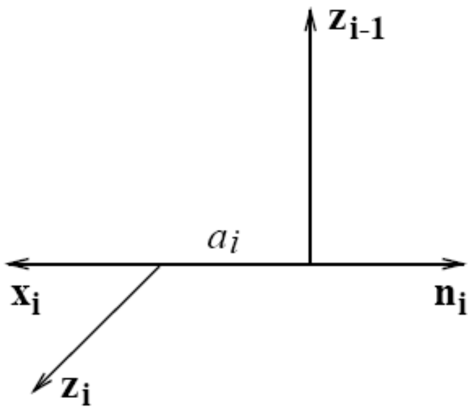
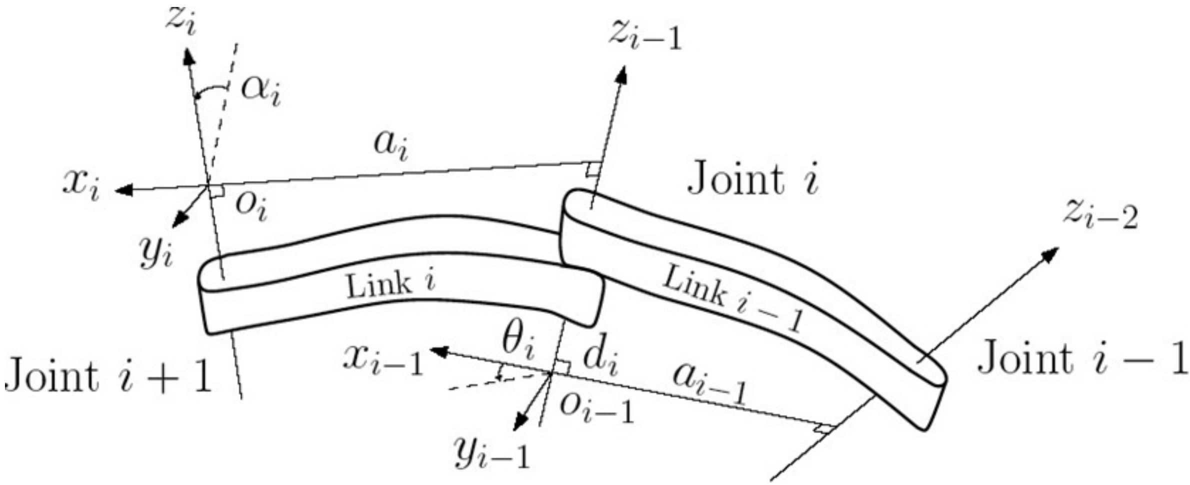
Intro to Robotics

