Intro to Robotics

Lecture 9

For Joint i

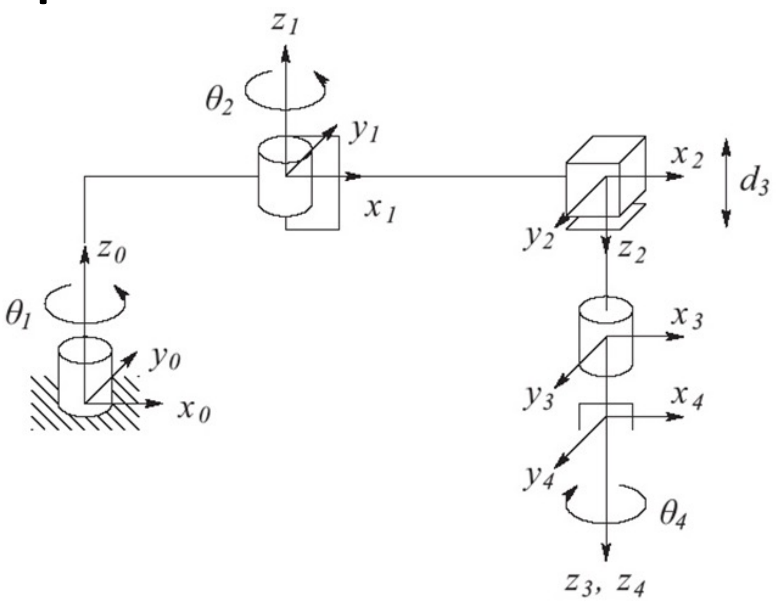
- If joint i is revolute

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

- If joint i is prismatic

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

Example



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta_1^\star$
2	a_2	180	0	$\theta_2^\star$
3	0	0	$d_3^\star$	0
4	0	0	d_4	$\theta_4^\star$

* joint variable

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} \qquad J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

