Intro to Robotics

Lecture 8

Big Picture

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$$

Forward

Kinematics

Inverse

$$\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \varphi \end{bmatrix}$$

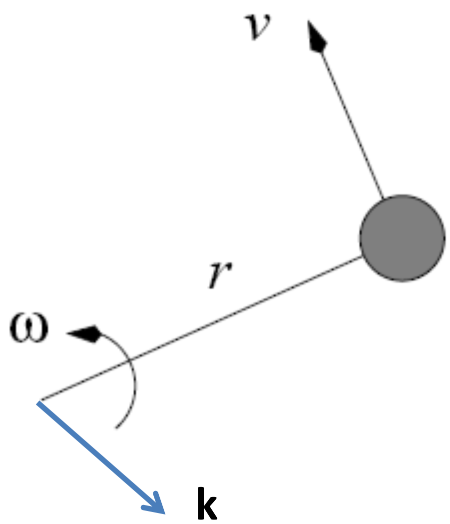
Joint Space

Task Space

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix} \quad \text{Jacobian Matrix} \quad \begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Joint space velocity vs. task space velocity

Angular Velocity



$$\omega = \dot{\theta} k$$

$$v = \omega \times r$$

Angular Velocities

$$\omega(t) = \dot{\theta}(t)k$$

For a robot, k is always align with z (joint)

$$\omega_{0,n}^{0}(t) = \sum \begin{cases} \omega_{0,1}^{0}(t) \\ \omega_{1,2}^{0}(t) = R_1^0 \omega_{1,2}^{1} = \dot{\theta}_{1,2}(t) R_1^0 k_1^1 = \dot{\theta}_{1,2}(t) z_1^0 \\ \omega_{2,3}^{0}(t) = R_2^0 \omega_{2,3}^{2} = \dot{\theta}_{2,3}(t) R_2^0 k_2^2 = \dot{\theta}_{2,3}(t) z_2^0 \\ \cdots\cdots\cdots \\ \omega_{n-1,n}^{0}(t) = R_{n-1}^0 \omega_{n-1,n}^{n-1} = \dot{\theta}_{n-1,n}(t) R_{n-1}^0 k_{n-1}^{n-1} = \dot{\theta}_{n-1,n}(t) z_{n-1}^0 \end{cases}$$

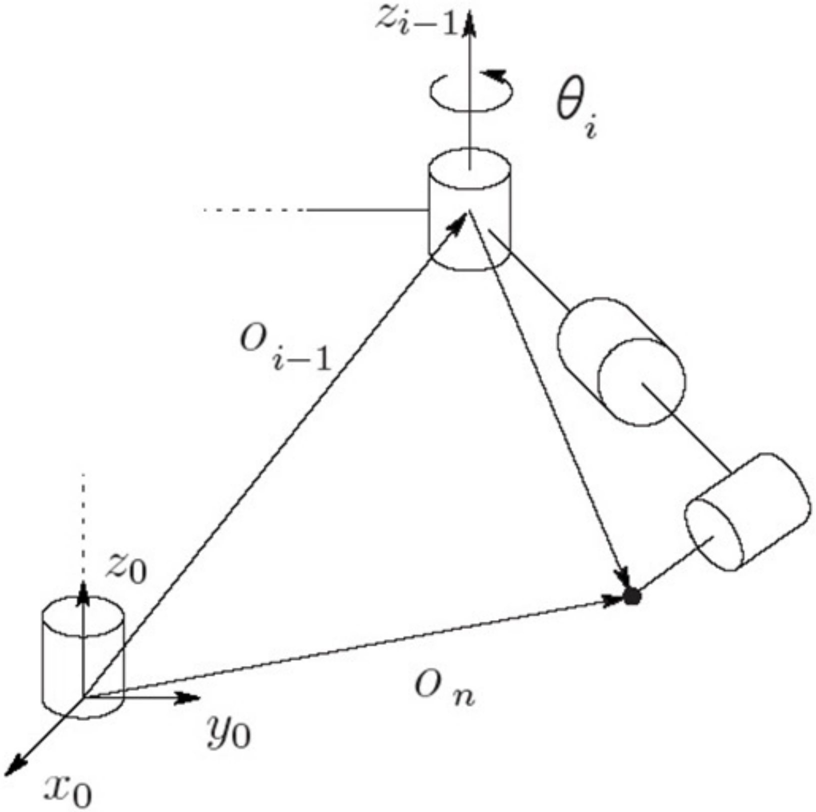
$\omega_{i-1,i}^{j}(t)$ -- Angular velocity of link i, relative to frame j,

