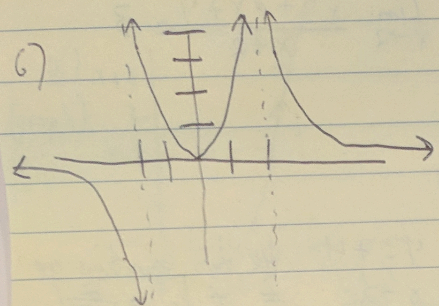
Module 3

Section 2.6

4) a) 2 b) -1 c) $-\infty$ d) $-\infty$ e) ∞ f) $x=0, y=2, y=-1$
$x=2$

6)



15) $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \lim_{x\to\infty} \frac{\frac{3x-2}{x}}{\frac{2x+1}{x}} = \lim_{x\to\infty} \frac{3-\frac{2}{x}}{2+\frac{1}{x}}$

$= \frac{\lim_{x\to\infty} 3 - 2\lim_{x\to\infty} \frac{1}{x}}{\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{1}{x}} = \frac{3-0}{2+0} = \boxed{\frac{3}{2}}$

16) $\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to\infty} \frac{\frac{1-x^2}{x^3}}{\frac{x^3-x+1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x^3}-\frac{1}{x}}{x-\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0}{x} = \boxed{0}$

22) $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}} = \lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{\sqrt{x^4+1}}{x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{x^4+1}{x^4}}}$ since $\sqrt{x^4} = x^{4 \cdot \frac{1}{2}} = x^2$ for $x>0$

$= \frac{1}{\lim_{x\to\infty} \sqrt{1+\frac{1}{x^4}}} = \frac{1}{\sqrt{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x^4}}} = \frac{1}{1+0} = \boxed{1}$

27) ~~$\lim_{x\to\infty} (\sqrt{9x^2+x} - 3x) = \lim_{x\to\infty} 9x^2+x-9x^2 = \lim_{x\to\infty} x = \infty$~~

$\lim_{x\to\infty} (\sqrt{9x^2+x} - 3x) = \lim_{x\to\infty} \sqrt{9x^2+x} - 3x \cdot \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} = \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x} =$

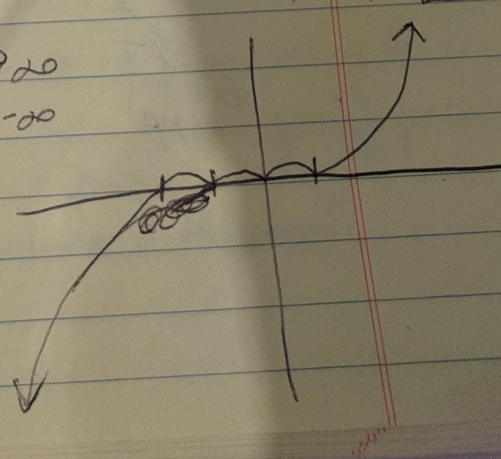
$= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{9x^2+x}+3x}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{9x^2+x}{x^2}}+3}$ since $\sqrt{x^2} = x$ when $x>0$

$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3} = \lim_{x\to\infty} \frac{1}{3+\frac{1}{\sqrt{x}}+3} = \lim_{x\to\infty} \frac{1}{6+\frac{1}{\sqrt{x}}} = \frac{1}{6+0} = \boxed{\frac{1}{6}}$

32) $\lim_{x\to\infty} (e^{-x} + 2\cos 3x) = \lim_{x\to\infty} e^{-x} + \lim_{x\to\infty} 2\cos 3x = 0 +$ DNE because cos oscillates btw -1, and 1 so $2\cos 3x$ oscillates btw -2, and 2 so $\boxed{\text{DNE}}$

64) $y = x^2(x^2-1)^2(x+2)$ when $x\to\infty, y\to\infty$
$x\to-\infty, y\to-\infty$

y int $= y(0) = 0$
x int $x=0, x=-1, x=1, x=-2$
doesnt change sign at $x=1$ $x=-1$



