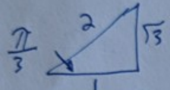
Module 3
7.6

1) $\int_0^{\pi/2} \cos 5x \cos 2x \, dx$ entry 80: $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

so $= \frac{\sin(3x)}{6} + \frac{\sin(7x)}{14} \Big]_0^{\pi/2} = \frac{-1}{6} + \frac{-1}{14} = \frac{-7}{42} + \frac{-3}{42} = \frac{-10}{42} = \boxed{\frac{-5}{21}}$

2) $\int_0^1 \sqrt{x - x^2} \, dx$ entry 113 $\int \sqrt{2au - u^2} du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

so $2a = 1$ $u = x$
$a = \frac{1}{2}$



$\frac{1}{4}\sqrt{x-x^2} + \frac{1}{8}\cos^{-1}\left(\frac{-1}{1}\right) - \left(\frac{1}{4}\right)\sqrt{x-x^2} + \frac{1}{8}\cos^{-1}(1)$

$\frac{1}{8}\pi - \frac{1}{8}0 = \boxed{\frac{\pi}{8}}$

so... $\frac{x-\frac{1}{2}}{2}\sqrt{x-x^2} + \frac{1}{8}\cos^{-1}\left(\frac{\frac{1}{2}-x}{\frac{1}{2}}\right)\Big]_0^1$

6) $\int_0^2 x^2\sqrt{4-x^2}\,dx$

$x = 2\sin u$
$dx = 2\cos u \, du$
when $x = 2$, $u = \frac{\pi}{2}$

$= \int_0^{\pi/2} 4\sin^2 u \cdot 2\cos u \cdot 2\cos u \, du$

because when $x = 2\sin u$
$\sqrt{4-x^2} = \sqrt{4 - 4\sin^2 u} = \sqrt{4(1-\sin^2 u)} = \sqrt{4\cos^2 u} = 2\cos u$

entry 86!

$= \int_0^{\pi/2} 16(\sin u \cos u)^2 \, du$ $\sin u \cos u = \frac{\sin 2u}{2}$

$= \int_0^{\pi/2} 4\sin^2(2u)\,du$ $t = 2u$ $dt = 2du$ so $\int_0^{\pi} 2\sin^2 t \, dt = \int_0^{\pi}(1 - \cos(2t))dt$

$= t - \frac{\sin 2t}{2}\Big]_0^{\pi} = \boxed{\pi}$

entry 86? on back

check = entry 86

7) $\cancel{\int \frac{e^x}{4-e^{2x}}dx}$ entry 86 looks at $\sin^n u \cos^m u$, lets evaluate: ~~...~~

$\int_0^{\pi/2} 16(\sin u \cos u)^2 du$ $= -\frac{\sin u \cos^3 u}{4} + \frac{1}{4}\int \sin^0 u \cos^2 u \, du \Big]_0^{\pi/2}$

$n=2$ entry 86
$m=2$

$\frac{1}{4}\int \cos^2$

$= \cancel{\frac{0\sin u\cos^3 u}{4} + \frac{4}{4} + \frac{\sin^3 u}{3}\Big]_0^{\pi/2}} \cancel{= 0 + \frac{1}{12} + (0 + 0)}$

$= -\frac{\sin u \cos^3 u}{4} + \frac{1}{4}\frac{1}{2}\int(1+\cos 2\alpha)\,dx$

$= \frac{-\sin u \cos^3 u}{4} + \frac{1}{8}\int(\alpha + \frac{1}{2}\sin(2\alpha))\,dx$

so $\int_0^{\pi/2} \sin^2 u \cos^2 u = \frac{-\sin u \cos^3 u}{4} + \frac{1}{8}u + \frac{1}{16}\sin 2u \Big]_0^{\pi/2} = 0 + \frac{\pi}{16} + 0 = \frac{\pi}{16}$

so $16\int_0^{\pi/2} \sin^2 u \cos^2 u = \boxed{\pi}$

8) $\int \frac{e^x}{4-e^{2x}}dx$ $u = e^x$, $du = e^x dx$ so $\int \frac{du}{4-u^2}$ entry 19 $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

