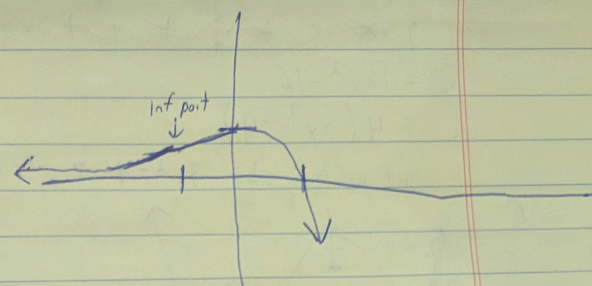
$(0, \infty)$ + | − | −

local max $= (0, 1)$

$y'' = -e^x + -xe^x = e^x(-x - 1)$

inf point: $x = -1$

	e^x	$(-x-1)$	y''
$-\infty, -1$	+	+	+
$-1, \infty$	+	−	−



Module 9

Section 4.7

8) $l \cdot w = 1000\,m^2$ since $l \cdot w = A$, $w = \frac{A}{l}$ so $L \cdot \frac{1000\,m^2}{L} = 1000\,m^2$

Trying to minimize perimeter which is $P(L) = 2L + 2\left(\frac{1000\,m^2}{L}\right)$ where $0 \le L \le \infty$

$P(L) = 2L + 2\left(\frac{1000}{L}\right)$

$P'(L) = 2 + 2\left(\frac{-1000}{L^2}\right)$ ← equals 0 when $-1 = \frac{-1000}{L^2}$ $L^2 = 1000$

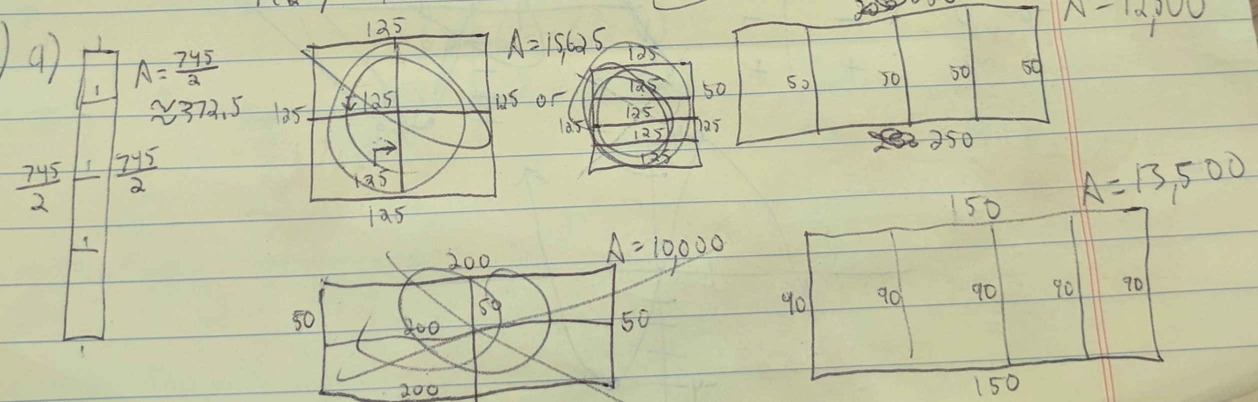
$L = 10\sqrt{10}$

So minimized perimeter but $A = 1000$ when $\boxed{L = 10\sqrt{10} \text{ and } W = \frac{100}{\sqrt{10}}}$

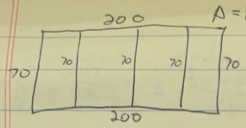
Quick check: $P(5) = 10 + 400 = 410$ $P(10\sqrt{10}) = 20\sqrt{10} + \frac{200}{\sqrt{10}} \approx 126$

$P(20) = 40 + 100 = 140$ $\boxed{W \text{ also} = 10\sqrt{10}}$ so square!

