Intro to Robotics

Lecture 3

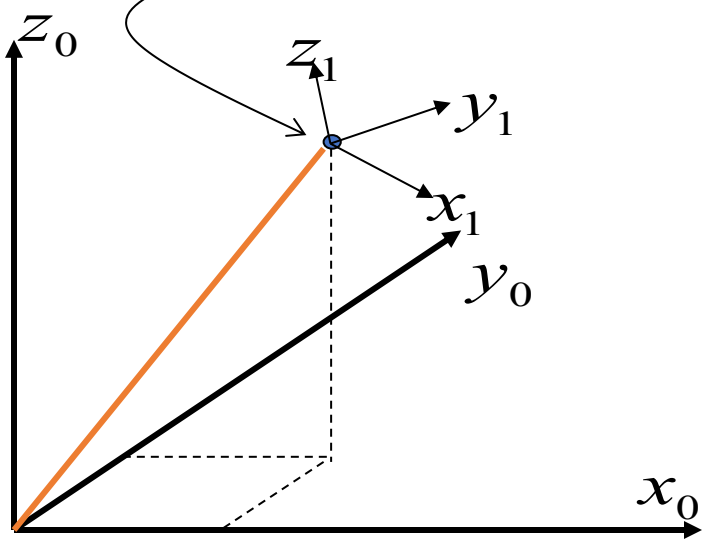
What about Translation?

$$d^0 = (d_x, d_y, d_z)^T$$

$$p^0 = R_1^0 p^1 + d^0$$

$$\begin{bmatrix} p^0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$$



$$H = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{3\times3} & d_{3\times1} \\ 0 & 1 \end{bmatrix}$$