28) a) We want $C(t)$ which is kind of like the position function for velocity problems

Starting amt = 5000L After 1 min, there are 30_g of salt in 5025L

Brine rate = 25 L/min $\frac{30(25)}{5025} = \frac{30}{201}$

Brine concentration = $\frac{30g}{L}$ which is = $\frac{30t}{200+t}$ when $t=1$

Better proof! salt conc = $\frac{30 \cdot 25 \cdot t}{5000 + 25t} = \frac{30 \cdot 25 \cdot t}{5000+25t} = \frac{\frac{30 \cdot 25 \cdot t}{25}}{\frac{5000+25t}{25}} = \frac{30t}{200+t}$ QED

Section 2.7 3i) a) $m = \lim_{x \to 1} \frac{f(x)-f(1)}{x-1} =$

$\frac{4x-x^2-3}{x-1} = \frac{-x^2+4x-3}{x-1} = \frac{-(x+3)\cancel{(x-1)}}{\cancel{x-1}} = -x+3 = \boxed{2}$

3ii) $m = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-(x+h)^2-(4x-x^2)}{h} = \frac{4x+4h-x^2-2xh-h^2-4x+x^2}{h}$

$= \lim_{h\to 0} \frac{4h-2xh-h^2}{h} = \lim_{x\to 0} 4-2x-h = \lim_{x \to 0} 4-2x$

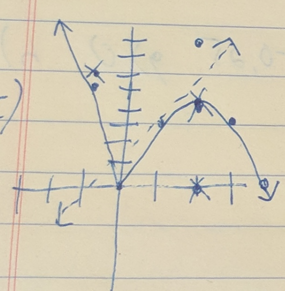
So at pt 1,3, $m = 4-2 = \boxed{2}$

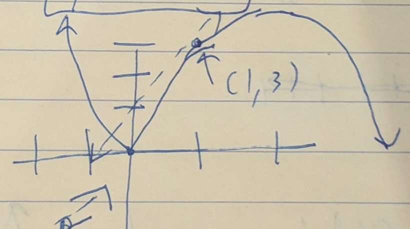
b) tangent line = $y-3 = 2(x-1)$

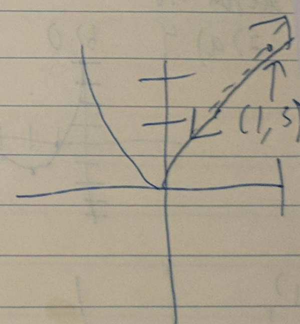
$y-3 = 2x-2$

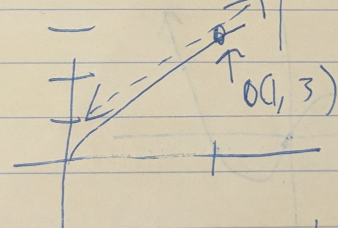
$\boxed{y = 2x+1}$

c)









12) a) A runs at constant speed of approx $\frac{60m}{8sec} = \frac{7.5m}{1sec}$

B accelerates as he/she runs, exponentially inc speed

b) approx 9 sec

c) $t=0$, $t=14$

17) $g'(-2)$: very +++ $g'(0)$: approx -1 $g'(2)$: approx $+1$ $g'(4)$: little +

so $\boxed{g'(0),\ 0,\ g'(4),\ g'(2),\ g'(-2)}$

20) at $x=5$, $y=-3$ $m=4$

$y+3 = 4(x-5)$ $y = 4(x-5)-3$

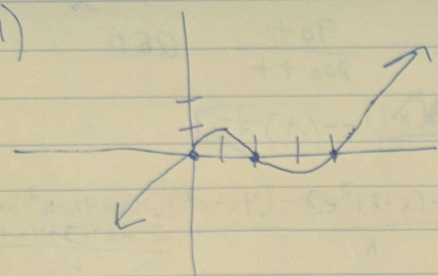
check: $x=5$ $y=-3$ satisfies above equation

