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Module 2 Math XII

Section 2.1

2) a) $t=36$, $t=42$ $\quad \dfrac{2948-2530}{42-36} = \dfrac{418}{6} = 69.66$

b) $t=38$, $t=42$ $\quad \dfrac{2948-2661}{42-38} = \dfrac{287}{4} = 71.75$

c) $t=40$, $t=42$ $\quad \dfrac{2806-2948}{40-42}$ (reverse) $= \dfrac{-142}{-2} = 71$

d) $t=42$, $t=44$ $\quad \dfrac{2948-3080}{42-44} = \dfrac{-132}{-2} = 66$

5) velocity $= 40$ ft/s initial

distance height $= 40t - 16t^2$ so when $t=2$, $h=16$

i) find avg velocity which is $\dfrac{\text{distance}}{\text{time elapsed}}$ when $i = 0.5$

$$\frac{h(2)-h(2.5)}{0.5} = \frac{16-0}{0.5} = \boxed{32 \text{ ft/s}}$$

ii) lasting 0.1 seconds

$$\frac{h(2)-h(2.1)}{0.1} = \frac{16-13.44}{0.1} = \frac{2.56}{0.1} = \boxed{25.6 \text{ ft/s}}$$

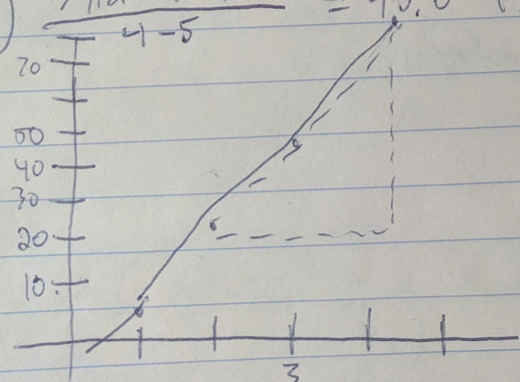
iii) lasting 0.05 sec $\Rightarrow \boxed{24.8 \text{ ft/s}}$

iv) lasting 0.01 sec $\Rightarrow \boxed{24.16 \text{ ft/s}}$

b) $\approx \boxed{24 \text{ ft/s}}$

7) i) i) $\dfrac{79.2-20.6}{2} = 29.3$ ft/sec $\qquad$ ii) $\dfrac{79.2-46.5}{1} = 32.7$

iii) $\dfrac{79.2-124.8}{4-5} = 45.6$ ft/sec $\qquad$ iv) $\dfrac{176.7-79.2}{2} = 48.75$



approx $\dfrac{76-20}{2} \approx 28$ ft/sec

Section 2.2 6) a) 4 b) 3.9 c) 4 d) DNE, there is a gap at $x=-3$

e) -1 f) 1 g) DNE $h(x)$ approaches 2 diff values from the left and right. Left limit $\neq$ right limit $\Rightarrow$ DNE

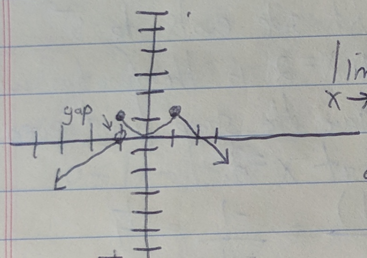
h) ~~1~~

i) 2 j) DNE, discontinuity/gap at $f(2)$ k) 3 l) 0

10) $\lim_{t \to 12^-} f(t) = \boxed{150}$ $\lim_{t \to 12^+} f(t) = \boxed{300}$

The fact that the one-sided limits are diff implies that the patient receives the injection at hour 12 when the drug level reaches 150 or when hour 12 is hit. The level of the drug at hour 12 can only be one number so that is why the two limits are diff numbers.

