1) $m = 5$ force $= 25$

$x = 1 - 0.75 = 0.25$

$k(0.25) = 25$

$k = 100 = \frac{F}{x}$ diff

$5\frac{d^2x}{dt^2} + 100x = 0$

$5r^2 + 100 = 0$

$r = \pm\sqrt{\frac{-100}{5}} = \pm\sqrt{\frac{100}{5}}\, i$

$x = c_1 \sin\sqrt{20}\,t + c_2 \cos\sqrt{20}\,t$

$x(0) = 0.35$ $\quad x'(0) = 0$

$x'(t) = \sqrt{20}\,c_1 \cos\sqrt{20}\,t - \sqrt{20}\,c_2 \sin\sqrt{20}\,t$

so $c_2 = 0.35$ $\quad$ so $c_1 = 0$

so $\boxed{x = 0.35 \cos\sqrt{20}\,t}$

3) $m = 2$ kg damping constant $= 14$, force 6 stretched 0.5 beyond natural length

$k(0.5) = 6$

$k = 12$

$2\frac{d^2x}{dt^2} + 12x + 14\frac{dx}{dt} = 0$

aux $= 2r^2 + 14r + 12 = 0$

$(2r + 2)(r + 6) = 0$

$r = \{-6, -1\}$

solution: $y = c_1 e^{-6x} + c_2 e^{-x}$

$y' = -6c_1 e^{-6x} - c_2 e^{-x}$

if $y(0) = 1$ and $y'(0) = 0$

$c_1 + c_2 = 1$ $\quad -6c_1 - c_2 = 0$

$-5c_1 = 1$

$c_1 = -\frac{1}{5}$

$c_2 = \frac{6}{5}$

$\boxed{y = -\frac{1}{5}e^{-6x} + \frac{6}{5}e^{-x} \text{ where } x = \text{time}}$

$y = $ location

5) discriminant: $c^2 - 4mk = 0$ critical damping

$4mk = c^2$

$m = \frac{c^2}{4k} = \frac{14^2}{48} = \frac{196}{48} = 4.083$

13) $L\frac{dI}{dt} + RI + \frac{Q}{C} = E(t)$ $\quad I = \frac{dQ}{dt}$

$\frac{d^2Q}{dt^2} + 20\frac{dQ}{dt} + 500Q = 12$

aux: $r^2 + 20 + 500 = 0$

$r = \frac{-20 \pm \sqrt{400 - 2000}}{2} = -10 \pm \frac{\sqrt{-1600}}{2}$

$= -10 \pm 20i$

$Q = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t)$

(continued on back)

17) contined) since 13 is just a poly w/ degree 0, $Q_p$ is in form A

$Q_p = A \qquad Q'_p = 0 \qquad Q''_p = 0$

sub $0 + 20(0) + 500A = 12$

$A = \frac{12}{500} = \frac{6}{250} = \frac{3}{125}$

$Q =$ so general: $e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + \frac{3}{125}$

Since $Q(0) = 0$ and $I(0) = Q'(0) = 0$

$Q' = -10e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + e^{-10t}(20c_2 \cos 20t - 20c_1 \sin 20t)$

$Q'(0) = -10(c_1) + 1(+20c_2) \qquad 10c_1 = 20c_2$

$c_2 = \frac{c_1}{2}$

$Q(0) = 1(c_1) + \frac{3}{125} \qquad c_1 = \frac{-3}{125} \qquad$ so $c_2 = \frac{-3}{250}$

so Q given initial conditions which give value to $c_1$ and $c_2$:

$$\boxed{Q = e^{-10t}\left(\frac{-3}{125}\cos 20t + \frac{-3}{250}\sin 20t\right) + \frac{3}{125}}$$

$Q'$ is also $= e^{-10t}\left(\left(\frac{30}{125} + \frac{-30}{125}\right)\cos 20t + \left(\frac{60}{125} + \frac{30}{250}\right)\sin 20t\right.$

$$Q' = \boxed{e^{-10t}\left(\frac{150}{250}\right)\sin 20t}$$

17.4) a) $y' = xy$ can't use normal methods because x is not a constant

$y' - xy = 0$ assume solution of form $y = \sum_{n=0}^{\infty} c_n x^n$

$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$ $\qquad \cancel{y'' = \sum_{n=2}^{\infty}(n-1)n c_n x^{n-2} = \sum_{n=0}(n+2)(n+1)c_n x^{n}}$

substitute in derivatives

$\sum_{n=1}^{\infty} n c_n x^{n-1} - x\sum_{n=0}^{\infty} c_n x^n = 0$

$= c_1 + \sum_{n=2}^{\infty} n c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^{n+1} = 0$

