41) $y = \cosh^{-1}\sqrt{x}$

$y' = \frac{1}{\sqrt{x-1}} \frac{d}{dx}\sqrt{x} = \boxed{\frac{1}{2\sqrt{x-1}\sqrt{x}}}$

Module 7
Section 3.9

2) a) Area of circle $= \pi r^2$

We want $\frac{dA}{dt}$ so $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

b) so $r = 30m$ and $\frac{dr}{dt} = 1$, then

$\frac{dA}{dt} = 2\pi(30m)\cdot 1\frac{m}{s} = \boxed{60\pi m^2/s}$

11) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$ find $\frac{dz}{dt}$

$4\cdot 5 + 4\cdot 4 + 2\cdot\frac{dz}{dt} = 0$

$2\frac{dz}{dt} = -36$

$\boxed{\frac{dz}{dt} = -18}$

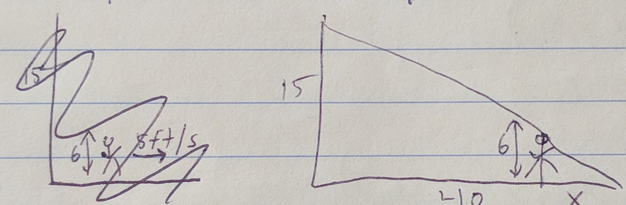
14) sphere area $= 4\pi r^2$

$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$

~~$-1 = 80\pi\frac{dr}{dt}$ so $\frac{dr}{dt} = \frac{-1}{80\pi}$~~

when $d = 10$, $r = 5$ so $-1 = 40\pi\frac{dr}{dt}$ so $\boxed{\frac{dr}{dt} = \frac{-1}{40\pi}}$

15)

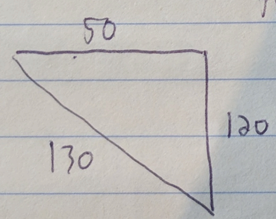


$\frac{x}{x+y} = \frac{6}{15} \Rightarrow 15x = 6x + 6y \Rightarrow 9x = 6y$

$\Rightarrow x = \frac{2}{3}y$ $\quad \frac{dx}{dt} = \frac{2}{3}\frac{dy}{dt} = \frac{2}{3}(5ft/s)$

tip moves at $\frac{10}{3} + \frac{15}{3} = \boxed{\frac{25}{3} ft/s}$ $\quad \boxed{\frac{dx}{dt} = \frac{10}{3} ft/s}$

17)



$w^2 + s^2 = d^2$

$2w\frac{dw}{dt} + 2s\frac{ds}{dt} = 2d\frac{dd}{dt}$

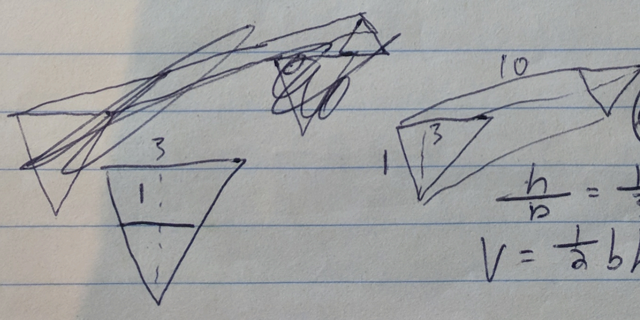
$2\cdot 50mi\cdot 25mi/h + 240\cdot 60mi/h = 260\frac{dd}{dt}$

$\frac{2500mi^2}{h} + \frac{14400mi^2}{h} = 260mi\frac{dd}{dt}$

$\frac{16900}{260mi}\frac{mi^2}{h} = \frac{dd}{dt}$

$\boxed{65\frac{mi}{hr} = \frac{dd}{dt}}$

26)



$\frac{h}{b} = \frac{1}{3} \Rightarrow b = 3h$

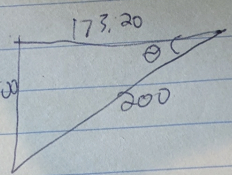
$V = \frac{1}{2}bhl = \frac{3}{2}h^2 l$

$$\frac{dV}{dt} = 3hl\,\frac{dh}{dt}$$

$$\frac{dV}{dt} = 3 \cdot 6 \cdot 10 \cdot 12 = \boxed{2160\ ft^3/min}$$

$$V = \frac{1}{3}\, b\, h\, l$$

$$\frac{dV}{dt} \quad \frac{db}{dt} \quad \frac{dh}{dt} \quad \frac{dL}{dt}$$

30) 

$\theta = \tan^{-1}\left(\frac{h}{x}\right)$

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{h}{x}\right)^2} \cdot \frac{-h}{x^2} \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{1+0.33} \cdot \frac{-100}{30000}\ 8 ft/s = -0.3556$

29) $b = h \quad 2r = h \Rightarrow r = \frac{h}{2}$ $\qquad V = \pi r^2 \frac{h}{3}$

