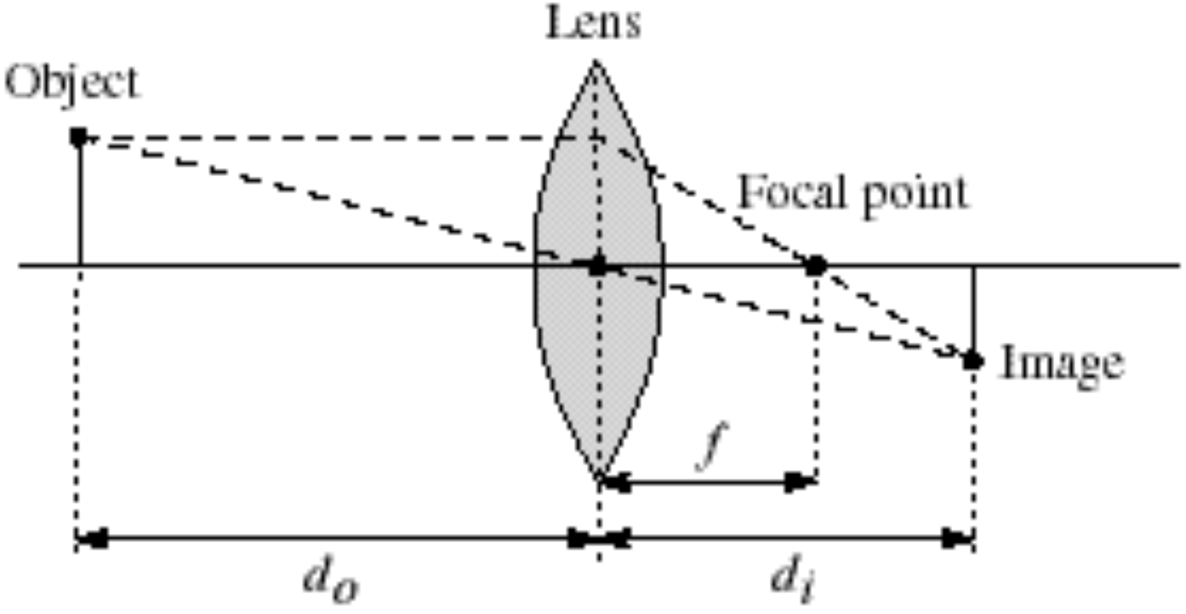
Intro to Robotics

Lecture 15

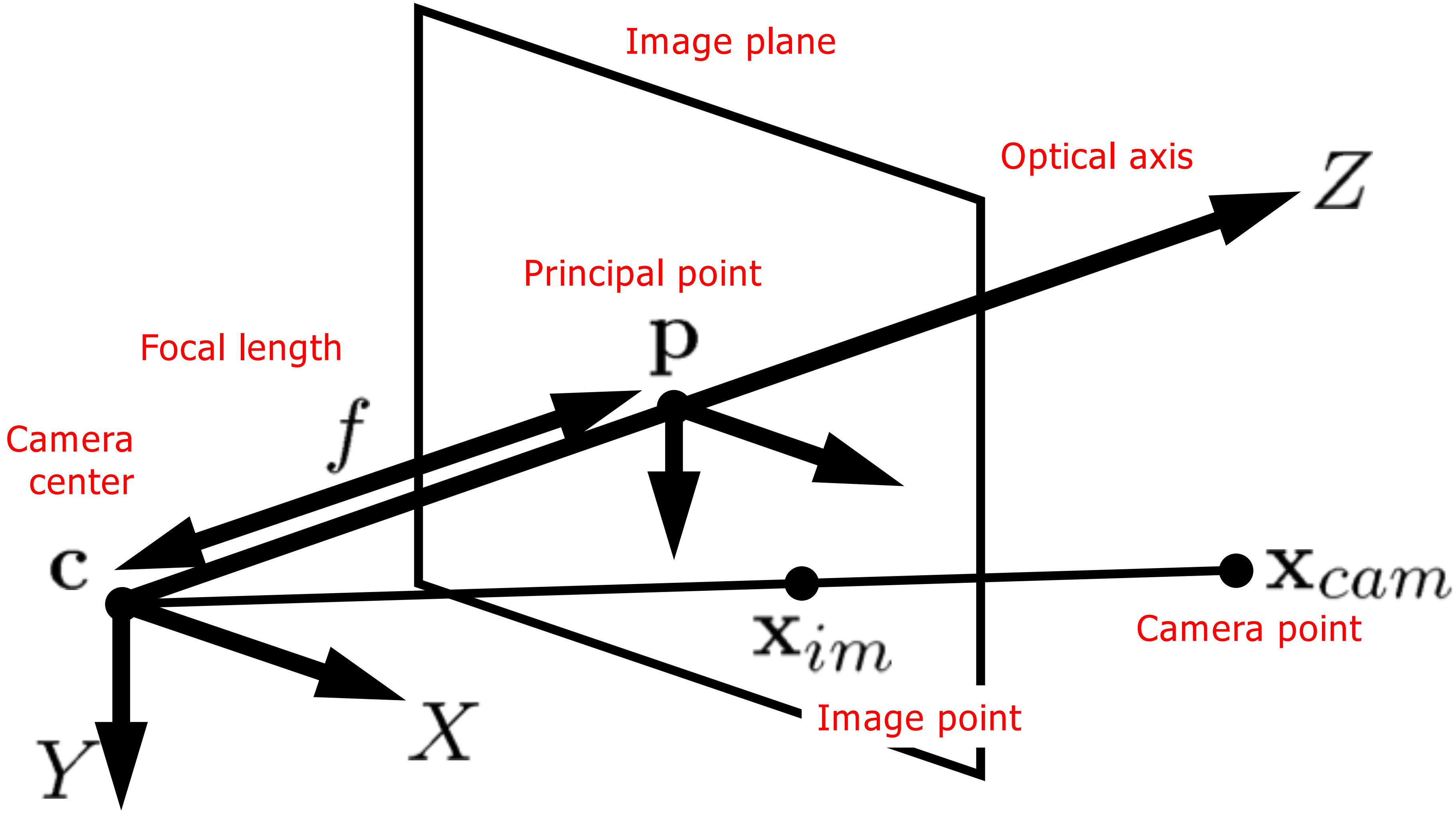
Thin Lens Model

- Thin lens equation:

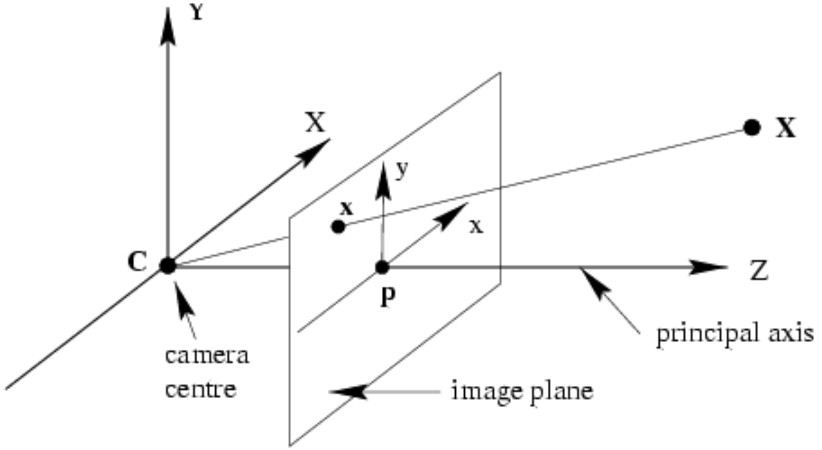
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

