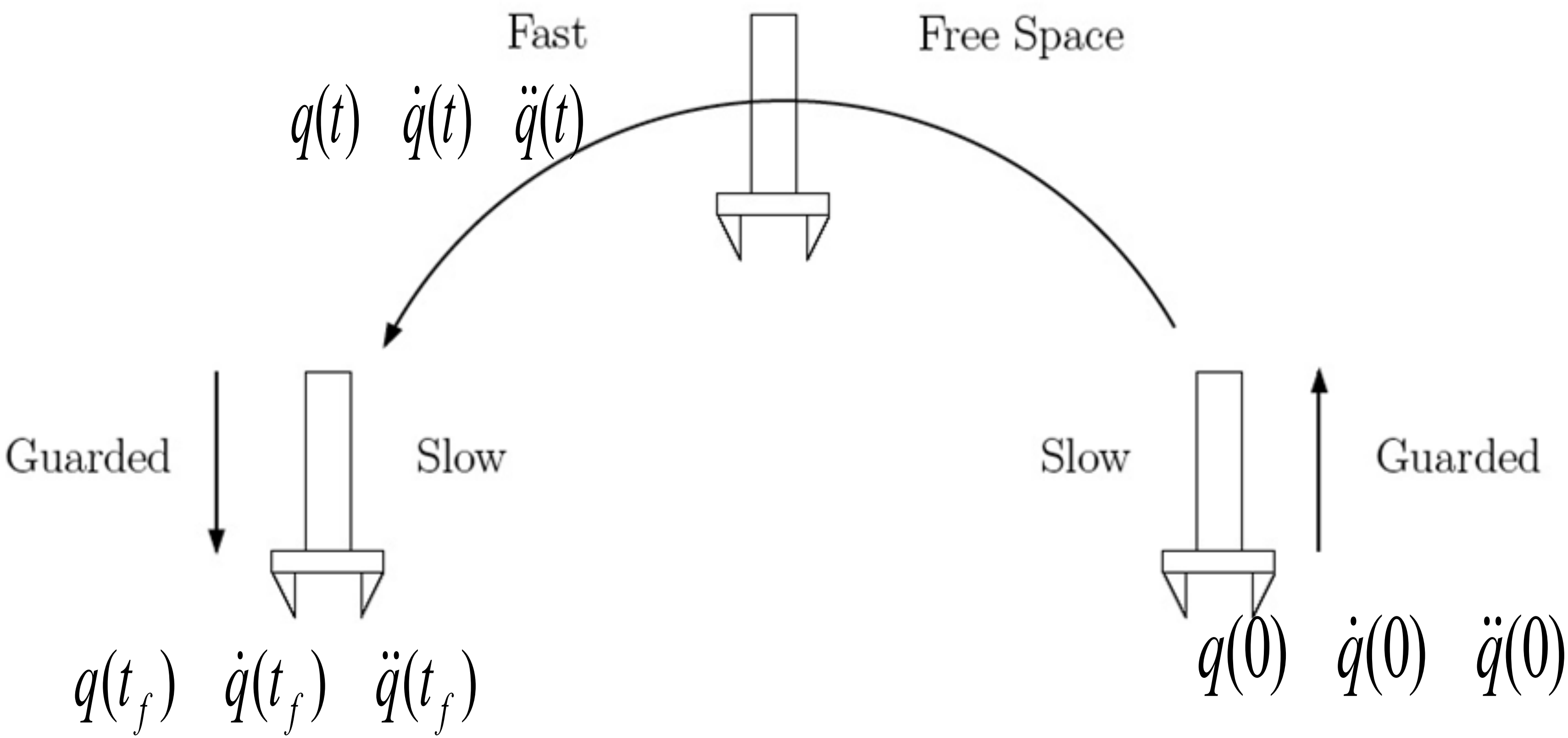
Intro to Robotics

Lecture 12

Motion Planning Algorithms

	Potential Field	PRM	Wave front	Cell Decomp	Visibility
Practical for low D	Y	Y	Y	Y	Y
Practical for high D>3	Y (tricky with random samples)	Y	N	N	N
Fast	Y	Y	Low-D	Low-D	Low-D
Efficient update (world, start, goal)	Y	N/Y	N	N/Y	N
Problem	Local min	Narrow path	Not practical for high D	Need to know the geometry	Need to know the geometry

