4) $\sum_{n=1}^{\infty} n^{-0.3}$ so use $\int_1^{\infty} n^{-3/10}\,dn$ positive and decreasing ✓

$\lim_{t\to\infty} \frac{10}{7} n^{7/10}\Big]_1^t = \lim_{t\to\infty} \frac{10}{7} t^{7/10} - \frac{10}{7} = \infty$ so $\sum_{n=1}^{\infty} n^{-0.3}$ $\boxed{\text{diverges}}$

5) $\sum_{n=1}^{\infty} \frac{2}{5n-1}$ so use $\int_1^{\infty} \frac{2}{5n-1}\,dn$ positive and decreasing

$\lim_{t\to\infty} \int_1^t \frac{2}{5n-1} = \lim_{t\to\infty} 2\int_1^t \frac{1}{5n-1}\,dn$ $\quad u = 5n-1$, $du = 5\,dn$ $\quad \boxed{\text{diverges}}$

$\lim_{u\to\infty} \frac{2}{5}\int_4^{u} \frac{1}{u}\,du = \frac{2}{5}\ln u\Big]_4^u$

6) $\sum_{n=1}^{\infty} \frac{1}{(3n-1)^4}$ so use $\int_1^{\infty} \frac{1}{(3n-1)^4}\,dn$ positive and decreasing $\quad \lim_{t\to\infty}\int_1^t \frac{1}{(3n-1)^4}\,dn$ $\quad u = 3n-1$, $du = 3\,dn$

$\lim_{t\to\infty} \frac{1}{3}\int_2^t \frac{1}{u^4}\,du = \lim_{t\to\infty} \frac{1}{3}\int_2^t u^{-4}\,du = \lim_{t\to\infty} \frac{-1}{9} u^{-3}\Big]_2^t = \frac{1}{72}$ $\boxed{\text{Converges}}$

7) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ so use $\int_1^{\infty} \frac{n}{n^2+1}\,dn$ positive and decreasing

$\lim_{t\to\infty} \int_1^t \frac{n}{n^2+1}\,dn$ $\quad u = n^2+1$, $du = 2n\,dn$

$\lim_{t\to\infty} \frac{1}{2}\int_2^t \frac{1}{u}\,du = \lim_{t\to\infty} \frac{1}{2}\ln u\Big]_2^t du = \lim_{t\to\infty} \frac{1}{2}(\ln t - \ln 2) = \infty$ $\boxed{\text{diverges}}$

10) $\sum_{n=3}^{\infty} n^{-0.9999} = \sum_{n=3}^{\infty} \frac{1}{n^{0.9999}}$ $\quad 0.9999 < 1$ p series so divergent. makes sense because it is > harmonic series.

check using integral test:

use $\int_3^{\infty} n^{-0.9999}$ so $\lim_{t\to\infty}\int_3^t n^{0.0001} = \lim_{t\to\infty} \frac{1}{1.0001} n^{1.0001}\Big]_3^t = \lim_{t\to\infty} \frac{1}{1.0001}\left(n^{1.0001} - 3^{1.0001}\right)$

$= \infty$

$\boxed{\text{diverges}}$ ✓

12) $\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11},\dots$

$a_n = \frac{1}{5+(2n-2)} = \frac{1}{3+2n}$ so $\sum_{n=1}^{\infty} \frac{1}{3+2n}$ positive and decreasing ✓

so $\int_1^{\infty} \frac{1}{3+2n}\,dn \to \lim_{t\to\infty}\int_1^t \frac{1}{3+2n}\,dn$ $\quad u = 3+2n$, $du = 2\,dn$

$= \frac{1}{2}\lim_{t\to\infty}\int_5^t \frac{1}{u}\,du = \frac{1}{2}\lim_{t\to\infty} \ln u\Big]_5^t = \frac{1}{2}\lim_{t\to\infty} \ln t - \ln 5 \Rightarrow \infty$ $\boxed{\text{diverges}}$

17) $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ this is $< \frac{1}{n^2}$ which is a p series with $p > 1$ so $\boxed{\text{converges}}$

