# Modeling Pedestrian Dynamics: The Force Based Approach

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# Who am I?

- Trained as a mathematician.
- Worked as an engineer in industry for 14 years.
- Teach modeling to computer scientists at Munich University of Applied Sciences.

➥ A typical mathematical modeler: a mixed breed.

# Outline

# Why model pedestrian dynamics?

- The engineer's view: Improve safety through better planning and control, make traffic more efficient.
- The empirical scientist's view: Better understand crowds.
- The game developer's view: Entertain.

➥ Different goals lead to different requirements.

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# Motivation

- Force based models use a physical analogy to model pedestrian motion.
- Where does the idea come from?

# The history

- The inspiration from psychology (Kurt Lewin, 1890-1947): Field theory in social science: Selected theoretical papers. (Lewin, 1951)
- A first cellular automaton based on repulsive forces: 'The model hypothesizes the existence of repulsive forces between pedestrians ...' (Gipps and Marksjö, 1985)
- The social force model: 'It is suggested that the motion of pedestrians can be described as if they would be subject to 'social forces' '. (Helbing and Molnár, 1995)

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# The physical analogy

- The idea is that pedestrians can be modeled like particles that are subject to physical forces, like Newtonian mechanics.
- The beauty is that all mathematical and numerical knowledge, all algorithms developed for physics are at our disposal!

➥ The price we pay is that our virtual pedestrians will behave like particles.

# Requirements for a pedestrian model

Minimum requirements:

- Agents move towards a target.
- At a speed that makes sense for humans.
- Agents avoid collisions with others.
- Agents avoid collisions with obstacles.

# Agent $j$ moves towards target $x^0$

- Let $x^j$ be the position of agent $j$ in two dimensional space: $x^j = (x_1^j, x_2^j)$
- Let $x^0$ be the position of the target.
- In this tutorial we'll mostly choose $x^0 = (0, 0)$

The velocity $v^j$ of agent $j$ is given by the derivative of its location $x^j$ with respect to time.

$$\frac{dx^j}{dt} = v^j. \tag{1}$$

# Agent $j$ moves towards target $x^0$

Instantaneous acceleration to the free-flow velocity $v_0^j$ is driven by a force that points towards the target $x^0$.

$$\frac{dv^j}{dt} = -\mu \frac{(x^j - x^0)}{\|x^j - x^0\|} v_0^j. \tag{2}$$

(Note: Constant $\mu$ has unit $\frac{1}{s}$. For simplicity, $\mu = 1\frac{1}{s}$. )

This is similar to how a satellite is accelerated towards Earth.
Gravitational force $F$ between masses $m_0, m_1$, Earth and satellite, at positions $x^0, x^1$ with gravitational constant $G$:

$$F = -G m_1 m_2 \frac{x_1 - x_0}{\|x_1 - x_0\|^2} \tag{3}$$

# Questions

- If $v^j$ points towards $x^0$, which path does our particle follow?
- What do you observe? Does it fit your expectations for pedestrians?

# Starting velocity points to target

Figure: Starting velocity: $v(0) = (-1, 0)$, , $\mu = 1$, free-flow velocity $v^0 = 1.34$.

# Questions

- What if $v^j$ is tangential to an orbit around the target?
- What do you observe? Does it fit your expectations for pedestrians?

# Starting velocity tangential to orbit

Figure: Starting velocity: $v(0) = (0, 1)$, , $\mu = 1$, free-flow velocity $v^0 = 1.34$.

# Starting velocity tangential to orbit

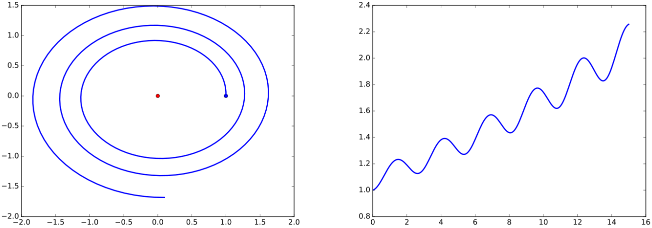



Figure: Left: Trajectory towards target. Right: speed $v$. $\mu = 1$, free flow velocity $v^0 = 1.34$

# Adding 'friction'

$$\frac{dv^j}{dt} = -\mu \left( \frac{(x^j - x^0)}{\|x^j - x^0\|} v_0^j - v^j \right) \tag{4}$$

- This time, the agent's change in velocity is attenuated by the velocity it already has. There is some 'inertia'.
- The agent decelerates when it exceeds its free-flow velocity.