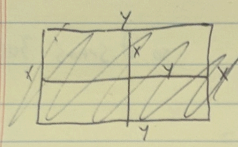
11) a)



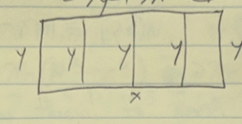
$A = 14000$

a) 

seems like max area is the ~~square~~ pen with $x \approx 200$ and $y \approx 70$

b) 

~~$3y + 3x = 750$~~ $2x + 5y = 750$



or $5y = 750 - 2x$

$y = \frac{750 - 2x}{5}$

$y = \frac{750 - 2x}{5}$

c) $x \cdot y = A$

d) $2x + 5y = 750$

$y = \frac{750 - 2x}{5}$

e) $x \cdot \frac{750 - 2x}{5} = A$

$x \cdot (150 - 0.4x) = A$

$x \cdot -0.4 + (150 - 0.4x) = A'$

$-0.4x + 150 - 0.4x = A'$

$150 - 0.8x = A'$

crit values:

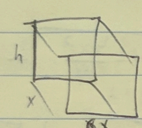
$150 - 0.8x = 0$

$-0.8x = -150$

$x = \frac{-150}{-0.8}$

$\boxed{x = 187.5 \quad y = 75}$

close!

14) 

$x^2 h = 32000$ cm   minimize perimeter

$h = \frac{32000}{x^2}$

$\text{side} = x \cdot \frac{32000}{x^2} = \frac{32000}{x} = \frac{32000}{x}$

$\text{perimeter} = \frac{4 \cdot 32000}{x} + x^2 = \frac{128000}{x} + x^2$ so $P' = \frac{-128000}{x^2} + 2x$

$P' = 0$ when

$-128000 = -2x \cdot x^2$

$-128000 = -2x^3$

$64000 = x^3 \rightarrow x = 40$

crit number: $\boxed{x = 40 \quad \text{so } h = 20}$

$0 < x < \infty$

$P(40) = 3200 + 1600 = 4800$

$P(80)$ when $h = 5$: 8000 so it seems P is minimized at $x = 40$

$P(0) = 0$ $P(2) = 64004$

15) $x^2 + 4xh = 1200 = P$ maximize $x^2 h = V$

$4xh = 1200 - x^2$

$\frac{1200 - x^2}{4x} = h$

so $x^2 \cdot \frac{1200 - x^2}{4x} = V$

if you want to get ...

$x\frac{(1200 - x^2)}{4} = V$

$\boxed{V' = 300 - \frac{3x^2}{4}} = \frac{-3x^2 + 1200}{4}$ $\frac{1}{4}\left(\frac{d}{dx}(x(1200 - x^2))\right) = V'$

critical values: $x = 20$

so max volume $= 20 \cdot h$ and $h = \frac{1200 - x^2}{4x} = \frac{800}{80} = 10$

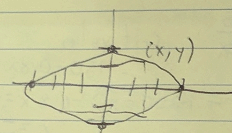
$\boxed{= 300}$

Quick check $V(0) = 0$ $V(15) = 243.75$ so it seems 300 is a max

$V(34) \approx 0$ $V(25) = 143.75$

26) $a = 3$, $b = 2$

$1 = \frac{x^2}{9} + \frac{y^2}{4} = 1$ Area of largest rectangle



Area $= 2x\,2y = 4xy$

also $\frac{y^2}{4} = 1 - \frac{x^2}{9}$ $y^2 = 4 - \frac{4x^2}{9} \rightarrow y = \pm\sqrt{4 - \frac{4x^2}{9}}$

$\text{Area}(x) = 4x\sqrt{4 - \frac{4x^2}{9}}$

$A' = \ldots + \sqrt{4 - \frac{4x^2}{9}} \cdot 4$ $4x \frac{d}{dx}\sqrt{4 - \frac{4x^2}{9}} + 4 \cdot \sqrt{4 - 4x^2/9}$

$\left(4 - \frac{4x^2}{9}\right)^{-1/2} \cdot \frac{9 \cdot 8x}{81}$

$\frac{d}{dx}\sqrt{4 - \frac{4x^2}{9}} = \frac{1}{2}\left(4 - \frac{4x^2}{9}\right)^{-1/2} \cdot \frac{d}{dx}\frac{4x^2}{9}$

$= \frac{1}{2\sqrt{4 - \frac{4x^2}{9}}} \cdot \frac{9 \cdot 8x}{81}$

$\frac{1}{2\sqrt{4 - \frac{4x^2}{9}}} \cdot \frac{8x}{9}$

THIS IS TOO HARD SO...