$a=2$

$= \boxed{\frac{1}{4}\ln\left|\frac{e^x+2}{e^x-2}\right| + C}$

9) $\int \frac{\sqrt{9x^2+4}}{x^2}dx$ entry 24

$= 3\int \frac{\sqrt{x^2+\frac{4}{9}}}{x^2}$ $= \boxed{3\left(-\frac{\sqrt{\frac{4}{9}+x^2}}{x} + \ln\left(x + \sqrt{\frac{4}{9}+x^2}\right)\right) + C}$

$u^2 = x^2$
$u = x$
$a^2 = \frac{4}{9}$
$a = \frac{2}{3}$

$3x, \ldots$

11) $\int_0^{\pi} \cos^6\theta \, d\theta$ entry 74 $= \frac{1}{6}\cos^5 u \sin u + \frac{5}{6}\int \cos^4 u \, du$

and $\frac{5}{6}\int \cos^4 u \, du = \frac{5}{6}\left(\frac{1}{4}\cos^3 u \sin u + \frac{3}{4}\int \cos^2 u \, du\right)$ entry 64

and $\frac{3}{4}\int \cos^2 u \, du = \frac{u}{2} + \frac{1}{4}\sin 2u + C$

so $= \frac{1}{6}\cos^5 u \sin u + \frac{5}{6}\left(\frac{1}{4}\cos^3 u \sin u + \frac{3}{4}\left(\frac{u}{2} + \frac{1}{4}\sin 2u\right)\right)\Big]_0^{\pi} =$

$= 0 + \frac{5}{6}\left(0 + \frac{3\pi}{8}\right) - \left(0 + \frac{5}{6}(0+0)\right) = \frac{15\pi}{48} = \boxed{\frac{5}{16}\pi}$

14) $\int_0^{\pi} x^3 \sin x\,dx$ entry 84 $\quad -x^3\cos x + 3\int x^2 \cos x\,dx\,dx$

$n=3$

and $3\int x^2 \cos x\,dx$ is entry 85 so $= x^2\cos x\,dx = x^2\sin x - 2\int x\sin x\,dx$

$n=2$

$2\int x\sin x\,dx$ is entry 82 $= \sin x - x\cos x$

so $= -x^3\cos x + 3(x^2 \sin x - 2(\sin x - x\cos x))\Big]_0^{\pi}$

$+\pi^3 \quad +3(\; 0 \quad - (2\pi \quad)) - (0 + (0 \; (\quad)$

$+\pi^3 - 6\pi = \boxed{\pi(\pi^2-6}$

23) $\int \sec^5 x\,dx$ entry 77 $= \frac{1}{4}\tan x\sec^3 x + \frac{3}{4}\int \sec^3 x\,dx$

$n=5$

$\int \sec^3 x\,dx$ is entry 71 $= \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|$

so $\boxed{\frac{1}{4}\tan x\sec^3 x + \frac{3}{4}\left(\frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|\right) + C}$

25) $\int \frac{\sqrt{4+(\ln x)^2}}{x}\,dx$ $\quad u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ $\quad du = \frac{dx}{x}$ $\quad$ so $\int \sqrt{4+u^2}\,du$ $\quad$ entry 21

$a=2$

$= \boxed{\frac{\ln x}{2}\sqrt{4+(\ln x)^2} + \frac{4}{2}\ln(\ln x + \sqrt{4+(\ln x)^2} + C}$

26) $\int_0^1 x^4 e^{-x}\,dx$ entry 97 so

$a=-1$

$u=x$

$n=4$

$= \frac{1}{-1}x^4e^{-x} - \frac{4}{-1}\int x^3 e^{-x}\,du + C$

$= -1(x^4)e^{-x} - -4\int x^3 e^{-x}\,du$

and $\int x^3 e^{-x}\,dx = -1x^3e^{-x} - -3\int x^2 e^{-x}$

and $\int x^2 e^{-x}\,dx = -1x^2e^{-x} - -2\int x^1 e^{-x}$

and $\int x^1 e^{-x}\,dx = -1x^1e^{-x} - -1\int e^{-x}$ $\quad$ and $\int e^{-x} = -e^{-x}$

so $-x^4e^{-x} + 4(-x^3e^{-x} + 3(-x^2e^{-x} + 2(-xe^{-x} - e^{-x})))\Big]_0^1$

$\frac{-x^4}{e} + 4\left(\frac{-x^3}{e} + 3\left(\frac{-x^2}{e} + 2\left(\frac{-x}{e} - \frac{1}{e}\right)\right]_0^1$ $\quad \frac{-65}{e} \; 24$

