Module 11

$\boxed{\text{Section 5.4}}$

2) $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

$\frac{d}{dx}\left(\frac{1}{2}x + \frac{1}{4}\sin 2x + C\right) = \frac{1}{2} + \frac{1}{4}\cos 2x \cdot 2$

$= \frac{1}{2} + \frac{1}{2}\cos 2x \quad = \frac{1}{2} + \frac{1}{2}(2\cos^2 x - 1)$

$= \frac{1}{2} + \cos^2 x - \frac{1}{2} = \cos^2 x$

6) $\int \sqrt[4]{x^5}\,dx = \int x^{5\cdot\frac{1}{4}}\,dx = \int x^{\frac{5}{4}}\,dx = \boxed{\frac{4}{9}x^{\frac{9}{4}} + C}$

8) $\int \left(v^6 - 2v^5 - v^3 + \frac{2}{7}\right)dv = \int v^6 dx - 2\int v^5 dx - \int v^3 dx + \int \frac{2}{7}dx$

$\boxed{= \frac{v^7}{7} - \frac{1}{3}v^6 - \frac{v^4}{4} + C}$

12) $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right)dx = \int x^2 dx + \int 1\,dx + \int (x^2+1)^{-1}dx$

$= \frac{x^3}{3} + x + \tan^{-1} x + C$

16) $\int \sec t\,(\sec t + \tan t)\,dt = \int \frac{1}{\cos t}\left(\frac{1}{\cos t} + \frac{\sin t}{\cos t}\right)dx$

$= \cancel{\int \left(\frac{1}{\cos^2 t} + \frac{\sin t}{\cos^2 t}\right)}$

$= \int \sec^2 t + \sec t \tan t\,dt = \int \sec^2 t\,dt + \int \sec t \tan t\,dt =$

$\boxed{\tan t + \sec t + C}$

22) $\int_1^2 (4x^3 - 3x^2 + 2x)\,dx$

$= 4\frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2}\Big]_1^2 = x^4 - x^3 + x^2\Big]_1^2 = (16 - 8 + 4) - (1 - 1 + 1)$

$= 12 - (1) = \boxed{11}$

27) $\int_0^\pi (5e^x + 3\sin x)\,dx =$

$5e^x - 3\cos x\Big]_0^\pi = 5e^\pi + 3 - (5 - 3) = \boxed{5e^\pi + 1}$

28) $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3}\right)dx = \int_1^2 x^{-2} - 4x^{-3}\,dx = -x^{-1} - \frac{4x^{-2}}{-2}\Big]_1^2 = -x^{-1} + 2x^{-2}\Big]_1^2$

$= -\frac{1}{2} + \frac{1}{2} - (-1 + 2) = \boxed{1}$

35) $\int_0^1 (x^{10} + 10^x)\,dx$

$\frac{x^{11}}{11} + \frac{10^x}{\ln 10}\Big]_0^1 = \frac{1}{11} + \frac{10}{\ln 10} - \left(0 + \frac{1}{\ln 10}\right) = \boxed{\frac{1}{11} + \frac{9}{\ln 10}}$

37) $\int_0^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} \tan^2\theta\,d\theta$ $\frac{d}{dt}\tan t = \sec^2 t = 1 + \tan t$

$= -\theta + \tan\theta\Big]_0^{\pi/4} = \boxed{\frac{-\pi}{4} + 1}$

51) It represents the difference in size of the child b/w year 10 and year 5. It is the net change from year 5 and year 10.

53) It is the net change in oil gallons in 2 hrs. $r(t)$ can be several different numbers between minute 0 and 120. The integral $\int_0^{120} r(t)\,dt$ represents the total oil lost in those 2 hrs.