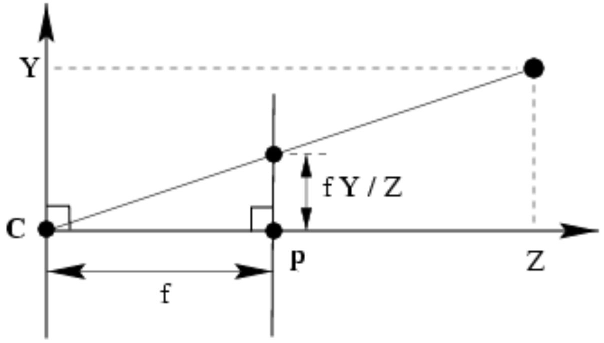


Pinhole Camera Model



Pinhole Camera Model

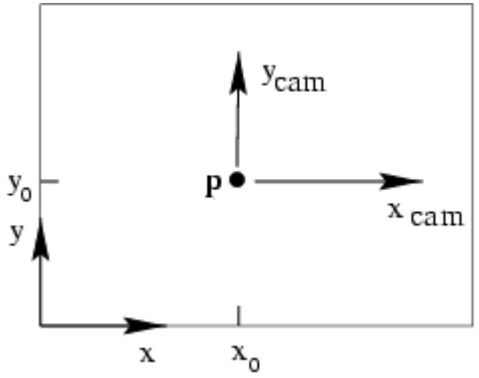




$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} \square \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