$= \boxed{\frac{-65}{e} + 24}$

7.7 | 7) $\int_1^2 \sqrt{x^3-1}\,dx$ $n=10$ $\Delta x = \frac{2-1}{10} = \frac{1}{10}$

7a) $\frac{1}{20}\left(f(1.0) + 2(f(1.1)) + 2(f(1.2)) + \dots f(2.0)\right)$

$\frac{1}{20}(0 + 1.150651 + 1.706458 + 2.188149 + 2.641211 + 3.082207 + 3.519090 + 3.956260 + 4.396362 + 4.841074 + 2.645751)$

$= \boxed{1.506360}$

7b) $\frac{1}{10}\left(f(1.05) + f(1.15) \dots f(1.95)\right) = \frac{1}{10}(0.397020 + 0.721716 + 0.976281 + 1.208459 + 1.431301 + 1.650416 + 1.868722 + 2.087911 + 2.309031 + 2.532760) = \boxed{1.518361}$

7c) $\frac{1}{30}\left(f(1) + 4(f(1.1)) + 2(f(1.2)) + 4(f(1.3)) + \dots 2(f(1.4)) + 4f(1.5) + 2f(1.6) + 4f(1.7) + 2f(1.8) + 4f(1.9)\right.$

$f(2.0) = \boxed{1.51151}$

9a) $\int_0^2 \frac{e^x}{1+x^2}\,dx$ $h=10$ $\Delta x = \frac{2}{10}$

trap $= \left(f(0) + 2f(0.2) + 2f(0.4) + 2f(0.6) \dots f(2)\right) = \boxed{2.660833}$

$\frac{1}{10}x = \frac{\Delta x}{2}$

9b) midpoint $= \frac{1}{5}\left(f(0.1) + f(0.3) + f(0.5) + \dots f(1.9)\right) = \boxed{2.664377}$

Simpsons $= \frac{1}{15}\left(f(0) + 4f(0.2) + 2(f(0.4)) + 4f(0.6) \dots f(2.0)\right) = \boxed{2.663244}$

22) $E_s \le \frac{K(b-a)^5}{180n^3}$ $K \ge f^{(4)}(x)$ $\int_0^1 e^{x^2}\,dx$ what is n so accurate to within 0.00001?

$f' = 2xe^{x^2}$ $u = x^2$ $\frac{du}{dx} = 2x$

$f'' = 2x(2xe^{x^2}) + e^{x^2}\cdot 2 = 4x^2e^{x^2} + 2e^{x^2} = e^{x^2}(4x^2+2)$

$f''' = e^{x^2}\cdot 8x + (4x^2+2)\cdot 2xe^{x^2} = 8xe^{x^2} + 8x^3e^{x^2} + 4xe^{x^2} = e^{x^2}(\overbrace{8x + 8x^3 + 4x}^{12x})$

$f^4 = e^{x^2}(24x^2+12) + (12x+8x^3)2xe^{x^2} = e^{x^2}(24x^2+12) + (24x^2+16x^4)e^{x^2}$

$= e^{x^2}(16x^4 + 48x^2 + 12)$

$f^4 = 76e$ when $x=1$ so $f^4 \le 76e$

$f^4 = 12$ when $x=0$

so for error less than 0.00001, $\frac{76e(1)^5}{180(n)^4} < 0.0001 \rightarrow$ 0.018

$\frac{76e1^5}{n^4} < 0.018 \rightarrow$ ← n

$11477.1899 < n^4$ $n \approx 10.35044$

so needs to be $\ge \boxed{12}$

56) $f(x_0)=0 \quad \Delta x = 2$

$\frac{2}{3}\left(0+4(6.2)+2(7.2)+4(6.8)+2(5.6)+4(5.0)+2(4.8)+4(4.8)+0\right)$

$=\boxed{84.266}$

module 7.8

5) $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty} \int_3^t \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty} -2(x-2)^{-\frac{1}{2}}\Big]_3^t$