54) $100 + \int_0^{15} n'(t)\,dt$ represents the total population of honey bees at time 15 (15 weeks?). 100 is the base population and $\int_0^{15} n'(t)\,dt$ is the net change so $100 + \int_0^{15} n'(t)\,dt$ is the total population at $t = 15$

55) If $R'(x)$ = marginal revenue then $\int_{1000}^{5000} R'(x)\,dx$ = total revenue of goods 1000 to 5000. These goods could sell for different prices.

64) $\int_0^{10} 200 - 4t\,dt = 200t - 4\frac{t^2}{2}\Big]_0^{10} = 200t - 2t^2\Big]_0^{10}$

$= 2000 - 200 = \boxed{1800}$

67) I assume we want to find the ↑ in total cost, not marginal cost?

$C'(x)$ = marginal cost = $3 - 0.01x + 0.000006x^2$

$\int_{2000}^{4000} 3 - 0.01x + 0.000006x^2\,dx = 3x - 0.01\frac{x^2}{2} + 0.000006\frac{x^3}{3}\Big]_{2000}^{4000}$

$= 12000 - 80000 + 128000 - (6000 - 20000 + 16000) = \boxed{58000}$

Section 5.5

3) $du = 3x^2\,dx$ so $\frac{du}{3} = x^2\,dx$ so $\int \frac{1}{3}\sqrt{u}\,du$

$= \frac{1}{3}\int u^{\frac{1}{2}}\,du$

$= \frac{2}{9} u^{\frac{3}{2}} = \boxed{\frac{2}{9}(x^3+1)^{\frac{3}{2}}} \boxed{+C}$

check: $\frac{d}{dx}\frac{2}{9}(x^3+1)^{\frac{3}{2}} = \frac{1}{3}(x^3+1)^{\frac{1}{2}} 3x^2 = x^2\sqrt{x^3+1}$

4) $\int \sin^2\theta \cos\theta \, d\theta$, $u = \sin\theta$ $\quad \frac{du}{dx} = \cos\theta$ so $du = \cos\theta \, dx$

$= \int u^2 \, du = \frac{u^3}{3} = \boxed{\frac{\sin^3\theta}{3} + C}$

8) $\int x^2 e^{x^3} dx$, $u = x^3$ so $\frac{du}{dx} = 3x^2$ so $du = 3x^2 dx$ so $\frac{du}{3} = x^2 dx$

$= \frac{1}{3}\int e^u \, du = \frac{1}{3} e^u \boxed{\frac{1}{3} e^{x^3} + C}$

12) $\int \sec^2 2\theta \, d\theta$ $\quad u = 2\theta$ $\quad \frac{du}{d\theta} = 2$ so $du = 2 d\theta$ so $\frac{du}{2} = d\theta$

$= \frac{1}{2}\int \sec^2 u \, du = \frac{1}{2}\tan u = \boxed{\frac{1}{2}\tan 2\theta + C}$

13) $\int \frac{dx}{5-3x}$, $u = 5-3x$ $\quad \frac{du}{dx} = -3$ so $du = -3dx$ so $\frac{du}{-3} = dx$

$= \int \frac{1}{5-3x} dx = \frac{-1}{3}\int \frac{1}{u} du = \frac{-1}{3}\ln|u| = \boxed{\frac{-1}{3}\ln|5-3x| + C}$

14) $\int y^2 (4-y^3)^{2/3} dy$, $u = 4-y^3$ $\quad \frac{du}{dy} = -3y^2$ so $du = -3y^2 dy$ so $\frac{du}{-3} = y^2 dy$

$= \frac{-1}{3}\int u^{2/3} du = \frac{-1}{3} \cdot \frac{3}{5} u^{5/3} = \boxed{\frac{-3}{15}(4-y^3)^{5/3} + C}$

17) $\int \frac{e^v}{(1-e^v)^2} dv$, $V = 1-e^v$ $\quad \frac{dV}{dv} = -e^v$ so $dV = -e^v dv$ so $-dV = e^v dv$

$= -\int \frac{1}{V^2} dV = -\int V^{-2} dV = V^{-1} = \boxed{\frac{1}{1-e^v} + C}$