Lecture 6

Denavit-Hartenberg Convention

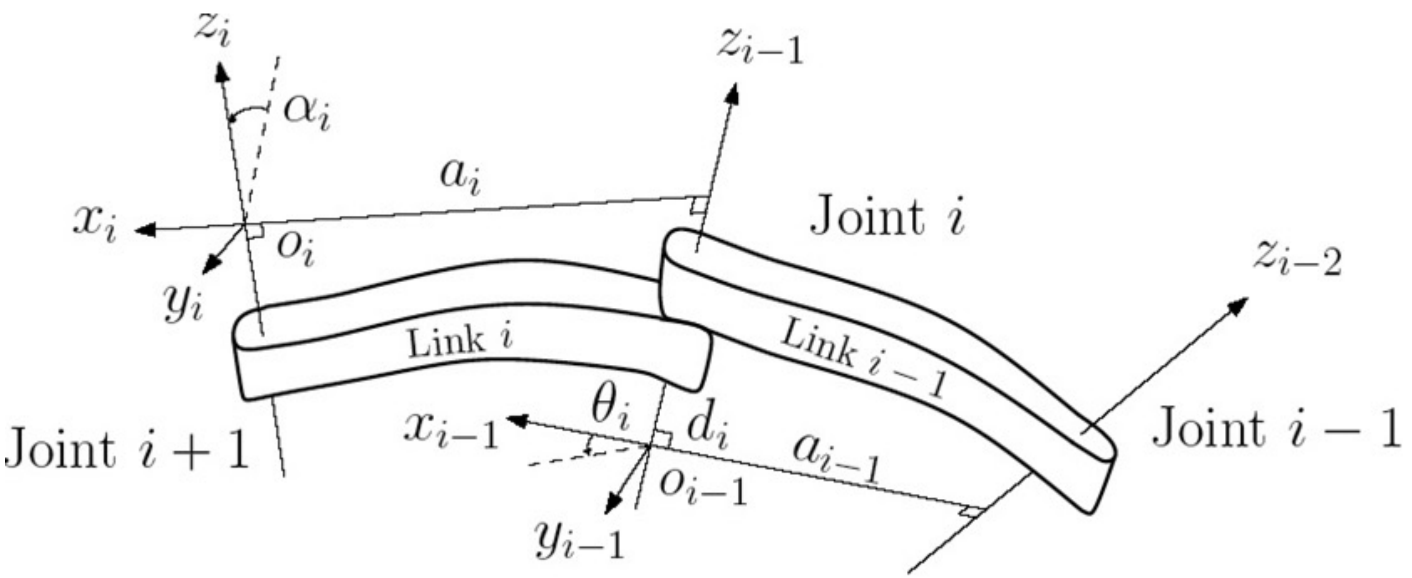
- Joint counts up from 1 at the base; axis counts up from 0
- Joint i connects link i - 1 to link i
- Align the Z_i with joint i+1
- *Base coordinate system:* Z_0 axis align with joint 1, origin is at the base
- *Origin of the coordinate system i:*
  - intersection of the Z_i & Z_{i-1} or
  - the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis
- *X_i axis:*
  - $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ pointing from Z_{i-1} to Z_i , or
  - along the common normal between the Z_{i-1} & Z_i axes when they are parallel
- *Y_i axis:* $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$