~~check w/ Integral Test too:~~

~~positive and decreasing~~ $\cancel{\int_1^{\infty} \frac{1}{n^2+4}\,dn = \lim_{t\to\infty}\int_1^t \frac{1}{n^2+4}\,dn}$ $\quad \cancel{u = n^2+4}$ $\quad \cancel{du = 2n\,dn}$

$\cancel{= \lim_{t\to\infty} \frac{1}{2}\int_1^t \frac{1}{u}\,du}$

22) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ so positive and decreasing so use $\int_2^{\infty} \frac{\ln n}{n^2} = \lim_{t\to\infty} \int_2^t \frac{\ln n}{n^2}\,dn =$ $u = \ln n$, $du = \frac{1}{n}\,dn$

IBP?
$u = \ln n$ $dv = n^{-2}$
$du = \frac{1}{n}\,dn$ $v = -n^{-1}$

$= \lim_{t\to\infty}$. usub doesn't work

$= \lim_{t\to\infty} -\frac{\ln n}{n}\Big]_2^{\infty} - \int_2^t -n^{-2}$

$= \frac{\ln 2}{2}$ $+ \lim_{t\to\infty} \frac{n^{-1}}{}\Big]_2^t$

$+ \frac{1}{2}$

$= \frac{\ln 2 + 1}{2}$ so $\boxed{\text{Converges}}$

36) a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $S_{10} = \frac{1}{1} + \frac{1}{16} + \dots \approx \boxed{1.0820}$

$R_{10} \le \int_{10}^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \int_{10}^t \frac{1}{x^4}\,dx = \lim_{t\to\infty} -\frac{1}{3x^3}\Big]_{10}^t = \lim_{t\to\infty} -\frac{1}{3t^3} + \frac{1}{3\cdot 10^3} = \frac{1}{3\cdot 10^3} = \frac{1}{3000}$

size of error is at most $\boxed{\frac{1}{3000}}$

b) $S_n + \int_{n+1}^{\infty} f(x)\,dx \le S \le S_n + \int_n^{\infty} f(x)\,dx$

$S_{10} = 1.082$

$\int_{11}^{\infty} \frac{1}{n^4} = \frac{1}{1331\cdot 3}$

$1.08225 \le S \le 1.082 + \frac{1}{3000} \approx 1.08233$

exact $= 1.0823232371$

new estimate $= \frac{1.08225 + 1.08233}{2} = \boxed{1.08229}$

c) new estimate is greater 1.08229 vs old $= 1.082 = S_{10}$

d) formula for error $= \frac{1}{3n^3}$

so $\frac{1}{3n^3} < 0.00001$

$\frac{1}{0.00003} < n^3$ $n \approx 32.18$ so $\boxed{n > 33}$

11.4) 3) $\sum_{n=1}^{\infty} \frac{1}{n^3+8}$ smaller than $\frac{1}{n^3}$ convergent

7) $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ look at dominant parts $b_n = \frac{9^n}{10^n} = \frac{9}{10}^n = \frac{9}{10}\cdot\frac{9}{10}^{n-1}$ geometric w $r = \frac{9}{10}$

Limit Comparison $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{9^n/3+10^n}{9^n/10^n} = \lim_{n\to\infty} \frac{10^n}{10^n+3} = 1$

so $\boxed{\text{convergent too}}$

8) $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$ this is ~~greater~~ than $\lim_{n\to\infty} \frac{6^n}{5^n} = \lim_{n\to\infty} \frac{6}{5}\cdot\frac{6}{5}^{n-1}$ $r=\frac{6}{5}$ so diverges

so this diverges too

10) $\sum_{k=1}^{\infty} \frac{k\sin^2 k}{1+k^3}$ $b=\frac{k}{1+k^3}$ which is $<\frac{1}{k^2}$ so b converges by Comparison test

$a=\frac{k\sin^2 k}{1+k^3}$ is $\le$ b because $\sin^2 k$ is always ≤ 1

so $\boxed{\text{a converges}}$

12) $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2} = \sum_{k=1}^{\infty} \frac{2k^3-2k-k^2+1}{(k+1)(k^4+8k^2+16)} = \sum_{k=1}^{\infty} \frac{2k^3-k^2-2k+1}{k^5+8k^3+16k+k^4+8k^2+16} = a$