1. Compute Z_{i-1}

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

2. Compute O_i

$$A_1^0 = A_1, A_2^0 = A_1^0 A_2, A_3^0 = A_2^0 A_3, A_4^0 = A_3^0 A_4$$

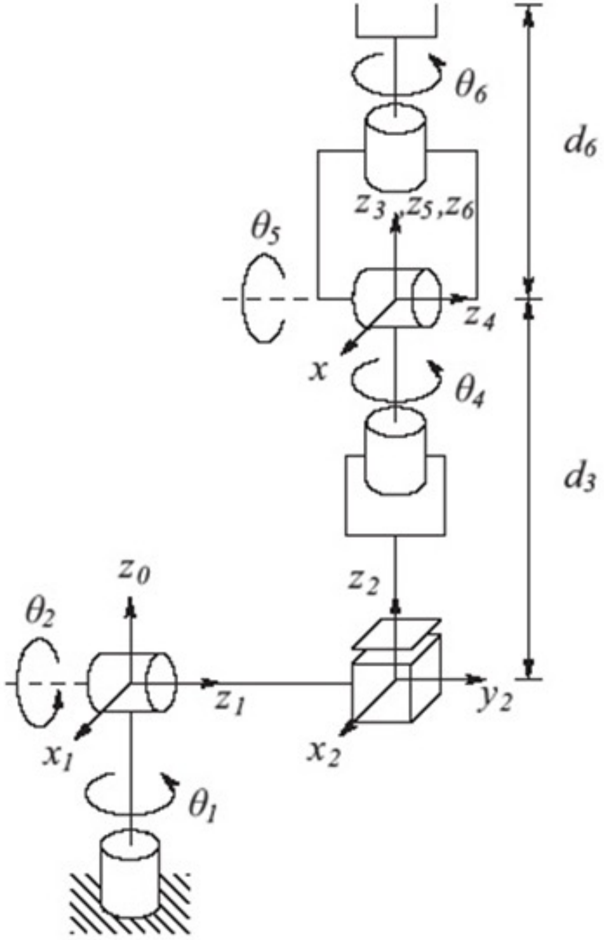
o_4

$$A_4^0 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ref. page 93

z_4

Example



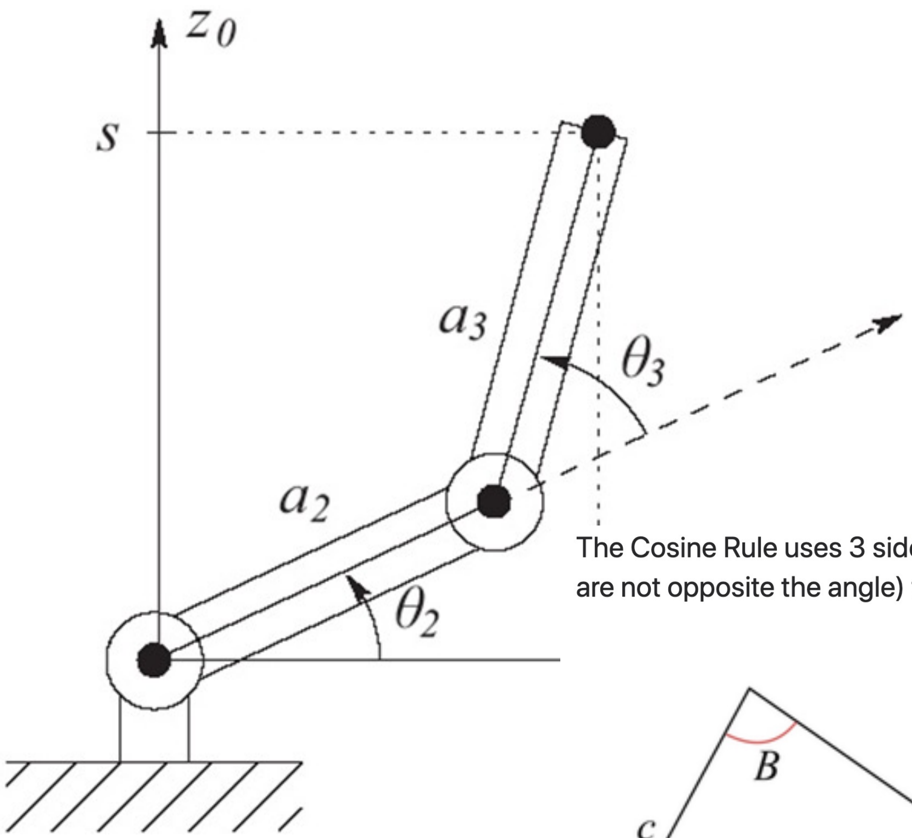
$$A_1^0 = \begin{bmatrix} c_1 & 0 & \boxed{\begin{matrix} -s_1 \\ c_1 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} s_1 \\ 0 \end{matrix} & \begin{matrix} 0 \\ -1 \end{matrix} & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 1 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \qquad z_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