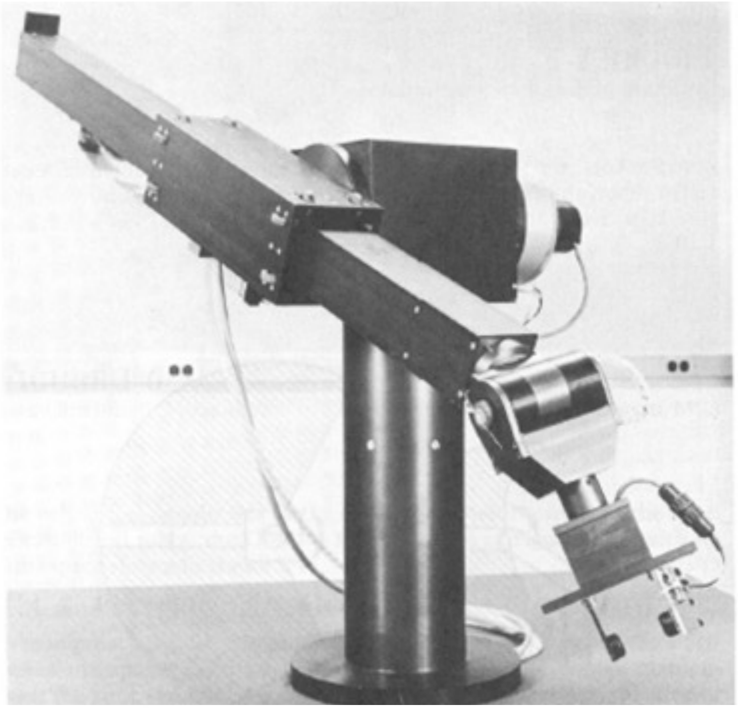




Link and Joint Parameters

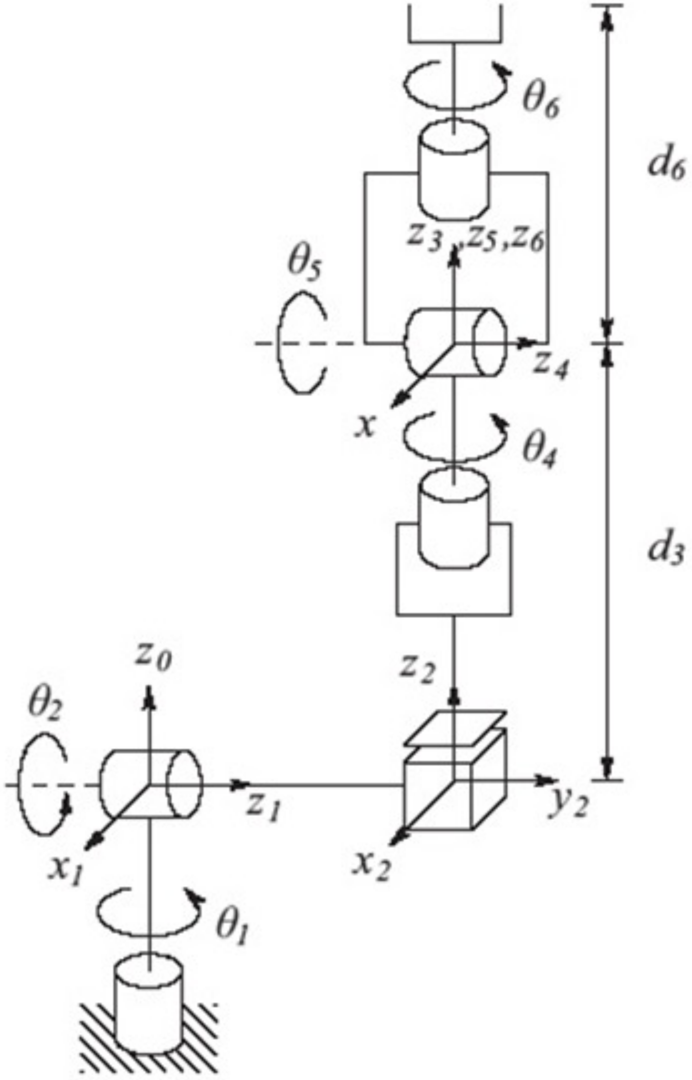
- *Joint angle* θ_i : the angle from X_{i-1} to X_i about the Z_{i-1}
- *Joint distance* d_i : the distance from X_{i-1} to X_i, as measured along Z_{i-1}. It could be negative
- *Link length* a_i: the distance from Z_{i-1} to Z_i , along X_i. It is always positive
- *Link twist angle* α_i: the angle from Z_{i-1} to Z_i about the X_i axis





