Design of Test Problem for Genetic Algorithm

Shish-Ming Wang
Jie-Wei Wu
Wei-Ming Chen
Tian-Li Yu

TEIL Technical Report No. 2013004
January, 2013

Taiwan Evolutionary Intelligence Laboratory (TEIL)
Department of Electrical Engineering
National Taiwan University
No.1, Sec. 4, Roosevelt Rd., Taipei, Taiwan

http://teil.ee.ntu.edu.tw

Design of Test Problem for Genetic Algorithm

Shish-Ming Wang r01921055@ntu.edu.tw
Jie-Wei Wu r01921056@ntu.edu.tw
Wei-Ming Chen r00921044@ntu.edu.tw
Tian-Li Yu tianliyu@cc.ee.ntu.edu.tw
National Taiwan University

Abstract

When the estimation of distribution algorithms are applied to real-world problems, two kinds of problem structures, overlapping and conflict structures, might be difficult to solve. To test different EDAs' capabilities of dealing with overlapping and conflict structures, some test problems have been proposed. However, the upper-bound of the degree of overlap and the effect of conflict have not been fully investigated. This paper investigates how to properly define the degree of overlap and the degree of conflict to reflect the difficulties of problems for the EDAs. A new test problem is proposed with the new definitions of the degree of overlap and the degree of conflict. A framework for building the proposed problem is presented, and some model-building genetic algorithms are tested by the problem. This test problem can be applied to further researches on overlapping and conflict structures.

1 Introduction

Researches on the estimation of distribution algorithms (EDAs) are currently important topics of genetic algorithms (GAs). EDAs work by ensuring proper growth and mixing of good sub-solutions. Real-world problems might be composed of non-fully separable sub-problems, which are sometimes called building blocks (BBs). Two BBs overlap on each other when some genes are shared by both of them. With an overlapping structure, optima of the sub-problems might not compose the global optimum of the problem. Such structures are called conflict structures. To test EDAs' performances on dealing with overlapping and conflict structures, a test problem with controllable degree of overlap and degree of conflict is needed.

Some test problems have been designed to test EDAs [2, 3, 11, 12, 14]. Some desired properties of a test problem have been proposed and are summarized as follows.

1. The problem should be composed of decomposable sub-problems.
2. The optimum solution of the problem should be either known or can be calculated in polynomial time for researchers to check whether an EDA is successful.
3. The degree of overlap and the degree of conflict should be controllable.
4. The problem should be equally difficult in different parts to reduce noises of experiments, and thus an homogeneous structure for the problem is desired.

For simplicity, in this paper, the degree of overlap is denoted as $\mathbb{O}$ and the degree of conflict is denoted as $\mathbb{C}$.

Some of the proposed problems are with controllable degree of overlap. However, there are few discussions about how to set a reasonable upper-bound of $\mathbb{O}$. This paper proposes a new definition of $\mathbb{O}$ from the point of view of BB-wise crossover and discusses how to decide the upper-bound of $\mathbb{O}$ by considering linkage model and crossover operator.

Chang *et al.* [2] introduced two different types of BBs and designed a problem with controllable $\mathbb{C}$. At the global optimum of the problem, one type of BBs satisfy their own optima and the other type of BBs do not satisfy their own optima. For simplicity, we call the first type of BB

	optimum	Controllability of $\mathbb{O}$	Controllability of $\mathbb{C}$
OneMax	$O(1)$	NA	NA
concatenated Trap	$O(1)$	NA	NA
Cyclic Trap	$O(1)$	NA	NA
2D Spin-glass Problem	$O(n^{3.5})$	No	No
3D Spin-glass Problem	NP-Complete		
NK-land scape	$O(2^k n)$	Yes	No
Problem of Tsuji *et al.*	$O(1)$	Yes	NA
Problem of Chang *et al.*	$O(1)$	Yes	Yes
Problem of Chen *et al.*	$O(1)$	Yes	Yes

Table 1: Comparison of Test Problems. The parameter n, k represents the problem size and the sub-problem size respectively. The NA term means no overlapping structures or conflict structures exist in the problem.

as BBSAT and call the second type of BB as BBUNSAT. Their definition of $\mathbb{C}$ considers only the number of BBUNSAT. This paper investigates the effect of the number of edges between BBSAT and BBUNSAT on the problem difficulty and propose a new definition of $\mathbb{C}$.