$\text{Area}^2 = 16x^2 \cdot \left(4 - \frac{4x^2}{9}\right)$

$= 64x^2 - \frac{64x^4}{9}$

$A^{2\prime} = 128x - \frac{256x^3}{9} = 128x\left(1 - \frac{2x^2}{9}\right)$

$\frac{2x^2}{9} = 1$ $2x^2 = 9$

crit value $= 0, \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}$

so $x = \frac{3}{2}$, $y = \ldots$, $A = \ldots = 2.59$

if $x = 1$, $y = \sqrt{\frac{32}{9}}$, $A = \frac{\sqrt{32}}{\ldots} = 1.88$

if $x = 2$, $y = \sqrt{\frac{20}{9}}$, $A = 2\sqrt{\frac{20}{9}} =$

$2.121 = x = \frac{3}{\sqrt{2}}$ $y = \sqrt{2}$

$x = 1$ $y = \sqrt{\frac{32}{9}}$ $A = \sqrt{\frac{32}{9}} = 1.88$

$x = 2$ $y = \sqrt{\frac{20}{9}}$ $A = 2.98$

so it seems like $\boxed{x = \frac{3}{\sqrt{2}} \quad y = \sqrt{2} \text{ is the biggest rectangle}}$

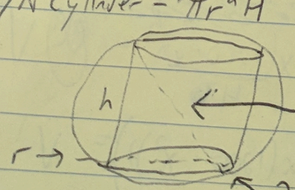
$x = 3$ $y = 0$ $A = 0$

$x = 2.5$ $y = 1.1055$ $A = 2.7638$

$\boxed{\text{Area} = 12}$ $\boxed{\text{Length} = \frac{6}{\sqrt{2}} \quad \text{Width} = 2\sqrt{2}}$

31) A of cylinder $= \pi r^2 H$ so lets try to redefine this w/out using H.

and $h^2 + 4r^2 = 4R^2$



$h^2 = 4R^2 - 4r^2$

$\boxed{\frac{h^2}{4} = R^2 - r^2}$ or $\longrightarrow$ (next page)

0

Volume of cylinder is $V = 2\pi r^2\sqrt{R^2-r^2}$

$h = 2\sqrt{R^2-r^2}$

$$V' = 4\pi r\sqrt{R^2-r^2} + 2\pi r^2 \frac{d}{dx}\sqrt{R^2-r^2}$$

$$= 4\pi r\sqrt{R^2-r^2} - \frac{2\pi r^3}{\sqrt{R^2-r^2}}$$

$$= 4\pi r(R^2-r^2) - 2\pi r^3$$

crit values: $= 4\pi r R^2 - 4\pi r^3 - 2\pi r^3 = 4\pi r R^2 - 6\pi r^3$

$4\pi r R^2 - 6\pi r^3 = 0 \rightarrow 4\pi r R^2 = 6\pi r^3$

~~$r(4\pi R^2 - 2\pi r^2)$~~

$4\pi R^2 = 6\pi r^2$

$\frac{4\pi R^2}{6\pi} = r^2$

$\frac{2}{3}R^2 = r^2 \rightarrow r = \sqrt{\frac{2}{3}}R$

So largest possible volume of cylinder is

$$= 2\pi\left(\sqrt{\tfrac{2}{3}}R\right)^2\sqrt{R^2-\left(\sqrt{\tfrac{2}{3}}R\right)^2}$$

$$= 2\pi\left(\tfrac{2}{3}R^2\right)\sqrt{R^2-\tfrac{2}{3}R^2} = \frac{4\pi}{3}R^2 \times \sqrt{\frac{R^2}{3}} = \boxed{\frac{4\pi R^3}{3\sqrt{3}}}$$