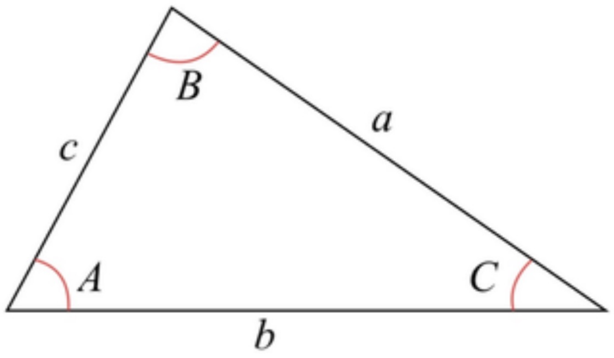
Ref. page 90

Inverse Kinematics



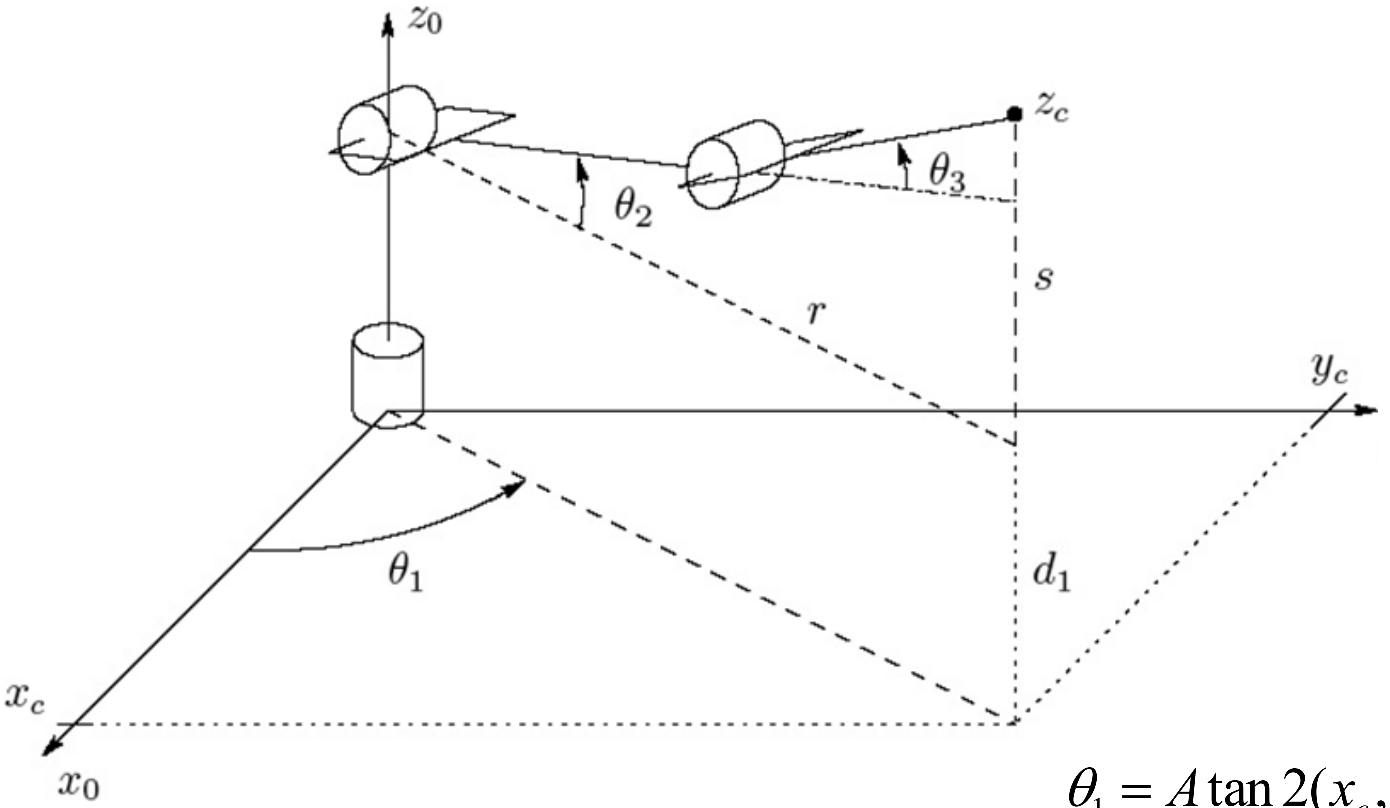
$$\cos\theta_3 = \frac{r^2 + s^2 - a_2^2 - a_3^2}{2a_2a_3}$$

The Cosine Rule uses 3 sides to find the angles, or and angle and 2 sides (where the 2 sides are not opposite the angle) to find the side opposite the angle