say $u = x-2 \quad du = dx$

so $\int \frac{1}{u^{3/2}}\,du = \int u^{-3/2}$

$= \frac{u^{-1/2}}{-\frac{1}{2}} = -2u^{-1/2} = -2(x-2)^{-\frac{1}{2}}$

~~diverges because~~

$\lim_{t\to\infty} -2(t-2)^{-\frac{1}{2}} - (-2(1)^{-\frac{1}{2}})$

$0 \quad - -2$

$\lim_{t\to\infty} -2(t-2)^{-\frac{1}{2}} = \frac{0-2}{(t-2)^{\frac{1}{2}}} = 0$

so $\int_3^{\infty} \frac{1}{(x-2)^{3/2}} = 0 + 2 = \boxed{2}$

6) $\int_0^{\infty} \frac{1}{\sqrt[4]{1+x}}\,dx = \int_0^{\infty} (1+x)^{-\frac{1}{4}}$ $\quad u = 1+x,\ du = dx \quad$ so $\int 1+x^{-\frac{1}{4}}dx = \int u^{-\frac{1}{4}}\,du = \frac{4}{3}u^{\frac{3}{4}}$

$= \frac{4}{3}(1+x)^{\frac{3}{4}}$

$= \frac{4}{3}(1+x)^{\frac{3}{4}}\Big]_0^{\infty} = \infty - \frac{4}{3} = \infty \quad \boxed{\text{diverges}}$

8) $\int_1^{\infty} \frac{1}{(2x+1)^3} = \int_1^{\infty} (2x+1)^{-3} \quad u = 2x+1,\ du = 2dx$

$= \int_1^{\infty} \frac{1}{2}u^{-3}\,du = \lim_{t\to\infty} \frac{1}{4}u^{-2}\Big]_1^t = \lim_{t\to\infty} \frac{-1}{4}(2x+1)^{-2}\Big]_1^t = 0 - \frac{-1}{4\cdot 9} = \frac{1}{36}$

$= \lim_{t\to\infty} \int_1^t \frac{1}{2}u^{-3}\,du$

$\boxed{+\frac{1}{36}}$

9) $\int_2^{\infty} e^{-5p}\,dp \quad$ let $u = 5p$, $du = 5dp$

$= \int_2^{\infty} \frac{1}{5}e^{-u}\,du = \lim_{t\to\infty} \frac{-1}{5}e^{-u}\Big]_2^t$

$z = -u$
$dz = -1du \quad$ so $\int \frac{1}{5}e^z\,dz = -\frac{1}{5}e^z\,dz = \frac{-1}{5}e^{-u}\,du$

$\lim_{t\to\infty} \frac{-1}{5e^{5p}}\Big]_2^t = \lim_{t\to\infty} \frac{-1}{5e^{5p}} - \frac{-1}{5e^{10}}\Big]_2^t$

$= 0 - \frac{-1}{5e^{10}} = \boxed{\frac{1}{5e^{10}}}$

13) $\int_{-\infty}^{\infty} xe^{-x^2}dx = \int_{-\infty}^{0} xe^{-x^2} + \int_{0}^{\infty} xe^{-x^2}$

so let's find $\int xe^{-x^2}$ let $u = x$ $du = dx$

maybe let $dv = e^{-x^2}$

IBP not working, is there a IBP solution?

$\int xe^{-x^2}dx$ $\quad u = x^2$

$du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$-\frac{1}{2}\int e^{-u}du =$

let $z = -u$

$dz = -du$

$= -\frac{1}{2}\int e^{z}dz = \frac{-1}{2}e^{z}$ so $\frac{-1}{2e^{-u}}$ so $\frac{-1}{2e}^{-x^2} + C$ (antiderivative)

so $\int_{-\infty}^{\infty} xe^{-x^2}dx = \lim_{t\to-\infty} \frac{-1}{2e}^{-x^2}\Big]_t^0 + \lim_{t\to\infty} \frac{-1}{2e}^{-x^2}\Big]_0^t$

left side: $\left(\frac{-1}{2} - 0\right) + \left(0 - \frac{-1}{2}\right) = \boxed{0}$

16) $\int_0^{\infty} \sin\theta\, e^{\cos\theta}$ $\quad u = \cos\theta$ $\quad du = -\sin\theta\, d\theta$