$\frac{dV}{dt} = 30 ft^3/min$ $\qquad V = \pi\left(\frac{h}{2}\right)^2 \frac{h}{3} = \pi \frac{h^2}{4}\left(\frac{h}{3}\right) = \pi \frac{h^3}{12}$

$h = 10$ $\qquad V' = \frac{\pi}{12} h^3 = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{4}{\pi} \cdot \frac{1}{h^2} = 30 ft^3/min \cdot \frac{4}{\pi} \cdot \frac{1}{100} = \boxed{0.3819\ ft/min}$

Section 3.10

2) $f(x) = \sin x, \quad a = \frac{\pi}{6}$

$f'(x) = \cos x$ so the slope at $a = \frac{\sqrt{3}}{2}$

so $L(x) = \frac{1}{2} + \cos x \left(x - \frac{\pi}{6}\right)$

3) $f(x) = \sqrt{x}, \quad a = 4$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

so $L(x) = 2 + \frac{1}{2\sqrt{x}}(x - 4)$

10) $f(x) = e^x \cos x \approx 1 + x$

$f'(x) = e^x - \sin x + \cos x\, e^x$

$L(x) = e^x \cos x + (e^x - \sin x + \cos x\, e^x)(x - a)$?

$L(0) = 1 + 1(x - 0) = 1 + x$

b) within 0.1 $\qquad -0.767 < x < 0.608$

$-0.1 < e^x \cos x - 1 - x < 0.1$

$0.9 < e^x \cos x - x < 1.1$

13) $y = \tan \sqrt{t}$

$y' = \sec^2 \sqrt{t} \cdot \frac{1}{2} t^{-\frac{1}{2}}$

so the differential is: $\boxed{dy = \sec^2 \sqrt{t} \cdot \frac{1}{2} t^{-\frac{1}{2}}\, dx}$

15) $y = e^{x/10}$

$y' = e^{x/10} \cdot \frac{1}{10}$ $\qquad$ so $dy = e^{x/10} \cdot \frac{1}{10} \cdot 0.1 = \boxed{\frac{1}{100}}$

18) $y = \frac{x+1}{x-1}, \quad x = 2, \quad dx = 0.05$

$y' = \frac{(x-1) - (x+1)}{(x-1)(x-1)} = \frac{-2}{(x-1)^2}$ $\qquad$ so $dy = \frac{-2}{1} \cdot 0.05 = \boxed{-0.1}$

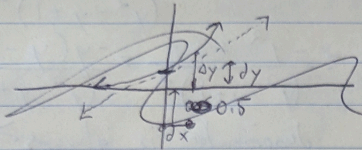
22) $y = e^x$, $x = 0$, $\Delta x = 0.5$

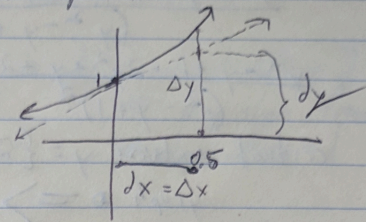
so $\Delta y = \left| 1 - e^{0.5} \right| \approx .648721$

find dy next:

$y' = e^x$ so differential $\Rightarrow dy = e^x \cdot 0.5 = e^0 \cdot 0.5 = 0.5$

0.5 as dy makes sense because the slope at $x=0$ is 1





23) $(1.999)^4$ use linear approximation

find linearization at $a=2$

$f(x) = x^4$

$f'(x) = 4x^3$

$L(x) = f(a) + f'(a)(x-a)$

if $a = 2$, then

$L(x) = 16 + 32(x-2) = 16 + 32x - 64 = -48 + 32x$

so $(1.999)^4 \approx 16 + 32(1.999 - 2) = \boxed{15.968}$

the linearization should undervalue and it does. $(1.999)^4 = 15.968023$

40) $F = kR^4$

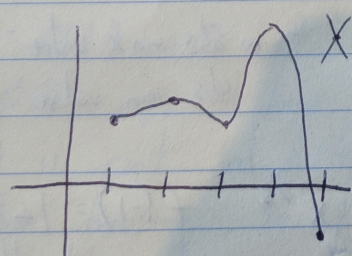
$\frac{dF}{dR} = 4kR^3$ relative change in $F = \frac{4kR^3 dR}{kR^4} = \boxed{\frac{4\,dR}{R}}$

$dF = 4kR^3 dR$

so relative change in F is about $4\times$ relative change in R

a 5% inc in radius would inc $\frac{\Delta F}{F}$ by 20%. $\frac{\Delta F}{F}$ is $\approx \frac{dF}{F}$

$\boxed{\text{Module 4.1}}$ 8)



30) $f(x) = x^3 + 6x^2 - 15x$

$f'(x) = 3x^2 + 12x - 15$

$= 3(x^2 + 4x - 5)$

$= 3(x+5)(x-1)$

critical nums: $x = \{-5, 1\}$

35) $g(y) = \frac{y-1}{y^2-y+1}$

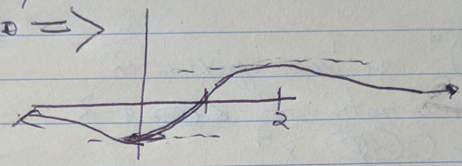
$g'(y) = \frac{\frac{d}{dx}(y-1)\cdot(y^2-y+1) - (y-1)(2y-1)}{(y^2-y+1)^2}$