$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

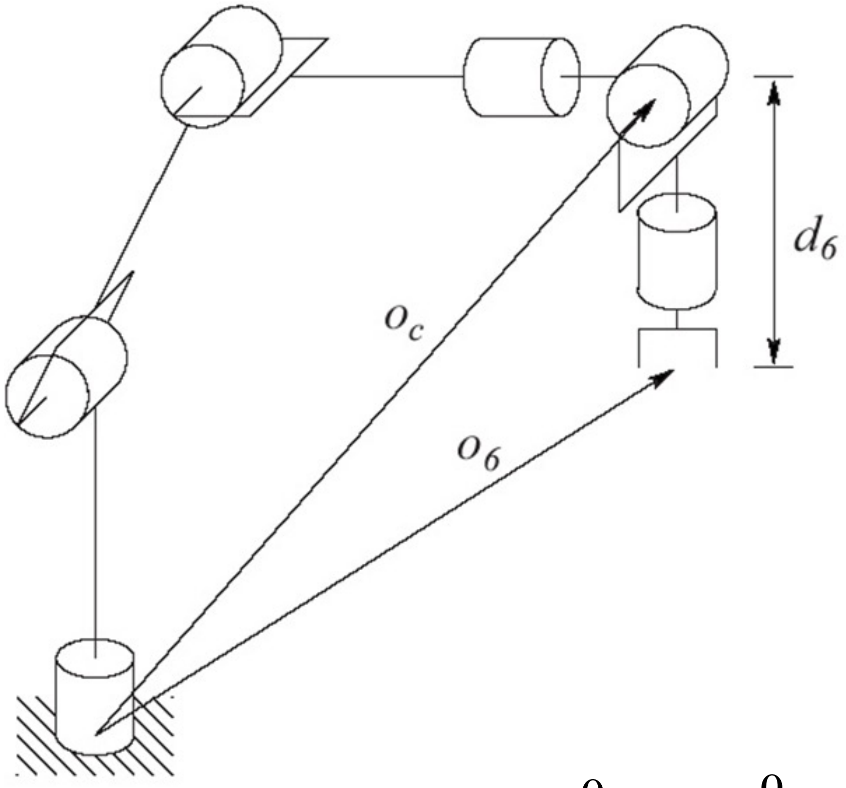
Geometric Approach



$$\theta_1 = A\tan 2(x_c, y_c)$$

$$\theta_1 = \pi + A\tan 2(x_c, y_c)$$

Example -- Decoupling



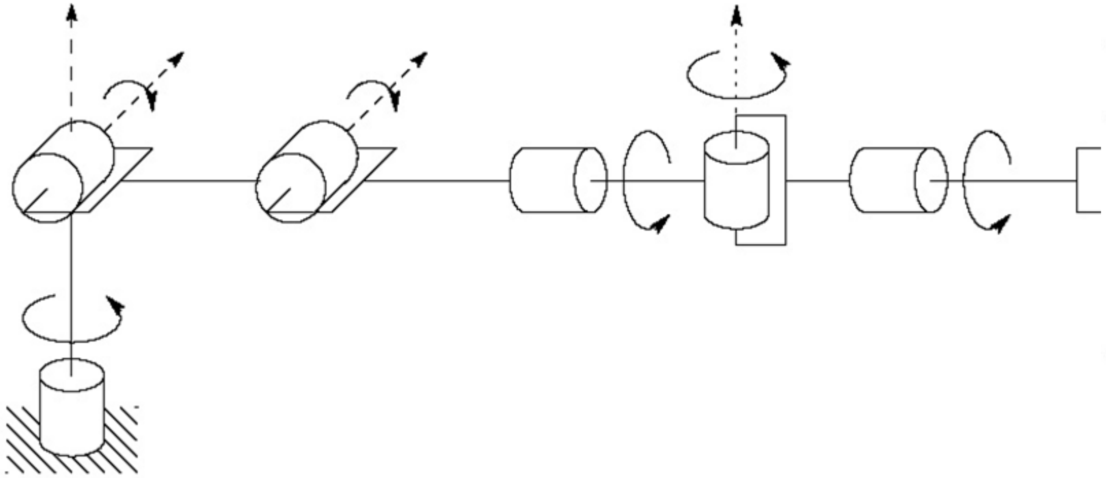
$$o_6^0 = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$o_c^0 = o_6^0 - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R = R_1^{\ 0} R_2^{\ 1} R_3^{\ 2} \ldots R_n^{\ n-1}$$

