Lesson 4 Sections 2.3 and 2.4

1) Inverse of $A = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix} \leftarrow$

Check: $\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

2) Inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ via Gauss Jordan $\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow$

$$\begin{array}{c} \\ -R1 \\ -R1 \end{array}\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right] \begin{array}{c} +R3 \\ \rightarrow +R3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right] \quad \boxed{A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}}$$

3) $x - 2y = 1$
$2x + 3y = 4$

$B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $A^{-1} = ?$

$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{7}\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$

$Ax = B$
$A^{-1}Ax = A^{-1}B$
$x = A^{-1}B$

$A^{-1}B = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = X =$

$X = \begin{bmatrix} \frac{11}{7} \\ \frac{2}{7} \end{bmatrix}$ so $\boxed{\begin{array}{l} x = \frac{11}{7} \\ y = \frac{2}{7} \end{array}}$

4) a) $R_3 \leftrightarrow R1$

R_3 and R_1 are swapped $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

b) $\begin{bmatrix}1&0&0\\0&1&0\\0&-3&1\end{bmatrix}$ $-3(R_2)+R_3 \to R_3$

5) $A^{-1} = \begin{bmatrix}1&0&0\\0&\frac{1}{3}&0\\0&0&1\end{bmatrix}$ check: $\begin{bmatrix}1&0&0\\0&3&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&\frac{1}{3}&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ ✓

b) $A^{-1} = \begin{bmatrix}1&0&0\\0&1&-2\\0&0&1\end{bmatrix}$ check: $\begin{bmatrix}1&0&0\\0&1&-2\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&2\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ ✓

$R_2 + -2(R_3) \to R_2$

6) $A = \begin{bmatrix}1&2\\3&1\end{bmatrix} \to R_2 + -3(R_1) \to R_2$ $\begin{bmatrix}1&2\\0&-5\end{bmatrix}$ $E_1 = \begin{bmatrix}1&0\\-3&1\end{bmatrix}$

$R_1 + \frac{2}{5}(R_2) \to R_1$ $\begin{bmatrix}1&0\\0&-5\end{bmatrix}$ $E_2 \begin{bmatrix}1&\frac{2}{5}\\0&1\end{bmatrix}$

$\frac{-1}{5}R_2 \to R_2$ $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $E_3 = \begin{bmatrix}1&0\\0&-\frac{1}{5}\end{bmatrix}$

$E_1^{-1} = \begin{bmatrix}1&0\\3&1\end{bmatrix}$ $E_2^{-1} = \begin{bmatrix}1&-\frac{2}{5}\\0&1\end{bmatrix}$ $E_3^{-1} = \begin{bmatrix}1&0\\0&-5\end{bmatrix}$

$E_1^{-1}E_2^{-1} = \begin{bmatrix}1&-\frac{2}{5}\\3&-\frac{1}{5}\end{bmatrix} = C$ $CE_3^{-1} = \begin{bmatrix}1&2\\3&1\end{bmatrix}$ ✓

7) $\begin{bmatrix}-2&1\\-6&4\end{bmatrix}$ $R_2 + -3(R_1) \to R_2$ $\begin{bmatrix}-2&1\\0&1\end{bmatrix}$ E_1 $\begin{bmatrix}1&0\\-3&1\end{bmatrix}$

$LU = \begin{bmatrix}1&0\\3&1\end{bmatrix}\begin{bmatrix}-2&1\\0&1\end{bmatrix}$ $E_1^{-1} = \begin{bmatrix}1&0\\3&1\end{bmatrix}$

$= \begin{bmatrix}-2&1\\-6&4\end{bmatrix}$ ✓

8) $Ax = B$

$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$LUx = B$ let $y = Ux$

$LU = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$

$Ly = B$

$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$y_1 = \cancel{0}1$ from forward sub

$y_2 = -1$

$Ux = y$

$\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\boxed{\begin{aligned} x_1 &= \cancel{0}-1 \\ x_2 &= -1 \end{aligned}}$

Check:

$$\begin{bmatrix} -2 & 1 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \overset{B}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \checkmark$$