$\int_0^4 \sqrt{1+\frac{9}{4}x}\,dx = \int_0^4 (1+\frac{9}{4}x)^{\frac{1}{2}}dx$ $\quad u = 1+\frac{9}{4}x$

$du = \frac{9}{4}dx$

$\frac{4}{9}du = dx$

$= \frac{4}{9}\int_{1}^{10} (u)^{\frac{1}{2}}du$

$= \frac{4}{9}\,\frac{2}{3}u^{\frac{3}{2}}\Big]_1^{10} = \frac{8}{27}u^{\frac{3}{2}}\Big]_1^{10} = \frac{8}{27}(10^{\frac{3}{2}}-1)$

Module 4 | 8.1).

2) Find length of curve $y=\sqrt{2-x^2}$, $0 \le x \le 1$ $\quad y^2 = 2-x^2$ or $y^2+x^2=2$

Formula for arc length is $\int_a^b \sqrt{1+f'(x)^2}\,dx$

$\frac{dy}{dx} = \frac{1}{2}(2-x^2)^{-\frac{1}{2}}\cdot -2x$

$= -x(2-x^2)^{-\frac{1}{2}}$

$\frac{dy}{dx}^2 = x^2(2-x^2)^{-1}$

so $\int_0^1 \sqrt{1+(x^2(2-x^2)^{-1})}$

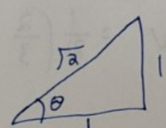
$= \int_0^1 \sqrt{1+\frac{x^2}{2-x^2}} = \int_0^1 \sqrt{\frac{2}{2-x^2}}$

$= \sqrt{2}\int_0^1 \frac{1}{2-x^2}$

$= \sqrt{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big]_0^1$

$= \sqrt{2}\sin^{-1}\frac{1}{\sqrt{2}}$

$= \boxed{\frac{\sqrt{2}\pi}{4}}$




radius $= \sqrt{2}$

when $x=0$ $y=\sqrt{2}$

when $x=1$ $y=1$

so $\theta$ is btw $\frac{\pi}{4}$ and $\frac{\pi}{2}$ which is $\frac{1}{8}$ of a circle.

$\frac{1}{8}2\pi r = \boxed{\frac{1}{4}\pi\sqrt{2}}$ ✓

3) $\int_0^{\pi}\sqrt{1+(\cos x)^2}\,dx$ ~~$= \int_0^{\pi}\sin x = -\cos x\Big]_0^{\pi} = -\cos\pi+\cos 0 = 2$~~

$\sin^2 x+\cos^2 x = 1$

$y = \sin x$

$\frac{dy}{dx} = \cos x$

$\boxed{3.8202}$

4) $y = xe^{-x}$

$\frac{dy}{dx} = -xe^{-x} + e^{-x}$

$= -xe^{-x} + e^{-x}$

$= e^{-x}(-x+1)$

$\int_0^2 \sqrt{1+e^{-x}(-x+1)^2} \approx \boxed{3.3876}$

7) $x=\sqrt{y}-y \quad 1\le y\le 4 \quad x=\sqrt{y}(1-\sqrt{y})$

$\frac{dx}{dy}=\sqrt{y}(-\frac{1}{2}y^{-\frac{1}{2}})+(1-\sqrt{y})(\frac{1}{2}y^{-\frac{1}{2}}) = \frac{1}{2}y^{-\frac{1}{2}}(-\sqrt{y}+1-\sqrt{y})=\frac{1}{2}y^{-\frac{1}{2}}(-2\sqrt{y}+1)$

$=\frac{1}{2}y^{-\frac{1}{2}}-1$ by product rule.

$\int_1^4\sqrt{1+(\frac{1}{2}y^{-\frac{1}{2}})^2}\,dy = \int_1^4\sqrt{1+\frac{1}{4}y^{-1}}\,dy \approx 3.8188$

8) $y^2=\ln x \quad -1\le y\le 1$ so define relative to $dy$ $\quad e^{y^2}=e^{\ln x}=x \quad \frac{dx}{dy}=e^{y^2}\cdot 2y$

$\cancel{2y\frac{dy}{dx}=\frac{1}{x} \rightarrow \frac{dy}{dx}=\frac{1}{x2y}=\frac{1}{2x\sqrt{\ln x}}}$ $\quad \int_{-1}^1\sqrt{4y^2e^{2y^2}+1}\,dy \approx \boxed{4.25523}$