homogeneous coordinates

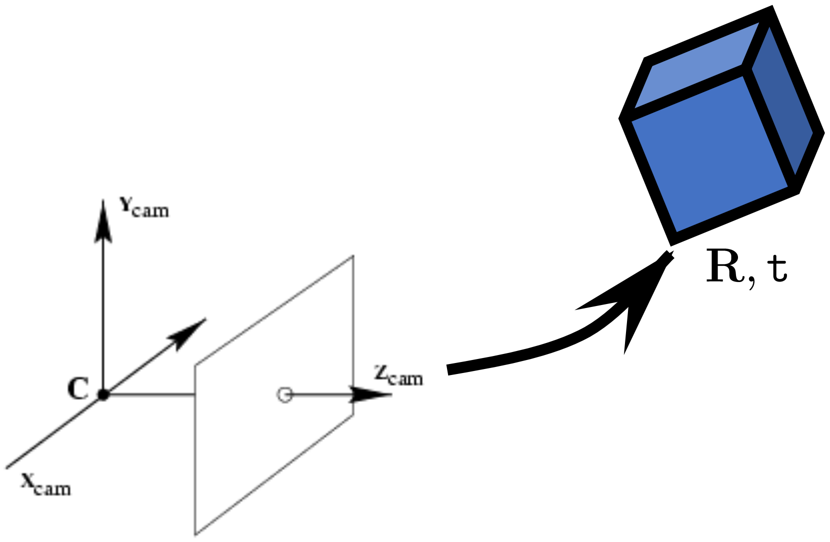
Principal Point Offset



$$(p_x, p_y)^T$$ principal point

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Object in Camera Coordinate System

