$$\frac{81}{n^4} \cdot \frac{n^2(n+1)^2}{4} = \frac{81}{n^4} \cdot \frac{n^2(n^2+2n+1)}{4}$$

$$\frac{81}{4} \cdot \frac{n^2(n+1)^2}{n^4} = \frac{81}{4} \cdot \frac{(n+1)^2}{n^2} \qquad \frac{6^2}{2^2} = 9$$

2) $\sum_{i=1}^{6} \frac{1}{i+1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$

4) $\sum_{i=4}^{6} i^3 = 4^3 + 5^3 + 6^3 = 64 + 125 + 216$

10) $\sum_{i=1}^{n} f(x_i)\Delta x_i = f(1)\Delta x_1 + f(1+1)\Delta x_{1+1} + f(1+2)\Delta x_{1+2}$
$\ldots f(n-1)\Delta x_{n-1} + f(n)\Delta x_n$

12) $\sum_{i=3}^{7} \sqrt{i}$ 14) $\sum_{i=3}^{23} \frac{i}{i+4}$ 22) $\sum_{i=3}^{6} = 3(5) + 4(6) + 5(7) + 6(8)$

$\sum_{i=3}^{6} i(i+2) = 15 + 24 + 35 + 48$
$= \boxed{122}$

26) $\sum_{i=1}^{100} 4 = \boxed{400}$ 31) $\sum_{i=1}^{n} i^2 + 3i + 4$

$(1+3+4) + (4+6+4) + (9+9+4) +$
$(16+12+4) = 8 + 14 + 22 + 32 + 44$

$n=5 : 120$ $n=3 : 44$ $n=1 : 8$
$n=4 : 76$ $n=2 : 22$

OR $= \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 4 = \frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2} + 4n$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{9n(n+1)}{6} + \frac{24n}{6}$$

$$\frac{(n+1)(2n^2+n)}{6} + \frac{9n(n+1)}{6} + \frac{24n}{6} =$$

$$\frac{(n+1)(2n^2+10n)}{6} + \frac{24n}{6}$$

$$= \frac{2n^3 + 10n^2 + 2n^2 + 10n + 24n}{6} = \frac{2n^3 + 12n^2 + 34n}{6} = \boxed{\frac{n(2n^2+12n+34)}{6}}$$

45) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{2i}{n}\right)^3 + 5\left(\frac{2i}{n}\right)\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{8i^3}{n^3}\right) + \left(\frac{10i}{n}\right)\right]$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{16}{n^4} i^3\right) + \left(\frac{20}{n^2} i\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{16}{n^4} \left(\frac{n(n+1)}{2}\right)^2 + \frac{20}{n^2}\left(\frac{n(n+1)}{2}\right) \right]$$

$$= \lim_{n\to\infty} \left[\frac{16}{n^4} \left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2}\right) + \frac{20}{n^2}\left(\frac{n(n+1)}{2}\right) \right]$$

$$= \lim_{n\to\infty} \left[\left(4 \cdot \frac{n^2}{n^2} \cdot \frac{n+1}{n} \cdot \frac{n+1}{n}\right) + \left(10 \cdot \frac{n}{n} \cdot \frac{n+1}{n}\right) \right]$$

$$= \lim_{n\to\infty} \left[\left(4 \cdot 1 \cdot \left(1+\frac{1}{n}\right) \cdot \left(1+\frac{1}{n}\right)\right) + \left(10 \cdot 1 \cdot \left(1+\frac{1}{n}\right)\right) \right] = \boxed{14}$$

$\boxed{\text{Section 5.2}}$ 4) $f(x) = \frac{1}{x}$ $\quad 1 \le x \le 2$ $\quad n=4$ find Riemann sum, using right endpoints