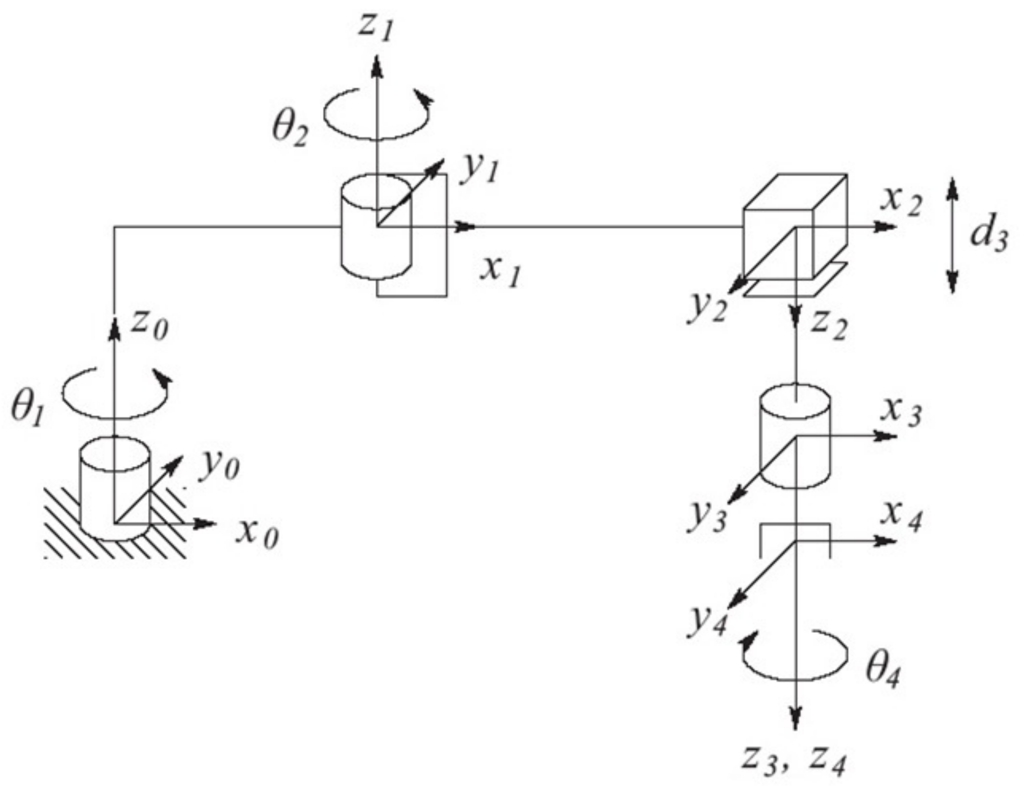
Link	d_i	a_i	α_i	θ_i
1	0	0	−90	$\theta_1^\star$
2	d_2	0	+90	$\theta_2^\star$
3	$d_3^\star$	0	0	0
4	0	0	−90	$\theta_4^\star$
5	0	0	+90	$\theta_5^\star$
6	d_6	0	0	$\theta_6^\star$

