\right]_0^9$$

$$=\pi[2y^2]_0^9 = \boxed{\pi 162}$$



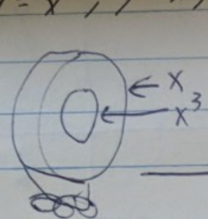


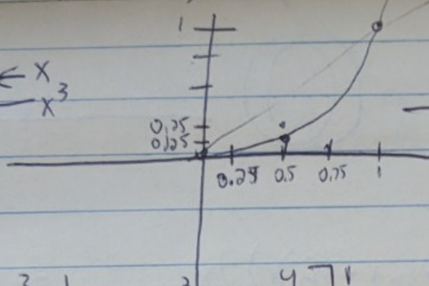
$2\int_0^6 36-x^2-x^2+36$

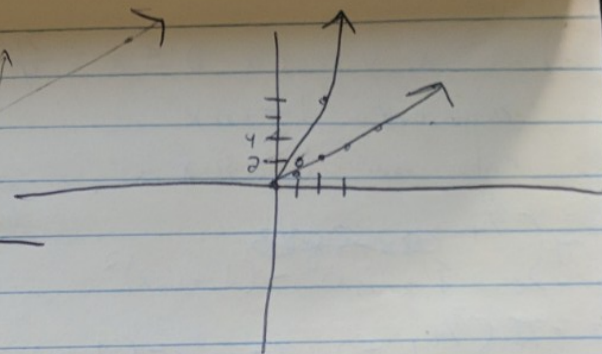
$2\int_0^6 72-2x^2 \rightarrow 2\left[72x-\frac{2}{3}x^3\right]_0^6$

$\rightarrow 2[432-144] = 288\cdot 2 = \boxed{576}$ ✓

7) $y=x^3,\ y=x,\ x\geq 0$, about x axis

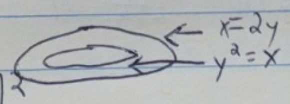






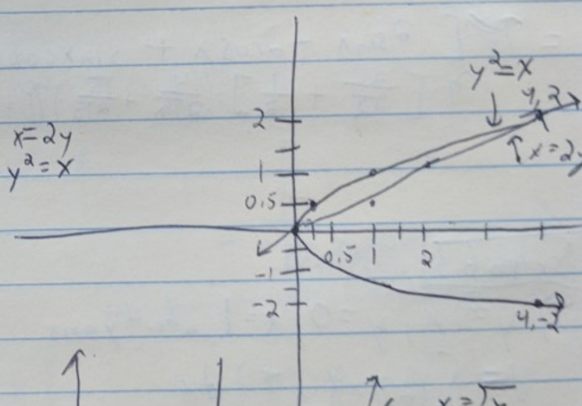
$$\int_0^1 x - x^3\,dx = \frac{x^2}{2} - \frac{x^4}{4}\Big]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

9) $y^2 = x,\ x = 2y$, about y



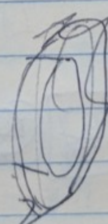
$$\int_0^2 2y - y^2\,dy = 2\frac{y^2}{2} - \frac{y^3}{3}\Big]_0^2$$

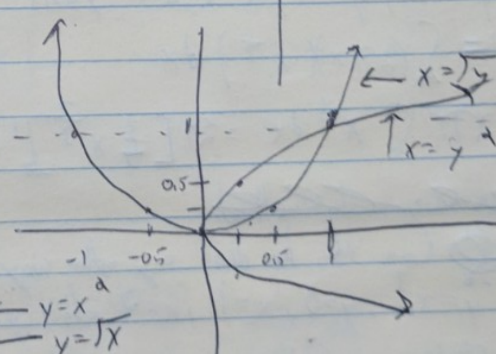
$$= y^2 - \frac{y^3}{3}\Big]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$



11) $y=x^2,\ x=y^2$, about $y=1$

$x=\sqrt{y}$

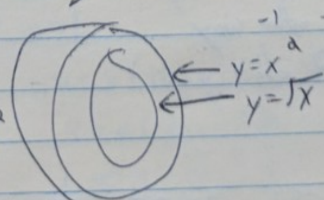




$\pi\int_0^1 \cancel{x^2 - \sqrt{x}}\,dx$

radius outer $= 1 - x^2$

radius inner $= 1 - \sqrt{x}$



$$\pi\int_0^1 (1-x^2)^2 - \pi\int_0^1 (1-\sqrt{x})^2$$

$$= \pi\int_0^1 1 - 2x^2 + x^4 - \pi\int_0^1 1 - 2\sqrt{x} + x$$

$$= \pi\int_0^1 -2x^2 + 2\sqrt{x} + x^4 - x = \pi\left[-2\frac{x^3}{3} + 2\frac{x^{3/2}}{\frac{3}{2}} + \frac{x^5}{5} - \frac{x^2}{2}\right]_0^1$$