20) $\int \frac{z^2}{z^3+1} dz$, $u = z^3+1$ so $\frac{du}{dz} = 3z^2$ so $du = 3z^2 dz$ so $\frac{du}{3} = z^2 dz$

$= \frac{1}{3}\int \frac{1}{u} du = \frac{1}{3}\ln|u| = \boxed{\frac{1}{3}\ln|z^3+1| + C}$

21) $\int \frac{(\ln x)^2}{x} dx$ $\quad u = \ln x$ so $\frac{du}{dx} = \frac{1}{x}$ so $du = \frac{1}{x} dx$

$= \int u^2 \, du = \frac{u^3}{3} = \boxed{\frac{(\ln x)^3}{3} + C}$

23) $\int \sec^2\theta \tan^3\theta \, d\theta$, $u = \tan\theta$ so $\frac{du}{d\theta} = \sec^2\theta$ so $du = \sec^2\theta \, d\theta$

$\int u^3 du = \frac{u^4}{4} = \boxed{\frac{\tan^4\theta}{4} + C}$

26) $\int \frac{dx}{ax+b}$ $(a \neq 0)$ $u = ax+b$ so $\frac{du}{dx} = a$ so $du = a\,dx$ so $\frac{du}{a} = dx$

$= \frac{1}{a}\int \frac{1}{u}\,du = \frac{1}{a}\ln|u| = \boxed{\frac{1}{a}\ln|ax+b| + C}$

28) $\int e^{\cos t}\sin t\,dx$, $u = \cos t$ $\frac{du}{dt} = -\sin t$ $du = -\sin t\,dt$

$= -\int e^u\,du = \boxed{-e^{\cos t} + C}$

44) $\int \frac{x}{1+x^4}\,dx$ ~~$u = 1+x^4$ $\frac{du}{dx} = 4x^3$ $du = 4x^3dx$ $\frac{du}{4} = x^3dx$~~

~~$= \frac{1}{4}\int \frac{1}{u}$~~ $u = x^2$ so $\frac{du}{dx} = 2x$ so $du = 2x\,dx$ so $\frac{du}{2} = x\,dx$

so $\frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\arctan u = \boxed{\frac{1}{2}\arctan(x^2) + C}$

67) $\int_1^2 x\sqrt{x-1}\,dx$ $u = x-1$ $\frac{du}{dx} = 1$ $du = dx$

$= \int (u+1)\sqrt{u}\,du = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big]_1^2 = \frac{2}{5}\sqrt{x-1}^{\frac{5}{2}} + \frac{2}{3}\sqrt{x-1}^{\frac{3}{2}} = \boxed{\frac{16}{15}}$

