$\frac{dV}{dt} = 3hl\frac{dh}{dt}$

$\frac{dV}{dt} = 3 \cdot 6 \cdot 10 \cdot 12 = \boxed{2160\ ft^3/min}$

Module 8

6) Local min: $x = \{2, 4\}$
Local max: $x = \{6\}$

8) a) The graph is inc on intervals: $(0,1) \cup (2,3) \cup (5,7)$. Reason: In these intervals, the tangent lines have positive slope, so $f'(x) > 0$. This is the I/D Test.

b) Local min: $x = \{2, 5\}$. Local max: $x = \{1, 3, 7\}$

c) Concave up: $\boxed{x = (1,3) \cup (4,6)}$ d) Inflection pts: $\boxed{x = \{1, 3, 4, 6\}}$

Concave down: $\boxed{x = (0,1) \cup (3,4) \cup (6, \infty)}$

The tangent lines are inc on the intervals where f is concave up. That means f'' is positive. When the tangent lines are inc, it creates this concave up shape. Similare reasoning for concave down intervals

- At the inflection points, the concavity changes its direction and it's continuous.

11) $x^4 - 2x^2 + 3 = f(x)$

$f'(x) = 4x^3 - 4x = 4x(x^2-1)$ so crit values = 0, 1, −1

$f''(x) = 12x^2 - 4 = 4(3x^2-1)$

Interval	$4x$	x^2-1	4	$3x^2-1$	f'	f''
$x < -1$	−	+			−	
$-1 < x < 0$	−	−			+	
$x > 1$	+	+			+	

!!!

Interval	$4x$	x^2-1	f'
$0 < x < 1$	+	−	−

a) inc: $\boxed{x = (-1, 0) \cup (1, \infty)}$ bec f' is positive so inc.

dec: $\boxed{x = (-\infty, -1) \cup (0, 1)}$ The f' is negative, so the f is dec.

b) $\boxed{\text{Local min: } x = \{-1, 1\} \quad \text{Local max: } x = \{0\}}$ because f' changes from negative to positive. First derivative Test!

f' changes from + to − so local max

pts: local min: $(-1, 2)$ $(1, 2)$
max: $(0, 3)$

c) $f''(x) = 12x^2 - 4 = 4(3x^2-1)$

so $3x^2 - 1 = 0 \rightarrow 3x^2 = 1 \rightarrow x^2 = \frac{1}{3} \rightarrow x = \pm\frac{1}{\sqrt{3}}$

	f''	
$-\infty, -\frac{1}{\sqrt{3}}$	+	concave up
$-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$	−	concave down
$\frac{1}{\sqrt{3}}, \infty$	+	concave up

Inflection pts: $-\frac{1}{\sqrt{3}}, 2.4444$

$\frac{1}{\sqrt{3}}, 2.4444$

12) $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{x^2+1 - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)(x^2+1)} = \frac{1-x^2}{x^2+1}$

crit numbers $= x = \{1, -1\}$

	$1-x^2$	x^2+1	f'
$-\infty < x < -1$	$-$	$+$	$-$
$-1 < x < 1$	$+$	$+$	$+$
$1 < x < \infty$	$-$	$+$	$-$

so dec: $(-\infty, -1) \cup (1, \infty)$ and inc: $(-1, 1)$

b) local min: $(-1, \frac{-1}{2})$ local max: $(1, \frac{1}{2})$

c) $f''(x) = \frac{(x^2+1)(-2x) - (1-x^2)(2x)}{(x^2+1)^2} = \frac{-2x^3 - 2x - 2x + 2x^3}{(x^2+1)^2}$

$= \frac{-4x}{(x^2+1)^2}$ so $f''(x) = 0$ at $x = \{0\}$

so Interval	$f''(x)$	Concavity
$(-\infty, 0)$	$+$	up
$(0, \infty)$	$-$	down

0,0 is inflection pt since curve changes from concave down to concave up

14) $f(x) = \cos^2 x - 2\sin x$, $0 \leq x \leq 2\pi$

$f'(x) = -2\sin x \cos x - 2\cos x = -2\cos x(\sin x + 1)$

crit values: $x = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

a) Intervals	$-2\cos x$	$\sin x + 1$	f'	
$0 < x < \frac{\pi}{2}$	$-$	$+$	$-$	dec
$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$+$	$+$	$+$	inc
$\frac{3\pi}{2} < x < 2\pi$	$-$	$+$	$-$	dec

b) local min: $(\frac{\pi}{2}, -2)$

max: $(\frac{3\pi}{2}, 2)$

c) $f''(x) = -2\cos x(\cos x) + (\sin x + 1)(2\sin x) = -2\cos^2 x + 2\sin^2 x + 2\sin x$