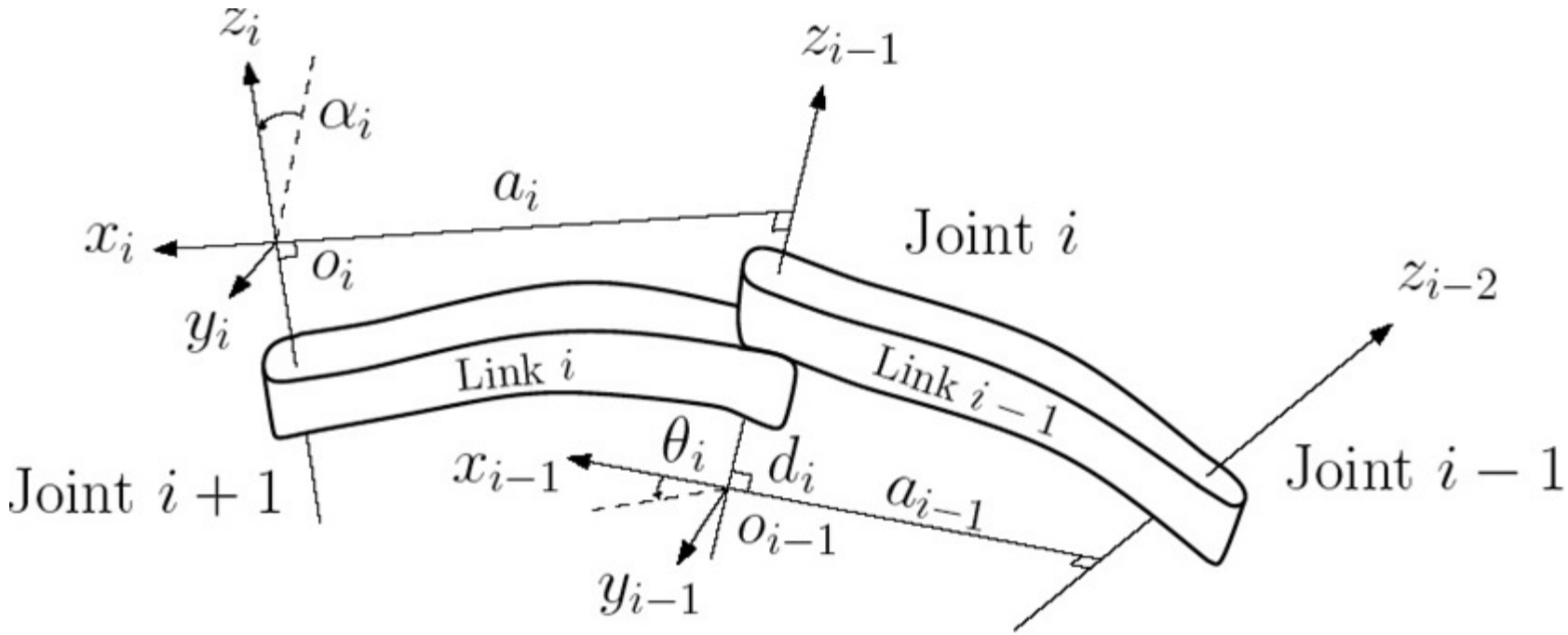
Rotation matrix

Position vector

Inverse

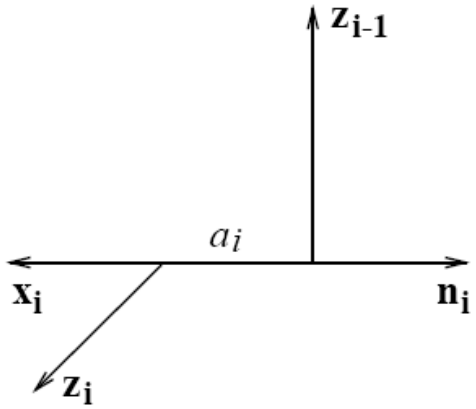
$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}$$

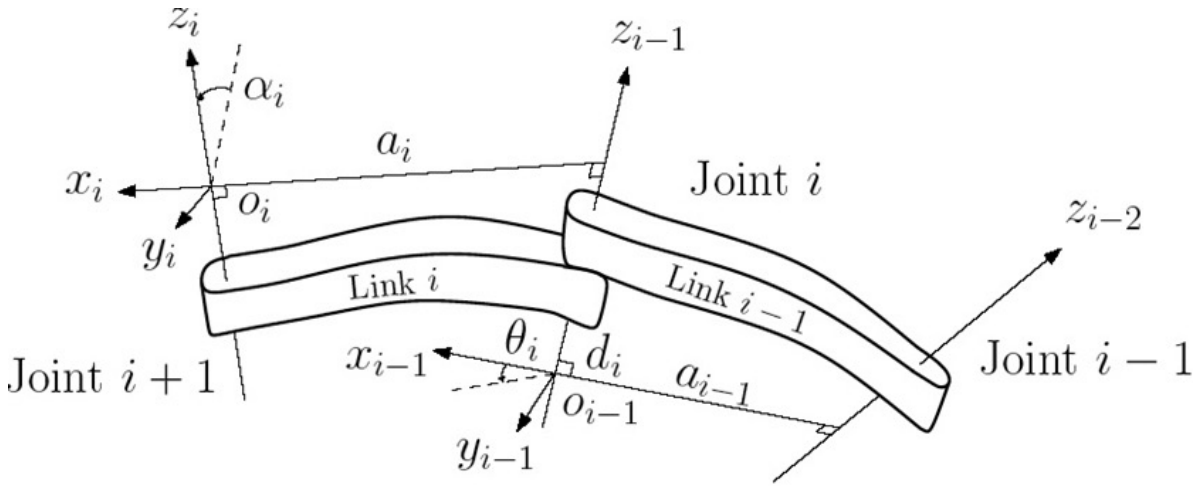
$$H^{-1}H = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R^T R & 0 \\ 0 & 1 \end{bmatrix} = I_{4\times 4}$$


z_i
α_i
z_{i-1}
a_i
x_i
o_i
Joint i
z_{i-2}
y_i
Link i
Link $i-1$
θ_i
d_i
x_{i-1}
Joint $i+1$
Joint $i-1$
a_{i-1}
o_{i-1}
y_{i-1}