$\cancel{\int_{-1}^1\sqrt{1+\left(\frac{1}{2x\sqrt{\ln x}}\right)^2}=\int_{-1}^1\sqrt{1+\frac{1}{4x^2(\ln x)^2}}}$

9) $y=1+6x^{3/2} \quad 0\le x\le 1$

$\frac{dy}{dx}=9x^{\frac{1}{2}} \qquad \int_0^1\sqrt{1+(9x^{\frac{1}{2}})^2}=\int_0^1\sqrt{1+81x}\,dx \quad u=1+81x \quad \frac{du}{dx}=81 \quad \frac{du}{81}=dx$

$=\frac{1}{81}\int_1^{82}u^{\frac{1}{2}}\,du=\frac{1}{81}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big]_1^{82}=\frac{2}{243}(82\sqrt{82})-\frac{2}{243}$

$\boxed{=\frac{2}{243}(82\sqrt{82}-1)}$

11) $y=\frac{x^3}{3}+\frac{1}{4x} \quad 1\le x\le 2 \qquad y=\frac{x^3}{3}+\frac{1}{4}x^{-1}$

$\frac{dy}{dx}=x^2-\frac{1}{4}x^{-2}$ so $L=\cancel{\int_1^2\sqrt{1+(x^2+\frac{1}{4})^2}\,dx}$

$\frac{dy}{dx}^2=(x^2-\frac{1}{4}x^{-2})^2 \qquad =\cancel{\int_1^2\sqrt{1+x^4+\frac{1}{16}}}$

$=x^4-\frac{1}{4}x^2x^{-2}-\frac{1}{4}x^2x^{-2}+\frac{1}{16}x^{-4}$

$=x^4-\frac{1}{2}+\frac{1}{16}x^{-4}$ $\qquad$ so $L=\int_1^2\sqrt{\frac{1}{2}+x^4+\frac{1}{16}x^{-4}}\,dx$

$=\int_1^2\sqrt{(x^2+\frac{1}{4}x^{-2})^2}\,dx$

$=\int_1^2 x^2+\frac{1}{4}x^{-2}=\frac{x^3}{3}+\frac{-1}{4}x^{-1}\Big]_1^2=\frac{8}{3}-\frac{1}{8}-\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{64}{24}-\frac{3}{24}-\frac{8}{24}+\frac{6}{24}=\boxed{\frac{59}{24}}$

13) $x = \frac{1}{3}\sqrt{y}(y-3) \quad 1 \le y \le 9$

$= \frac{1}{3}y^{\frac{3}{2}} - \sqrt{y}$ $\qquad \frac{dx}{dy} = \frac{1}{2}y^{\frac{1}{2}} - \frac{1}{2}y^{\frac{-1}{2}}$ $\quad$ so $\frac{dx^2}{dy} = \cancel{\ldots}$

$\frac{dx^2}{dy} = \frac{1}{4}y \quad -\frac{1}{2} \quad + \frac{1}{4}y^{-1}$

$L = \int_1^9 \sqrt{\frac{1}{4}y + \frac{1}{2} + \frac{1}{4}y^{-1}}$ same thing as $\frac{dx^2}{dy}$ but positive $\frac{1}{2}$ so the square root must be $\frac{dx}{dy}$ but with ~~an~~ + sign

$L = \int_1^9 \frac{1}{2}y^{\frac{1}{2}} + \frac{1}{2}y^{\frac{-1}{2}}$

$L = \frac{1}{3}y^{\frac{3}{2}} + y^{\frac{1}{2}}\Big]_1^9 = 9 + 3 - (\frac{1}{3} + 1) = 12 - \frac{4}{3} = 10\frac{2}{3}$

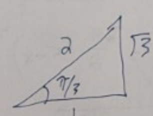
14) $y = \ln(\cos x) \quad 0 \le x \le \pi/3$

$\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x = -\tan x \quad \frac{dy^2}{dx} = \tan^2 x$ $\qquad \int_0^{\pi/3} \sqrt{1 + \tan^2 x} = \int_0^{\pi/3} \sec^2 x$

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $\qquad = \cancel{\sqrt{\tan x}\Big]_0^{\pi/3} = 2\sqrt{3}}$