$= 2(\sin^2 x + \sin x - \cos^2 x)$

How do I simplify? $f''(x) = 0$ at $x = \frac{\pi}{6}, \frac{3\pi}{2}$

Interval	f''	
$0 < x < \frac{\pi}{6}$	$-$	concave dow
$\frac{\pi}{6} < x < \frac{3\pi}{2}$	$+$	up
$\frac{3\pi}{2} < x < 2\pi$	$-$	down

$\boxed{\text{Inflection pts: } (\frac{\pi}{6}, 0), (\frac{3\pi}{2}, 0)}$

38) $f(x) = 36x + 3x^2 - 2x^3 = -2x^3 + 3x^2 + 36x$

$f'(x) = -6x^2 + 6x + 36 = -6(x^2 - x - 6) = -6(x-3)(x+2)$

so $f'(x) = 0$ when $x = \{3, -2\}$

a)

Interval	-6	$x-3$	$x+2$	f'	
$(-\infty, -2)$	$-$	$-$	$-$	$-$	dec
$(-2, 3)$	$-$	$-$	$+$	$+$	inc
$(3, \infty)$	$-$	$+$	$+$	$-$	dec

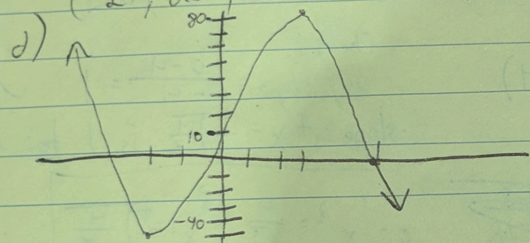
b) max: $(3, (-54+27+108)) = (3, 81)$

min: $(-2, -44)$

c) $f''(x) = -12x + 6$ 0 when $x = \frac{1}{2}$ Inflection pts: $($

$(-\infty, \frac{1}{2})$	$+$	up
$(\frac{1}{2}, \infty)$	$-$	down

d)



40) $g(x) = 200 + 8x^3 + x^4$

a) $g'(x) = 24x^2 + 4x^3$ $g'(x) = 0$ when $x = \{-6, 0\}$

$= x^2(24 + 4x)$

$= 4x^2(6 + x)$

Interval	$4x^2$	$6+x$	f'
$(-\infty, -6)$	$+$	$-$	$-$
$(-6, 0)$	$+$	$+$	$+$
$(0, \infty)$	$+$	$+$	$+$

b) local min: $(-6, -232)$

no local max. deriv never goes from $+$ to $-$

c) $g''(x) = 48x + 12x^2$

$= 12x(4 + x)$ so $g''(x) = 0$ when $x = \{0, -4\}$

Interval	$12x$	$4+x$	f''	
$-\infty, -4$	$-$	$-$	$+$	concave up
$-4, 0$	$-$	$+$	$-$	down
$0, \infty$	$+$	$+$	$+$	up

so inflection pts $= \boxed{(-4, -56), (0, 200)}$

d) (next page)

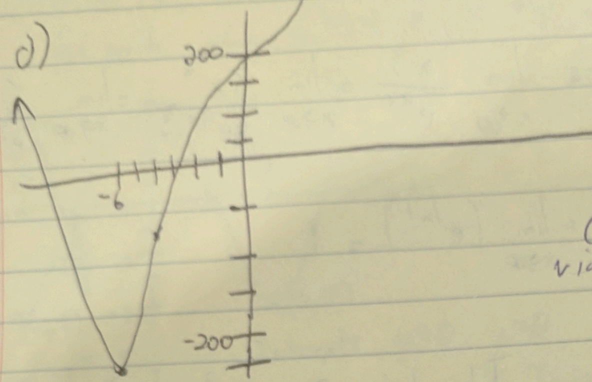
75
+5
80
-100
105

225
+100 Eric
395

500
+300
90
890

200
10
610

40) d)



8) $\lim_{x\to 3} \frac{x-3}{x^2-9}$ evals to $\frac{0}{0}$ so indeterminate

$= \lim_{x\to 3} \frac{x-3}{(x+3)(x-3)} = \frac{1}{x+3} = \boxed{\frac{1}{6}}$

Check: via L'Hospital $\lim_{x\to 3} \frac{1}{x+3+x-3} = \frac{1}{2x} \boxed{= \frac{1}{6}}$ ✓