$b=\frac{1}{k^2}$ and b converges

a has smaller numerator and bigger denom so $\boxed{\text{a converges too}}$

14) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}} = \sum_{n=1}^{\infty} (3n^4+1)^{-\frac{1}{3}} = \cancel{\int^{\infty} (3n^4+1)^{-\frac{1}{3}}\,dn}$ $\cancel{u=3n^4+1}$ $\cancel{du=12n^3\,dn}$ $b=\frac{1}{n^{4/3}}$ p series where $r=\frac{4}{3}$ $\boxed{\text{so converges}}$

$a<\frac{1}{n^{4/3}}$ so $\boxed{\text{Converges}}$

17) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ $b=\frac{1}{\sqrt{n^2}}=\frac{1}{n}$ $\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n^2}}} = 1 > 0$

$\boxed{\text{so diverges}}$

18) $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2} = 2\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+2} = a$ compare to $b=\frac{1}{n^{\frac{1}{2}}}$ p series p ~~<1~~ so ~~converges~~ diverges

~~$b<\frac{1}{n^2}$ so converges too.~~ can't just compare because ~~a>b~~ a<b

have to use Limit Comparison Test

$\lim_{n\to\infty} \frac{2}{\sqrt{n}+2}\cdot\frac{\sqrt{n}}{1} = \lim_{n\to\infty}\frac{2\sqrt{n}}{\sqrt{n}+2} = \lim_{n\to\infty} 2\,\frac{1}{1+\frac{2}{\sqrt{n}}} = 2 > 0$ so $\boxed{\text{a diverges too}}$

24) $\sum_{n=1}^{\infty} \frac{n+3^n}{n+2^n}$ compare to $b=\frac{3}{2}^n = \frac{3}{2}\cdot\frac{3}{2}^{n-1}$ geometric with $r=\frac{3}{2}$ so b diverges

~~sum = 3/2~~

divide by 2^n

$\sum_{n=1}^{\infty} \frac{1+\frac{3^n}{n}}{1+\frac{2^n}{n}}$

diverges, numerator is bigger so use Limit Comparison Test:

$\lim_{n\to\infty} \frac{n+3^n}{n+2^n}\cdot\left(\frac{2}{3}\right)^n = \lim_{n\to\infty}\frac{2^n(n+3^n)}{3^n(n+2^n)} = \lim_{n\to\infty}\frac{\frac{n+3^n}{3^n}}{\frac{n+2^n}{2^n}} = 1$

so $\boxed{\text{a diverges too}}$ (~~converges~~)

11.5)

4) $\frac{1}{\ln 3} - \frac{1}{\ln 4} + \frac{1}{\ln 5} - \frac{1}{\ln 6} \ldots$

$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\ln(n+2)}$ $\quad b = \frac{1}{\ln(n+1)}$

i) $b_{n+1} \leq b_n$ for all n ✓
can be equal!

ii) $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ ✓

$\boxed{\text{converges}}$

5) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n} = \sum_{n=1}^{\infty} (-1^{n-1}) \frac{1}{3+5n}$ $\quad b = \frac{1}{3+5n}$

i) $b_{n+1} < b_n$ for all n ✓

ii) $\lim_{n\to\infty} \frac{1}{3+5n} = 0$ ✓

$\boxed{\text{Converges by AST (Alternating series test)}}$

~~Use comparison test to $c = \frac{1}{n}$. diverges~~
~~$b > c$ so can't use comparison test~~

~~what about limit comparison test?~~

~~$\lim_{n\to\infty} \frac{\frac{1}{3+5n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{3+5n} = \lim_{n\to\infty} \frac{1}{\frac{3}{n}+5} = \frac{1}{5}$ so b diverges too!~~

~~Since b diverges, the series sum diverges too~~

~~Since $\lim_{n\to\infty} \frac{\frac{1}{3+5n}}{\frac{1}{n}} = \frac{1}{5}$, then $\sum b_n$ diverges too and so $\sum_{n=1}^{\infty} (-1)^{n-1} b$ diverges~~

7) $\sum_{n=1}^{\infty} -1^n \frac{3n-1}{2n+1}$ $\quad b = \frac{3n-1}{2n+1}$

i) decreasing?
use first derivative

$f'(x) = \frac{3(2n+1) - 2(3n-1)}{2n+1}$ for $n \geq 1$

$\lim_{n\to\infty} -1^n \frac{3n-1}{2n+1} = \lim_{n\to\infty} -1^n \frac{n - \frac{1}{2n}}{1 + \frac{1}{2n}} = \infty$

$\lim -1^n\, n$ so diverges by Divergence Test

so $\boxed{\text{series diverges}}$

$= \frac{6n+3-6n+2}{2n+1} = \frac{5}{2n+1}$ for $n \geq 1$ always positive

fails i) so does not converge, but does it diverge?
by Alternating test.