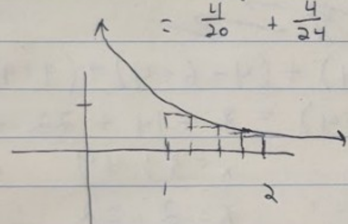
$\Delta x = \frac{2-1}{4} = \frac{1}{4}$ and $a = 1$

right endpoints are: $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$ so

$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x = f\left(\tfrac{5}{4}\right)\Delta x + f\left(\tfrac{6}{4}\right)\Delta x + f\left(\tfrac{7}{4}\right)\Delta x + f\left(\tfrac{8}{4}\right)\Delta x$$

$$= \frac{1}{4}\left(\frac{4}{5} + \frac{4}{6} + \frac{4}{7} + \frac{1}{2}\right)$$

$$= \frac{4}{20} + \frac{4}{24} + \frac{4}{28} + \frac{1}{8} = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \boxed{0.634523}$$



we use rectangles to approximate the area, using the right endpoints as heights and Δx as width.

b) using midpoints, Δx is still $\frac{1}{4}$ but the midpoints are

subintervals: $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$

midpts: $\frac{9}{8}, \frac{11}{8}, \frac{13}{8}, \frac{15}{8}$

$$M_4 = \sum_{i=1}^{4} f(\bar{x}_i)\Delta x = \Delta x\left[f(\bar{x}_1) + \dots + f(\bar{x}_n)\right]$$

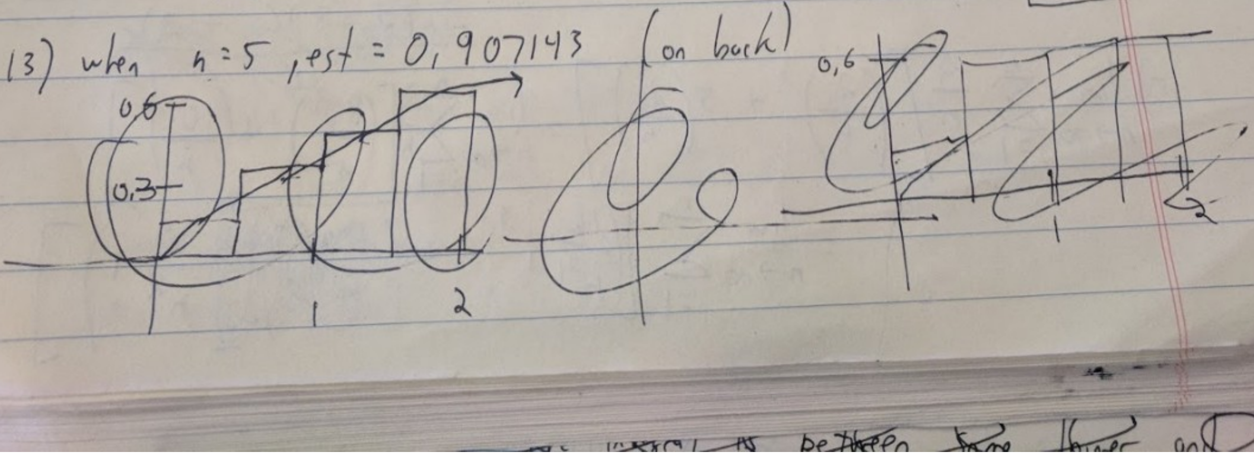
$$= \frac{1}{4}\left(\frac{8}{9} + \frac{8}{11} + \frac{8}{13}, \frac{8}{15}\right) = \frac{8}{36} + \frac{8}{44} + \frac{8}{52} + \frac{8}{60} = \boxed{0.691220}$$

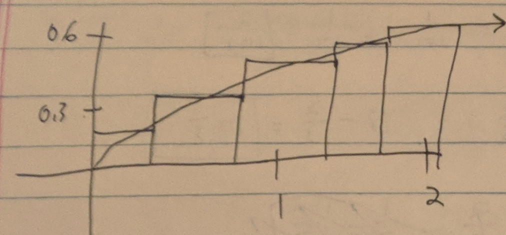
Just for kicks

$$L_4 = \sum_{i=0}^{3} f(x_i)\Delta x = \frac{1}{4}\left(\frac{4}{4}, \frac{4}{5}, \frac{4}{6}, \frac{4}{7}\right) = \frac{4}{16}, \frac{4}{20}, \frac{4}{24}, \frac{4}{28} = \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$$