This paper continues as follows. In Section 2, we describe some relevant researches on test problems and some shortcomings of them. In Section 3, we propose our definition of $\mathbb{O}$ and discuss the limit of $\mathbb{O}$. In Section 4, we propose our definition of $\mathbb{C}$. In Section 5, we propose our test problem with the new definitions of $\mathbb{O}$ an $\mathbb{C}$. In Section 6, we apply this problem to test some model-building GAs and present the experimental results. Finally, we conclude this paper with discussions and possible future work.

2 RELATED WORK

This section reviews and classifies some proposed test problems according to their properties and discusses some shortcomings of each of the test problems. Comparison of different test problems is shown in Table 1.

2.1 Test Problems with BBs

OneMax is probably the most common test problem for GAs. The fitness is given by the number of ones in a binary string. OneMax does not pose any problem for GAs but serves as a baseline of test problems. The trap function [5] is a deceptive function. A problem consists of concatenated trap function leads GAs toward converging to its sub-optimal solution. A generalization of the trap-function is the B-trap [6], which is with adjustable dependencies between variables in the BBs. Test problems composed of BBs with trap-like function can be solved well by current EDAs. However, overlapping BBs bring more challenges to EDAs.

2.2 Test Problems with Overlap

Yu *et al.* [18] used a test problem with cyclic overlapping traps for theoretical development. The problem is with a known optimum and a homogeneous structure. However, the overlap structure of the problem is not adjustable.

The Ising spin-glass problem was used to test the performances of different EDAs [11, 16]. A two-dimension spin-glass problem is as shown in Figure 1. The problem is described by an energy function, $H(S) = -\sum_i \sum_j J_{ij} S_i S_j$, where $S = \{S_1, S_2, \cdots, S_{\ell-1}\}$ is a set of spin variables, and J is a set of coupling constants. Each spin variable S_i can be either -1 or +1. Each coupling constant, $J_{ij} \in$ {-1, +1}, relates a pair of spins S_i and S_j. If $J_{ij} S_i S_j = 1$, the coupling of S_i and S_j is satisfied. Given a set of coupling constants J, the objective is to find a set of spin values S that minimizes the energy function H, that is, maximizes the number of satisfied couplings. The problem can be extended to higher dimension. Each spin in a three-dimension spin-glass problem interacts with six neighboring spins. Each pair of coupling spin variables can be regarded as a BB.

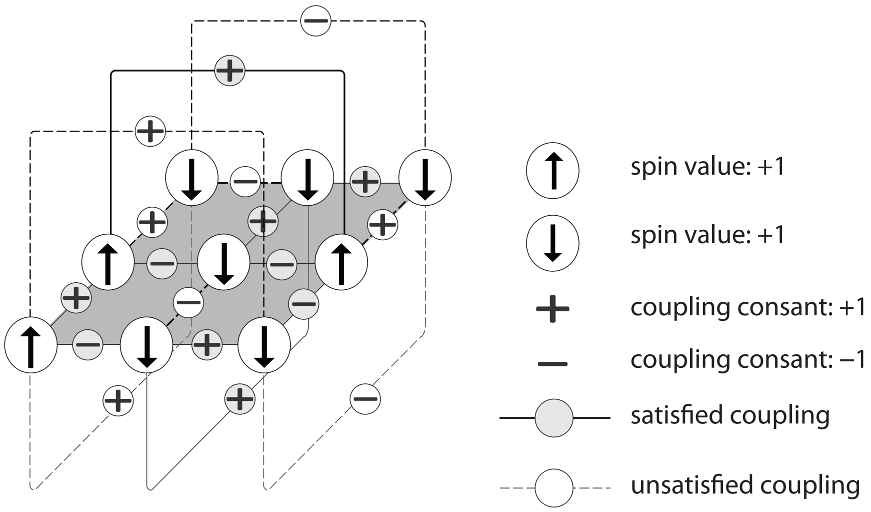


Figure 1: The 3 x 3 2D Ising spin-glass system.

$\mathbb{O}$ is not controllable although spin-glass problems are of highly overlapping structures. Finally, the problem of finding ground states of spin-glass problems with dimension equal to or higher than 3 is NP-complete [1].

2.3 Test Problems with Controllable Overlap

Tsuji *et al.* [14] proposed a test problem with a known optimum and controllable $\mathbb{O}$. The overlap structure of this problem is cased on samples from normal distributions and $\mathbb{O}$ is controlled by the variance of the distributions. Some experiments [2] showed that the difficulty of the problem is not positively proportional to variance when the variance is large. Moreover, the overlap structure of this problem is non-homogeneous, which induces noises in experiments.