13) $\lim_{t\to 0} \frac{e^{2t}-1}{\sin t}$ indeterminate $\frac{0}{0}$

$= \lim_{t\to 0} \frac{e^{2t}\cdot 2}{\cos t} = \frac{2}{1}$

16) $\lim_{x\to 0} \frac{x^2}{1-\cos(x)}$ indeterminate form $\frac{0}{0}$

$= \lim_{x\to 0} \frac{2x}{\sin x}$ still indeterminate

$= \lim_{x\to 0} \frac{2}{\cos x} = 2$

21) $\lim_{x\to 0^+} \frac{\ln x}{x} = -\infty$

24) $\lim_{t\to 0} \frac{8^t - 5^t}{t}$ indeterminate form $\lim_{t\to 0} \frac{8^t \ln 8 - 5^t \ln 5}{1} =$

$= \ln 8 - \ln 5$

27) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{e^x - 1}{2x} = \lim_{x\to 0} \frac{e^x}{2} = \boxed{\frac{1}{2}}$

indeterminate form of $\frac{0}{0}$

4) $y = x^4 - 8x^2 + 8$ Intercepts y int = 8

32) $\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \lim_{x\to\infty} 2\ln x \cdot \frac{1}{x} = \lim_{x\to\infty} \frac{2\ln x}{x} = \lim_{x\to\infty} \frac{2\frac{1}{x}}{1} \boxed{=0}$

indeterminate form of $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2}{x}$

54) $\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2} = \lim_{x\to 0} \frac{-\sin mx \cdot m + \sin mx \cdot m}{2x} = \lim_{x\to 0} \frac{-\cos mx \cdot m^2 + \cos mx \cdot m^2}{2}$

Indeterminate form $\frac{0}{0}$

$\boxed{=0}$

47) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{2\theta^2} = \lim_{\theta\to 0} \frac{-\sin\theta}{4\theta} = \lim_{\theta\to 0} \frac{-\cos\theta}{4} = \boxed{-\frac{1}{4}}$

indeterminate $\frac{0}{0}$

65

Oops, wrong 47 ↑

47) $\lim_{x\to\infty} x^3 e^{-x^2} = \lim_{x\to\infty} \frac{x^3}{e^{x^2}} = \lim_{x\to\infty} \frac{x^3}{e^{2x}} = \lim_{x\to\infty} \frac{3x^2}{e^{2x}\cdot 2} = \lim_{x\to\infty} \frac{6x}{e^{2x}\cdot 4} = \lim_{x\to\infty} \frac{6}{e^{2x}\cdot 8}$

indeterminate $\infty \cdot 0$

$= 0$

63) $\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} (e^{\ln x})^{\frac{1}{x}} = \lim_{x\to\infty} (e^{\ln 1}) = 1$

$\boxed{\text{Section 4.5}}$ 2) $y = 2 + 3x^2 - x^3$ Intercepts: $y = 2$ $3x^2 - x^3 = -2 \Rightarrow x^2(3-x) = -2$

Domain $= \mathbb{R}$

$x_{int} = (3.196, 0)$

$y' = 6x - 3x^2 = -3x^2 + 6x = -3x(x-2)$

crit values: $x = \{2, 0\}$

local min $= (0, 2)$

local max $= (2, 6)$

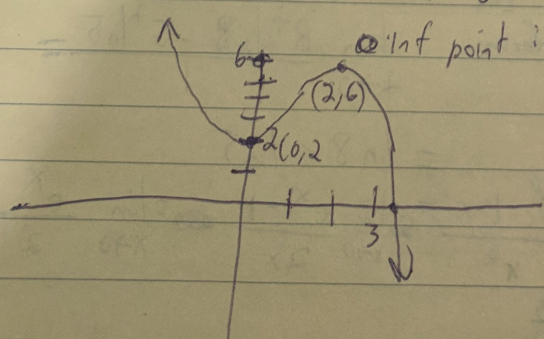
Interval	$-3x$	$x-2$	y'
$-\infty, 0$	+	−	−
$0, 2$	−	−	+
$2, \infty$	−	+	−