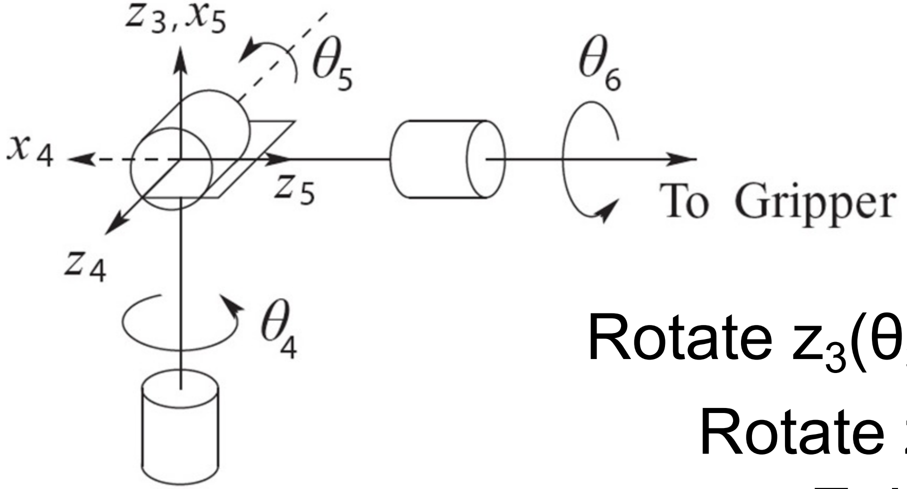
$$A_3^0 = A_1^{\ 0} A_2^{\ 1} A_3^{\ 2} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wrist



Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

* variable



$$R_6^3 = (R_3^0)^T R_6^0$$

Rotate $z_3(\theta_3)$, $x_4(-90)$, $z_4(\theta_4)$, $x_5(90)$, $z_5(\theta_5)$

Rotate $z(\theta_3)$, $y(\theta_4)$, $z(\theta_5)$

Euler angles!!!