$= c_1 + \sum_{n=0}^{\infty}(n+2)c_{n+2}x^{n+1} - \sum_{n=0}^{\infty} c_n x^{n+1} = 0 \qquad c_1 = 0$

(Continued on back)

$= c_1 + \sum_{n=0}^{\infty}\left((n+2)c_{n+2} - c_n\right)x^{n+1} = 0 \qquad$ so $(n+2)c_{n+2} - c_n = 0$

$(n+2)c_{n+2} = c_n$

2 continued) $C_1 = 0$

$C_{n+2} = \frac{C_n}{n+2}$

$C_0 = C_0$

$C_2 = \frac{C_0}{2}$

so $C_3 = \frac{C_1}{n+2} = \frac{C_1}{3} = 0$

$C_4 = \frac{C_2}{4} = \frac{C_0}{2\cdot 4}$

$C_5 = \frac{C_3}{5} = 0$

$C_6 = \frac{C_4}{6} = \frac{C_0}{6\cdot 4\cdot 2}$

$C_7 = \frac{C_5}{7} = 0$

$C_8 = \frac{C_6}{8} = \frac{C_0}{8\cdot 6\cdot 4\cdot 2}$

$C_{2n+1} = 0$

so $C_{2n} = \frac{C_0}{2^n \cdot n!}$

so no odd terms so $y = \sum_{n=0}^{\infty} \frac{C_0}{2^n \cdot n!} x^{2n}$

$$= C_0 \sum_{n=0}^{\infty} \frac{x^{2n}}{\frac{2^n}{n!}} = C_0 \sum_{n=0}^{\infty} \frac{\left(\frac{x^2}{2}\right)^n}{n!} = \boxed{C_0 e^{\frac{x^2}{2}}}$$

remember $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

4) $(x-3)y' + 2y = 0$

$y = \sum_{n=0}^{\infty} C_n x^n \qquad y' = \sum_{n=1}^{\infty} n C_n x^{n-1} = \sum_{n=0}^{\infty} n C_n x^{n-1}$

$$\sum_{n=0}^{\infty} n C_n x^n - 3\sum_{n=1}^{\infty} n C_n x^{n-1} + 2\sum_{n=0}^{\infty} C_n x^n$$

change $n=0$ to $n=1$

$$\cancel{\sum n C_n x^n - 3\sum_{n=1} n C_n x^{n-1}}$$

$$\sum_{n=0}^{\infty} n C_n x^n - 3\sum_{n=0}^{\infty} (n+1) C_{n+1} x^n + 2\sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left(n C_n - 3(n+1) C_{n+1} + 2C_n\right) x^n = 0$$

for this to be true, coeff has to be true

$C_n(n+2) = 3(n+1)C_{n+1}$

$\frac{C_n(n+2)}{3(n+1)} = C_{n+1}$

$= C_{n+1}$

so $C_0 = C_0$

$C_1 = \frac{2C_0}{3}$

$C_2 = \frac{3C_1}{6} = \frac{3\cdot 2 C_0}{6\cdot 3}$

$C_3 = \frac{4C_2}{9}$

$C_4 = \frac{5C_3}{12} = \frac{5\cdot 4\cdot C_0}{12\cdot 9}$

$C_n = \frac{(n+1)!\, C_0}{3\cdot 6\cdot 9 \cdots (3n)}$

$C_0 = C_0$

$\cancel{C_1 = \frac{C_0}{1}}$

$\cancel{C_2 = \frac{C_1}{2} = \frac{C_0}{2}}$

$\cancel{C_3 = \frac{C_2}{3} = \frac{C_0}{2\cdot 3}}$

$\cancel{C_4 = \frac{C_3}{4} = \frac{C_0}{4\cdot 3\cdot 2}}$

$\cancel{C_5 = \frac{C_4}{5} = \frac{C_0}{5!}}$

$\cancel{C_n = \frac{C_0}{n!}}$

$\cancel{y = \sum_{n=0}^{\infty} \frac{C_0}{n!} x^n}$

$$\boxed{y = \sum_{n=0}^{\infty} \frac{(n+1)!\, C_0}{3\cdot 6\cdot 9 \cdots 3n} x^n}$$