Denavit-Hartenberg Convention

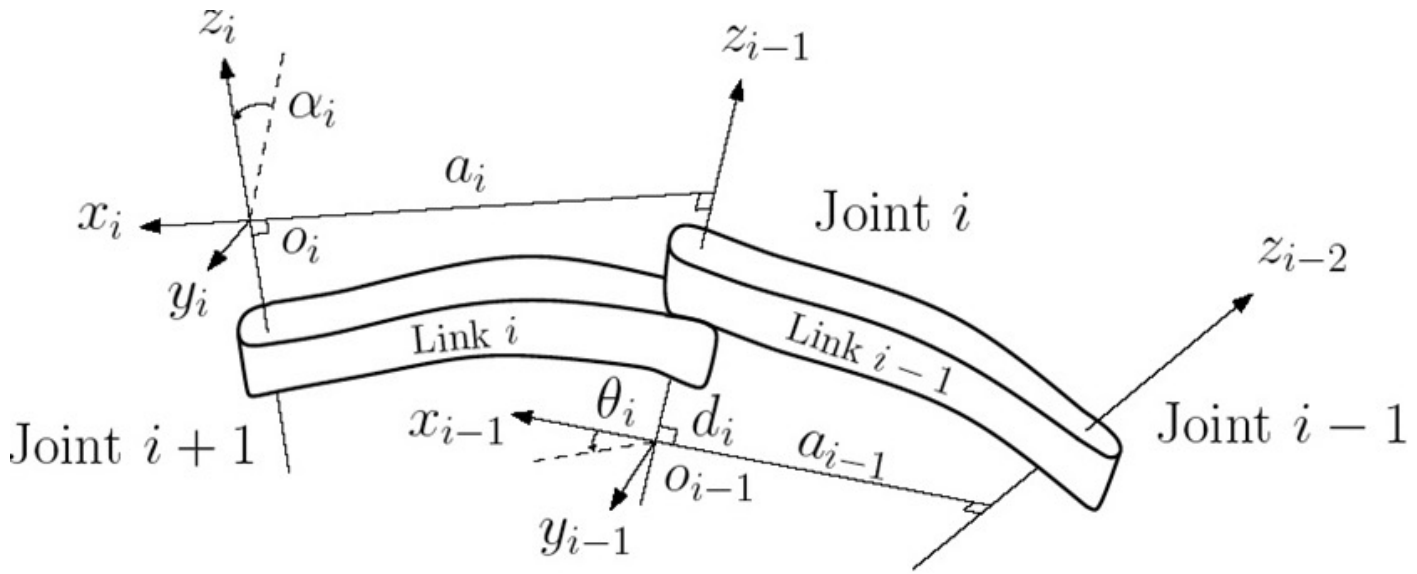
- Joint counts up from 1 at the base; axis counts up from 0
- Joint i connects link i - 1 to link i
- Align the Z_i with joint i+1
- *Base coordinate system:* Z_0 axis align with joint 1, origin is at the base
- *Origin of the coordinate system i:*
  - intersection of the Z_i & Z_{i-1} or
  - the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis
- *X_i axis:*
  - $X_i = \pm(Z_{i-1} \times Z_i) / \|Z_{i-1} \times Z_i\|$ pointing from Z_{i-1} to Z_i , or
  - along the common normal between the Z_{i-1} & Z_i axes when they are parallel
- *Y_i axis:* $Y_i = +(Z_i \times X_i) / \|Z_i \times X_i\|$