* joint variable



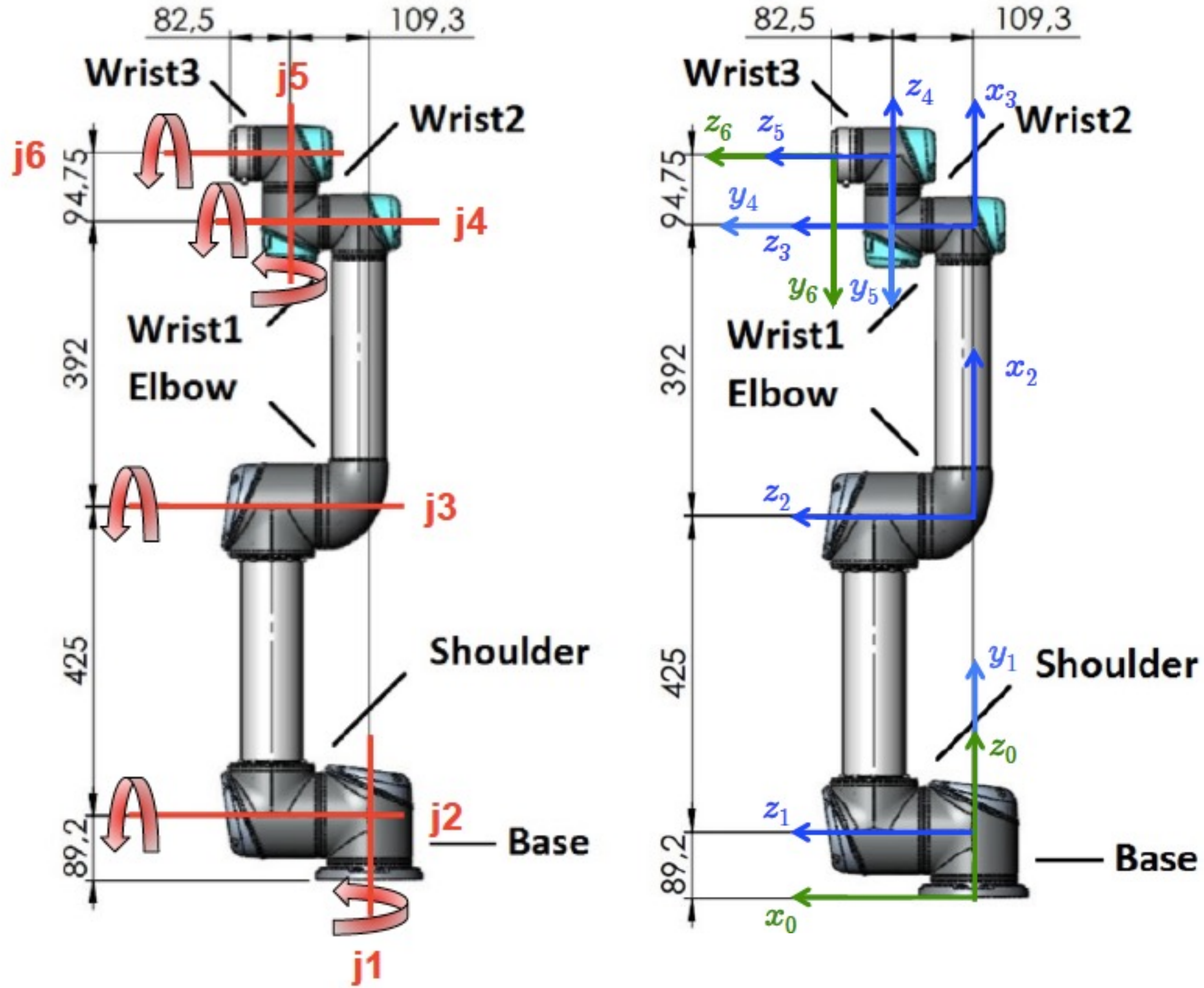
Stanford manipulator

SCARA manipulator

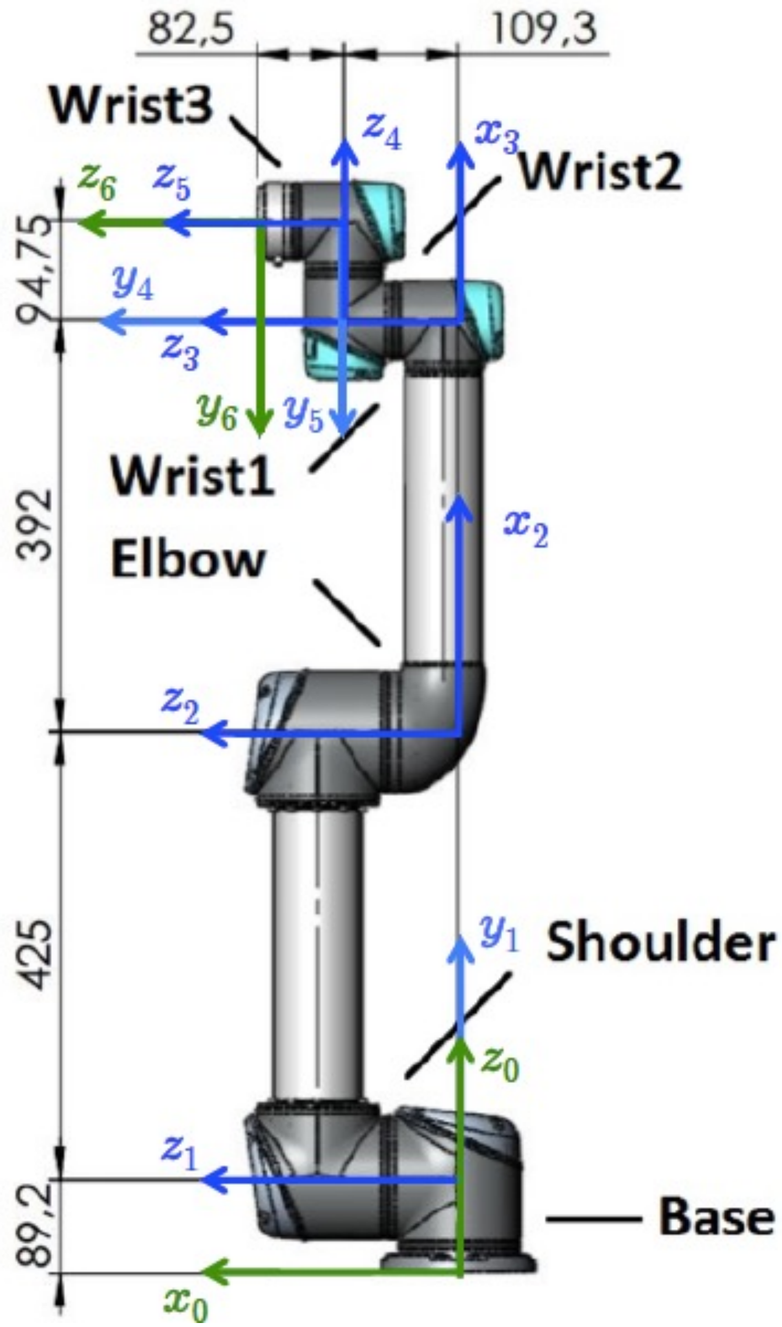


Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta_1^\star$
2	a_2	180	0	$\theta_2^\star$
3	0	0	$d_3^\star$	0
4	0	0	d_4	$\theta_4^\star$

* joint variable