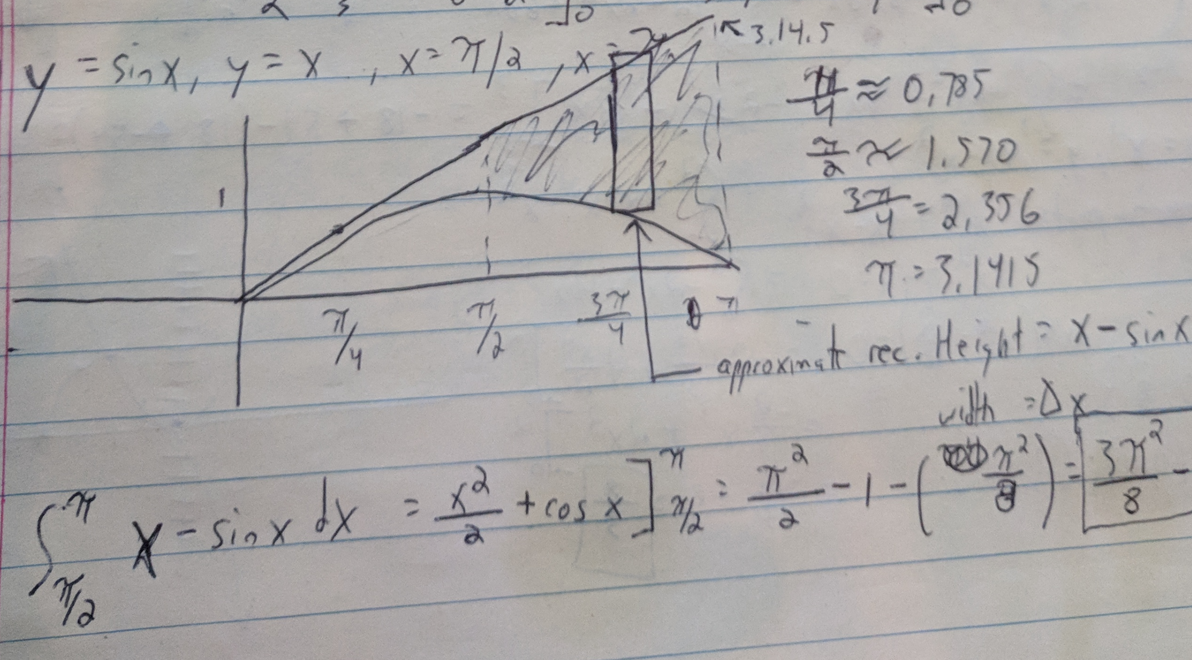
6.1) 3) $\int_{-1}^1 e^y - (y^2-2) = \int_{-1}^1 e^y - y^2 + 2 = e^y - \frac{y^3}{3} + 2y\Big]_{-1}^1$

$= e - \frac{1}{3} + 2 - (\frac{1}{e} + \frac{1}{3} - 2) = e - \frac{1}{e} - \frac{2}{3} + 4 = \boxed{e - \frac{1}{3} + \frac{10}{3}}$

4) $\int_0^3 2y - y^2 - (y^2 - 4y) = \int_0^3 -2y^2 + 6y$

$= -2\frac{y^3}{3} + 6\frac{y^2}{2}\Big]_0^3 = \frac{-2}{3}y^3 + 3y^2\Big]_0^3 = -18 + 27 = \boxed{9}$

6) $y = \sin x$, $y = x$, $x = \pi/2$, $x = \pi$



$\frac{\pi}{4} \approx 0.785$

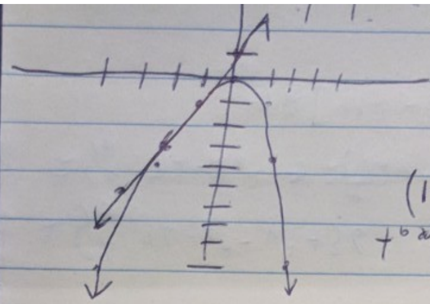
$\frac{\pi}{2} \approx 1.570$

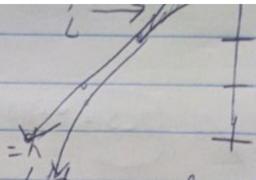
$\frac{3\pi}{4} = 2.356$

$\pi = 3.1415$

$\int_{\pi/2}^{\pi} x - \sin x\,dx = \frac{x^2}{2} + \cos x\Big]_{\pi/2}^{\pi} = \frac{\pi^2}{2} - 1 - \left(\frac{\pi^2}{8}\right) = \boxed{\frac{3\pi^2}{8} - 1}$

56) slope at (1,1) for $y=x^2$ is $2x$ so the slope at (1,1) is 2, the equation is $y-1=2(x-1)$
$y=2x-1$