$L = \ln|\sec x + \tan x|\Big]_0^{\pi/3}$




$= \boxed{\ln|2 + \sqrt{3}|}$ $\cancel{\ln|1|}$

module 8.2 $\quad$ 1)

a) $y = \tan x \quad 0 \le x \le \pi/3$ $\qquad$ a) $\int 2\pi \tan x \, ds \qquad ds = \sqrt{1 + (\sec^2 x)^2} = \sqrt{1 + \sec^4 x}$

$\frac{dy}{dx} = \sec^2 x \quad S = \int 2\pi y \, ds = \int_0^{\pi/3} 2\pi \tan x \sqrt{1 + \sec^4 x} = \boxed{2\pi \int_0^{\pi/3} \tan x \sqrt{1 + \sec^4 x} \, dx}$ rotated around x axis

a b) rotate around y axis $\quad y = \tan x$

$$\boxed{2\pi \int_0^{\pi/3} x \sqrt{1 + \sec^4 x} \, dx}$$

b) $2\pi \int_0^{\pi/3} \tan x \sqrt{1 + \sec^4 x} \, dx \approx \boxed{10.5017}$

$2\pi \int_0^{\pi/3} x \sqrt{1 + \sec^4 x} \, dx \approx \boxed{7.9352}$

3a) $y = e^{-x^2}, -1 \le x \le 1 \quad \frac{dy}{dx} = e^{-x^2} \cdot -2x \quad \frac{dy^2}{dx} = 4x^2 e^{-2x^2}$ or $\ln y = \ln e^{-x^2}$

$\ln y = -x^2 \quad \sqrt{\ln y} = -x$

i) $\boxed{S = \int_{-1}^{1} 2\pi e^{-x^2} \sqrt{1 + 4x^2 e^{-2x^2}} \, dx}$ $\qquad$ ii) $\boxed{\int_{-1}^{1} 2\pi y \sqrt{1 - \left(\frac{1}{2y\sqrt{\ln y}}\right)^2} \, dy}$

$-\sqrt{\ln y} = x \quad \frac{dx}{dy} = \frac{-1}{2y\sqrt{\ln y}}$

7) $y = x^3 \quad 0 \le x \le 2 \qquad \frac{dy}{dx} = 3x^2 \qquad \frac{dy}{dx}^2 = 9x^4 \qquad u = 1 + 9x^4 \qquad \frac{du}{dx} = 36x^3 \qquad du = 36x^3 dx$

$SA = 2\pi \int_0^2 x^3 \sqrt{1 + 9x^4}\, dx$

$= \frac{2\pi}{36} \int_1^{145} \sqrt{u}\, du = \frac{\pi}{18}\left(\frac{2}{3} u^{\frac{3}{2}}\right)\Big]_1^{145} = \frac{2\pi}{54}\left(1 + 9x^4\right)^{\frac{3}{2}}\Big]_0^2$

$= \frac{2\pi}{54}\left(145^{\frac{3}{2}} - 1\right) = \boxed{\frac{1}{27}\left(145^{\frac{3}{2}} - 1\right)}$

6) $y = \tan^{-1} x \quad 0 \le x \le 2 \qquad \frac{dy}{dx} = \frac{1}{1+x^2} \qquad \frac{dy}{dx}^2 = \frac{1}{x^4 + 2x^2 + 1}$

$\tan y = x$

$SA = \int_0^2 2\pi \tan^{-1}(x) \sqrt{1 + \frac{1}{x^4 + 2x^2 + 1}}$

$\boxed{\text{6 on back}}$

8) $y = \sqrt{5-x}, \quad 3 \le x \le 5 \qquad \frac{dy}{dx} = -\frac{1}{2}(5-x)^{-\frac{1}{2}} \qquad \frac{dy}{dx}^2 = \frac{1}{4}(5-x)^{-1}$

$SA = 2\pi \int_3^5 \sqrt{5-x} \sqrt{1 + \frac{1}{4}(5-x)^{-1}}\, dx = \qquad 2\pi \int_3^5 \sqrt{(5-x) + \frac{1}{4}}\, dx \qquad 2\pi \int_3^5 \sqrt{\frac{21}{4} - x}\, dx$