$= \frac{y^2-y+1 - (y-1)(2y-1)}{(y^2-y+1)^2} = \frac{y^2-y+1-(2y^2-y-2y+1)}{(y^2-y+1)^2}$

$= \frac{y^2-y+1-2y^2+y+2y-1}{(y^2-y+1)^2} = \frac{-y^2+2y}{(y^2-y+1)^2} = \frac{-y(y-2)}{(y^2-y+1)^2}$

critical values: $y = \{2, 0\}$ denominator is always > 0

graph $\Rightarrow$



40) $g(\theta) = 4\theta - \tan\theta$

$g'(\theta) = 4 - \sec^2\theta$

find crit: $4 - \frac{1}{\cos^2\theta} = 0$

$\frac{1}{\cos^2\theta} = 4 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$ $\boxed{\theta = \{\frac{\pi}{3}, -\frac{\pi}{3}\}}$

43) $f(x) = x^2 e^{-3x}$

$f'(x) = x^2 e^{-3x}\cdot -3 + e^{-3x}\cdot 2x = 3x^2 e^{-3x} + e^{-3x}\cdot 2x$

$= e^{-3x}(3x^2 + 2x)$

$= e^{-3x}(x(3x+2))$

crit values $= \{0, \frac{-2}{3}\}$

49) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2)$

crit points: $x = \{-1, 2\}$

$f(-1) = -2 - 3 + 12 + 1 = 8$

$f(2) = 16 - 12 - 24 + 1 = -19$ abs max value $= 8$

endpts $f(-2) = -16 - 12 + 24 + 1 = -3$ abs min value: -19

$f(3) = 54 - 27 - 36 + 1 = -8$

~~redo~~ Section 4.2

9) $f(x) = 1 - x^{2/3}$ $f(-1) = 1 - (1) = f(1) = 1 - 1 = 0$

$f'(x) = -\frac{2}{3}x^{-\frac{1}{3}}$

So you'd think that there would be a point c where $f'(c) = 0$ but...

does not exist at $x = 0$

at $x = 0$, see if it's diffentiable:

$\lim_{x\to 0^+} f'(x)$ is neg

$\lim_{x\to 0^-} f'(x)$ is pos

does not contradict bec $f(x)$ is continuous but not diffentiable

11) $f(x) = 2x^2 - 3x + 1$, $[0,2]$

f is a polynomial so it is continuous and differentiable for all x so it is definately both cont and diff on $[0,2]$

$f(2) - f(0) = (4x-3)(2)$ $\qquad f'(x) = 4x - 3$

$3 - 1 = (4x-3)(2)$

$2 = 8x - 6$

$\boxed{x=1}$ $\qquad x = 1$ is also in interval $[0,2]$

check: avg slope $= \frac{f(2)-f(0)}{2-0} = 1$ ✓ ← avg slope from $[0,2]$

and $4(1) - 3 = 1$ ✓ ← slope at $x=1$

12) $f(x) = x^3 - 3x + 2$, $[-2,2]$

again, $x^3 - 3x - 2$ is a polynomial so f is continuous and differentiable for all x, including on the interval $[-2,2]$

$f'(x) = 3x^2 - 3$

$f(-2) - f(2) = (3x^2 - 3)(-4)$

$0 - 4 = -12x^2 + 12$

$-16 = -12x^2$

$\frac{-16}{-12} = x^2$

Check:

avg slope $= \frac{0-4}{-4} = 1$ ✓

slope at $x = \pm\sqrt{\frac{4}{3}}$ is 1 too ✓

or $\boxed{x^2 = \frac{4}{3} \quad x = \pm\sqrt{\frac{4}{3}}}$

14) $f(x) = \frac{1}{x}$, $[1,3]$

even though $\frac{1}{x}$ is not continuous or differential at 0, the domain is $[1,3]$ and the function is indeed continuous and differentiable on this interval because $f'(x)$ exists for all x in $[1,3]$

$f'(x) = \frac{-x}{x^2} = \frac{-1}{x}$ ← exists for all x in $[1,3]$

$f(1) - f(3) = \frac{-1}{x}(-2)$ $\qquad$ slope at $x=3$: $\frac{-1}{3}$ ✓

$1 - \frac{1}{3} = \frac{2}{x}$ $\qquad$ avg slope $= \frac{\frac{2}{3}}{-2} = \frac{-1}{3}$ ✓

$\frac{2}{3} = \frac{2}{x}$ $\qquad \boxed{x=3}$

37) So $f(t) = 0$ at start and end. so application of Rolle's.

$f(t)$ reps distance btw the two runners

$f(t)$ is cont and differentiable since $g(t)$ and $h(t)$ are position functions. so there is a number c so that $f'(c) = 0$ meaning the two runners have the same speed