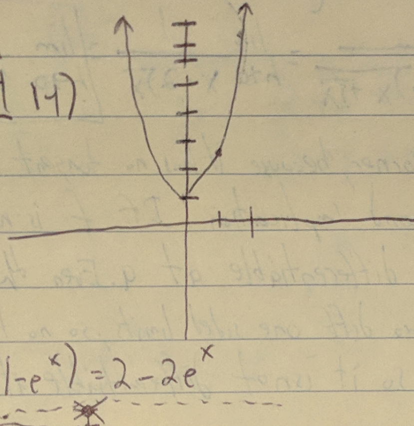
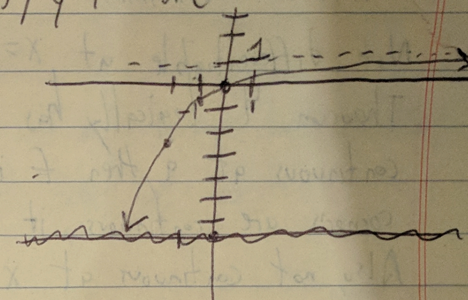
$\frac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$ | $1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$ | $-\tan\alpha\tan\beta$

$\boxed{\text{Module 4}}$

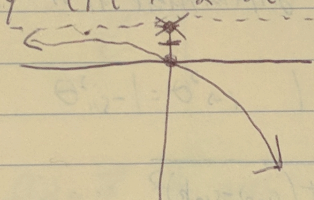
Section 1.4 14)



15) $y = 1 - \frac{1}{2}e^{-x}$



16) $y = 2(1-e^{x}) = 2 - 2e^{x}$



22) $\boxed{(2)\left(\frac{2}{3}\right)^{x}}$ $b = \frac{2}{3}$ $c = 2$ so

x	y
1	$\frac{4}{3}$ ✓
-1	3 ✓

24) probably II but I'll check

day	cents	total
1	1	1
2	2	3
3	4	7
4	8	15
⋮		
15	32768	
⋮		
28	268435456	= \$2,684,354

so... def II

$\boxed{\text{Section 3.1}}$

4) $f(x) = e^5$ so $f'(x) = \cancel{5e}\ \cancel{\ }\ \boxed{0}$ because $\lim_{h\to 0} 0 = 0$

7) $7t^4 - 5t = f'(t)$

12) $-6cy^{-7} = B'(y)$

15) $R(a) = (3a+1)^2$ $R'(a) = 2(3a+1) = 6a+2$

16) $h(t) = \sqrt[4]{t} - 4e^{t}$ $h'(t) = \frac{1}{4}t^{-\frac{3}{4}} - e^{t}$ (using power, diff, and exp rules)

17) $S(p) = \sqrt{p} - p$ $S'(p) = \frac{1}{2}p^{-\frac{1}{2}} - 1$

18) $y = \sqrt[3]{x}\,(2+x)$ (it's a product. Could use product rule but....)

$y = 2\sqrt[3]{x} + x^{\frac{1}{3}}x = 2x^{\frac{1}{3}} + x^{\frac{4}{3}}$ so $\boxed{y' = \frac{2}{3}x^{-\frac{2}{3}} + \frac{4}{3}x^{\frac{1}{3}}}$ ✓

using product rule: $x^{\frac{1}{3}} + (2+x)\cdot\frac{1}{3}x^{-\frac{2}{3}}$

$= x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{2}{3}} + \frac{1}{3}x^{\frac{1}{3}} = \frac{3}{3}x^{\frac{1}{3}} + \frac{1}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{2}{3}} =$

$y' = \boxed{\frac{4}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{2}{3}}}$ ✓

19) $y = 3e^{x} + \frac{4}{\sqrt[3]{x}} = 3e^{x} + 4x^{-\frac{1}{3}}$ $\boxed{y' = 3e^{x} + \frac{-4}{3}x^{-\frac{4}{3}}}$

52) $y = e^{x+1} + 1$ $\quad y' = e^{x+1}$ $\quad$? Is this right?