or $\boxed{\sum_{n=0}^{\infty} \frac{(n+1) C_0}{3^n} x^2}$

Remember: $12\cdot 9\cdot 6\cdot 3 = (3\cdot 4)\cdot(3\cdot 3)(3\cdot 2)$
$= n!\, 3^n$

5) $y'' + xy' + y = 0$ assume has solution $y = \sum_{n=0}^{\infty} C_n x^n$ find formula for coefficient $C_n$

so $y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$ $y'' = \sum_{n=2}^{\infty} (n-1) n C_n x^{n-2}$

subbing:

$$\sum_{n=2}^{\infty} (n)(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^n + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) C_{n+2} x^n + \sum_{n=0}^{\infty} n C_n x^n + \sum_{n=0}^{\infty} C_n x^n = 0$$

now $x^n$ ~~so~~ form so set coefficient of $x^n$ to 0

$$((n+2)(n+1) C_{n+2} + n C_n + C_n) x^n = 0$$

$$C_n(n+1)$$

$$C_{n+2} = \frac{-C_n(n+1)}{(n+2)(n+1)} = \frac{-C_n}{n+2}$$

$C_0 = C_0$ $C_1 = C_1$ $C_2 = \frac{-C_0}{2}$ $C_3 = \frac{-C_1}{3}$ $C_4 = \frac{-C_2}{4} = \frac{C_0}{4 \cdot 2}$

$C_5 = \frac{-C_3}{5} = \frac{C_1}{5 \cdot 3}$ $C_6 = \frac{-C_4}{6} = \frac{-C_0}{6 \cdot 4 \cdot 2}$

even: $C_{2n} = \frac{-1^{(n)} C_0}{2 \cdot 4 \cdot 6 \cdots (2n)}$ odd: $C_{2n+1} = \frac{-1^{n+1} C_1}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

$$\boxed{y = C_0 \left( \frac{-1^{(n)}}{2 \cdot 4 \cdot 6 \cdots (2n)} \right) x^{2n} + C_1 \left( \frac{-1^{n+1}}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \right) x^{2n-1}}$$

6) $y'' = y$ so assume $y = \sum_{n=0}^{\infty} C_n x^n$ $y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$ $y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) C_{n+2} x^n - \sum_{n=0}^{\infty} C_n x^n = 0 \rightarrow \sum_{n=0}^{\infty} \left((n+2)(n+1) C_{n+2} - C_n\right) x^n = 0$$

set coefficient to 0 $C_{n+2} = \frac{C_n}{(n+2)(n+1)}$

$C_0 = C_0$ $C_1 = C_1$

$C_2 = \frac{C_0}{2}$ $C_3 = \frac{C_1}{3 \cdot 2}$

$C_4 = \frac{C_2}{4 \cdot 3} = \frac{C_0}{4 \cdot 3 \cdot 2}$

$C_5 = \frac{C_3}{5 \cdot 4} = \frac{C_1}{5 \cdot 4 \cdot 3 \cdot 2}$

$C_6 = \frac{C_4}{6 \cdot 5} = \frac{C_0}{6!}$

even and odd behave differently

odd: $C_{2n+1} = \frac{C_1}{(2n+1)!}$

even: $C_{2n} = \frac{C_0}{(2n)!}$

odd terms $= C_1 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

even terms $= C_0 \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

so $\boxed{y = C_1 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} + C_0 \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}}$

7) $(x-1)y'' + y' = 0$ $y = \sum_{n=0}^{\infty} C_n x^n$ $y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$ $y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$

subbing in:

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-1} - \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} + \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$\sum_{n=1}^{\infty} (n+1)(n) C_{n+1} x^{n-1}$$

now run term off latter two so we all start at $n=2$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-1} - \sum_{n=2}^{\infty} (n+1)(n) C_{n+1} x^{n-1} + \sum_{n=2}^{\infty} n C_n x^{n-1} - 2C_2 + C_1 = 0$$

combine sums

$$\sum_{n=2}^{\infty} \left((n)(n-1) C_n - (n+1)(n) C_{n+1} + n C_n\right) x^{n-1} - 2C_2 + C_1 = 0$$

so matching coefficients!