~~similar to $\sum_{n=1}^{\infty} \frac{3}{2}n = \sum_{n=1}^{\infty} \frac{3}{2} n = \sum_{n=1}^{\infty} \frac{n}{1} = \infty$~~

~~Limit Comparison $\lim_{n\to\infty} \frac{3n-1}{2n+1} \cdot \frac{2n}{3n} = \frac{6n^2-2n}{6n^2+3n} = \lim_{n\to\infty} \frac{1-\frac{1}{3n}}{1+\frac{1}{2n}} = 1$ so $\frac{3n-1}{2n+1}$ diverges too~~

so $\sum_{n=1}^{\infty} -1^n \frac{3n-1}{2n+1}$ diverges too.

9) $\sum_{n=1}^{\infty} (-1)^n e^{-n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{e^n}$ $\quad \frac{1}{e^n} = b$

i) $b_{n+1} \le b_n$ ✓

ii) $\lim_{n\to\infty} \frac{1}{e^n} = 0$ ✓

$\boxed{\text{converges by AST}}$

12) $\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$

$f'(x)$ of $ne^{-n} = -ne^{-n} + e^{-n} = e^{-n}(-n+1)$
$= e^{-n}(1-n)$
which is ~~neg~~ $-$ if $n > 1$

i) so $b_{n+1} \le b$ ✓

ii) $\lim_{n\to\infty} ne^{-n} = 0$

is this true?

~~$= \lim_{n\to 0} n \frac{1}{e^n} = \lim_{n\to 0} \frac{n}{e} \frac{1}{e}$~~

L Hospital? $\lim_{n\to\infty} \frac{n}{e^n} = \frac{1}{e^n} = 0$ ✓

$\boxed{\text{So convergent}}$

13) $\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$ $\quad b = e^{2/n}$

i) $b_{n+1} \le b_n$ ✓

$f'(x) = e^{\frac{2}{n}} \cdot \frac{-2}{n^2}$ which is $-$ for all n

ii) $\lim_{n\to\infty} e^{\frac{2}{n}} \neq 0 = 1$ ~~geometric series~~ fails AST

Divergence test

$\lim_{n\to\infty} (-1)^{n-1} e^{\frac{2}{n}} = \lim_{n\to\infty} (-1)^{n-1} 1$

= does not = 0 so $\boxed{\text{diverges}}$

14) $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$

i) $b_{n+1} \le b_n$? increasing as $n\to\infty$, $\arctan n \to \frac{\pi}{2}^-$ from the left. so increasing

fails AST

ii) fails $\lim_{n\to\infty} \arctan x$ too. it converges to $\frac{\pi}{2}$

Divergence test shows divergence

$\lim_{n\to\infty} (-1)^{n-1} \arctan n = \lim_{n\to\infty} (-1)^{n-1} \frac{\pi}{2} \neq 0$ $\boxed{\text{So diverges}}$

24) $\sum_{n=1}^{\infty} \frac{(-\frac{1}{3})^n}{n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n3^n}$ $\quad |\text{error}| < 0.0005$ how many terms do we need?

i) $b_{n+1} \le b_n$ ✓ $\quad$ ii) $\lim_{n\to\infty} \frac{1}{n3^n} = 0$ ✓ So AST converges

$0.0005 = \frac{1}{2000}$

$S = -\frac{1}{3} + \frac{1}{18} - \frac{1}{81} + \frac{1}{324} - \frac{1}{1215} + \frac{1}{4374}$
$\qquad b_6$

$b_6 = \frac{1}{4374} < \frac{1}{2000}$

need $\boxed{> 6 \text{ terms}}$