Pelikan *et al.* [12] used NK landscapes with nearest neighbor interactions and controllable $\mathbb{O}$ for performance testing. The optimum of the problem can be computed in polynomial time using a dynamic programming algorithm. The problem is with three parameters, n, k and *step* which respectively represents the number of bits, the number of neighbors per bit and the *step* with which the basis bits are selected. The size of of each BB is $k + 1$ and the number of the bits two neighboring BBs share is $k + 1 - step$. Therefore, $\mathbb{O}$ of the problem is controlled by the *step* parameter. Since the possible values of the *step* are integers between 1 and $k+1$, the granularity of $\mathbb{O}$ is coarse. The upper-bound of $\mathbb{O}$ is reached when the *step* is equal to 1. Each BB of the problem contributes to the overall fitness according to a lookup table with $2^{(k+1)}$ random generated real values. Each real value corresponds to the fitness of a combination of values of $k+1$ bits. However, dependent genes with random generated fitness sub-function are not always identified as a BB by GAs according to Lee *et al.* [9]. This makes overlapping of BBs less meaningful to some extent and might induce noises in experiments.

2.4 Test Problems with controllable Overlap and Conflict

Chang *et al.* [2] designed a test problem with a known global optimum, homogeneous structures and controllable $\mathbb{O}$. They defined $\mathbb{O}$ as the average number of BBs to which a gene belongs. They also investigated the phenomenon that the optima of sub-problems do not compose the global optimum and called this phenomenon conflict. They considered two different type of BBs with the optimum equal to all ones and all zeros respectively. When two different type BBs share some bits, the optima can not be achieved at the same time. Their experiment results showed that the problem difficulty increases as $\mathbb{C}$ increases. Complicated conflict structures might exist in the Ising spin-glass problem.

Chen *et al.* [3] proposed a test problem with a known global optimum, homogeneous structures and controllable $\mathbb{O}$ and $\mathbb{C}$. They designed two different types of BBs, which we refer to as BBsat and BBunsat in this paper. The fitness function V_1 and V_0 for BBsat and BBunsat are:

$$V_1 = \begin{cases} \frac{t-u}{t} \cdot q & \text{if } u \leq t \\ \frac{u-t}{k-t} & \text{if } u > t \end{cases} \tag{1}$$

and

$$V_0 = \begin{cases} \frac{t-u}{t} \cdot (1-\epsilon) & \text{if } u \leq t \\ \frac{u-t}{k-t} \cdot q & \text{if } u > t \end{cases}, \tag{2}$$

where k is the number of bits in each sub-problem, t is an integer satisfying $0 < t < k$, and q and ϵ are real numbers satisfying $0 < q < 1$ and $0 < \epsilon < (1 - q)$. Instances for V_1 and V_0 are shown in Figure 2. The functions indicate that the optimum of BBsat is all ones, while the optimum of BBunsat is all zeros. The global optimum is designed to be all ones. Thus, BBunsat does not achieve its optimum at global optimum. To ensure the global optimum be all ones, each BBunsat should be paired with a different BBsat.

The definition of $\mathbb{O}$ was also defined as the average number of BBs to which a gene belongs, but was remapped to start from 0. $\mathbb{C}$ was defined as the ratio of the number of BBunsat to the number of all BBs. The definitions of $\mathbb{O}$ and $\mathbb{C}$ can be written as follows:

$$\mathbb{O} = \frac{mk}{\ell} - 1 \tag{3}$$

and

$$\mathbb{C} = \frac{m_0}{m}, \tag{4}$$

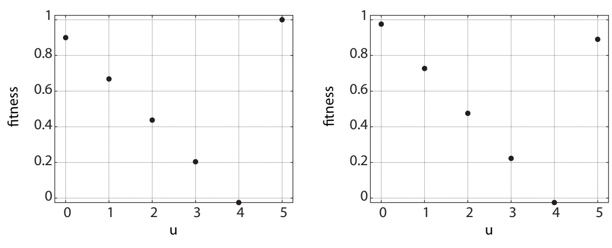
where m is the number of all BBs, ℓ is the problem length, and m_0 is the number of BBunsat. Though an upper-bound of $\mathbb{O}$ was set in their experiments, no theoretical reason evidenced that the upper-bound is reasonable. Moreover, overlap concept origins from BBs' sharing of bits and thus is an inter-BB relation. The definition with respect to the number of BBs which a gene belongs to may not directly reflect the problem difficulty.