$$= \pi\left[-\frac{2}{3} + \frac{4}{3} + \frac{1}{5} - \frac{1}{2}\right]_0^1$$

$$= \pi\left[\frac{2}{3} + \frac{1}{5} - \frac{1}{2}\right] = \pi\left[\frac{1}{6} + \frac{1}{5}\right] = \boxed{\pi\frac{11}{30}}$$

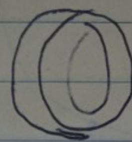
6) $xy = 1$, $x=0$, $y=1$, $y=3$ about x-axis

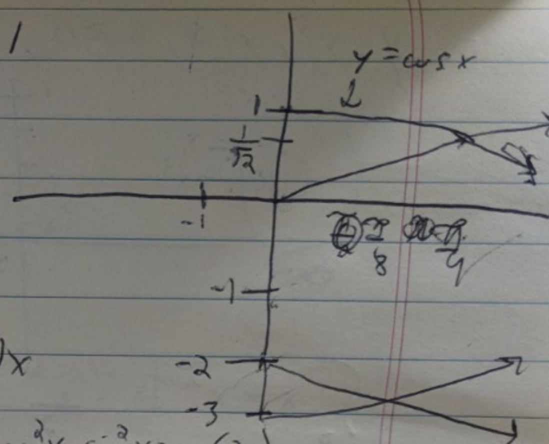
$x = \frac{1}{y}$

14) $y = \sin x$, $y = \cos x$, $0 \le x \le \frac{\pi}{4}$, about $y = -1$

radius outer : $1 + \cos x$

radius inner : $1 + \sin x$





$$\pi \int_0^{\pi/4} (1+\cos x)^2 - (1+\sin x)^2$$

$$= \pi \int_0^{\pi/4} 1 + 2\cos x + \cos^2 x - (1 + 2\sin x + \sin^2 x)\, dx$$

$$= \pi \int_0^{\pi/4} (2\cos x - 2\sin x + \cos^2 x - \sin^2 x)\, dx \rightarrow \cos^2 x - \sin^2 x = \cos(2x)$$

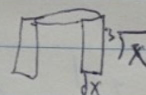
$$= \pi \left[2\sin x + 2\cos x + \sin x \cos x \right]_0^{\pi/4}$$

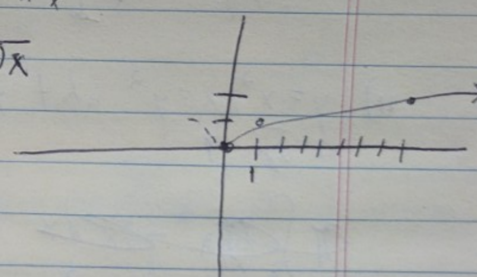
$$\pi \left[\frac{2}{\sqrt{2}} + \frac{1}{2} \right] = \left(\frac{4}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}} \right)\pi = \frac{4+\sqrt{2}}{2\sqrt{2}}\pi$$

Section 6.3

3) $y = \sqrt[3]{x}$, $y = 0$, $x = 1$, about y axis

cir $= 2\pi x$





$$V = \int_0^1 (2\pi x)(x^{\frac{1}{3}})\, dx$$

$$2\pi \int_0^1 x^{\frac{4}{3}}\, dx = 2\pi \left[\frac{3}{7} x^{\frac{7}{3}} \right]_0^1 = \boxed{\frac{6\pi}{7}}$$

~~Check using washer method~~

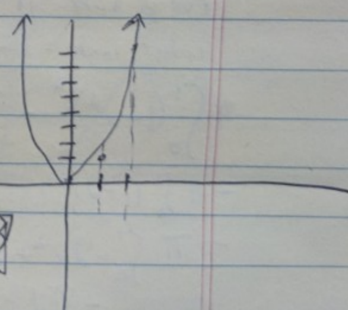
Can this be done any other way?

