Lesson 11

1) - say $v=(-2,-2)$ and $v=(2,2)$ $\quad T: R^2 \to R^2 \quad T(x,y)=(x+y,\ xy)$

$T(v)=(-4,\ 4)$ $\quad T(v)=(4,4)$ $\quad T(v)+T(v)=0,8$

$T(v+v)=T(0,0)=(0,0)$ $\quad$ so $\underline{T(v)+T(v) \neq T(v+v)}$

- example 2: Say $a=(2,-2)$ $\quad b=(1,1)$

$T(a)=(0,-4)$ $\quad T(b)=(2,1)$ $\quad T(a)+T(b)=(2,-3)$

$T(a+b)=T(3,-1)=(2,-3)$ eh, this works, oops

- scalar multiplication? let's try $c=(0,3)$ and $s=2$

$T(sc)=T(0,6)=(6,0)$

$sT(c)=2(3,0)=(6,0)$ $\to$ dammit

what about $c=(-1,3)$ and $s=-1$

$T(sc)=T(1,-3)=(-2,-3)$

$sT(c)=-1(2,-3)=(-2,3)$ oh hah!

so $\underline{T(sc) \neq sT(c)}$

2) $T(1,2)=\ 2(T(2,1))-3(T(1,0))$

$=2(0,1,4)-3(-1,1,2)=(0,2,8)-(-3,3,6)=\underline{(3,-1,2)}$

3) $A=\begin{bmatrix} 2 & -1 & 0 & 1 \\ 1 & 2 & 1 & -3 \end{bmatrix}$ find v such that $T(v)=Av=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 1 & 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$2v_1 - v_2 + v_4 = 1$

$v_1 + 2v_2 + v_3 - 3v_4 = 2$

$$\begin{bmatrix} 1 & 2 & 1 & -3 & 2 \\ 2 & -1 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & -3 & 2 \\ 0 & -5 & -2 & 7 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & -3 & 2 \\ 0 & 1 & \frac{2}{5} & -\frac{7}{5} & \frac{3}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{5} & -\frac{1}{5} & \frac{4}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{7}{5} & \frac{3}{5} \end{bmatrix}$$

$v_4 = t$

$v_3 = s$

$v_2 = -\frac{2}{5}s + \frac{7}{5}t - \frac{3}{5}$

$v_1 = -\frac{1}{5}s + \frac{1}{5}t + -\frac{4}{5}$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{matrix} -\frac{1}{5}s + \frac{1}{5}t + -\frac{4}{5} \\ -\frac{2}{5}s + \frac{7}{5}t + -\frac{3}{5} \\ s + 0 + 0 \\ 0 + t + 0 \end{matrix}$$

$$v = s\begin{bmatrix} -\frac{1}{5} \\ -\frac{2}{5} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{1}{5} \\ \frac{7}{5} \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \\ 0 \\ 0 \end{bmatrix}$$

LI?

4) $T(x,y,z) = (2x+y-z,\ x+z,\ -x+y-4z) = (0,0,0)$

$$\begin{aligned} 2x+y-z &= 0 \\ x+z &= 0 \\ -x+y-4z &= 0 \end{aligned}$$

solve $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ -1 & 1 & -4 \end{bmatrix} \to$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

Let $z = t$
$y = 3t$
$x = -t$

kernel $= t\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ so not one-to-one

5) $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 0 \\ 1 & 7 & -6 \end{bmatrix}$

a) find a basis for kernel of T

$\to \begin{bmatrix} 1 & 2 & -2 \\ 0 & -5 & 4 \\ 0 & 5 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -2 \\ 0 & -5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \to \cancel{x_2} = \left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

let $x_3 = t$
$x_2 = \frac{4}{5}t$
$x_1 = 2t - \frac{8}{5}t$
$= \frac{2}{5}t$

basis $= \cancel{\phi} \begin{bmatrix} 1 \\ \frac{4}{5} \\ \frac{2}{5} \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{4}{5} \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -\frac{2}{5} \\ 0 & 1 & -\frac{4}{5} \\ 0 & 0 & 0 \end{bmatrix}$

$\cancel{y_1}\ y_2$

y_1, y_2 are linearly independent

so basis for range$(T) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}$

6) equations represent this matrix:

$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ It's 3×2 which means it represents a transformation from $R^3 \to R^2$

rank T: $\to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -\frac{1}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{3}{5} \\ 0 & 1 & -\frac{1}{5} \end{bmatrix}$

rank 2
nullity $= 1$