Trajectory Design



Polynomial Trajectories

$$\mathrm{q}(0) = \mathrm{q}_0 \qquad \mathrm{q}(t_f) = \mathrm{q}_t$$

$$\dot{\mathrm{q}}(0) = 0 \qquad \dot{\mathrm{q}}(t_f) = 0$$

$$\ddot{\mathrm{q}}(0) = 0 \qquad \ddot{\mathrm{q}}(t_f) = 0$$

Cubic Polynomial Trajectories:

$$\mathrm{q}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\mathrm{q}}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$a_0 + a_1 t_0 + a_2 t_0^{\;2} + a_3 t_0^{\;3} = q_0$$

$$a_1 + 2a_2 t_0 + 3a_3 t_0^{\;2} = v_0$$

$$a_0 + a_1 t_f + a_2 t_f^{\;2} + a_3 t_f^{\;3} = q_f$$

$$a_1 + 2a_2 t_f + 3a_3 t_f^{\;2} = v_f$$

$$\Rightarrow \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^2 \\ 0 & 1 & 2t_0^2 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f^2 & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix}$$

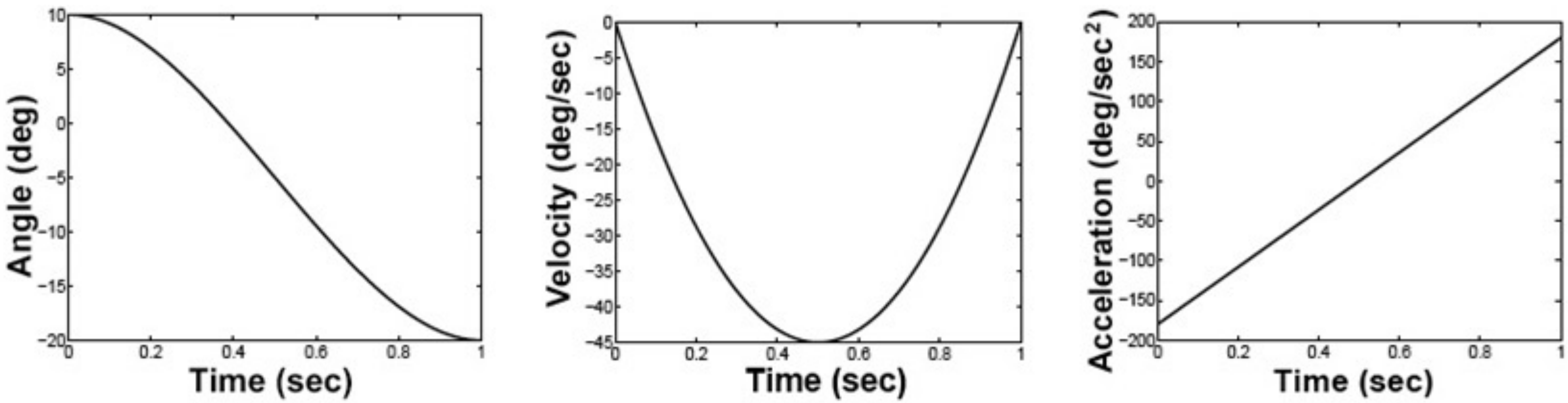
Example

$$\mathrm{q}(0)=\mathrm{q}_0 \quad \dot{\mathrm{q}}(0)=0 \quad \mathrm{q}(1)=\mathrm{q}_t \quad \dot{\mathrm{q}}(1)=0$$

$$\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^2 \\ 0 & 1 & 2t_0^2 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f^2 & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ 0 \\ q_f \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad a_0 = q_0 \qquad a_1 = 0 \qquad a_2 = 3(q_f - q_0) \qquad a_3 = -2(q_f - q_0)$$