# Questions for the practical session

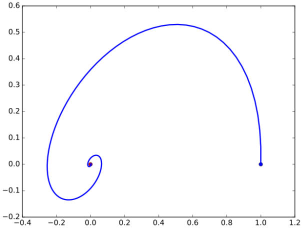
- What is different?

With 'friction':

Figure: Starting velocity: $v(0) = (0, 1)$, , $\mu = 1$, free-flow velocity $v^0 = 1.34$.

# Starting velocity tangential to orbit

With 'friction':



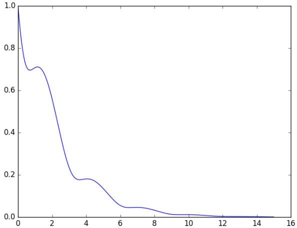




Figure: Left: Trajectory towards target. Right: speed $v$. $\mu = 1$, free flow velocity $v^0 = 1.34$

# Simplified motion to the target

$$\frac{dx^j}{dt} = v^j, \quad (5)$$

$$\frac{dv^j}{dt} = \frac{1}{\tau}\left(-\frac{x^j - x^0}{\|x^j - x^0\|} v_0^j - v^j\right). \quad (6)$$

$\tau = \frac{1}{\mu}$ can be interpreted meaningfully as an agent's reaction time.

This is almost the motion towards a target of the full social force model.

$$\frac{dv^j}{dt} = \frac{d^2x^j}{dt^2} = \frac{1}{\tau}\left(-\frac{x^j - x^0}{\|x^j - x^0\|}v_0^j - v^j\right). \tag{7}$$

$$= \frac{1}{\tau}\left(-\frac{x^j - x^0}{\|x^j - x^0\|}v_0^j - \frac{dx^j}{dt}\right). \tag{8}$$

strongly resembles the harmonic oscillator

$$m\ddot{x} = -(\omega_0^2 x - 2\zeta\omega_0\dot{x}) \tag{9}$$

with $\omega_0$ angular frequency, *zeta* damping ratio.

- Oscillations are a natural part of force based models!
- Are they also natural for humans?

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## Similarity to harmonic oscillator

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- Are they also natural for humans?

# Question: What is still unrealistic?

- Besides the oscillations?
- With respect to motion towards a target?

➥ Agents can still have any (unrealistically fast) speed.

➥ The social force model cuts off at a speed a little over free-flow.

- We'll neglect this for the moment.

➥ What about collisions?

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This is where 'social' forces comes in.

- Agent $j$ 'repulses' agent $k$ by a force $F_{j,k}$.
- Repulsive forces of all other agents are added (superposition principle).

$$\frac{dv^j}{dt} = \frac{1}{\tau}\left(-\frac{x^j - x^0}{\|x^j - x^0\|} v_0^j - v^j\right) + \sum_{k=1, k\neq j}^{n} F_{jk} \tag{10}$$

The force is given as the gradient $F_{j,k} = -\nabla P_{j,k}$ of a 'potential' $P_{j,k}$. Following Helbing and Molnár (1995) for an agent of circular shape:

$$F_{j,k} = \frac{x^j - x^k}{\|x^j - x^k\|} \frac{V^0}{\sigma} e^{-\frac{\|x^j - x^k\|}{\sigma}} \tag{11}$$

You get it from a potential of type $e^{-\|x^j - x^k\|}$.

Parameters $V^0$ und $\sigma$ give the strenght and reach of the repulsion. Helbing and Molnár (1995): $V^0 = 2.1$ ( with unit $m^2/s^2$) and $\sigma = 0.3$.

# Question: What is still missing?

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➥ Agents can still collide with obstacles.

➥ Add repulsive forces from obstacles.

- We'll neglect obstacles for the moment.