$= 2\pi \int_3^5 \sqrt{5-x} \sqrt{\frac{21-4x}{4(5-x)}}$

$2\pi \int_3^5 \sqrt{\frac{21}{4} - \frac{4x}{4}} = \pi \int_3^5 \sqrt{21 - 4x}\, dx \qquad u = 21 - 4x \qquad du = -4\, dx$

$= -\frac{\pi}{4} \int_9^1 \sqrt{u}\, du = -\frac{\pi}{4} \cdot \frac{2}{3} u^{\frac{3}{2}}\Big]_9^1 = \frac{-2\pi}{12}(1 - 27) = \frac{-52\pi}{12} = \boxed{\frac{13}{3}\pi}$

9) $y^2 = x + 1 \quad 0 \le x \le 3$

$y = \sqrt{x+1} \qquad \frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}} \qquad \frac{dy}{dx}^2 = \frac{1}{4}(x+1)^{-1}$

$SA = 2\pi \int_0^3 \sqrt{x+1} \sqrt{1 + \frac{1}{4(x+1)}} = 2\pi \int_0^3 \sqrt{x + 1 + \frac{1}{4}}$

$= 2\pi \int_0^3 \sqrt{x + \frac{5}{4}}\, dx \qquad u = x + \frac{5}{4} \qquad du = dx$

$= 2\pi \int_{5/4}^{17/4} \sqrt{u}\, du = 2\pi\left[\frac{2}{3} u^{\frac{3}{2}}\right]_{5/4}^{17/4} = \frac{4\pi}{3}\left(\frac{17}{4}\sqrt{\frac{17}{4}} - \frac{5}{4}\sqrt{\frac{5}{4}}\right)$

$= \frac{4\pi}{3}\left(\frac{17\sqrt{17}}{8} - \frac{5\sqrt{5}}{8}\right)$

$= \boxed{\frac{1}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)\pi}$

10) $y=\sqrt{1+e^x}$ $\quad 0\le x\le 1 \quad \frac{dy}{dx}=\frac{1}{2}(1+e^x)^{-\frac{1}{2}}e^x \quad \frac{dy^2}{dx}=\frac{1}{4}(1+e^x)^{-1}e^{2x}$

$$2\pi\int_0^1\sqrt{1+e^x}\sqrt{1+\frac{e^{2x}}{4(1+e^x)}}\,dx = 2\pi\int_0^1\sqrt{1+e^x+\frac{e^{2x}}{4}}\,dx = 2\pi\int_0^1\sqrt{\frac{1}{4}(4+4e^x+e^{2x})}\,dx = \pi\int_0^1\sqrt{(4+4e^x+e^{2x})}\,dx$$

$$=\pi\int_0^1\sqrt{(e^x+2)^2}\,dx = \pi\left[e^x+2x\right]_0^1 \quad \pi((e+2)-1) = \boxed{\pi(e+1)}$$

13) $x=\frac{1}{3}(y^2+2)^{\frac{3}{2}}$, $1\le y\le 2$ ~~don't change equation~~

rotate around x-axis so I need what y equals

still gonna use $ds=\sqrt{1+\frac{dx}{dy}}\,dy$

$$\frac{dx}{dy}=\frac{1}{2}(y^2+2)^{\frac{1}{2}}2y \to = y(y^2+2)^{\frac{1}{2}} \to \frac{dx^2}{dy}=y^2(y^2+2)$$

around x-axis

$$SA=2\pi\int_1^2 y\sqrt{1+y^2(y^2+2)}\,dy = 2\pi\int_1^2 y\sqrt{1+y^4+2y^2}$$

$$=2\pi\int_1^2 y\sqrt{(y^2+1)^2} = 2\pi\int_1^2 y(y^2+1) = 2\pi\int_1^2 y^3+y\,dy = 2\pi\left[\frac{y^4}{4}+\frac{y^2}{2}\right]_1^2$$

$$=2\pi\left[4+2-\left(\frac{1}{4}+\frac{1}{2}\right)\right] = 2\pi\left[6-\frac{3}{4}\right] = 2\pi\left(5\tfrac{1}{4}\right) = 2\pi\left(\frac{21}{4}\right)$$