12) $4x+y^2=12$, $x=y$

$4x=12-y^2$

$x=\frac{12-y^2}{4}$

intersection pts: $\frac{12-y^2}{4}=y$

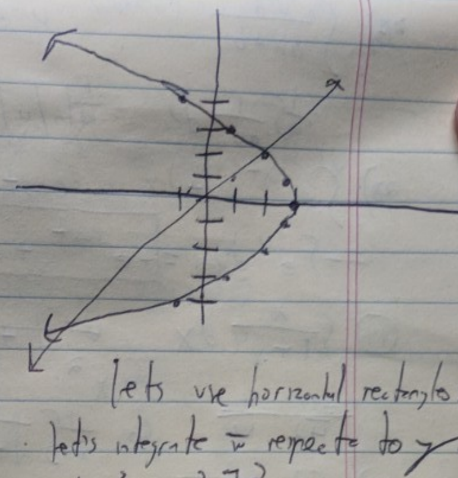
$12-y^2=4y$

$y^2+4y-12=0$

$(y-2)(y+6)$

$y=\{2,-6\}$

points: $(2,2)$, $(-6,-6)$



lets use horizontal rectangles so let's integrate w respect to y.

$$\int_{-6}^{2}\frac{12-y^2}{4}-y\,dy=\int_{-6}^{2}3-\frac{1}{4}y^2-y\,dy=3y-\frac{1}{4}\frac{y^3}{3}-\frac{y^2}{2}\Big]_{-6}^{2}$$

$$=6-\frac{2}{3}-2-(-18+18-18)=6-\frac{2}{3}-2+(18)=\boxed{21\tfrac{1}{3}}$$

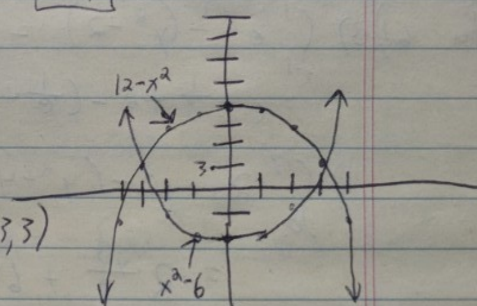
13) $y=12-x^2$, $y=x^2-6$

Intersection: $12-x^2=x^2-6$

$0=2x^2-18$

$0=(2x-6)(x+3)$

$x=\{3,-3\}$ points $=(-3,3),(3,3)$



$$\int_{-3}^{3}12-x^2-(x^2-6)\,dx=\int_{-3}^{3}12-2x^2+6\,dx$$

$$=\int_{-3}^{3}-2x^2+18\,dx=-2\frac{x^3}{3}+18x\Big]_{-3}^{3}=-18+54-(18-54)=36+36=\boxed{72}$$

14) $y=x^2$, $y=4x-x^2$

intersection points: $x^2=4x-x^2$

$2x^2-4x=0$

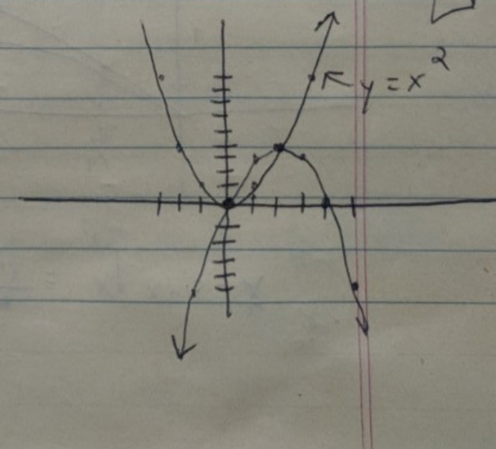
$2x(x-2)=0$

$x=\{0,2\}$

points: $(0,0)$, $(2,4)$

$$\int_{0}^{2}4x-x^2-x^2=\int_{0}^{2}-2x^2+4x=-2\frac{x^3}{3}+4\frac{x^2}{2}\Big]_{0}^{2}$$