37) $y = x^4 + 2e^x$ so $y' = 4x^3 + 2e^x$

point $= (0,2)$ slope $=$ ~~tangent~~ $0 + 2 = 2$ so tangent $=$ $y - 2 = 2(x)$ $\quad \boxed{y = 2x + 2}$

normal line: $y - 2 = -\frac{1}{2}x$

$= y = -\frac{1}{2}x + 2$

55) $y = 2x^3 + 3x^2 - 12x + 1$

$y' = 6x^2 + 6x - 12$ $\quad$ horizontal when $y' = 0$ so

$(3x - 3)(2x + 4) = 0$ $\quad x = \{1, -2\}$

$\boxed{x = 1,\ x = -2}$

62) $y = x^2 - 1$ so $y' = 2x$

the normal line will have a slope of $\frac{-1}{2x}$ so at $x = -1$, $y' = \frac{1}{2}$

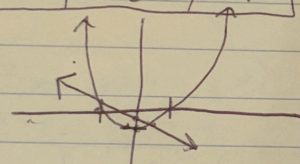
normal line $=$ $y = \frac{-1}{2}(x + 1) = \frac{-1}{2}x - \frac{1}{2}$

$\frac{-1}{2}x - \frac{1}{2} = x^2 - 1 \rightarrow x^2 + \frac{1}{2}x - \frac{1}{2} = 0$

$= 2x^2 + x - 1 = 0$

$= (2x - 1)(x + 1)$ so $x = \{\frac{1}{2}, -1\}$

$\boxed{\text{other pt} = \{\frac{1}{2}, \frac{-3}{4}\}}$



Section 3.2

4) $g(x) = (x + 2\sqrt{x})e^x$ $\quad g'(x) = (x + 2\sqrt{x})e^x + e^x(1 + x^{-\frac{1}{2}})$

$= e^x((x + 2\sqrt{x}) + (1 + x^{-\frac{1}{2}}))$

5) $y = \frac{x}{e^x}$ quotient rule $y' = \frac{e^x \cdot 1 - xe^x}{e^{2x}} = \frac{e^x(1 - x)}{e^{2x}} = \boxed{\frac{1 - x}{e^x}}$

→ product rule? $y = x \cdot e^{-x}$ so $y' = x \cdot e^{-x} + e^{-x} = e^{-x}(x + 1) = \frac{x + 1}{e^x}$

apparently this does not work, supposed to use chain rule?

6) $y = \frac{e^x}{1 - e^x}$ quotient rule: $\frac{(1 - e^x)e^x - e^x(-e^x)}{(1 - e^x)^2} = \frac{(1 - e^x)e^x + e^{2x}}{(1 - e^x)^2}$

$= \boxed{\frac{e^x}{(1 - e^x)^2}}$

8) $G(x) = \frac{x^2 - 2}{2x + 1}$ $\quad G'(x) = \frac{(2x + 1)(2x) - (x^2 - 2)2}{(2x + 1)^2} = \frac{4x^2 + 2x - (2x^2 - 4)}{4x^2 + 4x + 1}$

$= \frac{2x^2 + 2x + 4}{4x^2 + 4x + 1} = \frac{2x^2 + 2x + 4}{4x^2 + 4x + 1} =$

$= \frac{2x^2 + 2x + 4}{4x^2 + 4x + 1}$

9) $H(v) = (v-\sqrt{v})(v+\sqrt{v}) = v^2 + v$ $\boxed{H'(v) = 2v+1}$

24) $F(t) = \frac{At}{Bt^2+Ct^3}$ (at the end)

26) $f(x) = \frac{ax+b}{cx+d}$ $f'(x) = \frac{cx+d(a) - (ax+b)(c)}{(cx+d)^2} = \frac{acx+ad-acx-cb}{(cx+d)^2} =$ $\boxed{\frac{ad-cb}{(cx+d)^2}}$

34) $y = \frac{2x}{x^2+1}$ $y' = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} = \frac{-2(x^2-1)}{(x^2+1)^2}$ ~~at (1,1), the slope =~~

at $\{1,1\}$, the slope $= 0$

$\boxed{\text{tangent}: y = 1 \qquad \text{normal}: x = 1}$

46) $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} = \frac{x \cdot h'(x) - h(x)}{x^2} = \frac{2\cdot(-3) - 4}{4} = \frac{-10}{4} = \boxed{-2.5}$