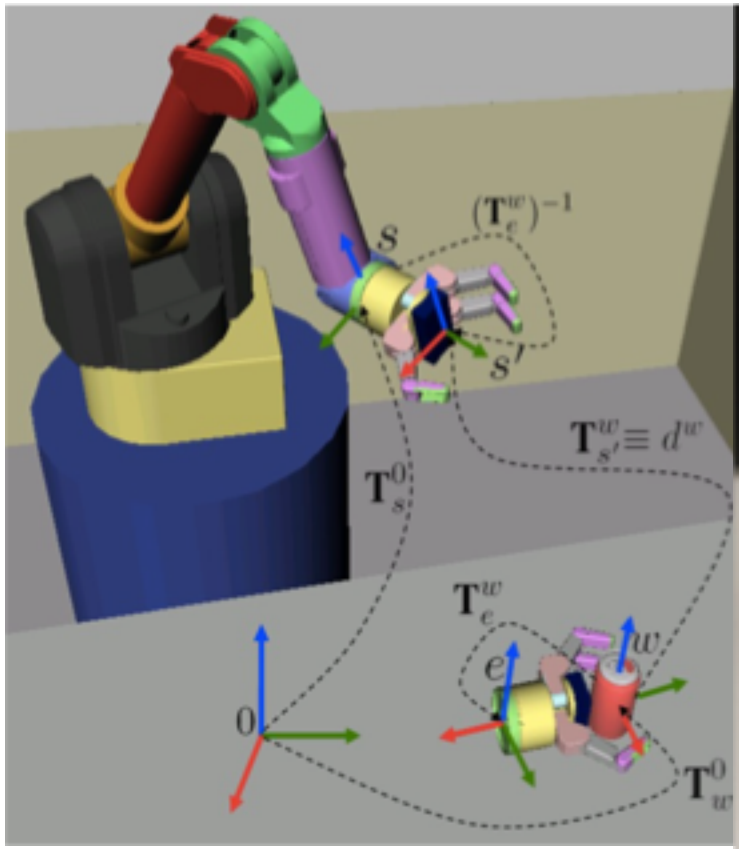


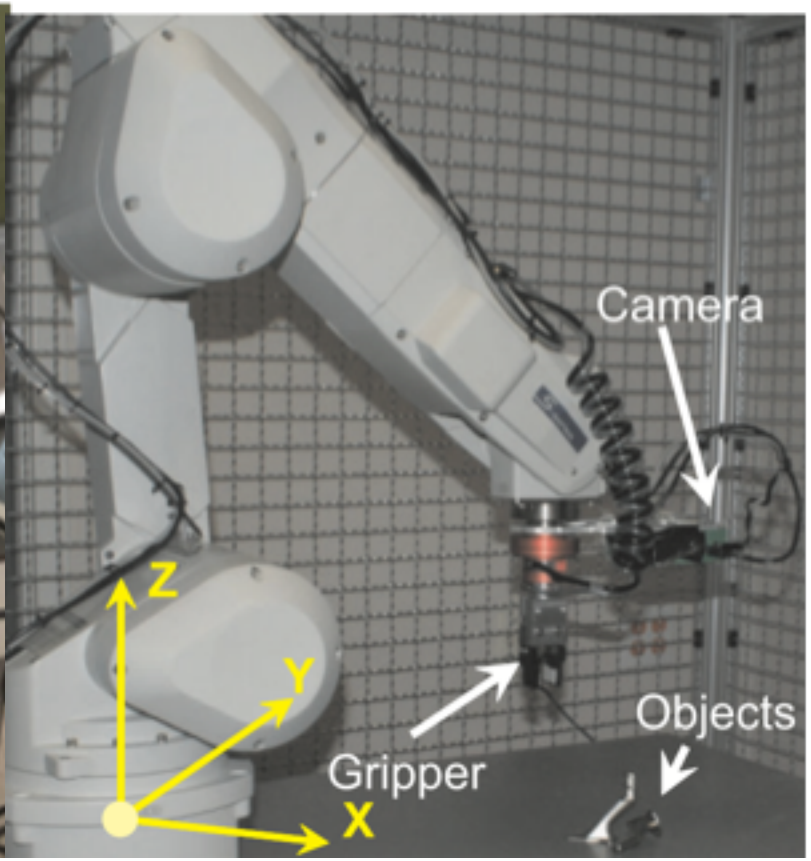
$$\mathtt{X}_{\mathtt{cam}} = \begin{bmatrix} \mathbf{R} & \mathtt{t} \\ 0 & 1 \end{bmatrix} \mathtt{X}_{\mathtt{obj}}$$

$$\mathtt{x} = \mathbf{K} \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathtt{t} \\ 0 & 1 \end{bmatrix} \mathtt{X}_{\mathtt{obj}}$$

K – intrinsic parameter matrix

Hand-Eye: Eye-in-Hand Eye-to-Hand





General Projective Camera Model

- Simple ideal lens

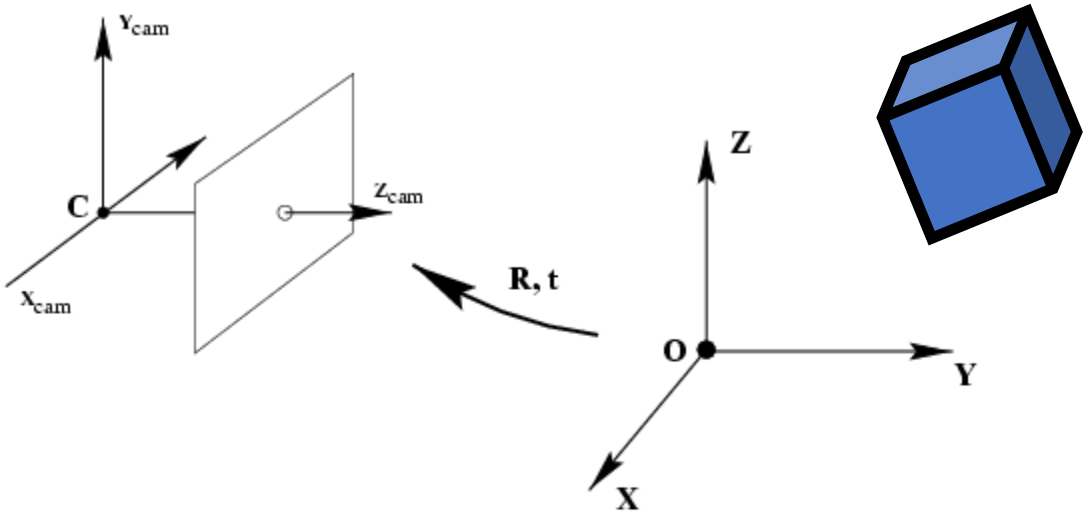
$$K = \begin{bmatrix} \alpha_x & & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix}$$