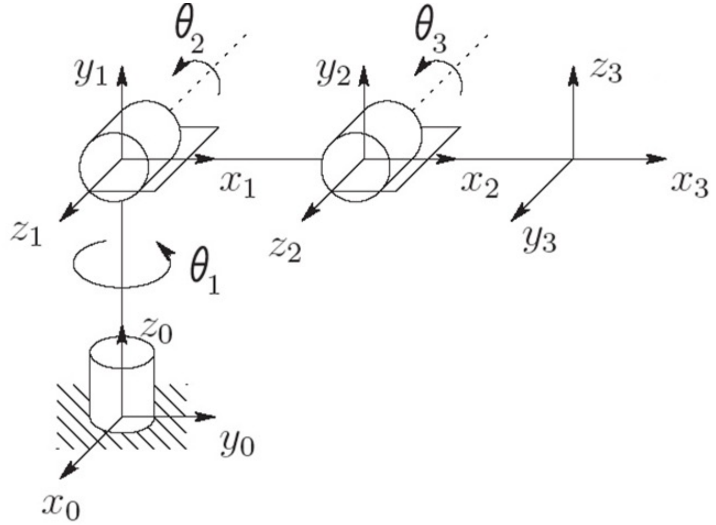


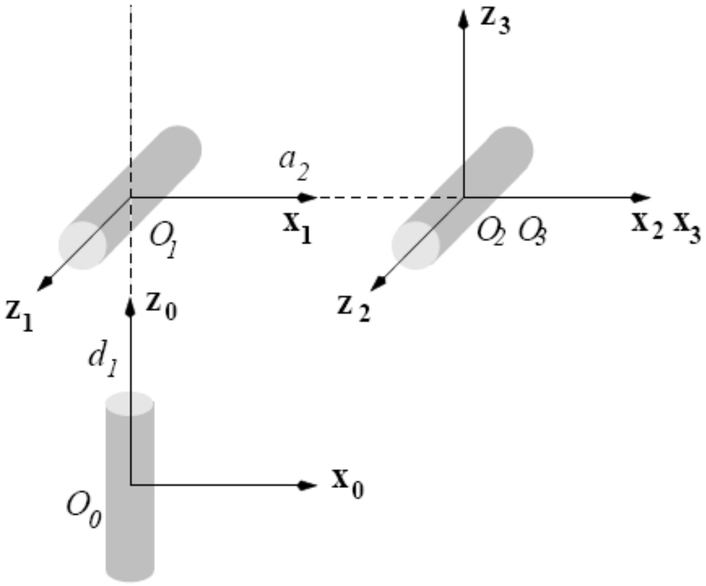
82,5
109,3
Wrist3
j5
Wrist2
j6
94,75
j4
Wrist1
392
Elbow
j3
425
Shoulder
j2
Base
89,2
j1
82,5
109,3
Wrist3
z_4
x_3
Wrist2
z_6
z_5
94,75
y_4
z_3
y_6
y_5
Wrist1
392
x_2
Elbow
z_2
425
y_1
Shoulder
z_0
z_1
Base
89,2
x_0