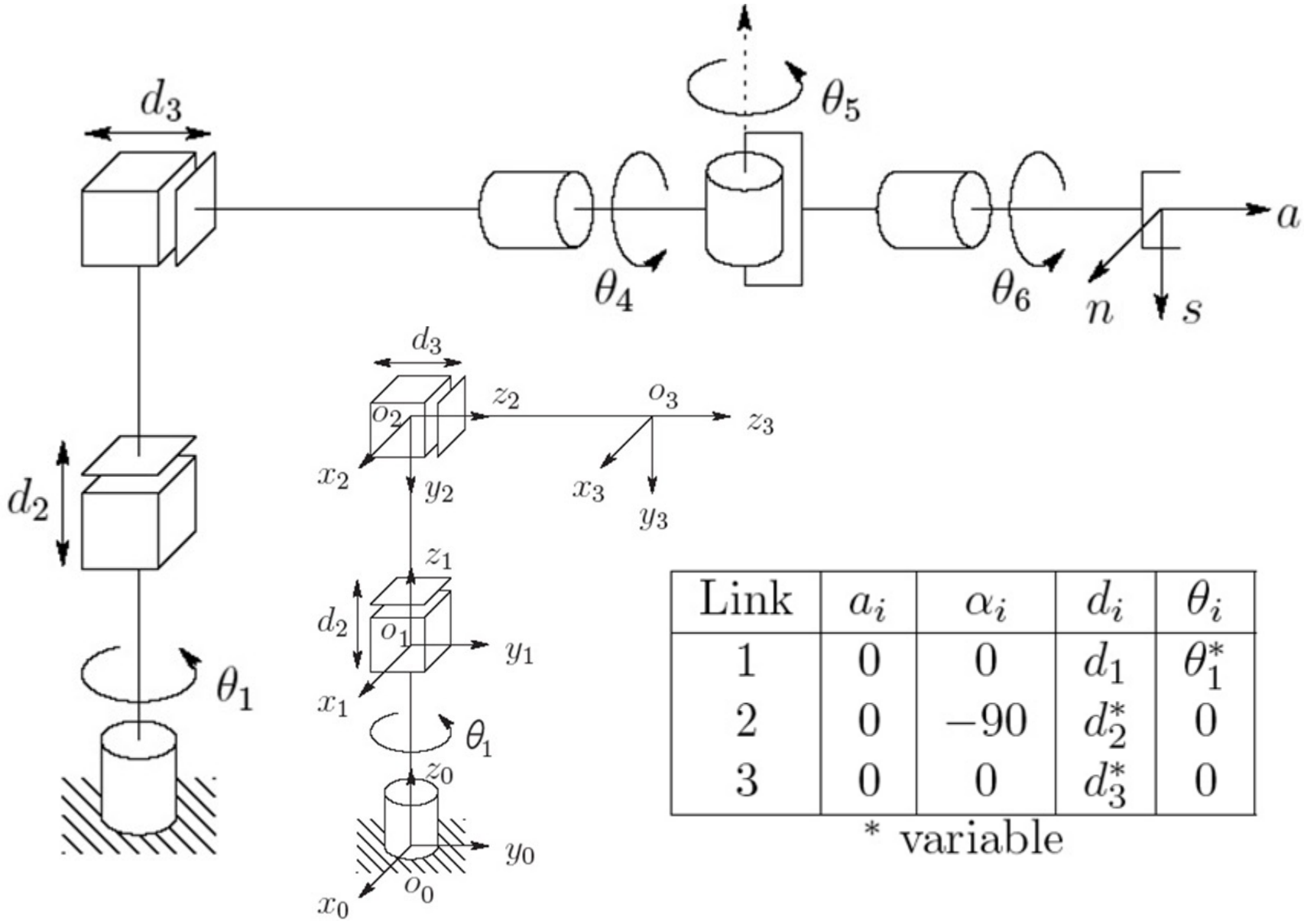




Link and Joint Parameters

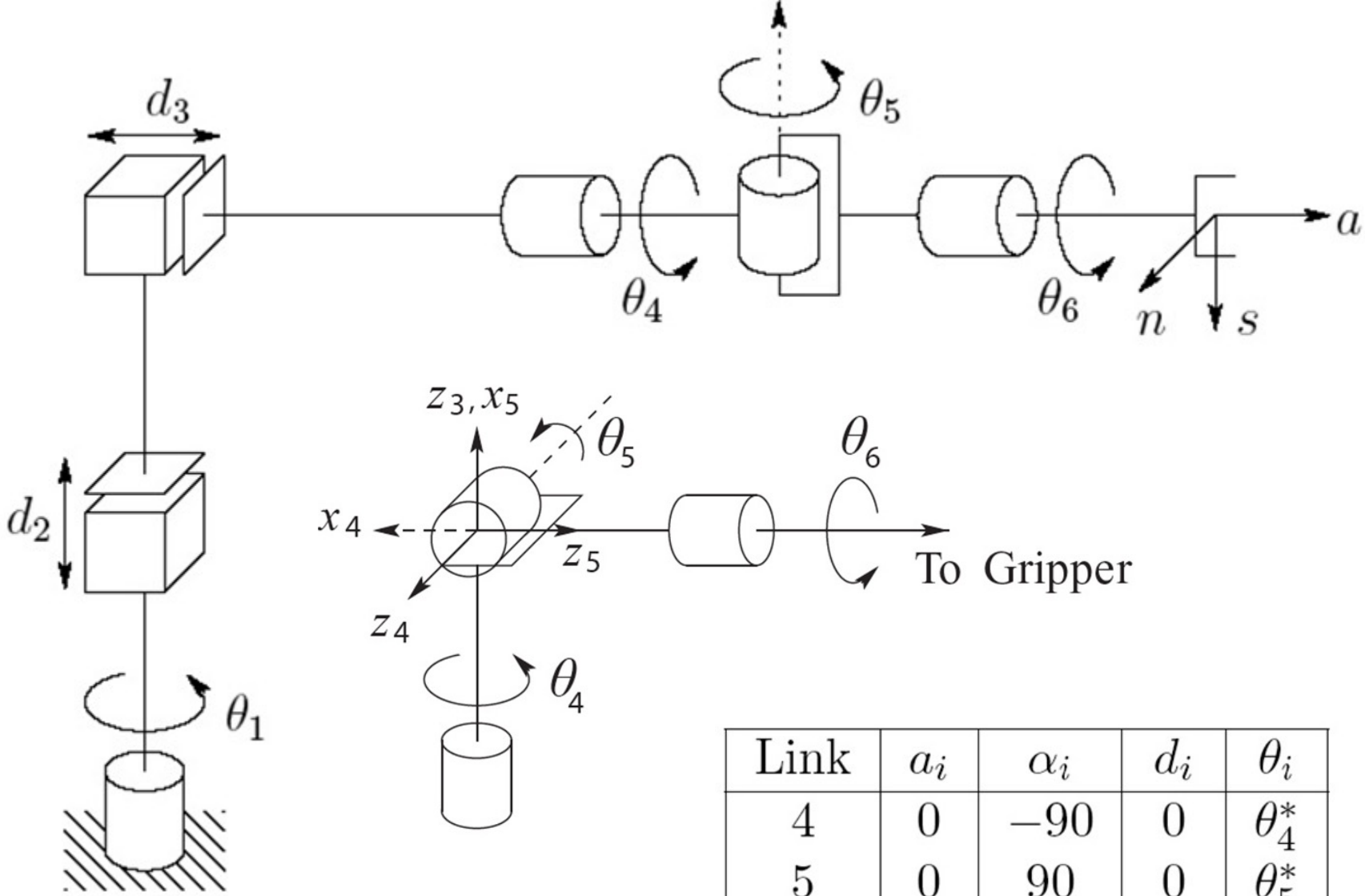
- *Joint angle* θ_i : the angle from X_{i-1} to X_i about the Z_{i-1}
- *Joint distance* d_i : the distance from X_{i-1} to X_i, as measured along Z_{i-1}. It could be negative
- *Link length* a_i: the distance from Z_{i-1} to Z_i , along X_i. It is always positive
- *Link twist angle* α_i: the angle from Z_{i-1} to Z_i about the X_i axis





Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

* variable



The spherical wrist

Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

* variable