Practice Problems $\boxed{\text{Sec 3.2}}$

3. $f(x) = (3x^2 - 5x)e^x$

$f'(x) = (3x^2 - 5x)e^x + e^x(6x - 5)$

5) $y = \frac{x}{e^x}$ $\quad y' \quad \frac{e^x - x(e^x)}{e^{2x}}$

Module 5

Section 1.5

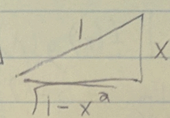
63) $\cos^{-1}(-1) = \pi$ $\qquad$ b) $\sin^{-1}(0.5) = \frac{\pi}{6}$

68) a) $\csc^{-1}\sqrt{2} = \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ $\qquad$ b) $\arcsin 1 = \frac{\pi}{2}$

70) $\tan(\sin^{-1}x) = \frac{\sin(\sin^{-1}x)}{\cos(\sin^{-1}x)} = \boxed{\frac{x}{\sqrt{1-x^2}}}$

so $\sin y = \sin x$

so $\sin y = x$



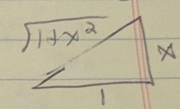
say $x = \frac{1}{2}$

$\sin^{-1}x = \frac{\pi}{6}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

71) $\sin(\tan^{-1}x)$ $\qquad$ so... $\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$



$\boxed{\frac{x}{\sqrt{1+x^2}}}$

How do we do this algebraically?

Section 3.4

2) $y = (2x^3 + 5)^4$ $\qquad$ $\frac{dy}{dx} = 4(2x^3+5)^3 \cdot 6x^2$

$u = 2x^3 + 5$ $\qquad$ $= \boxed{24x^2(2x^3+5)^3}$

$y = f(u) = u^4$

7) $F(x) = (5x^6 + 2x^3)^4$

$F'(x) = \boxed{4(5x^6 + 2x^3)^3 \cdot (30x^5 + 6x^2)}$

8) $F(x) = (1 + x + x^2)^{99}$

$F'(x) = 99(1 + x + x^2)^{98} \cdot (1 + 2x)$

9) $f(x) = \sqrt{5x+1}$ $\qquad$ $f'(x) = \frac{1}{2}(5x+1)^{-\frac{1}{2}} \cdot 5$

$= 2.5(5x+1)^{-\frac{1}{2}} = \boxed{\frac{2.5}{\sqrt{5x+1}}}$

17) $f(x)=(2x-3)^4(x^2+x+1)^5$

use product rule: $f'(x) = (2x-3)^4\cdot 5(x^2+x+1)^4\cdot(2x+1) + (x^2+x+1)^5\cdot 4(2x-3)^3\cdot(2)$

$= (2x-3)^4\cdot 5(x^2+x+1)^4\cdot(2x+1) + (x^2+x+1)^5\cdot 8(2x-3)^3$

$= (2x-3)^3(x^2+x+1)^4\left((2x-3)(5)(2x+1) + 8(x^2+x+1)\right)$

$= (2x-3)^3(x^2+x+1)^4\left((10x-15)(2x+1) + 8x^2+8x+8\right)$

$= \dots (20x^2+10x-30x-15+8x^2+8x+8)$

$= \dots (28x^2-12x-7)$

$\boxed{(2x-3)^3(x^2+x+1)^4(28x^2-12x-7)}$

18) $g(x)=(x^2+1)^3(x^2+2)^6$

$g'(x)=(x^2+1)^3\, 6(x^2+2)^5 + (x^2+2)^6\, 3(x^2+1)^2$

$=3(x^2+1)^2(x^2+2)^5\cdot\left(2(x^2+1) + (x^2+2)\right)$

$= \dots \cdot(2x^2+2+x^2+2)$

$= 3(x^2+1)^2(x^2+2)^5\cdot(3x^2+4)$

19) $h(t)=(t+1)^{\frac{2}{3}}(2t^2-1)^3$

$h'(t)=(t+1)^{2/3}\cdot\left(3\cdot(2t^2-1)^2\cdot 4t\right) + (2t^2-1)^3\cdot\frac{2}{3}(t+1)^{-\frac{1}{3}}\cdot 1$

$(t+1)^{\frac{2}{3}}\cdot\left(12t(2t^2-1)^2\right) + (2t^2-1)^3\cdot\frac{2}{3}(t+1)^{-\frac{1}{3}}$