i	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	$d_1 = 89.2$	$\theta_1 = \pi/2$
2	0	$a_2 = 425$	0	$\theta_2 = \pi/2$
3	0	$a_3 = 392$	0	$\theta_3 = 0$
4	$\pi/2$	0	$d_4 = 109.3$	$\theta_4 = \pi/2$
5	$-\pi/2$	0	$d_5 = 94.75$	$\theta_5 = 0$
6	0	0	$d_6 = 82.5$	$\theta_6 = 0$

Example



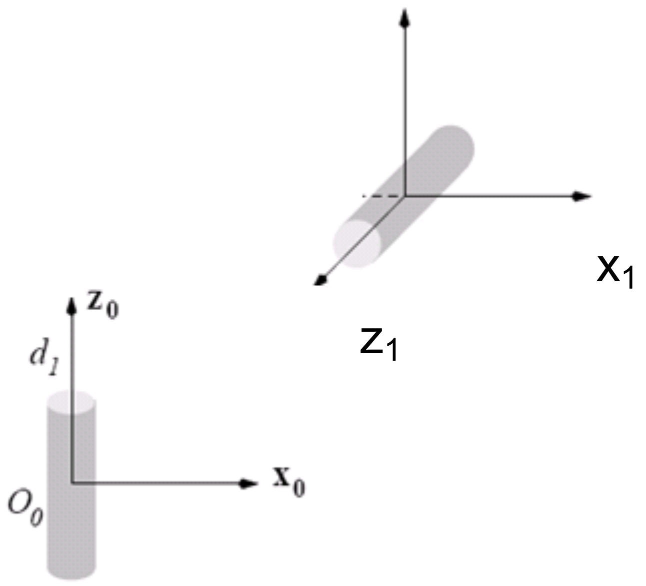


$$H_1^0 = R^0_{z_0,\theta_1} T_{z_0,d_1} R_{x_1,\alpha_1}$$

Transformation between i-1 and i

$$A_i^{i-1} = R(z_{i-1}, \theta_i) T(z_{i-1}, d_i) T(x_i, a_i) R(x_i, \alpha_i)$$

- Rotate about the Z $_{i-1}$ axis an angle of θ_i
- Translate along the Z $_{i-1}$ axis a distance of d_i
- Translate along the X_i axis a distance of a_i
- Rotate about the X_i axis an angle of α_i



Transformation between i-1 and i

- D-H transformation matrix for adjacent coordinate frames, *i* and *i-1.*
  - The position and orientation of the *i*-th frame coordinate can be expressed in the (*i-1)*th frame by the following homogeneous transformation matrix:

$$A_i^{i-1} = R(z_{i-1}, \theta_i) T(z_{i-1}, d_i) T(x_i, a_i) R(x_i, \alpha_i)$$

$$\begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

* variable

$$A_{i-1}^{i} = R(z_{i-1},\theta_i)T(z_{i-1},d_i)T(x_i,a_i)R(x_i,\alpha_i)$$

$$\begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = A_1^0 = \begin{bmatrix} C\theta_1 & -C0S\theta_1 & S0S\theta_1 & 0C\theta_1 \\ S\theta_1 & C0C\theta_1 & -S0C\theta_1 & 0S\theta_1 \\ 0 & S0 & C0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^1 = A_1 A_2 A_3 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & -S\theta_1 d_3 \\ S\theta_1 & 0 & -C\theta_1 & C\theta_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$