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# The full model (without obstacles)

The social force model and its extensions (e.g. other potential types) are probably the most wide spread model type for pedestrian simulations:

$$
\begin{aligned}
\frac{dx^j}{dt} &= v^j, && (12)\\
\frac{dv^j}{dt} &= \frac{1}{\tau}\left(-\frac{x^j - x^0}{\|x^j - x^0\|} v_0^j - v^j\right) + \sum_{k=1, k\neq j}^{n} F_{jk} && (13)
\end{aligned}
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# Criticism

But is a force based model always the best choice?

- Game developer: Does it look cool?
- Engineer: Does it work?
- Scientist: Does it describe and explain the truth?

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# Criticism

- Engineer: Collisions, oscillations and deadlocks.
- Engineer: Numerical issues.
- Scientist: Fundamental doubts.

A collision with oscillations and a deadlock:

Figure: Deadlock. Starting velocity: left person Person $v(0) = (1, 1)$, right person $v(0) = (-1, 1)$, $\mu = 1$, free-flow velocity $v^0 = 1.34$.

# Crossing paths?

## Deadlock

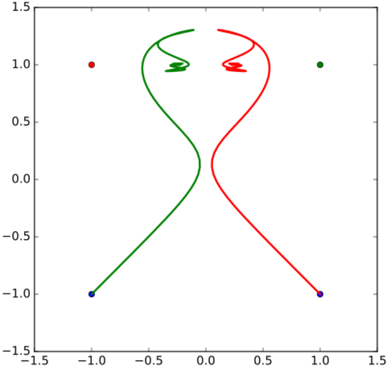



Figure: Trajectories of two agents caught in a deadlock. $\mu = 1$, free flow velocity $v^0 = 1.34$.

# Collisions versus oscillations

(Chraibi et al., 2011) '... discuss some intrinsic problems of this approach, like penetration of particles, unrealistic oscillations and velocities.'

- There is a trade-off between overlaps and oscillations!

Please look at

- (Chraibi, 2014; Dietrich et al., 2014; Chraibi, 2012) for more detailed arguments.
- (Yu et al., 2005; Chraibi et al., 2010) for extensions that take relative velocities into account.

# Numerical issues

The ordinary differential equations of force based models together with starting locations and velocities form an initial value problem.

$$\begin{aligned} \dot{y} &= \mathcal{F}(y) && (14) \\ y(0) &= y_0. && (15) \end{aligned}$$

The simplest numerical method to solve an initial value problem is Euler's scheme.

$$y_n = y_{n-1} + \Delta t \mathcal{F}(y_{n-1}). \quad (16)$$

# Why Euler's scheme is a poor choice

- Euler's scheme is slow: only first order convergence.
- ➥ Halving the step sizes only halves the error $\|$True solution - numerical approximation$\|$ .
- ➥ To get acceptable resolution we need a very small step size: $\Delta t < 10^{-3}$.

# Why typical solution strategies fail

Use higher order schemes:

- Fifth order Runge-Kutta: Halving the step size reduces the error by $2^{-5} = \frac{1}{32}$.
- But: The right hand side of the social force model is discontinuous, e.g. at target positions, and thus the solution is not smooth.

➥ Higher order schemes perform even worse than Euler.

➥ Even Euler's method must blow up when an agent 'steps' on a target.

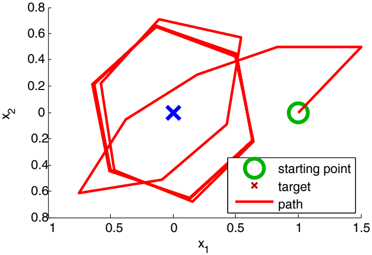
➥ Look at (Köster et al., 2013) for a mollified social force model.

Parallelize:

- Works well, but makes changes tedious.

# Euler's method close to a target

- Try in the practical session: stable 'orbit' at full speed.
- This agent will never reach the target, but circle it like the moon circles Earth.
- Reducing $\Delta t$ does not help.