$\boxed{= 0.759524}$

13) when $n = 5$, est $= 0.907143$ (on back)





$n=10$, est $= .902858$

$n=20$, est $= .901757$

14) $n=100$, est $= .901403$

0.8946 is the est using the left endpoint and $n=100$

0.9081 is the est using the right endpoint and $n=100$

The graph is increasing and so the right Riemann sum overestimates and the left endpoint underestimates, so $\int_0^2 \frac{x}{x+1}\,dx$ is in btw the two because the error it makes on the upside is < than the Right Riemann sum but > the left Riemann sum.

16) e^{-x^2} dec function so Left should be > right

L	R	n
1.07746	0.68479	5
0.980007	0.783670	10
0.901705	0.862437	50
0.89189	0.872262	100

must be btwn $0.872262 < A < 0.89189$

(further explanation on back)

If we were evaluating $\int_{-1}^{2} e^{-x^2}\,dx$, we cannot make a similar statement.

Reason: Estimation methods wouldn't consistently under estimate or overestimate the curve area.

Section 5.3

9) $g(s) = \int_5^s (t-t^2)^8\,dt$

$\boxed{g'(s) = (s-s^2)^8}$

14) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$

$h'(x) = \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}^2}{\sqrt{x}^4+1} = \boxed{\frac{1}{2\sqrt{x}} \cdot \frac{x}{x^2+1}}$

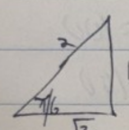
20) $\int_{-1}^{1} x^{100}\,dx = \frac{1^{101}}{101} - \frac{-1^{101}}{101} = \frac{1}{101} + \frac{1}{101} = \boxed{\frac{2}{101}}$

23) $\int_1^9 \sqrt{x}\,dx = \cancel{\frac{2x^{3/2}}{3}}\ \frac{2}{3}\cdot 9^{\frac{3}{2}} - \frac{2}{3} = 18 - \frac{2}{3} = \boxed{17\frac{1}{3}}$

24) $\int_1^8 x^{\frac{-2}{3}}\,dx$

$F(x) = 3x^{\frac{1}{3}} \quad 3\cdot 8^{\frac{1}{3}} - 3\cdot 1^{\frac{1}{3}} = 6 - 3 = \boxed{3}$

25) $\int_{\frac{\pi}{6}}^{\pi} \sin\theta\,d\theta = -\cos\pi - -\cos\frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2} = \boxed{\frac{2+\sqrt{3}}{2}}$



28) $\int_0^4 (4-t)\sqrt{t}\,dt$ What is the antiderivative of $(4-t)\sqrt{t}$?

well $(4-t)\sqrt{t} = 4\sqrt{t} - t^{\frac{3}{2}}$

$= \int_0^4 4\sqrt{t}\,dt - \int_0^4 t^{\frac{3}{2}}\,dt$

$= 4\int_0^4 \sqrt{t}\,dt - \int_0^4 t^{\frac{3}{2}}\,dt$

$= 4\ \frac{2t^{\frac{3}{2}}}{3} = \frac{8}{3}t^{\frac{3}{2}} = 4\int \sqrt{t}\,dt$

$\uparrow$

$\frac{2}{5}t^{\frac{5}{2}}$

$4\int_0^4 \sqrt{t}\,dt = \frac{64}{3} \qquad \int_0^4 t^{\frac{3}{2}}\,dt = \frac{64}{5}$

$\frac{64}{3} - \frac{64}{5} = \frac{320}{15} - \frac{192}{15} = \boxed{\frac{128}{15}}$

