Tarea 5

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Ejercicios Bernoulli

1.

$$xy' + x^5 y = x^5 y^{\frac{1}{2}}$$

Multiplicamos por el recíproco de y^r, donde $r = \frac{1}{2}$

$$\frac{1}{y^{\frac{1}{2}}}(xy' + x^5 y) = x^5$$

Hacemos cambio de variable $u = y^{\frac{1}{2}}$

$$u = y^{\frac{1}{2}} \Rightarrow u' = \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx} \Rightarrow u' = \frac{1}{2} y^{-\frac{1}{2}} y'$$

Sustituimos en la ecuación

$$2xu' + x^5 u = x^5$$

La transformamos a una ecuación de la forma $\frac{dx}{dy} + P(x)y = Q(x)$

$$\frac{dx}{du} + \frac{x^4 u}{2} = \frac{x^4}{2}$$

$$P(x) = \frac{x^4}{2},\ Q(x) = \frac{x^4}{2},\ I = e^{\int \frac{x^4}{2} dx} = e^{\frac{x^5}{10}}$$

Multiplicamos por el factor integrante

$$e^{\frac{x^5}{10}}[u' + \frac{ux^4}{2}] = \frac{x^4}{2} e^{\frac{x^5}{10}}$$

$$e^{\frac{x^5}{10}} u' + \frac{x^4}{2} u e^{\frac{x^5}{10}} = \frac{x^4}{2} e^{\frac{x^5}{10}}$$

Observamos que el lado izquierdo es la derivada del producto de las funciones $e^{\frac{x^5}{10}}$ y u

$$(e^{\frac{x^5}{10}} u)' = \frac{x^4}{2} e^{\frac{x^5}{10}}$$

$$\int (e^{\frac{x^5}{10}} u)' dx = \int \frac{x^4}{2} e^{\frac{x^5}{10}} dx$$

$$e^{\frac{x^5}{10}} u = \int \frac{x^4}{2} e^{\frac{x^5}{10}} dx$$

$$e^{\frac{x^5}{10}} u = e^{\frac{x^5}{10}} + C$$

$$u = 1 + \frac{C}{e^{\frac{x^5}{10}}}$$

$$y^{\frac{1}{2}} = 1 + \frac{C}{e^{\frac{x^5}{10}}}$$

2.

$$5y^3dx - y^2(-2x + y^2x^4)dy = 0$$

Suponemos $dy \neq 0$ y dividimos la ecuación entre $\frac{1}{dy}$

$$5y^3x' - y^2(-2x + y^2x^4) = 0$$

Desarrollamos

$$5y^3x' + 2xy^2 - y^4x^4 = 0$$
$$5y^3x' + 2xy^2 = y^4x^4$$

Observamos que $r = 4$, multiplicamos por el recíproco de x^r, *i.e.* x^{-4}

$$x^{-4}[5y^3x' + 2xy^2] = y^4$$
$$\frac{5y^3x'}{x^4} + \frac{2y^2}{x^3} = y^4$$
$$u = \frac{1}{x^3} \Rightarrow u' = -\frac{3}{x^4}\frac{dx}{dy} \Rightarrow u' = -\frac{3}{x^4}x' \Rightarrow -\frac{1}{3}u' = x^{-4}x'$$
$$-\frac{5}{3}y^3u' + 2y^2u = y^4$$

Multiplicamos por $-\frac{3}{5}y^{-3}$

$$\Rightarrow u' - \frac{6}{5}y^{-1}u = -\frac{3}{5}y$$

Tenemos que $P(x) = -\frac{6}{5}y^{-1}$, $Q(x) = -\frac{3}{5}y$ y $I = e^{\int -\frac{6}{5}y^{-1}} = e^{-\frac{6}{5}\ln|y|} = y^{-\frac{6}{5}}$

$$\Rightarrow y^{-\frac{6}{5}}[u' - \frac{6}{5}y^{-1}u] = -\frac{3}{5}y^{-\frac{1}{5}}$$
$$u'y^{-\frac{6}{5}} - \frac{6}{5}uy^{-\frac{11}{5}} = -\frac{3}{5}y^{-\frac{1}{5}}$$