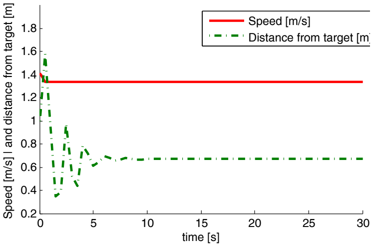




Figure: $\Delta t = 0.5\,s$, free-flow velocity $1.34\,\frac{m}{s}$. Figure taken from (Köster et al., 2013)

# Fundamental doubts

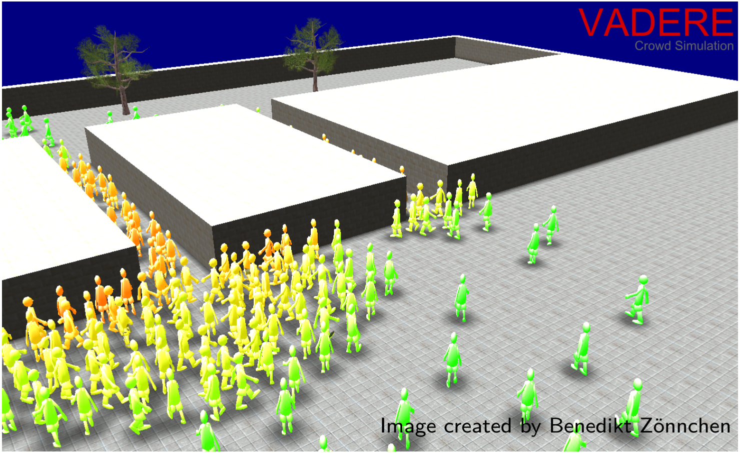
- Are humans particles?
- ➥ Particle physics works to reproduce some observed phenomena in crowd dynamics. It fails with others.
- ➥ Force based models are very useful for engineering and games development!
- But do they suffice to explain human behavior?

- Getting rid of inertia: Velocity based models (Dietrich and Köster, 2014; Tordeux and Seyfried, 2014; Tordeux et al., 2015).
- Getting rid of differential equations: Cellular automata, e.g. (Gipps and Marksjö, 1985; Blue and Adler, 2001; Burstedde et al., 2001; Kirik et al., 2009; Zhang et al., 2012; Davidich and Köster, 2013; Was and Lubaś, 2013; Bandini et al., 2014; Qiang et al., 2014; Hu et al., 2014; Hsu and Chu, 2014; Fu et al., 2015; Wei et al., 2015; Lubaś et al., 2016)
- Getting rid of differential equations: Utility and the Optimal Steps Model (Seitz and Köster, 2012; Seitz et al., 2014, 2015b; von Sivers and Köster, 2015; von Sivers and Köster, 2015; von Sivers et al., 2016).
- Towards modeling true decision making: (Seitz et al., 2015a; Seitz, 2016).

# Form your own opinion!

- Play with our open source implementation: openVADERE.
- www.vadere.org.
- License: LGPL.
- Motion models: Optimal Steps Model, Gradient Navigation Model, Social Force Model, Reynold's steering, ...

# Questions?


VADERE
Crowd Simulation
Image created by Benedikt Zönnchen

# Practical session: Social Force Model

- The Python implementation of the Social Force Model was programmed by Mario Parente in my research group.
- It is free under the CC-BA-SA license `https://en.wikipedia.org/wiki/Creative_Commons_license`:
- ‘Licensees may copy, distribute, display and perform the work and make derivative works and remixes based on it only if they give the author or licensor the credits (attribution) in the manner specified by these.’ ‘Licensees may distribute derivative works only under a license identical ("not more restrictive") to the license that governs the original work.’

➥ You can use and change and redistribute, but you must always mention where you got the original code from.

- Get your computers.
- Store the Python code on your computer.
- Let's play!

# Literature I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Parameters for the SFM:

| Param. | Description | Value |
|---|---|---|
| $v^0$ | free-flow velocity | 1.34 $m$ |
| $\tau$ | reaction time | 1 $s$ |
| $\sigma$ | reach of repulsion in social force | 0.3 |
| $V^0$ | strength of repulsion in social force | 2.1 |

Try to reproduce the following deadlock. Play with the force parameters and target locations to get overlaps and oscillations.

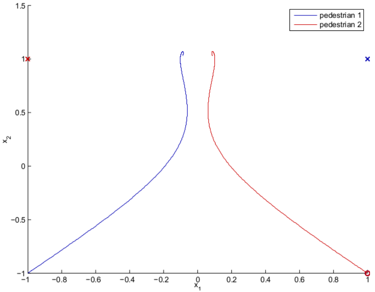


Figure: Figure from Köster et al. (2013): Trajectories of two agents with opposing targets (marked red and blue in $(-1, 1)$ und $(1, 1)$ (unit $m$)).