Angular Velocity with Revolute Joints

$$\omega_n^0 = \dot{\theta}_{0,1} z_0^0 + \dot{\theta}_{1,2} z_1^0 + \ldots + \dot{\theta}_{n-1,n} z_{n-1}^0$$

$$\omega_n^0 = \begin{bmatrix} z_0^0 & z_1^0 & \ldots & z_{n-1}^0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{0,1} \\ \dot{\theta}_{1,2} \\ \ldots \\ \dot{\theta}_{n-1,n} \end{bmatrix} = \begin{bmatrix} z_0^0 & z_1^0 & \ldots & z_{n-1}^0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ldots \\ \dot{\theta}_n \end{bmatrix}$$

Linear velocity with Revolute Joints



$$v = \omega \times r = \dot{\theta}_i z_{i-1} \times (o_n - o_{i-1})$$

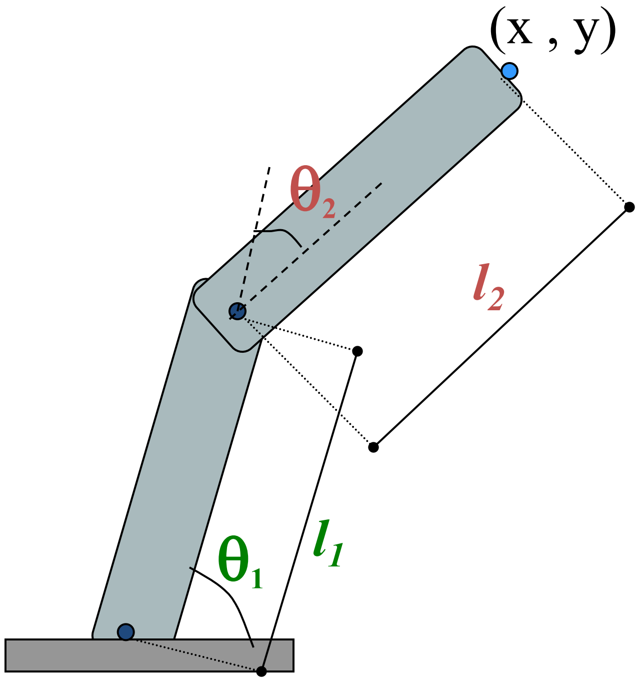
Combined Velocity for Revolute Joints

$$v_n^0 = \begin{bmatrix} z_0 \times (o_n - o_0) & z_1 \times (o_n - o_1) & \dots & z_{n-1} \times (o_n - o_{n-1}) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dots \\ \dot{\theta}_n \end{bmatrix}$$

$$\omega_n^0 = \begin{bmatrix} z_0^0 & z_1^0 & \dots & z_{n-1}^0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{0,1} \\ \dot{\theta}_{1,2} \\ \dots \\ \dot{\theta}_{n-1,n} \end{bmatrix} = \begin{bmatrix} z_0^0 & z_1^0 & \dots & z_{n-1}^0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dots \\ \dot{\theta}_n \end{bmatrix}$$