- Different x and y focal length
- Lens distortion
- Center in pixels

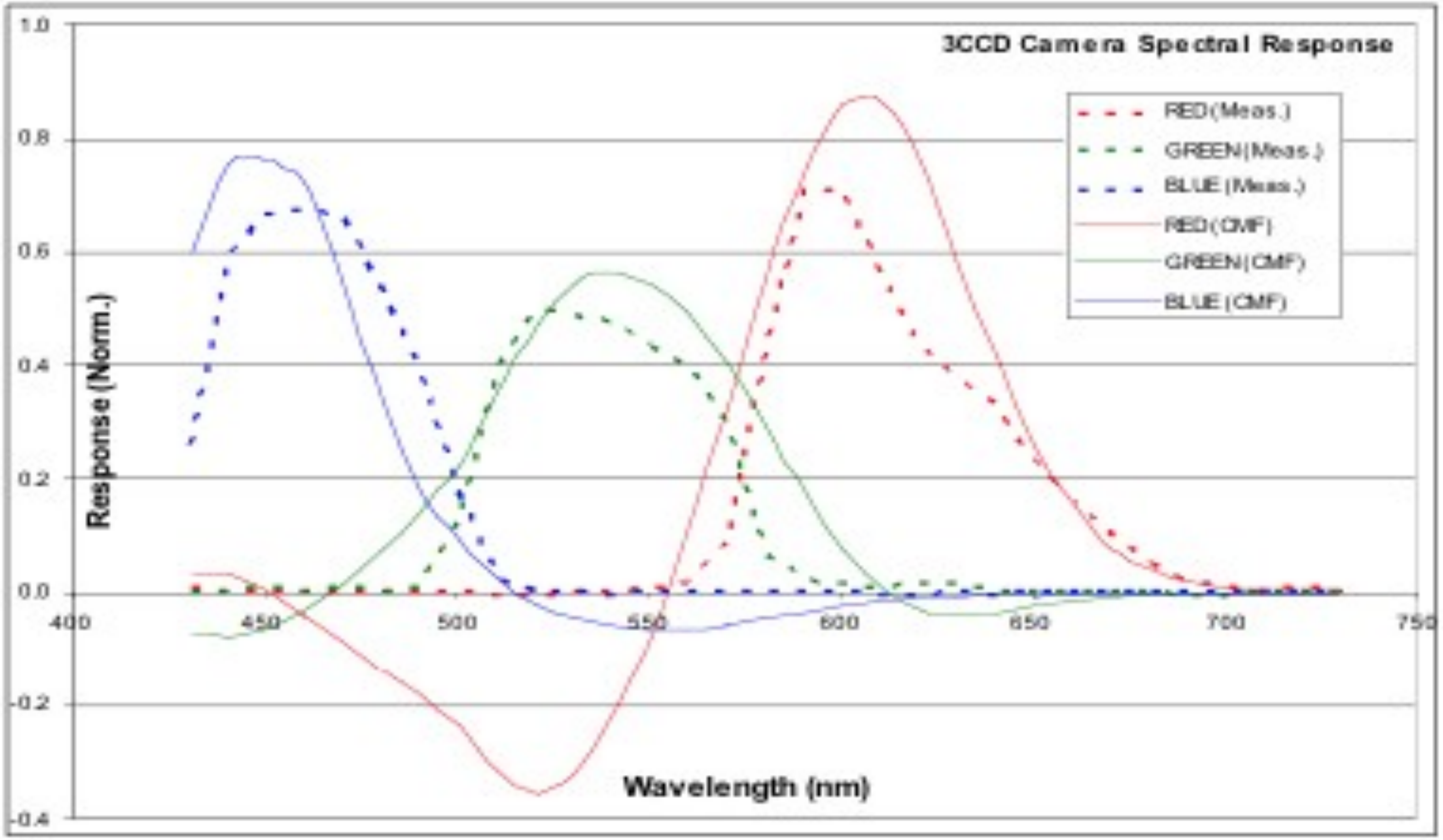
$$\mathrm{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