3 Overlap Definition

This section introduces our definition of $\mathbb{O}$ and then shows the reason why a non-trivial upper-bound of $\mathbb{O}$ exists. After that, the relation between the upper-bound and problem size is shown.

3.1 Definition of Overlap on Building Block

As mentioned in the earlier section, $\mathbb{O}$ was usually controlled by the number of BBs to which a gene belongs. However, this definition may not directly reflect the problem difficulty since overlap is an inter-BB relation. Consider a pair of overlapping BBs,(Figure 3). If the crossover operator exchanges only one of them, the information carried by the other is disrupted. However, if the two



(a) V_1 (b) V_0

Figure 2: Functions V_1 and V_0 with $k = 5$, $t = 4$, $q = 0.9$ and $\epsilon = 0.01$. The x-axis is the number of ones in a BB and the y-axis is the fitness value.

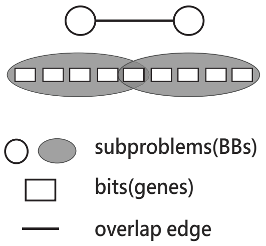


Figure 3: Illustration of problem as graph. A node represents a sub-problem. An edge between two nodes indicates that the two corresponding sub-problems share one bit.

BBs are exchanged together, there is no chance to recombine the information carried by the two BBs.

For illustration, we transfer a problem into a graph $G = (V, E)$, where V is the set of nodes representing the BBs and E is the set of edges representing the overlapping relation between the BBs. Disruption happens when the crossover operator exchanges two overlapping BBs asynchronously. The number of disrupted BBs that the crossover operator produces depends on the number of total edges of the graph but not the number of overlapping bits between the BBs. Thus, it is reasonable to define $\mathbb{O}$ as the average number of edges connected from a node. The value of $\mathbb{O}$ of a graph with $|E|$ edges and $|V|$ nodes is then equal to $|E|/|V|$. A trivial upper-bound of $|E|$ is $E_{max} = \frac{|V|(|V|-1)}{2}$ as a function of $|V|$, and thus the upper-bound of $\mathbb{O}$ is equal to $E_{max}/|V|$. However, the bound is not practical for GAs and a more reasonable upper-bound is shown in the following subsection. Dividing $\mathbb{O}$ by its upper-bound, the normalized overlap ratio ranged between 0 and 1 can then be written as:

$$\omega = \frac{|E|}{E_{max}}. \tag{5}$$

This paper considers only problems with homogeneous topologies with all nodes in the representing graph having the same degree, or the number of connected edges. If the average degree of nodes is d, $|E|$ is equal to $\frac{|V|d}{2}$. Thus, the definition of $\mathbb{O}$ is simplified as $d/2$, and the overlap ratio is simplified as:

$$\omega = \frac{d}{d_{max}}, \tag{6}$$

where d_{max} is a function of $|V|$.

3.2 Upper-bound of Overlap Degree

Yu *et al.* [15] derived the result that for GAs to work properly, the number of BBs misidentified by a model building method should be bounded by:

$$e_{critical} = I'\sqrt{m}, \tag{7}$$

where I' is the selection intensity and m is the number of BBs. Consider crossover of two overlapping BBs. When deciding which BB should be preserved, the overlapping bit is assigned to one of them. This is equivalent to misidentifying the disrupted BB. Thus, if the best crossover operator produces more than $I'\sqrt{m}$ disrupted BBs in a kind of problem, we can conclude that this kind of problem should not be solved by GAs since GAs will not work.

Yu *et al.* [18] proposed a crossover strategy dealing with overlapping BBs. The strategy was designed to minimize the number of disrupted BBs during crossover. By the strategy, all BBs are grouped into two sets for exchange. By representing the problem structure as a graph, the number of disrupted BBs is positively proportional to the number of overlap edges between the two groups, or the cut size of the partition. To minimize disrupted BBs, the partition with minimum cut size is preferred. The number of the nodes of the two sub-graphs is not fixed, but we can assume that there is a ratio between them. Specifically, when the two sub-graphs are of equal size, the maximal information exchange is achieved.

We consider the following crossover operator to investigate the upper limit of $\mathbb{O}$. The crossover operator always finds the partition with minimum cut size from all partitions that separate the graph into two equal-size sub-graphs. The size of the minimum cut for a graph with $\frac{|V|(|V|-1)}{2}$ edges is $|V|^2/4 = \Omega(\sqrt{|V|})$. Problems with this kind of structure are then not suitable for GAs.