59) a) Avg cost $= \frac{C(x)}{x}$ Marg cost $= C'(x)$

Avg cost' $= \frac{x \cdot C'(x) - C(x)}{x^2}$ $\rightarrow x C'(x) - C(x) = 0$

$= \frac{C'(x)}{x} - \frac{C(x)}{x^2}$ $= x C'(x) = C(x)$

$= \frac{xC'(x)}{x^2} - \frac{C(x)}{x^2} = 0$ when

$$\boxed{\begin{array}{l} C'(x) = \frac{C(x)}{x} \\ \text{Marg Cost} = \text{Avg cost} \end{array}}$$

b) i) $C(x) = 16000 + 200x + 4x^{3/2}$ $C(1000) = 342491.11$

Avg Cost$(1000) = 342491.11/1000 = 342.49$

Marg cost function $= 200 + 6x^{\frac{1}{2}}$

$M(1000) = 389.74$

ii) $A(x) = \frac{16000 + 200x + 4x^{3/2}}{x}$

$$A'(x) = \frac{x(200+6x^{\frac{1}{2}}) - (16000+200x+4x^{3/2})}{x^2} = \frac{200x + 6x^{\frac{3}{2}} - (16000 + 200x + 4x^{3/2})}{x^2}$$

$$= \frac{2x^{3/2} - 16000}{x^2}$$

$= 0$ when $2x^{\frac{3}{2}} = 16000 \rightarrow x^{\frac{3}{2}} = 8000$

$x = \sqrt[3]{8000}^{\,2} = 400$

iii) 320

$x_3 = 1.5555 - \frac{f(1.5555)}{f'(1.5555)} \approx 1.5315$

60) a) $P(x) = R(x) - C(x)$

$P'(x) = R'(x) - C'(x)$

At max if $P'(x) = 0$ so $R'(x) - C'(x) = 0 \rightarrow R'(x) = C'(x)$

b) $R(x) = 1700x - 7x^2$

$R'(x) = 1700 - 14x$

$C'(x) = 500 - 3.2x + 0.012x^2$

When $500 - 3.2x + 0.012x^2 = 1700 - 14x$, profit is max

$10.8x + 0.012x^2 = 1200$

$0.012x^2 + 10.8x - 1200 = 0$

$\frac{-10.8 \pm \sqrt{10.8^2 - 4(0.012)(-1200)}}{0.024} = 100 = x$

$\boxed{A\ \ x = 100,\ p(100) = 1000}$

61) for a \$2 dec in price, attendance ↑ 5000

so \$1 ↓, 3000 attendance ↑ (on back)

Max price when attendance = 0 $\quad 0 = 55000 - 3000x$

$x = 22$

$p(x) = 22 - \frac{1}{2500}(x)$

$R(x) = 22x - \frac{1}{2500}x^2$

$R'(x) = 22 - \frac{1}{1250}x \qquad R'(x) = 0$ when $22 = \frac{1}{1250}x$

$27500 = x$

so $p(x)$ should be $p(27500) = \boxed{11}$

$\boxed{\text{Section 4.8}}$

6) $f(x) = 2x^3 - 3x^2 + 2 = 0$, $x_1 = -1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad f'(x) = 6x^2 - 6x$

$x_2 = -1 - \frac{-2-3+2}{12} = -1 - \frac{-3}{12} = \frac{-3}{4}$

$x_3 = \ldots \approx -0.6825$

$= \frac{-3}{4} - \frac{f(\frac{-3}{4})}{f'(\frac{-3}{4})} \approx -0.6825$

7) $\frac{2}{x} - x^2 + 1 = 0$, $x_1 = 2$

$2x^{-1} - x^2 + 1 \qquad f'(x) = -2x^{-2} - 2x$

$x_2 = 2 - \frac{1-4+1}{-0.5-4}$

$= 2 - \frac{-2}{-4.5} = 1.5555$

$x_3 = 1.555 - \frac{f(1.555)}{f'(1.555)} \approx 1.5215$

$\boxed{\text{Section 4.9}}$

7) $7\frac{x^{7/5}}{7/5} + 8\frac{x^{1/5}}{1/5} + C$

8) $x^{3.4} - 2x^{\sqrt{2}-1} \rightarrow$ antideriv $= \boxed{\frac{x^{4.4}}{4.4} - 2\frac{x^{\sqrt{2}}}{\sqrt{2}} + C}$