$=\frac{2}{3}(t+1)^{-1/3}(2t^2-1)^2\left[(18t(t+1)) + (2t^2-1)\right]$

$=\boxed{\frac{2}{3}(t+1)^{-\frac{1}{3}}(2t^2-1)^2\left[20t^2+18t-1\right]}$

20) $F(t)=(3t-1)^4(2t+1)^{-3}$

$F'(t) = (3t-1)^4\cdot -3(2t+1)^{-4}\cdot 2 + (2t+1)^{-3}\cdot 4(3t-1)^3\cdot 3$

$=(3t-1)^4\cdot -6(2t+1)^{-4} + (2t+1)^{-3}\cdot 12(3t-1)^3$

$=-6(3t-1)^3(2t+1)^{-3}\left[(3t-1)(2t+1)^{-1} + (-2)\right]$

$=-6(3t-1)^3(2t+1)^{-3}\left[\left(\frac{3t+1}{2t+1}\right) -2\right]$

$=\frac{6(3t-1)^3}{(2t+1)^3}\left[2-\left(\frac{3t+1}{2t+1}\right)\right]$

$=\frac{6(3t-1)^3}{(2t+1)^3}\left[\frac{2(2t+1)}{2t+1}-\frac{3t+1}{2t+1}\right]$

$\dots \left[\frac{4t+2-3t+1}{2t+1}\right] \to \left[\frac{t+1}{2t+1}\right] = \boxed{\frac{6(3t-1)^3(t+1)}{(2t+1)^4}}$

24) $f(t) = 2^{t^3}$ or $u = t^3$ so $\frac{df}{dt} = 2^u \ln u 3t^2$

$f'(t) = 2^{t^3} \ln 2 \frac{d}{dt}(t^3)$ and replace $2^{t^3} \ln 2 \; 3t^2$

$= 2^{t^3} \ln 2 \; 3t^2$

28) $f(z) = e^{z/(z-1)}$

$f'(z) = e^{z/(z-1)} \frac{dy}{dz}(z/z-1) = e^{\frac{z}{z-1}} \cdot \frac{z-1-z}{(z-1)^2} = e^{\frac{z}{z-1}} \cdot \frac{-1}{(z-1)^2}$

36) $y = x^2 e^{-1/x}$

product rule and chain and quotient.

$y' = x^2 \frac{dy}{dx} e^{-1/x} + e^{-1/x} \frac{dy}{dx} x^2$

$= x^2 \frac{dy}{dx} e^{-1/x} + e^{-1/x} 2x$

$= x^2 e^{-1/x} \frac{dy}{dx} \frac{-1}{x} + e^{-1/x} 2x$

$= x^2 e^{-1/x} \frac{-1}{x^2} + e^{-1/x} 2x$

$= e^{-1/x} \cdot -1 + e^{-1/x} 2x$

$\boxed{\frac{1}{\sqrt[x]{e}}(-1 + 2x)}$

50) $y = e^{e^{x}}$

$y' = e^{e^x} \frac{dy}{dx} e^x$

$y' = e^{e^x} e^x$

$y'' = e^{e^x} \frac{dy}{dx} e^x + e^x \frac{dy}{dx} e^{e^x}$

$= e^{e^x} e^x + e^x e^{e^x} \frac{dy}{dx} e^x$

$= e^{e^x} e^x + e^x e^{e^x} e^x$

$= e^{e^x} e^x + e^{2x} e^{e^x} = e^{e^x}(e^x + e^{2x})$

52) $y = \sqrt{1 + x^3}$, $(2, 3)$

$y = (1 + x^3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1 + x^3)^{-\frac{1}{2}} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3+1}}$

slope at $x = 2$ $= \frac{1}{2} \cdot \frac{1}{\sqrt{9}} \cdot 12 = 2$

tan line $= y - 3 = 2(x-2) = y - 3 = 2x - 4$

$\boxed{y = 2x - 1}$

63) $h = f(g(x))$

$h' = f'(g(x)) \cdot g'(x)$

so if $h(x) = f\{(g(x)\}$ then $h'(1) = 5 \cdot 7 = 35$

b) $H(x) = g(f(x))$

so $H'(x) = g'f(x) \cdot f'(x)$ so $H'(1) = 9 \cdot 4 = 36$

a) $y = 4^x$

$\boxed{y' = 4^x \ln 4}$

b) $7^x = y$

$\boxed{y' = 7^x \ln 7}$

c) $3^{2x} = y$

$\boxed{y' = 3^{2x} \ln 3 \cdot 2}$

d) $5^{6x} = y$

$\boxed{y' = 5^{6x} \cdot \ln 5 \cdot 6}$

Section 3.5) find $\frac{dy}{dx}$ of:

9) $x^2 - 4xy + y^2 = 4$ $\quad 4\left(x\frac{dy}{dx} + y\right)$

$\frac{dy}{dx}x^2 - \frac{dy}{dx}4xy + \frac{dy}{dx}y^2 = 0 \Rightarrow 2x - \cancel{(\ )} + (2y)\frac{dy}{dx}$

$\Rightarrow x - 2x\frac{dy}{dx} + 2y + y\frac{dy}{dx} = 0$

$y\frac{dy}{dx} - 2x\frac{dy}{dx} = -2y - x$

$\frac{dy}{dx}(y - 2x) = -2y - x$

$\frac{dy}{dx} = \boxed{\frac{-2y - x}{y - 2x}}$

10) $xe^y = x - y$

$1 - \frac{dy}{dx} = xe^y\frac{dy}{dx} + e^y$

$1 - e^y = xe^y\frac{dy}{dx} + \frac{dy}{dx}$

$1 - e^y = \frac{dy}{dx}(xe^y + 1)$

$\boxed{\frac{1 - e^y}{xe^y + 1} = \frac{dy}{dx}}$

11) $y\cos x = x^2 + y^2$

$y(-\sin x) + \cos x\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} \Rightarrow 2y\frac{dy}{dx} - \cos x\frac{dy}{dx} = y\sin x - 2x$