Camera in World Coordinate System

- Calibrate where the camera is in the world coordinate system
- Extrinsic parameter calibration



Camera Spectral Response



http://www.definitionmagazine.com/journal/2010/5/7/capturing-colour.html

3 chip CCD

Images

- An image is a matrix of pixels $\mathbf{I}(x, y)$
- Grayscale
  - 8 bits per pixel $\rightarrow$ Intensities in range [0…255]
  - 0-1 for intensity representation
- RGB color
  - Three 8-bit color channels $\mathbf{I}_R, \mathbf{I}_G, \mathbf{I}_B$

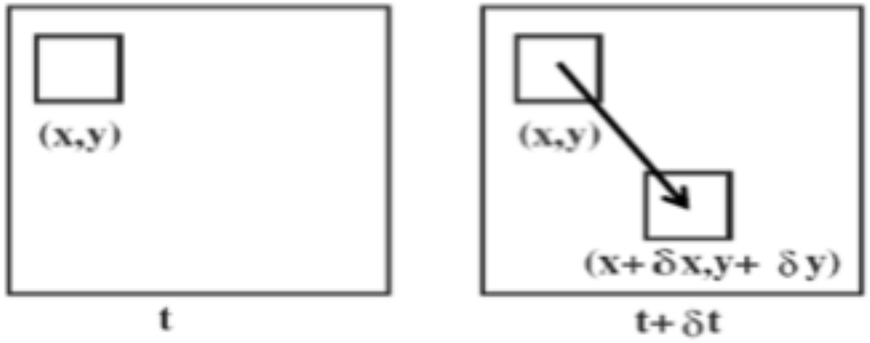
Color Representation

- RGB, HSV (hue, saturation, value), YUV, and other
- Adjust contract, brightness, gammar
- Chrominance: Perceived color
  - HS(V), (Y)UV, etc.
  - Normalized RGB removes some illumination dependence:

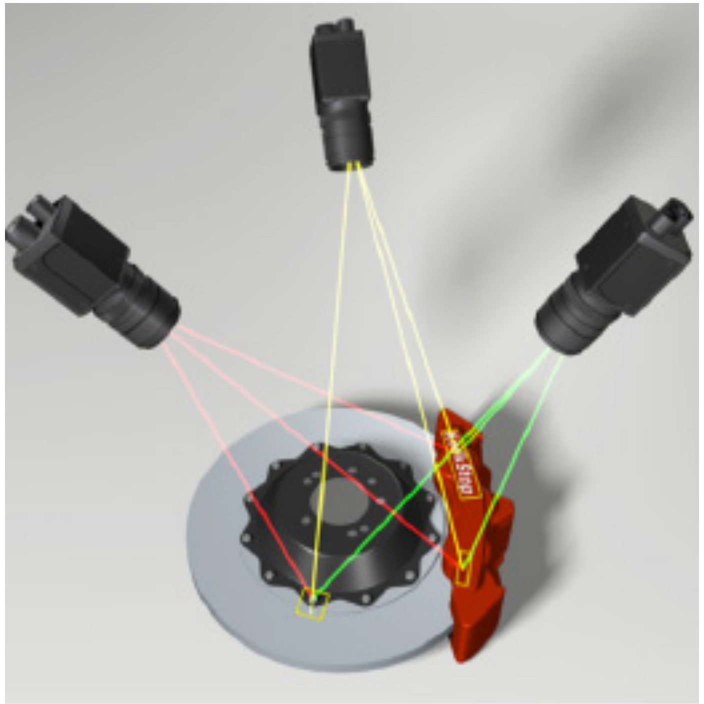
$$r = \frac{R}{R+G+B}, g = \frac{G}{R+G+B}$$

Optical Flow

- Aperture problem: Can only determine optical flow component in gradient direction
- Brightness constancy insufficient to solve for general optical flow vector field
- Need other assumptions:
  - Assume flow field is smoothly varying (Horn, 1986)
  - Assume low-dimensional function describes motion



