$$\begin{bmatrix} -6 & 4 \end{bmatrix} \begin{bmatrix} -1 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$$

Lesson 6 Sections 3.1 and 3.2

1) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & -1 \\ -3 & 1 & 0 \end{bmatrix}$

$M_{11} = 1$ $\quad M_{12} = -3$ $\quad M_{13} = 4$

$M_{21} = 0$ $\quad M_{22} = 0$ $\quad M_{23} = -1$

$M_{31} = 1$ $\quad M_{32} = -2$ $\quad M_{33} = 3$

$C_{11} = 1$ $\quad C_{12} = 3$ $\quad C_{13} = 4$

$C_{21} = 0$ $\quad C_{22} = 0$ $\quad C_{23} = 1$

$C_{31} = 1$ $\quad C_{32} = 2$ $\quad C_{33} = 3$

2) Expansion using row 2:

$\cancel{0 \cdot C_{12} + 0 \cdot C_{22} + 0 \cdot C_{23}}$

$1 \cdot C_{21} + 1 \cdot C_{22} - 1 \cdot C_{23} = 0 + 0 - 1 = \boxed{-1}$

Expansion $\bar{w}$ row 3:

$-3 \cdot C_{31} + 1 \cdot C_{32} + 0 \cdot C_{33} = -3 + 2 = \boxed{-1}$

3) a) $\boxed{6}$ b) $\boxed{24}$

for b) check $\bar{w}$ row expansion of first row!

$4 \cdot C_{11} + 0 \cdot C_{12} + 0 \cdot C_{13} =$

$4 \cdot 6 + 0 + 0 = 24$ ✓

$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$ $\quad \frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

4) a) multiplying a row by nonzero constant. Factored out the 2 from R1

b) added multiple of R1 to R3 so the determinant is equal.

$$R_3 + (-1)R_1 \to R3$$

5) a) $\boxed{1}$ using cofactor, it's the same via triangular matrix theorem

b) Interchange R_1 and R_3 so $|A| = -|B|$ and $|B| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1$ so $|A| = \boxed{-1}$

6) Lets use row 2 since it has a 0.

$$\begin{vmatrix} -2 & 1 & 3 \\ 0 & 2 & 2 \\ 1 & -1 & -4 \end{vmatrix} = -\underset{R_1 \Leftrightarrow R_3}{\begin{vmatrix} 1 & -1 & -4 \\ 0 & 2 & 2 \\ -2 & 1 & 3 \end{vmatrix}} = -\underset{R_3 + 2(R_1) \to R_3}{\begin{vmatrix} 1 & -1 & -4 \\ 0 & 2 & 2 \\ 0 & -1 & -5 \end{vmatrix}} = -\underset{R_3 + \frac{1}{2}(R_2) \to R_3}{\begin{vmatrix} 1 & -1 & -4 \\ 0 & 2 & 2 \\ 0 & 0 & -4 \end{vmatrix}} = \boxed{-8}$$

via triangular

Check: $\begin{vmatrix} -2 & 1 & 3 \\ 0 & 2 & 2 \\ 1 & -1 & -4 \end{vmatrix} = \cancel{\underset{R_2 + 2(R_3) \to R_2}{\begin{vmatrix} -2 & 1 & 3 \\ 0 & 6 & -6 \\ 1 & -1 & -4 \end{vmatrix}}}$ ~~cofactor expansion to R_2~~ $\cancel{0 \cdot C_{21} + 0 \cdot C_{22} + \dots C_{23}}$ $\cancel{-6 \cdot (2-1) = 6}$

$$= \underset{C_2 + -C_3 \to C_2}{\begin{vmatrix} -2 & -2 & 3 \\ 0 & 0 & 2 \\ 1 & 3 & -4 \end{vmatrix}} = 0 \cdot R_{21} + 0_{R_{22}} + 2 \cdot R_{23} =$$

$$2 \cdot (-6+2) = \boxed{-8}$$