Section 3.3 | 2) $f(x) = x\cos x + 2\tan x$ $f'(x) = \frac{d}{dx}(x\cos x) + \frac{d}{dx}(2\tan x)$

$= x \cdot -\sin x + \cos x + 2\sec^2 x + \tan x$

3) $f(x) = e^x \cos x$ $f'(x) = e^x \cdot -\sin x + \cos x e^x$

$= \boxed{e^x(-\sin x + \cos x)}$

4) $y = 2\sec x - \csc x$ $y' = \frac{d}{dx}(2\sec x) - \frac{d}{dx}(\csc x) = \boxed{2\sec x\tan x + \csc x\cot x}$

5) $y = \sec\theta\tan\theta$ $y' = \sec\theta\sec^2\theta + \tan\theta\sec\theta\tan\theta$

$= \sec^3\theta + \tan^2\theta\sec\theta$

$= \boxed{\sec\theta(\sec^2\theta + \tan^2\theta)}$

11) $f(\theta) = \frac{\sin\theta}{1+\cos\theta}$ $f'(\theta) = \frac{1+\cos\theta(\cos\theta) - \sin\theta(-\sin\theta)}{(1+\cos\theta)^2}$

$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2} = \frac{\cos\theta + 1}{(1+\cos\theta)^2} = \boxed{\frac{1}{1+\cos\theta}}$ Identity

12) $y = \frac{\cos x}{1-\sin x}$ $y' = \frac{(1-\sin x)\cdot -\sin x - \cos x(-\cos x)}{(1-\sin x)^2} = \frac{(\sin^2 x - \sin x) + \cos^2 x}{(1-\sin x)^2}$

$= \frac{1-\sin x}{(1-\sin x)^2} = \boxed{\frac{1}{1-\sin x}}$

$\frac{\cos t + \sin t - \frac{\sin t}{\cos^2 t}}{(1+\tan t)^2}$

14) $y = \dfrac{\sin x}{1 + \tan x}$ so... $(1 + \tan x)^2 = \left(1 + \dfrac{\sin x}{\cos x}\right)^2 = \left(\dfrac{\sin x + \cos x}{\cos x}\right)^2$

$= \dfrac{1 + 2\sin x \cos x}{\cos^2 x}$

so $y' = \dfrac{\cos x + \sin x - \frac{\sin x}{\cos^2 x}}{(1 + \tan x)^2} = \dfrac{(\cos^2 x)\left(\cos x + \sin x - \frac{\sin x}{\cos^2 x}\right)}{(\cos^2 x)(1 + \tan)^2}$

$= \dfrac{\cos^3 x + \sin x \cos^2 x - \sin x}{1 + 2\sin x \cos x} = \dfrac{\cos^3 x - \sin x(1 - \cos^2 x)}{1 + 2\sin x \cos x} = \boxed{\dfrac{\cos^3 x - \sin^3 x}{1 + 2\sin x \cos x}}$

22) $y = e^x \cos x$ at point $(0,1)$ so $y' = e^x \cdot -\sin x + \cos x\, e^x$

$= e^x(\cos x - \sin x)$

slope at $x = 0$ is $= 1$

line equation $= y - 1 = x$

$\boxed{y = x + 1}$

30) $f(t) = \sec t$

$f'(t) = \sec x \tan x$

$f''(t) = \sec x \sec^2 x + \tan \sec x \tan x$

$= \sec^3 x + \tan^2 x \sec x$

$\boxed{= \sec x(\sec^2 x + \tan^2 x)}$

24) (from section 3.2)

$F(t) = \dfrac{At}{Bt^2 + Ct^3} = \dfrac{(Bt^2 + Ct^3)A - At(2Bt + 3Ct^2)}{(Bt^2 + Ct^3)^2}$

$= \dfrac{ABt^2 + ACt^3 - 2ABt^2 - 3ACt^3}{(Bt^2 + Ct^3)^2}$

$= \dfrac{-ABt^2 - 2ACt^3}{(Bt^2 + Ct^3)^2}$

$= \dfrac{-At^2(B + 2Ct)}{(t^2(B + Ct))^2} = \dfrac{-At^2(B + 2Ct)}{t^4(B + Ct)^2}$

$= \boxed{\dfrac{-A(B + 2Ct)}{t^2(B + Ct)^2}}$