Cubic polynomial Trajectory Example



$$q_0 = 10 \qquad q_t = -20$$

Quintic Polynomial Trajectories

$$q(0) = q_0 \qquad q(t_f) = q_t$$

$$\dot{q}(0) = 0 \qquad \dot{q}(t_f) = 0$$

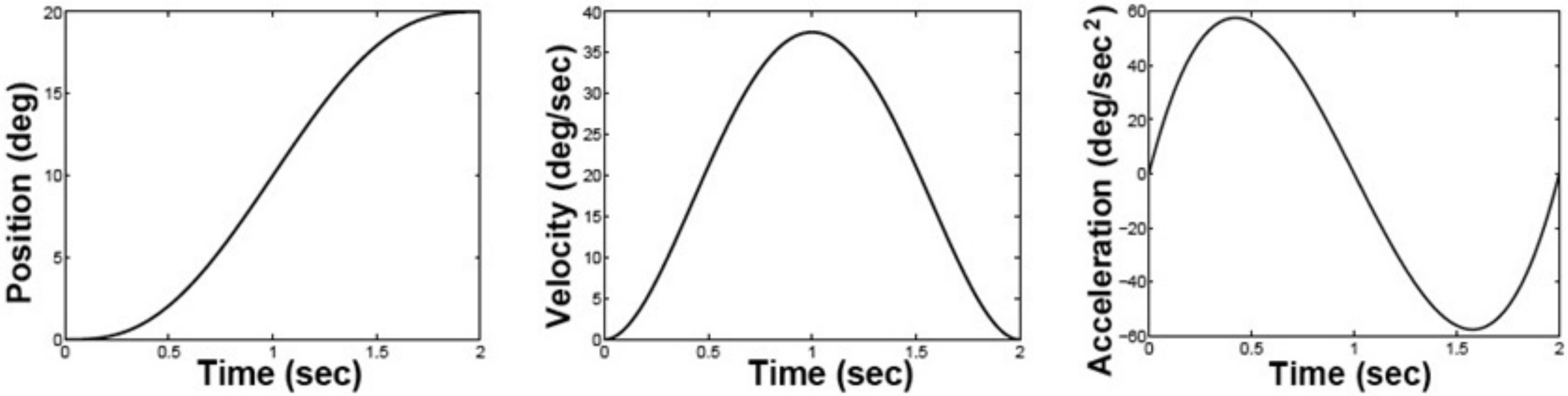
$$\ddot{q}(0) = 0 \qquad \ddot{q}(t_f) = 0$$

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

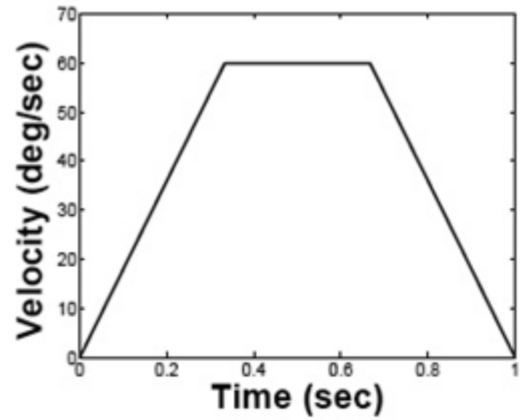
$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

Quintic polynomial Trajectory Example

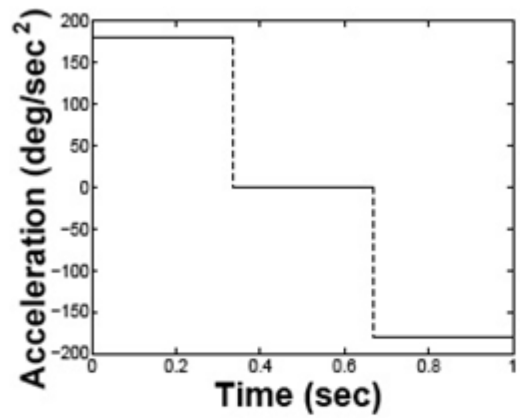


Linear Segments with Parabolic Blends (LSPB)

Speed ramped up/down
Limit maximum speed
Limit maximum acceleration



Blend:
Speed up to maximum speed with max acc
Keep the max speed for a while
Slow down with the max acc