34) $\int_0^3 (2\sin x - e^x)\,dx$

$= \int_0^3 2\sin x\,dx - \int_0^3 e^x\,dx \longrightarrow$ note: $\int_0^3 2\sin x\,dx = 2\int_0^3 \sin x\,dx =$

$= 2(-\cos 3 - -\cos 0)$

and: $\int_0^3 e^x\,dx = e^3 - e^0 = e^3 - 1$

$= 2(-\cos 3 + \cos 0) - e^3 + 1$

$= \boxed{3 - e^3 - 2\cos(3)}$

$20) \int_{-1}^{1} x^{100}\,dx = \frac{x^{101}}{101}\Big]_{-1}^{1} = \frac{1}{101} - \frac{-1}{101} = \frac{1}{101} + \frac{1}{101} = \boxed{\frac{2}{101}}$

36) $\int_1^{18} \sqrt{\frac{3}{z}}\,dz = \cancel{\int_1^{18} \left(\frac{3}{z}\right)^{\frac{1}{2}} dz = \frac{2}{3}\cdot\frac{3}{z}^{\frac{3}{2}}\Big]_1^{18} = \frac{2}{3}\cdot\frac{1}{6}^{\frac{3}{2}} - \frac{2}{3}\cdot 3^{\frac{3}{2}}}$

$= \sqrt{3} \int_1^{18} \sqrt{\frac{1}{z}}\,dz = \sqrt{3}\int_1^{18} \frac{1}{z}^{\frac{1}{2}}\,dz = \cancel{\sqrt{3}\int_1^{18} \frac{2}{3}\cdot\frac{1}{z}^{\frac{3}{2}} dz}$

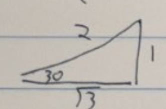
$= \sqrt{3}\int_1^{18} z^{-\frac{1}{2}}\,dz = \sqrt{3}\,\frac{z^{-\frac{1}{2}+1}}{\frac{1}{2}}\Big]_1^{18} = \sqrt{3}\cdot 2z^{\frac{1}{2}}\Big]_1^{18} =$

$\sqrt{3}\cdot 2\cdot 18^{\frac{1}{2}} - 2(1)^{\frac{1}{2}} = \boxed{2\sqrt{3}\,[3\sqrt{2}-1]}$

$= \sqrt{3}\,(6\sqrt{2}-2) \longrightarrow$

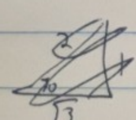
39) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx = 8\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = 8\tan^{-1} x\Big]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$8\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right) = 8\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 8\left(\frac{2\pi}{6} - \frac{\pi}{6}\right) = \boxed{\frac{4\pi}{3}}$



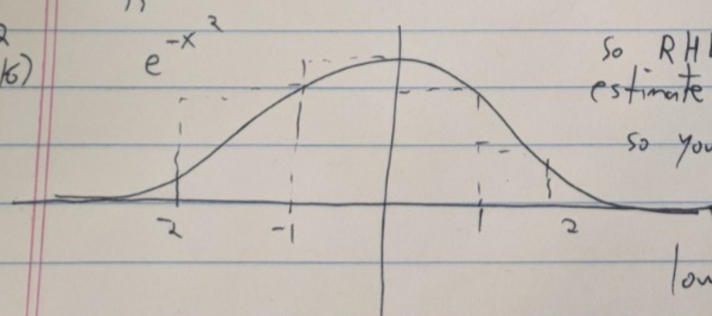
42) $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}}\,dx = 4\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}\,dx = 4\sin^{-1} x\Big]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}$

$4\left(\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{2}\right) = 4\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = 4\left(\frac{6\pi}{24} - \frac{4\pi}{24}\right) = \frac{8\pi}{24} = \boxed{\frac{\pi}{3}}$



Section 5.2

16)



So RHE would both over and under estimate the area btw −1 and 2. So you couldn't necessarily use RHE and LHE to find the lower and upper bounds.

But wouldn't the actual area still be inbetween the two numbers?