Note that being under the overlap upper-bound is just a necessary condition for problems to be GA-solvable. Consider a graph composed of two cliques with equal-size $|V|/2$. The crossover operator disrupts no BBs, but it does not recombine information of the BBs belonging to different cliques. GAs fail to solve such problem.

3.3 Scaling of Upper-bound with Problem Size

To find a reasonable upper-bound of $\mathbb{O}$, we should know the scaling of the value of E_{max} or d_{max} with the problem size. Since only homogeneous graphs with nodes of equal degree are discussed, only d_{max} is shown. E_{max} is equal to $\frac{|V|\cdot d_{max}}{2}$. Before discussing homogeneous graphs with all nodes of exactly the same degree, we discuss random graphs with all nodes of equal expectation of degree.

3.3.1 Random Graphs

A random graph is sampled from a probability model. Each edge in a random graph exists with probability $d/(m-1)$, where d is the expected degree of a node and m is the number of all nodes. Given a random graph, there are $C(m, \frac{m}{2})/2$ partitions to separate m nodes into two equal-size sub-graphs.* However, the probability distribution of the cut sizes of the $C(m, \frac{m}{2})/2$ partitions are not independent. Since the probability of each edge between two nodes is independent and there are $C(m, 2)$ possible combinations of two nodes in different sub-graphs, the degree of freedom of partition a graph into two equal-size sub-graphs is $C(m, 2)$. Therefore, there are $C(m, 2)$ independent partitions.

Denote the i^{th} cut size as X_i. We then have $C(m, 2)$ independent and identically distributed random variables $X_1, X_2, \ldots X_{C(m,2)}$. We denote the probability density function of X_i as $f_X(x)$ and the cumulative density function (CDF) as $F_X(x)$. Since the cut size is at most $\frac{m^2}{4}$ edges and each edge exists with the same probability $\frac{d}{m-1}$, $f_X(x)$ can then be represented as a binomial distribution with parameters $(n, p) = (\frac{m^2}{4}, \frac{d}{m-1})$.†

The size of the minimum cut found by the crossover operator is $X_{(1)} = \min_i X_i$, where $X_{(1)}$ is the first order statistic with the CDF $F_{X_{(1)}}(x) = (1 - F_X(x))^{C(m,2)}$. Approximate $F_{X_{(1)}}$ as a constant $z < 1$. With Chernoff's inequality [4], we derive $F_X(x) = 1 - z^{\frac{1}{N}} \le \exp(\frac{-1}{2p} \cdot \frac{(np-x)^2}{n})$, where n, p are the parameters of binomial distribution described above. By some approximation under the assumption that m is a large value, we derive $x \le \frac{md}{4} + \sqrt{\frac{md}{2}}$, which shows an upper-bound of the cut size. The term $F_X(x) = z$ approaches one and disappears at high order and this coincides with the approximation of $F_X(x)$ as a constant less than one. The derived upper-bound should be bounded by the upper-bound of Equation 7. Finally, we derive that $d_{max} = \theta(1/\sqrt{m})$ for random graphs.

3.3.2 Homogeneous Graphs

For homogeneous graphs, consider symmetric structures such as platonic solid or sphere with nodes spraying uniformly on the surface of the structure (Figure 4). The crossover operator should cut the structure into two symmetric parts. The cut size is $\frac{d}{2}V_c$, where d is the degree of the nodes and V_c is the number of nodes on the cut contour. With Equation 7, we get $d < \frac{2I'\sqrt{m}}{V_c}$. Under the assumption that I' is constant, we derive the result that $d_{max} = O(1)$ for both sphere and tetrahedron. The same result can be derived for other symmetric structures.

Since nodes of spin-glass problems' structure are of same degree, spin-glasses problems are also homogeneous. Applying the crossover operator, a 2D spin-glass problem has cut size $2\sqrt{m}$ while

*$C(n, k)$ is equal to $\frac{n!}{(n-k)!k!}$.

†The binomial distribution with parameter (n, p) is defined as $C(n, k)p^k(1-p)^{n-k}$.

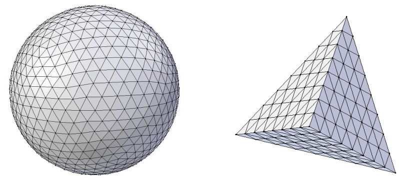


Figure 4: Nodes spray homogeneously on sphere and tetrahedron.

a 3D spin-glass problem has cut size $2m^{\frac{2}{3}}$. The cut size of 3D spin-glass problem exceeds the upper-bound of Equation 7. This result corresponds to the fact that 3D spin-glass problem is NP-complete and is not GA-solvable.