4) $y = x^3$, $y = 0$, $x = 1$, $x = 2$, about y axis

$$\int_1^2 (2\pi x)(x^3)\, dx$$

$$2\pi \int_1^2 x^4\, dx = 2\pi \left[\frac{x^5}{5} \right]_1^2 = \left(\frac{32}{5} - \frac{1}{5} \right) 2\pi$$

$$= \boxed{\frac{62\pi}{5}}$$



14) $y = \sin x,\ y = \cos x,\ 0 \le x \le \frac{\pi}{4}$, about $y = -1$

radius outer: $1 + \cos x$

radius inner: $1 + \sin x$

9) $xy = 1,\ x = 0,\ y = 1,\ y = 3$, about x-axis

$x = \frac{1}{y}$

radius = y

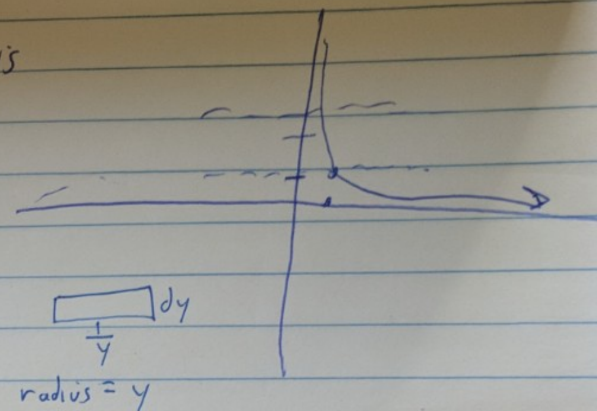
height = $\frac{1}{y}$

$V = \int_1^3 2\pi(y)\left(\frac{1}{y}\right) dy$

$= \int_1^3 2\pi(y)(y^{-1})\, dy$

$= 2\pi \int_1^3 (y)(y^{-1})\, dy$

$= 2\pi \int_1^3 y^0 = 2\pi[x]_1^3 = \boxed{4\pi}$



10) $y = \sqrt{x},\ x = 0,\ y = 2$, about the x-axis

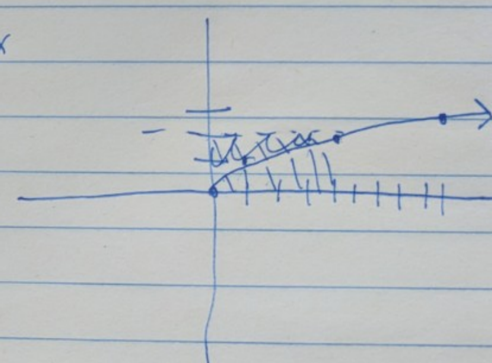
radius $= y$ $\qquad y = \sqrt{x} \to y^2 = x$

height $= y^2$

$\int_0^2 2\pi y (y^2)\, dy$

$= 2\pi \int_0^2 y^3\, dy = 2\pi\left[\frac{y^4}{4}\right]_0^2$

$= \pi\left[\frac{y^4}{2}\right]_0^2 = \boxed{8\pi}$



Check w/ disks

$\int_0^4 \pi x\, dx = \left[\pi \frac{x^2}{2}\right]_0^4 = 8\pi$

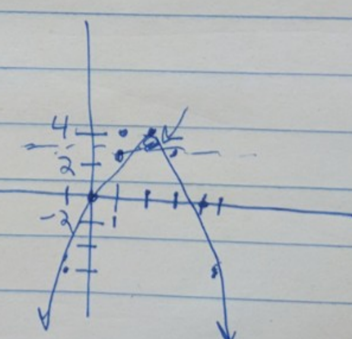
17) $y = 4x - x^2,\ y = 3$, about $x = 1$

vertical rects

$y = x(4 - x)$ $\quad -(x-1)(x-3)$ $\quad -(x^2 - 4x + 3)$

radius $= x - 1$ $\qquad 3 = x(4 - x) \to x^2 + 4x - 3$

height $= 4x - x^2 - 3$ $\qquad$ at $x = \{1, 3\}$



$\int_1^3 2\pi(x-1)(4x - x^2 - 3)$

$= 2\pi \int_1^3 4x^2 - x^3 - 3x - 4x + x^2 + 3$

$= 2\pi \int_1^3 -x^3 + 5x^2 - 7x + 3 = 2\pi\left[-\frac{x^4}{4} + 5\frac{x^3}{3} - 7\frac{x^2}{2} + 3x\right]_1^3 =$

$= 2\pi\left[\frac{-81}{4} + 45 - \frac{63}{2} + 9 - \left(\frac{-1}{4} + \frac{5}{3} - \frac{7}{2} + 3\right)\right] = \left[2\pi \cdot \left(\frac{-80}{4} + 51 + \frac{-56}{2} - \frac{5}{3}\right)\right]$

$\frac{8\pi}{3} = \boxed{8\pi/3}$