Segment 1

$\mathrm{q}(0) = \mathrm{q}_0 \quad \dot{\mathrm{q}}(0) = 0$ Acc = a Speed = V

$$\mathrm{q}(t) = a_0 + a_1 t + \frac{a}{2} t^2 \Rightarrow a_0 = q_0$$

$$\dot{\mathrm{q}}(t) = a_1 + at \qquad a_1 = 0$$

$$\Rightarrow \mathrm{q}(t) = q_0 + \frac{a}{2} t^2 \qquad 0 \le t \le t_b$$

$$(t_b = V / a)$$

Segment 2

$$\mathrm{q}(t_b) = \mathrm{q}_0 + \frac{V^2}{2a}$$

$$\mathrm{q}(t) = q_0 + \frac{V^2}{2a} + V(t - \frac{V}{a})$$

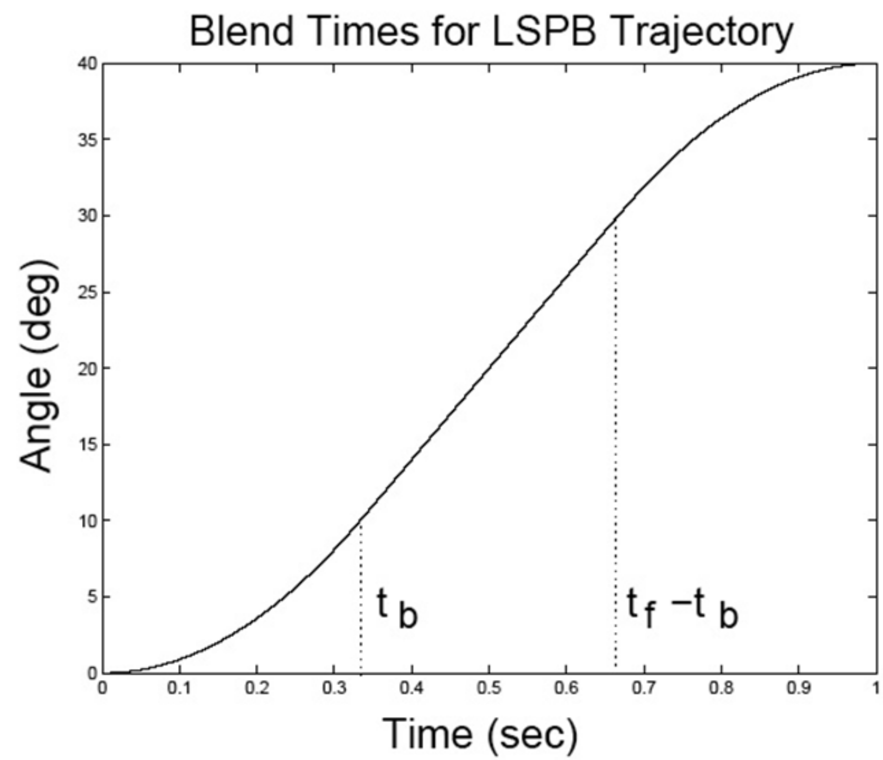
$$\mathrm{q}(t) = q_0 - \frac{V^2}{2a} + Vt$$

Segment 3

$q(t_f) = q_f \quad \dot{q}(t_f) = 0$ Acc = -a Speed = V

$$q(t) = a_0 + a_1 t + \frac{a}{2} t^2$$

$$\dot{q}(t) = a_1 + at$$



Segment 3 Continue

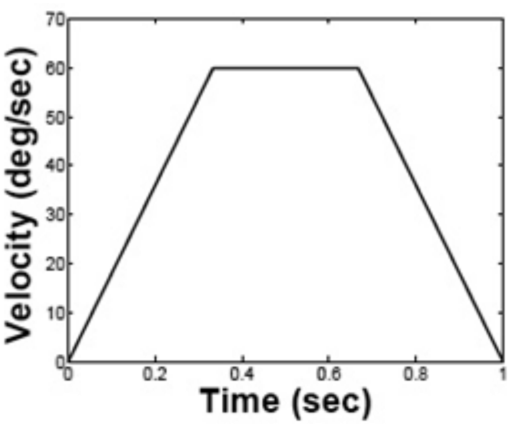
$$\mathrm{q}(t) = b_0 + b_1(t - (t_f - t_b)) + \frac{-a}{2}(t - (t_f - t_b))^2$$

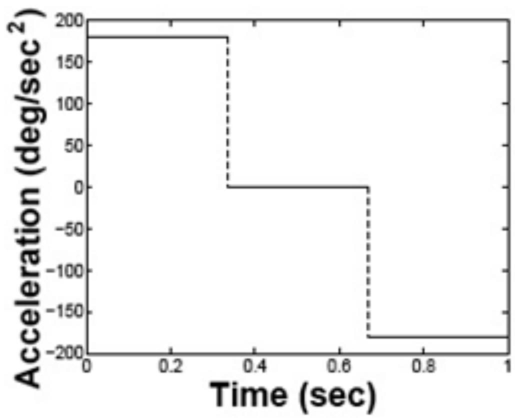
With

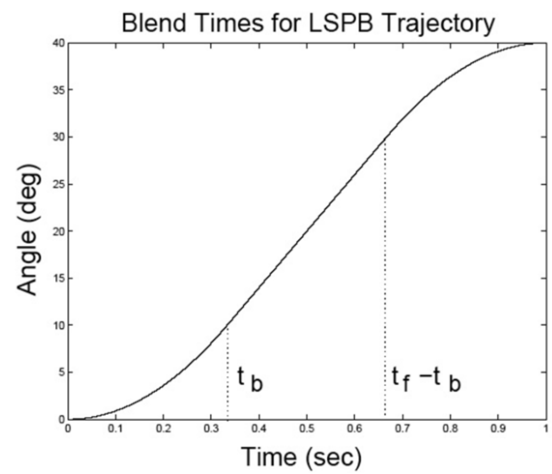
$$b_1 = at_b$$

$$\mathrm{q}(t_f) = \mathrm{q}_f$$

$$\Rightarrow \quad b_0 = q_f - \frac{a}{2}(t_b)^2$$


Velocity (deg/sec)
Time (sec)
0
10
20
30
40
50
60
70
0.2
0.4
0.6
0.8
1


Acceleration (deg/sec^2)
Time (sec)
200
150
100
50
0
-50
-100
-150
-200
0.2
0.4
0.6
0.8
1


Blend Times for LSPB Trajectory
Angle (deg)
Time (sec)
t b
t f −t b
0
5
10
15
20
25
30
35
40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

End