3.3.3 Practical Upper-bound

To sum up, the upper-bound d_{max} for random graphs is inversely proportional to the problem size, and d_{max} for homogeneous graphs is constant. Thus, in homogeneous graphs, the overlap edges of a sub-problem should be bounded by a constant. In practice, since 2D spin-glass problem is solvable by GAs, and the degree of BBs of a 2D spin-glass problem is 6, we fix d_{max} at 6 as a practical upper-bound in our problem.

4 Conflict Definition

This section presents our definition of conflict. For simplicity, we define edges between two BBsat as overlap edges and define edges between a BBsat and a BBunsat as conflict edges. We investigate how the numbers of the two types of edges affect the problem difficulty. Specifically, two independent variables are investigated. The first is the average number of conflict edges per BBunsat and the second is the average number of conflict edges per BBsat. Some experiments are conducted to investigate the effect of the two independent variables on the problem difficulty.

In the experiments, the functions of the BBunsat and BBsat are shown in Figure 2. For different conflict structures, we fix problem length and repeat different conflict patterns (Figure 5) until the problem length is matched. The problem length is fixed to compare different patterns in equal-size search space. We compare the number of function evaluation (NFE) required by different patterns using hBOA [10]. The NFE required by different conflict structure are shown in Table 2. The result shows that the average number of conflict edges per BBunsat do not have great effect on the problem difficulty. The reason might be as the following. The function for BBunsat is like a zero-max function in the range of $u < t$, where u is the number of ones and t is the lowest fitness point of the function. Since the value of each bit in a zero-max function is independent, and each bit of a BBunsat is shared by no more than one BBsat, each bit of a BBunsat might independently affect the BBsat overlapping on it.

Since only the second factor matters, we conclude that a reasonable definition of $\mathbb{C}$ can be defined as $|E_c|/m_s$, where $|E_c|$ is the number of conflict edges and m_s is the number of BBsat. In our problem, each gene of a BBsat is limited to be shared with no more than one BBunsat. Therefore, the maximum number of conflict edges for each BBsat is equal to the size of a BB, k, and the maximum number of conflict edges in the problem is $m_s \cdot k$. The normalized conflict ratio ranging between 0 and 1 can then be written as:

$$c = \begin{cases} 0 & if\ m_s = 0 \\ \frac{|E_c|}{m_s \cdot k} & otherwise. \end{cases} \tag{8}$$

Our definition of overlap ratio is $\omega = |E|/E_{max}$. In Section 3, we refer to $|E|$ as the number of edges in a graph. However, we separate the challenges that the two types of edges bring to a problem. Thus, $|E|$ is equal to the number of overlap edges instead of all edges. Different from previous work [3], under our definitions, the conflict ratio c is not limited by the overlap ratio ω. A problem with $\omega = 0$ can be with a non-zero c.

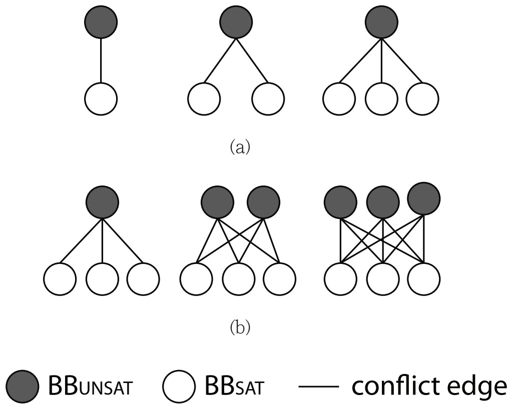


Figure 5: Different conflict structure. The independent variables in (a) is the number of conflict edges per BBsat. The independent variables in (b) is the number of conflict edges per BBunsat.

$\frac{Number\ of\ conflict\ edge}{Number\ of\ BB_{UNSAT}}$	1	1/2	1/3
Required NFE	$2.56 \cdot 10^5$	$2.33 \cdot 10^5$	$2.41 \cdot 10^5$
$\frac{Number\ of\ conflict\ edge}{Number\ of\ BB_{SAT}}$	1	2	3
Required NFE	$2.41 \cdot 10^5$	$3.31 \cdot 10^5$	$4.86 \cdot 10^5$

Table 2: NFE required by problems of different conflict structures with problem length fixed to 120.