11)

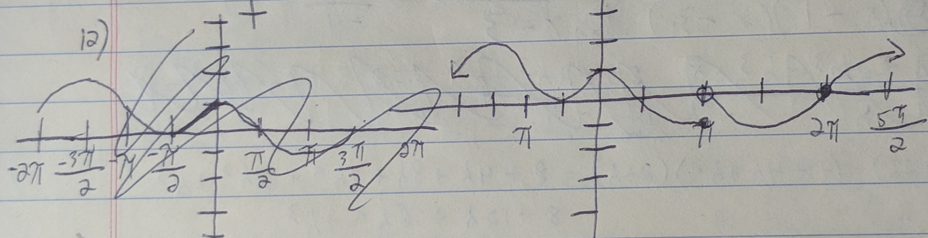


$\lim_{x \to a} f(x)$ exist for all a except -1 so

$a = (-\infty, 1) \cup (1, \infty)$

or $\{a \mid a \neq -1\}$

12)



19) $\lim_{x \to 3} \frac{x^2-3x}{x^2-9}$ at $x=3.1$, $f(x) = 0.508196$

$x = 3.05$, $f(x) = 0.504132$

$x = 3.01$, $f(x) = 0.500831$

$x = 3.001$, $f(x) = 0.500083$

$x = 3.0001$, $f(x) = 0.500008$

x	$f(x)$
2.9	0.491524
2.95	0.495798
2.99	0.499165
2.999	0.499916
2.9999	0.499991

So... $\lim_{x \to 3} \frac{x^2-3x}{x^2-9} = \boxed{0.5}$

can be written $= \frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \frac{x}{x+3} = \frac{3}{3+3} = \boxed{\frac{1}{2}}$

32) $\boxed{-\infty}$

38) $\cot x = \frac{\cos x}{\sin x}$. when $x \to \pi^-$, $\cos x$ is close to -1 and $\sin x$ is close to 0, but negative. so answer is $\boxed{\infty}$

$\lim_{x \to \pi^-} \cot x$

40) as $x \to 2^-$, $x^2 - 2x \to$ negative small number. actually... $\frac{x\cancel{(x-2)}}{(x-2)\cancel{(x-2)}} = \frac{x}{x-2} = \boxed{-\infty}$

Section 2.3

6) $\lim\limits_{v\to -2} \sqrt{v^4 + 3v + 6} = \sqrt{\lim\limits_{v\to -2} (v)^4 + 3\lim\limits_{v\to -2} (v) + \lim\limits_{v\to -2} 6} = \sqrt{16 + -6 + 6} = 4$

big rule is rule 11

10) $\dfrac{x^2+x-6}{x-2} =$

$\dfrac{(x-2)(x+3)}{x-2} = x+3$ they are two different equations. The two equations are equal only when $x \neq 2$ the left side is in indeterminate form when $x=2$ but the right side evals to 5

b) we don't actually care about when $x=2$ in the function when evaluating the limit, the two functions have the same limit even though the left side is undefined at $x=2$.

16) $\lim\limits_{x\to -1} \dfrac{(2x+1)(x+1)}{(x+1)(x-3)} = \dfrac{(2x+1)}{\lim\limits_{x\to -1} x-3} = \dfrac{-1}{-4} = \dfrac{1}{4}$

18) $\cancel{\lim\limits_{h\to 0} \dfrac{8+8h+2h^3+h^3-8}{h} = \lim\limits_{h\to 0} \dfrac{8h+2h^3+h^3}{h} = \lim\limits_{h\to 0} 8+8h+h^2 = 8}$

$(2+h)(2+h) = (4+4h+h^2)(2+h) = 8+4h+8h+4h^2+2h^2+h^3 =$
$8+12h+6h^2+h^3$

cubing a polynomial: $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

so...