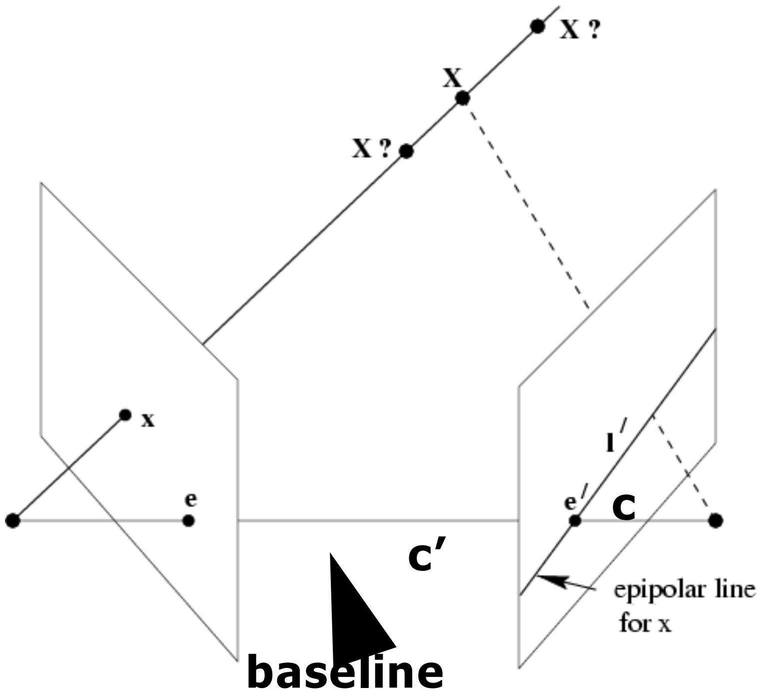
3D Vision

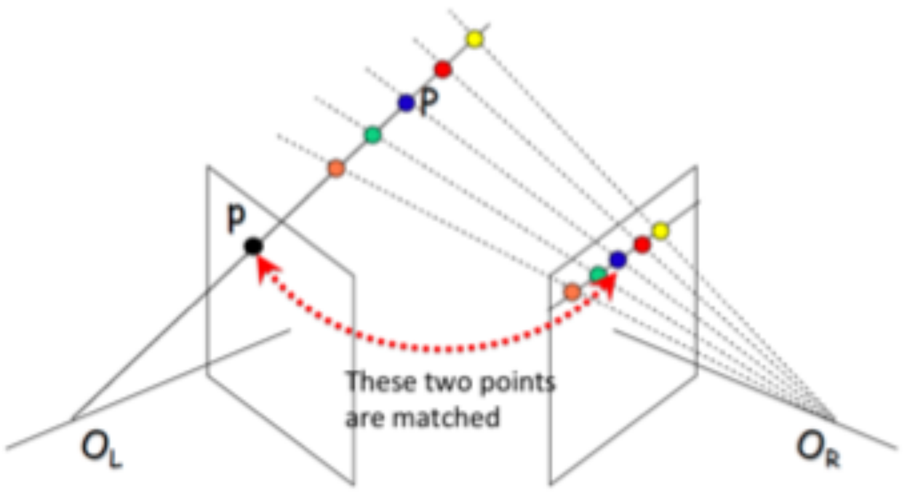


3D vision systems typically employ multiple cameras.