5 Proposed Test Problem

In this section, we first introduce our settings of overlap and conflict among sub-problems. The algorithm to construct the problem is then presented. This test problem consists of normalized overlap and conflict ratio, yet still keeps the desired properties described in previous sections.

5.1 Test Problem Setting

Following Equation 1, we define two functions as the fitness functions of BBsat and BBunsat. The argument settings are shown in Figure 2. The global optimum of the problem is all ones if each BBsat can be paired with one different BBunsat. This constraint implies that the number of BBsat should always be greater or equal to the number of BBunsat. The number of overlapping bits of any two BB is set to 1.

Our problem has four arguments: the problem length, ℓ, the size of BBs, k, the overlap ratio, ω and the conflict ratio, c. The number of BBs and edges can not be derived with Equation 5 and 8 given l, k, ω, and c Therefore we do not know how many BBs should be in the problem structure before the problem is constructed and thus we design an iterative approach to construct the problem.

5.2 Constructing the Problem

The thought of the iterative approach is as follows. Initially, the problem structure is empty and all genes belong to 0 BB. In each iteration, k genes are chosen to form a new BB and the new BB is added into the problem structure until all genes are used and belong to at least one BB. Each of the chosen gene either already belongs to some BBs or is still unused.

In order to satisfy the given overlap and conflict ratio, some edges are connected between the added BB and the existing BBs. Thus, some of the k bits are chosen from bits which belong to the existing BBs. In each iteration, we should decide which kind of BB should be added to the problem structure. Denote the desired overlap and conflict ratio as ω_d and c_d and denote overlap and conflict ratio in current structure as ω_{cur} and c_{cur} respectively. To satisfy the constraint of the designed global optimum, when the numbers of BBsat and BBunsat are the same, a BBsat should be added. In other cases, a BBsat is added if $|\omega_d - \omega_{cur}| \geq |c_d - c_{ccur}|$, and a BBunsat is added otherwise.

5.2.1 Add BBsat

Consider that there are m_s BBsat and $|E_o|$ overlap edges. To achieve the given overlap ratio, e_r overlap edges are required to connected from the new BB to the existing BBs, where

$$e_r = \omega \cdot (m_s + 1) \cdot \frac{d_{max}}{2} - |E_o|. \tag{9}$$

When a gene is chosen for the new BB, BBs that originally contain the gene connect with the new BB. Thus, the number of overlap edges added into the structure is equal to the number of BBsat that originally contain the chosen gene. On average, each chosen gene should originally belongs to $\bar{e} = e_r/k$ BBsat. All genes belonging to $\bar{e}$ BBs are candidates. However, in order to keep the graph homogeneous, we choose the gene with the minimum $f(g)$ which is defined as

$$f(g) = \min_{g} \sum_{v \in S(g)} e(v), \tag{10}$$

where $S(g)$ is the set of BBs containing the gene g, and $e(v)$ represents the number of overlap edges connected from BB v. If no gene belongs to $\bar{e}$ BBs, we keep decreasing $\bar{e}$ until a gene belonging to $\bar{e}$ BBs can be found. After choosing one gene for the new BB, $\bar{e}$ overlap edges are added and e_r is updated. Thus, e_r and $\bar{e}$ are recalculated for the rest of the genes that are going to be chosen for the new BB. An example of adding BBsat is shown in Figure 6.

5.2.2 Add BBunsat

Unlike adding BBsat, we do not choose genes for a new BBunsat in each iteration. Instead, we add genes for all the BBunsat at the end of the process. Consider there are m_s BBsat and $|E_c|$ conflict edges. To achieve the given conflict ratio, e_r conflict edges are required to connected from a new BBunsat to the existing BBs, where

$$e_r = c \cdot m_s \cdot k - |E_c|. \tag{11}$$

A BBunsat is then added to the problem and there is now $|E_c| + e_r$ conflict edges. Originally, e_r genes that belong to one BBsat and $k - e_r$ genes belonging to 0 BB should be chosen for the new BB. Hoever, since the genes are chosen at the end of the process, only $k - e_r$ genes belonging to 0 BB are reserved for later use. The value of e_r is limited to be between 1 and k. A value of e_r less than 1 is set to 1 and a value larger than k is set to k.

At the end of the process, we choose genes for all BBunsat. If there are E_c conflict edges and m_u BBunsat, each BBunsat should owns $|E_c|/m_u$ conflict edges on average. Thus, for each BBunsat, $|E_c|/m_u$ genes should be chosen from genes of BBsat and $k - (|E_c|/m_u)$ genes should be chosen from the preserved genes. Since the global optimum of the problem is designed to be all ones, each BBunsat should be paired with a different BBsat. An easy way to achieve this goal is to choose the first gene of each BBunsat from a different BBsat.