Example

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$



$$J = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix}$$

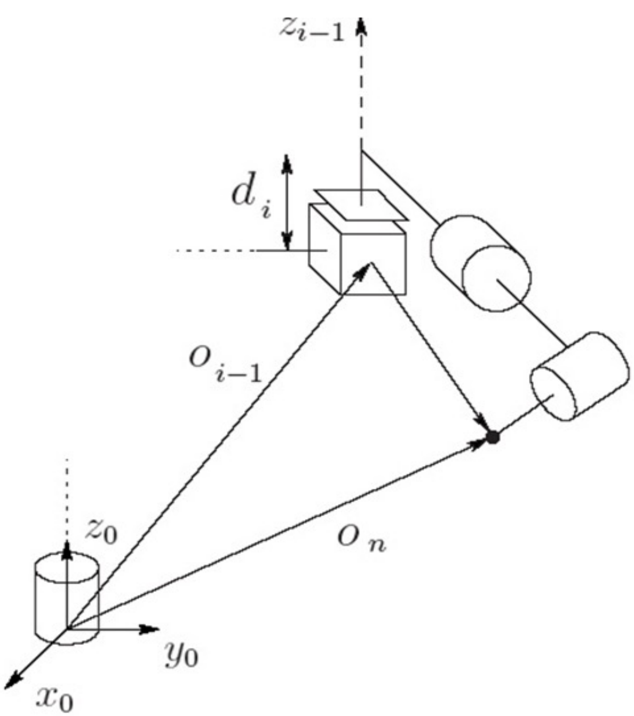
$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} l_1 c\theta_1 \\ l_1 s\theta_1 \\ 0 \end{bmatrix} \quad o_2 = \begin{bmatrix} l_1 c\theta_1 + l_2 c(\theta_1 + \theta_2) \\ l_1 s\theta_1 + l_2 s(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1 + \theta_2) & -l_2 s(\theta_1 + \theta_2) \\ l_1 c\theta_1 + l_2 c(\theta_1 + \theta_2) & l_2 c(\theta_1 + \theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

Prismatic Joints

- For prismatic joint i
- Contribute NULL to angular velocity of the end-effector
- Contribute $\dot{d}_i z_{i-1}^0$ to linear velocity of the end-effector



$$v_n^0 = \begin{bmatrix} z_0 & z_1 & \dots & z_{n-1} \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dots \\ \dot{d}_n \end{bmatrix}$$

$$\omega_n^0 = \begin{bmatrix} 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dots \\ \dot{d}_n \end{bmatrix}$$

For Joint i

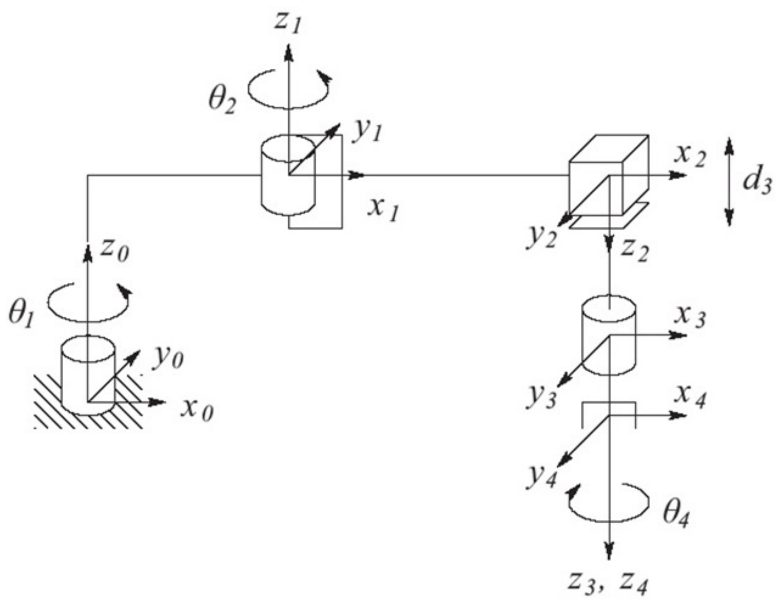
- If joint i is revolute

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

- If joint i is prismatic

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

Example



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta_1^\star$
2	a_2	180	0	$\theta_2^\star$
3	0	0	$d_3^\star$	0
4	0	0	d_4	$\theta_4^\star$

* joint variable

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} \qquad J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

1. Compute Z_{i-1}

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

2. Compute O_i

$$A_1^0 = A_1, A_2^0 = A_1^0 A_2, A_3^0 = A_2^0 A_3, A_4^0 = A_3^0 A_4$$

o_4

$$A_4^0 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ref. page 93

z_4