22) $m = \frac{3-2}{4-0} = \frac{1}{4}$ so line is: $y-3 = \frac{1}{4}(x-4)$

so $f(4) = 3$

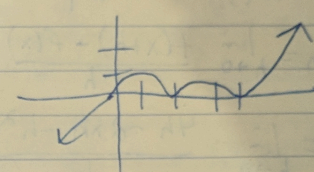
$f'(4) = \frac{1}{4}$

24)



← this is my answer, but can the graph look like this?

↓

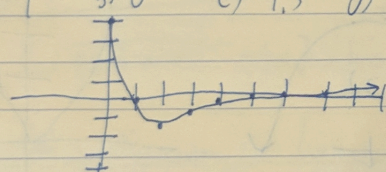


37) $f(x) = \sqrt{x}$ at $a=3$

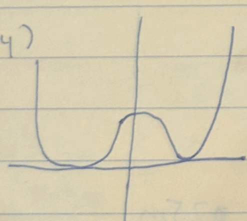
41) $f(x) = \cos a$ at $a = \pi$

Section 2.8

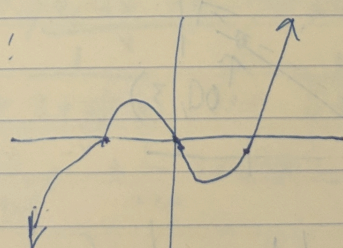
2) a) 4 b) 0 c) -1.5 d) -1 e) -0.5 f) -0.25 g) 0 h) 0.25



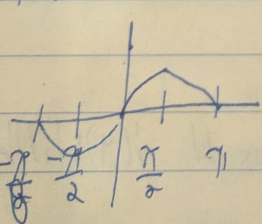
4)

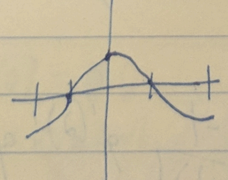


$f'(x)$:



16) $f(x) = \sin x$ $f'(x) = \cos x$





22) $f'(x) = \lim_{h \to 0} \frac{m(x+h)+b-mx-b}{h} =$

$\lim_{h \to 0} \frac{mx+mh+b-mx-b}{h} =$

$\lim_{h \to 0} \frac{mh}{h} = \boxed{m}$

20) at x=5, y=-3, m=4
y=4(x-5)-3

26) $g(t)=\frac{1}{\sqrt{t}}$ $\quad g'(t)=\lim_{h\to 0}\frac{\frac{1}{\sqrt{t+h}}-\frac{1}{\sqrt{t}}}{h} = g'(t)=\lim_{h\to 0}\frac{\frac{\sqrt{t}}{\sqrt{t}\sqrt{t+h}}-\frac{\sqrt{t+h}}{\sqrt{t+h}\sqrt{t}}}{h} =$

$\lim_{h\to 0}\frac{\sqrt{t}-\sqrt{t+h}}{h\cdot\sqrt{t}\cdot\sqrt{t+h}} = \lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0}\frac{x-x+h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

$=\lim_{h\to 0}\frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$ $\lim_{h\to 0}\frac{-1}{\sqrt{x}\cdot\sqrt{x}\cdot\sqrt{x}+\sqrt{x}} = \lim_{h\to 0}\frac{-1}{x\cdot 2\sqrt{x}} = \boxed{\lim_{h\to 0}\frac{-1}{2x^{3/2}}}$

42) Not differentiable at $x=2$, a corner, because it has no tangent line. Theorum 4 logically has a second implication; If f is not continuous at a, then f is not differentiable at a. Even though corner's are continuous, it has two diff one sided limits, so no tangent. Also, not continuous at $x=-1$, so it isnot differentiable at $x=-1$