Euler Angles

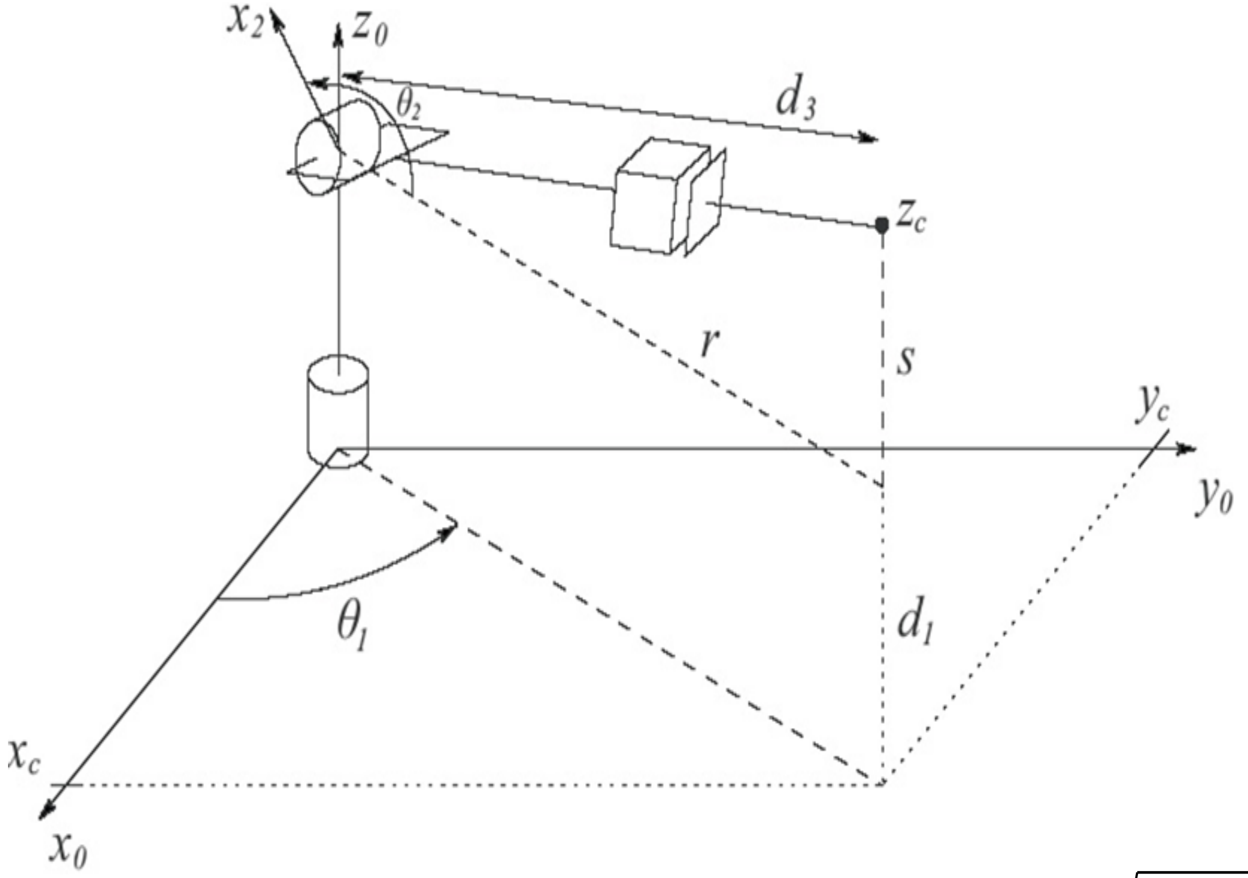
$$R_{ZYZ} = R_{z,\phi} R_{y,\theta} R_{Z,\psi}$$

$$= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc} \mathrm{c}\phi\mathrm{c}\theta\mathrm{c}\psi - \mathrm{s}\phi\mathrm{s}\psi & -\mathrm{c}\phi\mathrm{c}\theta\mathrm{s}\psi - \mathrm{s}\phi\mathrm{c}\psi & \mathrm{c}\phi\mathrm{s}\theta \\ \mathrm{s}\phi\mathrm{c}\theta\mathrm{c}\psi + \mathrm{c}\phi\mathrm{s}\psi & -\mathrm{s}\phi\mathrm{c}\theta\mathrm{s}\psi + \mathrm{c}\phi\mathrm{c}\psi & \mathrm{s}\phi\mathrm{s}\theta \\ -\mathrm{s}\theta\mathrm{c}\psi & \mathrm{s}\theta\mathrm{s}\psi & \mathrm{c}\theta \end{array} \right]$$

$$\phi = \theta_4 \quad \theta = \theta_5 \quad \varphi = \theta_6$$

Example



$$\theta_1 = A\tan 2(x_c, y_c)$$

$$\theta_1 = \pi + A\tan 2(x_c, y_c)$$

$$\theta_2 = \frac{\pi}{2} + A\tan 2(r, s)$$

$$d_3 = \sqrt{r^2 + s^2} = \sqrt{{x_c}^2 + {y_c}^2 + (z_c - d_1)^2}$$

Iterative Solutions of Inverse Kinematics

- Only holds for high sampling rates or low Cartesian velocities
- “a local solution” that may be “globally” inappropriate
- Problems with singular postures
- Can be used in two ways:
  - As an instantaneous solutions of “which way to take “
  - As an “batch” iteration method to find the correct configuration at a target

$$\dot{\mathbf{x}} = J(\theta)\dot{\theta}$$

$$\dot{\theta} = J(\theta)^{\#}\,\dot{\mathbf{x}}$$