$\frac{dy}{dx}(2y - \cos x) = y\sin x - 2x$

$\boxed{\frac{dy}{dx} = \frac{y\sin x - 2x}{2y - \cos x}}$

12) $\cos(xy) = 1 + \sin y$

$-\sin(xy)\cdot\left(x\frac{dy}{dx} + y\right) = \cos y\frac{dy}{dx}$

$x\cdot -\sin xy\frac{dy}{dx} - y(\sin xy) = \cos y\frac{dy}{dx}$

$-y(\sin xy) = \cos y\frac{dy}{dx} + \sin xy\frac{dy}{dx}\cdot x$

$\frac{dy}{dx}(\cos y + x\sin xy)$

$\boxed{\frac{-y(\sin xy)}{\cos y + (\sin xy)\cdot x}} = \frac{dy}{dx}$

26) $\sin(x+y) = 2x - 2y \Rightarrow \cos(x+y)\left(1 + \frac{dy}{dx}\right) = 2 - \left(2\frac{dy}{dx}\right)$

$\Rightarrow \cos(x+y) + \cos(x+y)\frac{dy}{dx} = 2 - \left(2\frac{dy}{dx}\right)$

$\Rightarrow \cos(x+y) - 2 = -\cos(x+y)\frac{dy}{dx} - 2\frac{dy}{dx}$

$= \frac{dy}{dx}(-\cos(x+y) - 2) = \cos(x+y) - 2$

$$\frac{dy}{dx} = \boxed{\frac{\cos(x+y)-2}{-\cos(x+y)-2}}$$

So at (π, π), tangent $= \frac{\cos(2\pi)-2}{\cos(2\pi)-2} = -1$

$y - \pi = -1(x - \pi)$

$y - \pi = -x + \pi$

$\boxed{y = -x + 2\pi}$

33(a) $y^2 = 5x^4 - x^2$

$2y\frac{dy}{dx} = 20x^3 - 2x$

$\frac{dy}{dx} = \frac{20x^3 - 2x}{2y}$ slope at 1,2 $= \frac{20-2}{4} = 4.5$

tangent line $= y - 2 = \frac{9}{2}(x-1)$

$y = \frac{9}{2}x - \frac{9}{2} + 2$

$\boxed{y = \frac{9}{2}x - \frac{5}{2}}$

35) Find y''

$x^2 + 4y^2 = 4$

$y' = 2x + 4 \cdot 2y \cdot \frac{dy}{dx} = 0$

$= 2x + 8y\frac{dy}{dx} = 0 \Rightarrow 8y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{8y} = \frac{-x}{4y}$

$y'' = \frac{-4y - (-x \cdot 4\frac{dy}{dx})}{16y^2} = \frac{-4y + (4x\frac{dy}{dx})}{16y^2} = \frac{-4y + (\frac{-4x^2}{4y})}{16y^2}$

$= \frac{-16y^2 + -4x^2}{64y^3} = \boxed{\frac{-4y^2 - x^2}{16y^3}}$

50) $y = \tan^{-1}(x^2)$ $\tan y = x^2$ $\cos^2 x + \sin^2 x = 1$

$\sec^2 y \frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{\sec^2 y} = \frac{2x}{1 + \tan^2 y} = \boxed{\frac{2x}{1+x^4}}$

51) $y = \sin^{-1}(2x+1)$

$\sin y = 2x+1$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\cos y \frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{\cos y} = \frac{2}{\sqrt{1-\sin^2 y}} = \frac{2}{\sqrt{1-(2x+1)^2}} = \frac{2}{\sqrt{1-4x^2+4x+1}} = \boxed{\frac{2}{\sqrt{4(-x^2+x)}}}$

$\boxed{\frac{1}{\sqrt{-x^2+x}}}$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

74a) First find tangent line:

$x^2 - xy + y^2 = 3$

$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + y$

or better: $2x - y = x\frac{dy}{dx} - 2y\frac{dy}{dx}$

$2x - y = \frac{dy}{dx}(x - 2y)$

$\frac{2x - y}{x - 2y} = \frac{dy}{dx}$ so at $(-1, 1)$, slope: $\frac{-3}{-3}$ or 1

normal line $= (y - 1) = -(x + 1)$ so slope of normal $= -1$

$y = -x$

Plug: $x^2 + x^2 + x^2 = 3$

$3x^2 = 3$

$x^2 = 1$ $x = \pm 1$ so another pt is $\boxed{(1, -2)}$

Plug $1 + y + y^2 = 3$

$y + y^2 = 2$ or $y^2 + y - 2 = 0$

$\cancel{y(y+1) = 2}$ $(y - 1)(y + 2)$