$$=\frac{42}{4}\pi = \boxed{\frac{21}{2}\pi}$$

14) $x=1+2y^2$, $1\le y\le 2$ rotate around x axis

$$\frac{dx}{dy}=4y \quad dx=4y\,dy \quad \frac{dx^2}{dy}=16y^2$$

$$SA=2\pi\int_1^2 y\sqrt{1+16y^2}\,dy \quad u=1+16y^2 \quad du=32y\,dy$$

$$SA=\frac{2\pi}{32}\int_{17}^{65}\sqrt{u}\,du = \frac{\pi}{16}\left[\frac{2}{3}u^{3/2}\right]_{17}^{65}$$

$$=\boxed{\frac{\pi}{24}\left(65\sqrt{65}-17\sqrt{17}\right)}$$

15) ~~rotate around y-axis, find SA~~

$y=\frac{1}{3}x^{3/2}$, $0\le x\le 12$

so express x in terms of y $\quad 3y=x^{3/2}$ and $(3y)^{2/3}=x$ so $\frac{dx}{dy}=\frac{2}{3}y^{-\frac{1}{3}}$

new limits for $x=0$, $y=0$

$4\sqrt{12}$ for $x=12$, $y=4\sqrt{12}=8\sqrt{3}$

15) $y = \frac{1}{3}x^{3/2}$   $0 \le x \le 12$   rotate around y

new limits are: when $x = 0$, $y = 0$; when $x = 12$, $y = 8\sqrt{3}$

$u = x + 4$ and so although you can't get rid of the x on the left by differentiation, you can say $x = u - 4$, $du = dx$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} \qquad \frac{dy^2}{dx} = \frac{1}{4}x$$

$$SA = 2\pi \int_0^{12} x\sqrt{1 + \frac{1}{4}x} = 2\pi \int_0^{12} x\sqrt{\frac{4+x}{4}} = \pi \int_0^{12} x\sqrt{4+x}$$

$u = x + 4 \qquad x = u - 4$

$$= \pi \int_0^{12} \sqrt{u}(u-4) = \pi \int_4^{16} (u^{3/2} - 4\sqrt{u})\,du = \pi\left[\frac{2}{5}u^{5/2} - \frac{2}{3}4u^{\frac{3}{2}}\right]_4^{16} = \pi\left[\frac{2}{5}u^{5/2} - \frac{8}{3}u^{\frac{3}{2}}\right]_4^{16}$$

$$= \pi\left[\frac{2}{5}16^{5/2} - \frac{8}{3}16^{3/2} - \left(\frac{2}{5}4^{\frac{5}{2}} - \frac{8}{3}4^{\frac{3}{2}}\right)\right] = \pi\left(\frac{1024 \cdot 2}{5} - \frac{64 \cdot 8}{3} - \left(\frac{64}{5} - \frac{64}{3}\right)\right)$$

$$\frac{2048}{5} - \frac{512}{3} - \frac{64}{5} - \frac{64}{3}$$

is this right? $\longrightarrow$ $= \boxed{\frac{3712\pi}{15}}$

6 a) = $y = \tan^{-1} x$, $0 \leq x \leq 2$

$\tan y = x$ $\quad \frac{dx}{dy} = \sec^2 y$, $dx = \sec^2 y\, dy$

when $x=0, y=0$

$x=2, y=\tan^{-1} 2$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

x axis around

i) $SA = 2\pi \int_0^{\tan^{-1} 2} y \sqrt{1 + (\sec^2 y)^2}\, dy$

ii) y axis

$SA = 2\pi \int_0^2 x \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2}\, dx$

b) $\boxed{\begin{array}{l} ii) \approx 13.721 \\ i) \approx 9.79564 \end{array}}$

supposed to be different because rotating around diff axis gives different areas!

what am i doing wrong?

27) $y = \frac{1}{x}$ $\quad \frac{dy}{dx} = -\frac{1}{x^2}$ $\quad SA = 2\pi \int_1^\infty y \sqrt{1 + \left(\frac{-1}{x^2}\right)^2}\, dx$

get rid of y so $\boxed{SA = 2\pi \int_1^\infty \frac{1}{x} \sqrt{1 + \frac{1}{x^4}}\, dx}$

already know $\int_1^\infty \frac{1}{x}$ does not converge and so ↑ does not converge.

by comparison theorem of improper integrals.

$\sqrt{1 + \frac{1}{x^4}}$ is always $> 1$ when $x \geq 1$