$= -\int_0^{\infty} e^{u} = \lim_{t\to\infty} e^{u}\Big]_0^t$ ~~diverges~~

$= \lim_{t\to\infty} e^{\cos\theta}\Big]_0^t = \quad + e$

oscillates so $\boxed{\text{diverges}}$

21) $\int_1^{\infty} \frac{\ln x}{x}dx$

$= \int_1^{\infty} \ln x\, x^{-1}$

IBP $u = \ln x$ $\quad dv = \frac{1}{x}$

$du = \frac{1}{x}dx$ $\quad v = \ln x$

$\ln x^2 - \int \frac{\ln x}{x} = \int \frac{\ln x}{x}$ so $2\int \frac{\ln x}{x}dx = (\ln x)^2$

$\int \frac{\ln x}{x}dx = \frac{(\ln x)^2}{2}$

also u-sub:

$u = \ln x$

$du = \frac{1}{x}dx$

$\int_1^{\infty} u\,du = \lim_{t\to\infty} \frac{u^2}{2}\Big]_1^{\infty} = \lim_{t\to\infty} \frac{u^2}{2}\Big]_1^{\infty}$ $\rightarrow$ ~~diverges~~ $= \lim_{t\to\infty} \frac{\ln x^2}{2}\Big]_1^{\infty}$

$= \lim_{t\to\infty} \frac{\ln x^2}{2} - \frac{\ln 1}{2} \rightarrow \boxed{\text{diverges}}$

27) $\int_0^1 \frac{1}{x}dx = \lim_{t\to 0^+} \ln x\Big]_0^1 = \ln 1 - \ln 0 = 0 - -\infty$

diverges

29) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} =$

what is $\int \frac{dx}{\sqrt[4]{x+2}}$ $\quad u = x+2$ so $du = dx$

$= \int u^{-\frac{1}{4}} = \frac{4}{3}u^{3/4} = \frac{4}{3}(x+2)^{3/4}$

$\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \lim_{t\to -2^+} \frac{4}{3}(x+2)^{3/4}\Big]_{-2}^{14} = \lim_{t\to-2^+} \frac{4}{3}(16)^{3/4} - \lim_{t\to-2^+} \frac{4}{3}(x+2)^{3/4}$

$\boxed{\frac{32}{3}}$

34) $\int_0^5 \frac{w}{w-2}\,dw = \int_0^2 \frac{w}{w-2}\,dw + \int_2^5 \frac{w}{w-2}\,dw$

left side $= \lim_{t\to 2^-} \int_0^t \frac{w}{w-2} = \lim_{t\to 2^-} \int_0^t \frac{(w-2)+2}{w-2} = \lim_{t\to 2^-} \int_0^t 1 + \frac{2}{w-2}\,dw =$

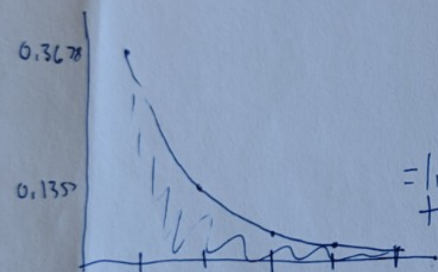
$\lim_{t\to 2^-} \int_0^t 1\,dw + \lim_{t\to 2^-} \int_0^t \frac{2}{w-2}\,dw \leftarrow$ eval this side $\quad u = w-2 \quad du = dw \quad \lim_{t\to 2^-} 2\int_{-2}^{t-0} u^{-1}\,du$

$= \lim_{t\to 2^-} 2\ln|u|\Big]_{-2}^{0} = \lim_{t\to 2^-} 2\ln|w-2|\Big]_0^2 = \lim_{t\to 2^-} 2(\ln(2-w) - \ln 2)$

$\lim_{t\to 2^-} w\Big]_0^t +$

$= -\infty$

41)



$\int_1^\infty e^{-x}\,dx \quad u=-x \quad du=-dx \quad -du=dx$

$= \lim_{t\to\infty} -\int_{-1}^{-t} e^u\,du = \lim_{t\to\infty} e^u\Big]_{-1}^{-t} = \lim_{t\to\infty} -e^{-x}\Big]_1^{t}$

$= \lim_{t\to\infty} -e^{-t} - \lim_{t\to\infty} -e^{-1}$

$0 - \frac{-1}{e} = \frac{1}{e} = \boxed{e^{-1}}$