$(2+h)^3 = 8+6h^2+12h+h^3$ so, $\lim\limits_{h\to 0} \dfrac{(2+h)^3-8}{h} =$

$\lim\limits_{h\to 0} = \dfrac{8+6h^2+12h+h^3-8}{h} = \lim\limits_{h\to 0} \dfrac{6h^2+12h+h^3}{h} = \lim\limits_{h\to 0} 6h+12+h^2 = \boxed{12}$

19) $\lim\limits_{x\to -2} \dfrac{x+2}{x^3+8} = \dfrac{x+2}{(x+2)(x^2-2x+4)} = \lim\limits_{x\to -2} \dfrac{1}{(x^2-2x+4)} = \boxed{\dfrac{1}{12}}$

26) $\lim\limits_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right) = \lim\limits_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t(t+1)}\right) = \lim\limits_{t\to 0} \left(\dfrac{t+1}{t(t+1)} - \dfrac{1}{t(t+1)}\right) =$

$\lim\limits_{t\to 0} \left(\dfrac{t}{t(t+1)}\right) \Rightarrow \dfrac{\lim\limits_{t\to 0} 1}{\lim\limits_{t\to 0} t+1} = \boxed{1}$

30) $\lim_{x\to-4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to-4} \frac{\sqrt{x^2+9}-5}{x+4} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} = \lim_{x\to-4} \frac{x^2+9-25}{x+4(\sqrt{x^2+9}+5)}$

$= \lim_{x\to-4} \frac{x^2-16}{x+4(\sqrt{x^2+9}+5)} = \lim_{x\to-4} \frac{(x+4)(x-4)}{\text{same}} = \lim_{x\to-4} \frac{x-4}{\sqrt{x^2+9}+5}$

$= \lim_{x\to-4} \frac{-8}{10} = \boxed{\frac{-4}{5}}$

38) Squeeze theorum

so the $\lim_{x\to1} 2x = 2$ and the $\lim_{x\to1} x^4-x^2+2 = 2$ so

if $2x \le g(x) \le x^4-x^2+2$, then $\lim_{x\to1} g(x) = 2$ as well

Section 2.4 2) $\delta = 0.4$, the smaller of $|2.6-3|$ and $|3-3.8|$

14) say $\lim_{x\to2}(5x-7) = 3$ find δ for:

a) $\varepsilon = 0.1$

$0 < |x-2| < \delta$ $\quad$ $0 < 5x-7-3 < 0.1 \rightarrow 0 < |5x-10| < 0.1$

but $|5x-10| = 5|x-2|$

so $0 < |x-2| < \delta$ and $5|x-2| < 0.1$ or $|x-2| < 0.5$

$\boxed{\delta = 0.5}$

b) $\varepsilon = 0.05$ $\quad$ $0 < |5x-10| < 0.5 \rightarrow 0 < |5(x-2)| < 0.05$

so $|x-2| < 0.25$

so $\boxed{\delta = 0.25}$

c) $\varepsilon = 0.01$ $\quad$ $0 < |5x-10| < 0.01 \rightarrow 0 < |5(x-2)| < 0.01$

so $|x-2| < 0.05$

so $\delta = 0.05$

16) $\lim_{x\to4}(2x-5) = 3$ We want a number δ such that $0 < |x-4| < \delta$

so then $|2x-5-3| < \varepsilon$

Lets relate the two equations: $|2x-5-3| = |2x-8| = |2(x-4)|$

so $|x-4| < \frac{\varepsilon}{2}$

so $\boxed{\delta = \frac{\varepsilon}{2}}$ (graph on next page)

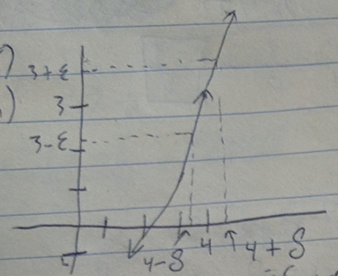
17) $\lim_{x\to-3}(1-4x) = 13$ We want a number δ such that $0 < |x+3| < \delta$

so then $|1-4x-13| < \varepsilon$

relate the two: $|1-4x-13| = |-12-4x| = |-4(x+3)|$ so $|x+3| < \frac{\varepsilon}{-4}$

so $\boxed{\delta = \frac{\varepsilon}{-4}}$ (graph on next page)