$y'' = -3x + (x-2)\cdot -3 = -3x - 3x + 6$

$= -6x + 6 = -6(x-1)$

Inf point: $x = \{1\}$

Interval	$x-1$	-6	y''
$-\infty, 1$	−	−	+
$1, \infty$	+	−	−



4) $y = x^4 - 8x^2 + 8$ Intercepts y int = 8 x-int = btw 1,2 and -1,-2

$\cancel{(x^2 \quad)(x^2 \quad)}$

$y(-2) = -8$

$y(1) = 1$ $y(3)$

$y(0) = 8$

$y(-1) = 1$

$y(2) = -8$

$y' = 4x^3 - 16x$

$= 4x(x^2 - 4)$

crit values $= 0, 2, -2$

Intervals	$4x$	x^2-4	y'
$-\infty, -2$	−	+	−
$-2, 0$	−	−	+
$0, 2$	+	−	−
$2, \infty$	+	+	+

local mins: $(-2,-8), (2,-8)$

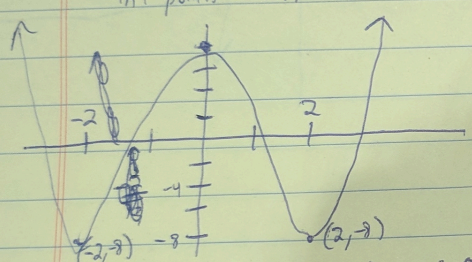
local maxes: $(0,8)$

$y'' = 12x^2 - 16$

$= 4(3x^2 - 4)$

inf points when $3x^2 = 4 \Rightarrow x^2 = \frac{4}{3} \rightarrow \boxed{x = \pm\frac{2}{\sqrt{3}}}$ $x = \pm\frac{2}{\sqrt{3}} \approx$

Interval	4	$3x^2-4$	y''
$-\infty, -\frac{2}{\sqrt{3}}$	+	+	+
$-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$	+	−	−
$\frac{2}{\sqrt{3}}, \infty$	+	+	+



6) $y = x^5 - 5x$ Intercepts $y=0$ $x=0$

$y' = 5x^4 - 5$

$= 5(x^4 - 1)$

crit values $= +1, -1$

local min: $1, -4$

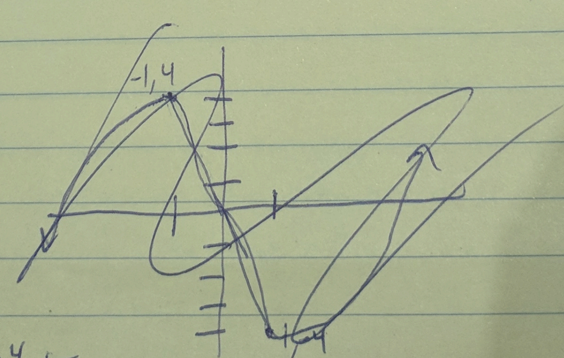
max: $-1, 4$

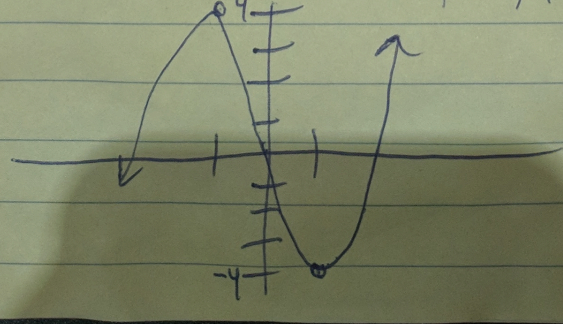
Intervals	5	x^4-1	y'
$-\infty, -1$	+	+	+
$-1, 1$	+	−	−
$1, \infty$	+	+	+

$y'' = 20x^3$

inflection $= (0,0)$

Interval	$20x^3$	y''
$-\infty, 0$	−	−
$0, \infty$	+	+





4a) $y=(1-x)e^x$ y int = 1 x int = 1

$y' = (1-x)e^x - e^x$

$= e^x(1-x-1) = e^x(-x)$

crit value : $x=0$ ~~but indeterminate $-\infty \cdot 0$ as $x \to -\infty$~~

~~using L'Hospital to eval this as $x \to 0$~~ ~~$\lim_{x\to 0^+} \frac{e^x}{-\frac{1}{x}} = \frac{e^x}{\frac{1}{x^2}} = -\infty$~~ ~~$\lim_{x\to 0} \frac{e^x}{-\frac{1}{x}} = \infty$~~

	e^x	$-x$	y'
$(-\infty, 0)$	+	+	+
$(0, \infty)$	+	−	−

local max = (0, 1)

$y'' = -e^x + -xe^x = e^x(-x-1)$

inf point : $x=-1$

	ex	$(-x-1)$	y''
$-\infty, -1$	+	+	+
$-1, \infty$	+	−	−