Quick check of $\frac{d}{dx} 2\frac{x^{\sqrt{2}}}{\sqrt{2}} = 2x^{\sqrt{2}-1}$ $\quad \frac{\sqrt{2} \cdot 2\sqrt{2}\, x^{\sqrt{2}-1}}{2} = 2x^{\sqrt{2}-1}$ ✓

10) $e^2 = f(x)$

antideriv: $e^2 x + C$

12) $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$

$= x^{2\cdot\frac{1}{3}} + x\,x^{\frac{1}{2}} = x^{\frac{2}{3}} + x^{\frac{3}{2}}$ $\quad$ antideriv: $\frac{x^{5/3}}{5/3} + \frac{x^{5/2}}{5/2}$

antideriv: $\boxed{\frac{3}{5}x^{\frac{5}{3}} + \frac{2}{5}x^{\frac{5}{2}} + C}$

16) $r(\theta) = \sec\theta\tan\theta - 2e^{\theta}$

antideriv: $\sec\theta - 2e^{\theta}$

17) $h(\theta) = 2\sin\theta - \sec^2\theta$

$-2\cos\theta - \tan\theta$: antiderivative

25) $f''(x) = 20x^3 - 12x^2 + 6x$

$f'(x) = 20\frac{x^4}{4} - 12\frac{x^3}{3} + 6\frac{x^2}{2} = 5x^4 - 4x^3 + 3x^2 + C$

$f(x) = 5\frac{x^5}{5} - 4\frac{x^4}{4} + 3\frac{x^3}{3} = \boxed{x^5 - x^4 + x^3 + Cx + D}$

26) $f''(x) = x^6 - 4x^4 + x + 1$

$f'(x) = \frac{x^7}{7} - 4\frac{x^5}{5} + \frac{x^2}{2} + x + C$

$f(x) = \frac{1}{7}\frac{x^8}{8} - 4\frac{x^6}{6} + \frac{1}{2}\frac{x^3}{3} + \frac{x^2}{2} + Cx + D$

$\boxed{= \frac{1}{56}x^8 - \frac{2}{3}x^6 + \frac{1}{6}x^3 + \frac{1}{2}x^2 + Cx + D}$

28) $f''(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -x^{-1} = -\frac{1}{x} + C$

$\boxed{f(x) = -\ln x} \boxed{+Cx + D}$

32) $f'(x) = 5x^4 - 3x^2 + 4$, $f(-1) = 2$

$f(x) = 5\frac{x^5}{5} - 3\frac{x^3}{3} + 4x = x^5 - x^3 + 4x + C$

$f(-1) = 2$ so $-1 + 1 - 4 + C = 2$

$C = 6$

so $\boxed{f(x) = x^5 - x^3 + 4x + 6}$

40) $f''(x) = 8x^3 + 5$, $f(1) = 0$, $f'(1) = 8$

$f'(x) = 8\frac{x^4}{4} + 5x = 2x^4 + 5x + C$

since $f'(1) = 8 \rightarrow 2 + 5 + C = 8$ so $C = 1$

$f'(x) = 2x^4 + 5x + 1$

$f(x) = 2\frac{x^5}{5} + 5\frac{x^2}{2} + x = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + D$

since $f(1) = 0 \rightarrow \frac{2}{5} + \frac{5}{2} + 1 + D = 0$

$\frac{29}{10} + 1 + D = 0$

$\frac{39}{10} + D = 0$

$D = \frac{-39}{10}$

$\boxed{f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}}$

61) each Δ \$1 $\downarrow$, attendence $\uparrow$ 3000

Section 4.7

$y - 27000 = -3000(x - 10)$

$y - 27000 = -3000x + 30000 \rightarrow 3000x = 57000$

$x = 19$

a) $\boxed{p(x) = 19 - \frac{x}{3000}}$ is the demand function

check $p(27000) = 10$ ✓

$p(33000) = 8$

$R(x) = 19x - \frac{x^2}{3000}$

$= 19x - \frac{1}{3000}x^2$

$R'(x) = 19 - \frac{1}{1500}x$

$R'(x) = 0$ when $x = 28500$

$R''(x) = \frac{-1}{1500}$ negative so $P(28500)$ is a max.

$\boxed{= 9.5}$

max revenue price