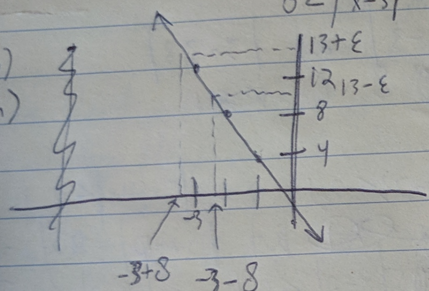
16) (graph)



and for whatever ε we pick, δ is $\frac{\varepsilon}{2}$

if we pick $\delta = \frac{\varepsilon}{2}$, then any x such that $0 < |x-3| < \frac{\varepsilon}{2}$ then $|(4x-5)-7| < \varepsilon$

17) (graph)



whatever ε we pick, δ is $\frac{\varepsilon}{-4}$

if $\delta = \frac{\varepsilon}{-4}$ then

if $0 < |x+4| < \frac{\varepsilon}{-4}$ then

$|1-4x-13| < \varepsilon$

19) $\lim\limits_{x \to 1} \frac{2+4x}{3} = 2$ We want to find a δ so that if $0 < |x-1| < \delta$

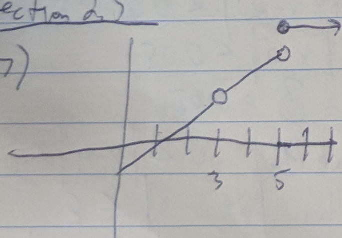
then $\left|\frac{2+4x}{3} - 2\right| < \varepsilon$

so $\left|\frac{2+4x}{3} - 2\right| = |4x-4| = |4(x-1)|$ so $|4(x-1)| < \varepsilon$

or $\boxed{|x-1| < \frac{\varepsilon}{4}}$

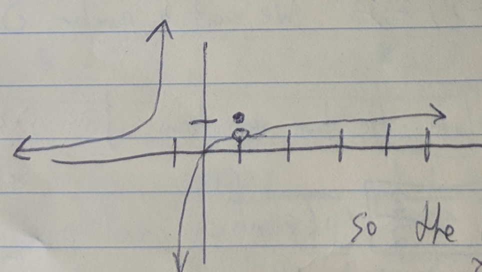
Section 2.5

7)



20) $f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

$\frac{x^2-x}{x^2-1} = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \frac{x}{x+1}$



so the $\lim\limits_{x \to a} f(x) \neq f(a)$

$\frac{1}{2} \neq 1$

Due to definition 3, a function is continuous if it is continuous at every number in the interval

24) $f(x) = \frac{x^3-8}{x^2-4}$ undefined at 2 and -2

since $x^3-8 = (x-2)(x^2+2x+4)$

and $x^2-4 = (x+2)(x-2)$

but $\frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}$ so...

$\lim_{x \to 2} \frac{x^2+2x+4}{x+2} = 3$

to make $f(x)$ continuous, define $f(x)$ to $=3$ at $x=2$

45) $f(x) = \begin{cases} cx^2+2x & \text{if } x<2 \\ x^3-cx & \text{if } x \geq 2 \end{cases}$

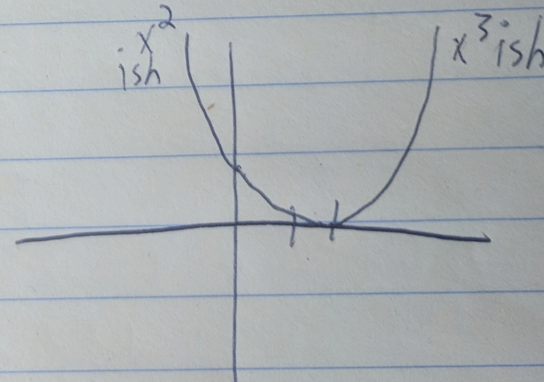
unsure... lets say $x=2$? then $4c+4$ the two equations ge $8-2c$ = if $\boxed{c = \frac{2}{3}}$

$4c+4 = 8-2c$

$6c = 4$

$c = \frac{2}{3}$

It'd look something like this: if $c = \frac{2}{3}$



Corrections for module 1

29 33 40