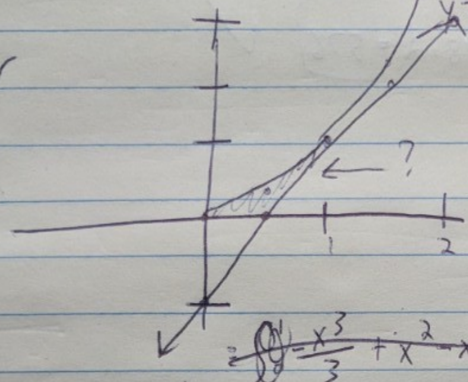
$$=\frac{-2}{3}x^3+2x^2\Big]_{0}^{2}=\frac{-16}{3}+8=\boxed{\frac{8}{3}}$$

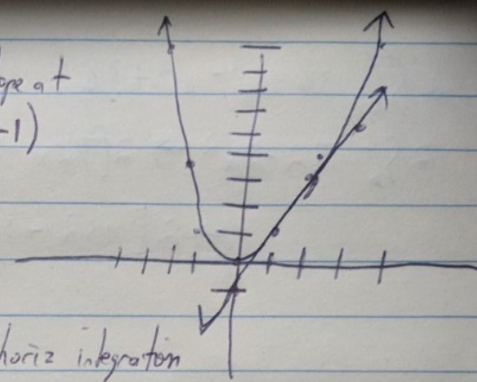


56) slope at $(1,1)$ for $y=x^2$ is $2x$ so the slope at $(1,1)$ is 2. The equation is $y-1=2(x-1)$

$y=2x-1$

or





$\leftarrow$?

let's use horiz integration

$$\int_0^1 2x-1-x^2\,dx=\int_0^1 -x^2+2x-1\,dx$$

$$=-\frac{x^3}{3}+x^2-x\Big]_0^1=-\frac{1}{3}+1-1=\frac{-1}{3}$$

$y=2x-1 \rightarrow \frac{y+1}{2}=x$

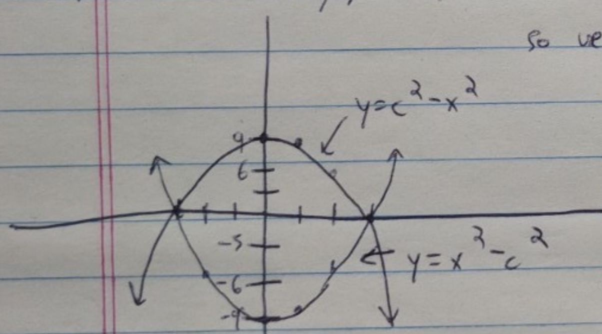
$y=x^2 \rightarrow x=\sqrt{y}$

$$\int_0^1 \frac{y+1}{2}-\sqrt{y}=\int_0^1 \frac{y}{2}+\frac{1}{2}-y^{\frac{1}{2}}$$

$$=\frac{1}{4}y^2+\frac{1}{2}y-\frac{2}{3}y^{\frac{3}{2}}\Big]_0^1=\frac{1}{4}+\frac{1}{2}-\frac{2}{3}=\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\boxed{\frac{1}{12}}$$

54) lets see what graphs look like when $c=3$

so we do vertical integration, curve also is <u>even</u>



$$2\int_0^c c^2-x^2-(x^2-c^2)\,dx$$

$$2\int_0^c 2c^2-2x^2\,dx$$

$$2\left[2c^2x-2\frac{x^3}{3}\right]_0^c=576$$

when $c=3$
area $=72$
so $c>3$

$$2\int_0^3 9-x^2-x^2+9$$

$$2\int_0^3 18-2x^2$$

$$2\left[18x-\frac{2}{3}x^3\right]_0^3$$

$$2[54-18]=72$$

$$4c^2x-\frac{4}{3}x^3\Big]_0^c=576$$

$$4c^3-\frac{4}{3}c^3=576$$

$$c^3-\frac{1}{3}c^3=144$$

$$\frac{2}{3}c^3=144,\ c^3=216$$

$c=\boxed{6}$