$-2C_2 + C_1 = 0$

and $(n(n-1) C_n - (n+1)(n) C_{n+1} + n C_n = 0$

$= n^2 C_n - (n+1)(n) C_{n+1} = 0$

$C_{n+1} = \frac{n^2 C_n}{(n+1)n} = \frac{n C_n}{n+1}$

$C_0 = C_0$ $C_1 = C_1$

$C_2 = \frac{C_1}{2}$ $C_3 = \frac{2C_2}{3} = \frac{2C_1}{3 \cdot 2}$

$C_4 = \frac{3C_3}{4} = \frac{3 \cdot 2 \cdot C_1}{4!}$

$C_n = \frac{C_1}{n}$

7) continued $\boxed{y = C_0 + C_1 \sum_{n=1}^{\infty} \frac{x^n}{n}}$

$$y = \sum_{n=0}^{\infty} C_n x^n \qquad y' = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) C_n x^{n-2}$$

9) $y'' - xy' - y = 0$ , $y(0) = 1$ , $y'(0) = 0$

subbing

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=0}^{\infty} n C_n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} n+2(n+1) C_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) C_n x^n = 0$$

so coefficient of $x^n$ is 0

$$(n+2)(n+1) C_{n+2} - (n+1) C_n = 0$$

$$C_{n+2} = \frac{C_n(n+1)}{(n+2)(n+1)} = \frac{C_n}{(n+2)}$$

$C_0 = C_0$ $\quad C_1 = C_1$

$C_2 = \frac{C_0}{2}$ $\quad C_3 = \frac{C_1}{3}$

$C_4 = \frac{C_2}{4} = \frac{C_0}{4\cdot 2}$

$C_5 = \frac{C_3}{5} = \frac{C_1}{5\cdot 3}$

$C_6 = \frac{C_4}{6} = \frac{C_0}{6\cdot 4\cdot 2}$

even: $\frac{C_0}{2\cdot 4\cdot 6 \ldots 2n} = \frac{C_0}{2^n n!}$ $\qquad 2\cdot 4\cdot 6\cdot 8 = 2(1)\cdot 2(2)\cdot 2(3)\cdot 2(4) = 2^n n!$

odd: $\frac{C_1}{1\cdot 3\cdot 5 \ldots 2n+1} =$

$$\text{so } y = \sum_{n=0}^{\infty} \left(\frac{C_0}{2^n n!}\right) x^{2n} + \sum_{n=0}^{\infty} \left(\frac{C_1}{1\cdot 3\cdot 5 \ldots (2n+1)}\right) x^{2n+1}$$

ex: $C_1$ is the $n=0$ term for the odd. so $\frac{C_1}{2n+1} x^0 = C_1$

$$= y = \sum_{n=0}^{\infty} \left(\frac{C_0}{2^n n!}\right) x^{2n} + \sum_{n=0}^{\infty} \frac{C_1 \, x^{2n+1}}{(2n+1)!/2^n n!}$$

$$= \sum_{n=0}^{\infty} \frac{C_0 \left(\frac{x^2}{2}\right)^n}{n!}$$

$$= \boxed{C_0 e^{\frac{x^2}{2}} + \sum_{n=0}^{\infty} \frac{C_1 x^{2n+1} \cdot 2^n n!}{(2n+1)!}}$$

so when $y(0) = 1$, implies $C_0 = 1$

$$\sum_{n=0}^{\infty} \frac{C_1 x^{2n+1} \cdot 2^n n!}{(2n+1)!} = C_1 x + \frac{C_1 x^3 \cdot 2}{3}, \ldots$$

so $y'(0) = 0$ implies $C_1 = 0$

$$y' = \frac{2\cdot 2x}{2^2} C_0 e^{\frac{x^2}{2}} + C_1 = 0$$

0 when $x = 0$

so $C_1 = 0$

so $\boxed{y = \cdot e^{\frac{x^2}{2}}}$ given initial conditions