Observamos que el lado izquierdo es la derivada del producto de las funciones u y $y^{-\frac{6}{5}}$

$$\Rightarrow (uy^{-\frac{6}{5}})' = -\frac{3}{5}y^{-\frac{1}{5}}$$
$$\Rightarrow \int (uy^{-\frac{6}{5}})'dy = -\frac{3}{5}\int y^{-\frac{1}{5}}dy$$
$$\Rightarrow uy^{-\frac{6}{5}} = -\frac{3}{5}\int y^{-\frac{1}{5}}dy$$
$$\Rightarrow uy^{-\frac{6}{5}} = -\frac{3}{4}y^{\frac{4}{5}} + C$$

Despejamos u

$$\Rightarrow u = -\frac{3}{4}y^2 + Cy^{\frac{6}{5}}$$
$$\Rightarrow x^{-3} = -\frac{3}{4}y^2 + Cy^{\frac{6}{5}}$$

Ejercicios Ricatti

1. $$y' = y^2 - 2xy + 1 + x^2$$

   $>$ Con solución particular $y = x,\ y' = 1$ Sustituimos

   $$y' = x^2 - 2x^2 + 1 + x^2$$
   $$1 = 1\checkmark$$

   Hacemos cambio de variable $y = x + \frac{1}{z}$

   $$y' = 1 - z^{-2}z'$$

   Sustituímos en la ecuación

   $$1 - z^{-2}z' = (x + \frac{1}{z})^2 - 2x(x + \frac{1}{z}) + 1 + x^2$$
   $$z^{-2}z' = x^2 + \frac{2x}{z} + \frac{1}{z^2} - 2x^2 - \frac{2x}{z} + x^2$$
   $$-z^{-2}z' = \frac{1}{z^2}$$
   $$z' = -1$$

   Integramos de ambos lados

   $$\int z'dx = -\int dx$$
   $$z = -x + C$$
   $$z = \frac{1}{y - x}$$
   $$\Rightarrow \frac{1}{y - x} = -x + C$$

   Suponemos $y \neq x$

   $$1 = (-x + C)(y - x)$$
   $$\frac{1}{-x + C} = y - x$$
   $$y = x + \frac{1}{C - x}$$

2. $$y' = y^2 - 2y - 15$$

   Solución particular $y = -3,\ y' = 0$

   $$y' = 9 + 6 - 15$$
   $$y' = 0\checkmark$$
   $$y = -3 + \frac{1}{z}$$
   $$y' = -\frac{1}{z^2}z'$$

Sustituimos en la ecuación

$$-z^{-2}z' = (-3+\frac{1}{z})^2 - 2(-3+\frac{1}{z}) - 15$$
$$-z^{-2}z' = 9 - \frac{6}{z} + \frac{1}{z^2} + 6 - \frac{2}{z} - 15$$
$$-z^{-2}z' = -\frac{6}{z} + \frac{1}{z^2} - \frac{2}{z}$$
$$z' = -6z + 1 - 2z$$
$$z' = -8z + 1$$
$$z' + 8z = 1$$

Tenemos que $P(x) = 8$, $Q(x) = 1$ y $I = e^{\int 8dx} = e^{8x}$ Multiplicamos por el factor integrante

$$\Rightarrow e^{8x}z' + 8e^{8x}z = e^{8x}$$
$$(e^{8x}z)' = e^{8x}$$

Integramos ambas partes

$$\int (e^{8x}z)'dx = \int e^{8x}dx$$
$$e^{8x}z = \frac{1}{8}e^{8x} + C$$
$$z = \frac{1}{8} + \frac{C}{e^{8x}}$$

Sustituimos $z = \frac{1}{y+3}$

$$\frac{1}{y+3} = \frac{1}{8} + \frac{1}{e^{8x}}$$

Despejamos y

$$1 = (y+3)(\frac{1}{8} + \frac{1}{e^{8x}})$$
$$y + 3 = \frac{1}{\frac{1}{8} + \frac{C}{e^{8x}}}$$
$$y = \frac{1}{\frac{1}{8} + \frac{C}{e^{8x}}} - 3$$
$$y = \frac{e^{8x}}{e^{8x} + 8C} - 3$$
$$y = \frac{e^{8x} - 3(e^{8x} + 8C)}{e^{8x} + 8C}$$
$$y = \frac{5e^{8x} - 24C}{e^{8x} + 8C}$$