Epipolar Geometry

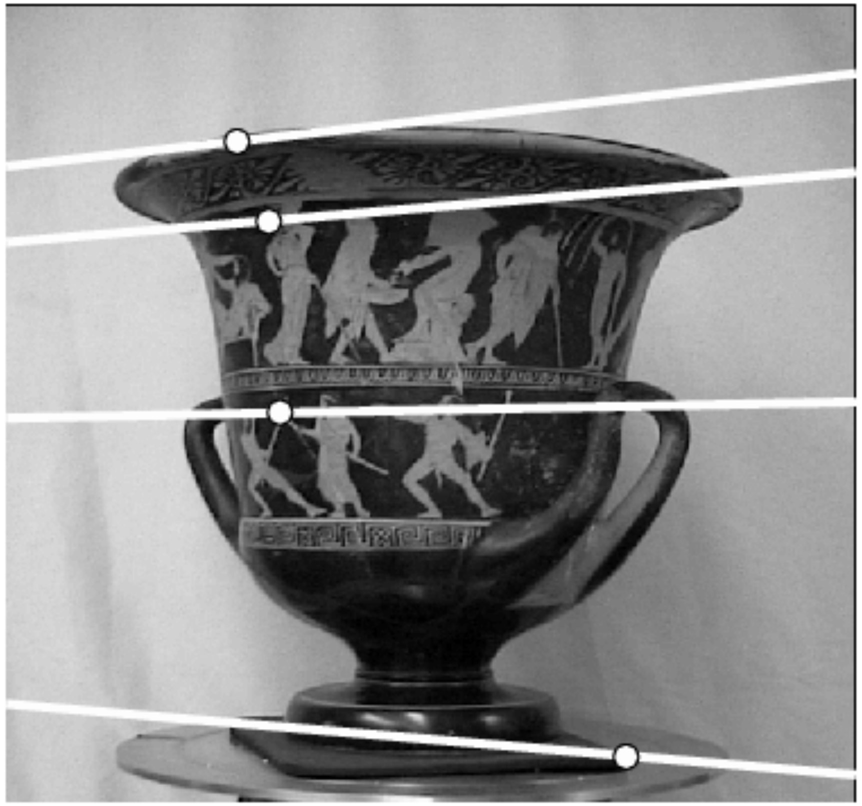
- **Epipoles**: Where baseline intersects image planes
- **Epipolar plane**: Any plane containing baseline
- **Epipolar line**: Intersection of epipolar plane with image plane





Example: Epipolar Lines

Left view



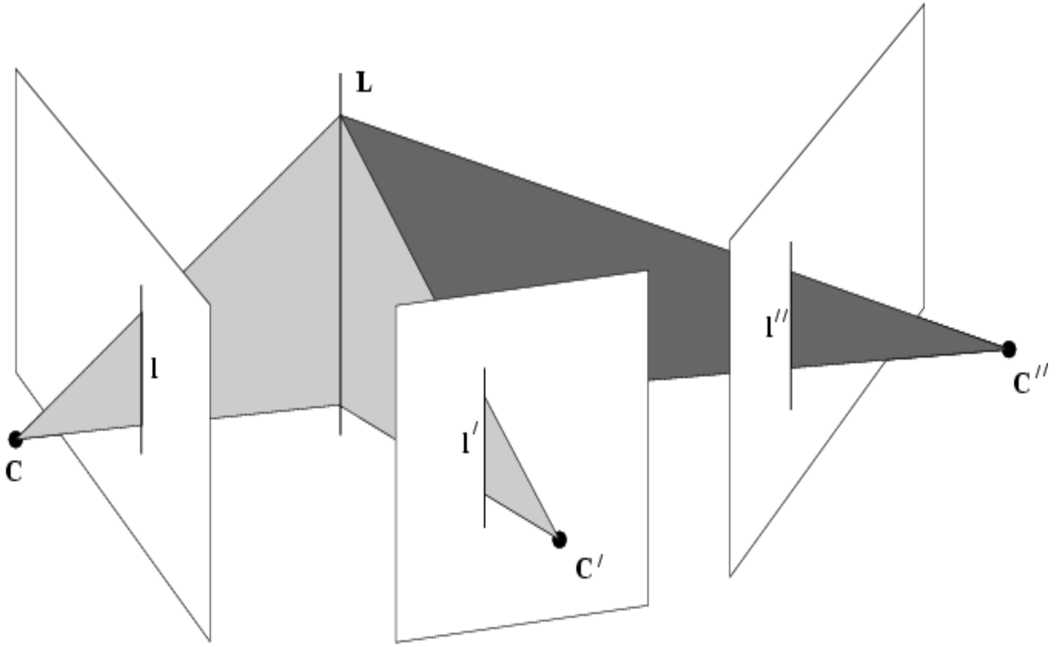
from Hartley & Zisserman

Right view

Known epipolar geometry constrains search for point correspondences

More Than Two Views

- Reconstruction methods
  - Bundle adjustment: Projective reconstruction from *n* views taking all into account simultaneously
  - Factorization: Affine reconstruction for *n* affine cameras (Tomasi & Kanade, 1992)



Depth Camera