There might be insufficient genes to form the new BB at the last iteration. In such case, we abandon all unused genes and finish the procedure. The pseudo code of the iterative approach is listed in Algorithm 1.

6 EXPERIMENTS

In this section, we first show the problem difficulties with different upper-bound of overlap. After that, we use the problem to test the performance of three model building GAs, DSMGA [17], hBOA [10], and ECGA [8]. DSMGA with the crossover method SBS [16] and hBOA could deal with overlapping structure. ECGA, which has no ability to store information about overlapping BBs in its model, is tested as comparison.

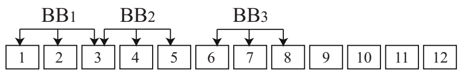
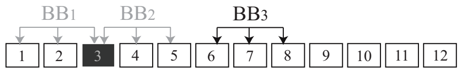
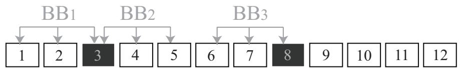
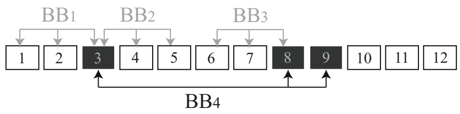
Step	Current problem structure	e_r	$\bar{e}$	Genes chosen
1		5-1 = 4	$\lceil 4/3 \rceil = 2$	{3}
2		4-2 = 2	$\lceil 2/2 \rceil = 1$	{3,8}
3		2-1 = 1	$\lceil 1/1 \rceil = 1$	{3,8,9}
4				

Figure 6: An example of adding a BBsat with $\omega = 0.4$. There are 3 BBs and 1 overlap edge in the initial structure. The value of e_r is calculated by Equation 9. Genes in dark color are the chosen genes for the new BBsat. BBs in light color are BBs that already link with the new BBsat, and genes belong to those BBs should not be chosen again.

In all the experiments, binary tournament selection without replacement is used to select promising solutions, and RTR [7,10] is used as the replacement strategy where the window size is $w = \min\{\text{problem size, population size}\}$. The convergent criterion is that the GA finds out the global optimum. The maximum number of function evaluations (NFE) in our experiments is set to 10^7. For each setting, we run bisections [13] with 10 successive runs to get the reliable population size. With the reliable population size, we record the average NFE required over 10 successive runs.

6.1 Experiments on Upper-bound of Overlap Degree

As we described in Section 3, for homogeneous problem, d_{max} should be bounded by a constant. Fixing overlap ratio equal to 1, for different value of d_{max}, we record the NFE required by hBOA scaling with respect to problem size as in Figure **??TBA**. We can see that the problem becomes exponentially hard if d_{max} is greater than **TBA**.

6.2 Performances of EDAs

The purpose of the experiments is to find the relation between the performance of GAs under different settings of $\mathbb{O}$ and $\mathbb{C}$. Thus, we conduct some experiments to find the NFE required for each of the three GAs to solve the test problem.

For each of the GAs, the following experiments are done. We fix problem size ℓ to 100. For each setting of $\mathbb{O}$ and $\mathbb{C}$, ten different problem structures are generated. For each of these problems, with the reliable population size, the average NFE over 10 successive runs is recorded. Since there is randomness in the problem building process, the above process is repeated 10 times for each setting. We then compute the average NFE, with the maximum and the minimum eliminated, as the results of the experiment.

The experimental results are shown in Tables 3, 4 and 5. The results show that ECGA, which lacks the ability to deal with an overlapping structure, can only solve problems of small $\mathbb{O}$ and $\mathbb{C}$. For the other two GAs the results show that the difficulty of the problem increases as $\mathbb{O}$ or $\mathbb{C}$ increases.

7 Conclusion

In this paper, a new test problem with overlapping and conflict structures is proposed. The upper-bound of the degree of overlap and the effect of conflict are reconsidered. We derive the result that the upper-bound of homogeneous problems is bounded by a constant. For conflict structure, we discuss the effect of the two types of edges on the problem difficulty. An iterative way of constructing the problem is also presented. Finally, three model-building GAs are tested with this new test problem. The experimental results show that our test problem becomes more difficult when either the degree of overlap or the degree of conflict becomes higher. Thus, the proposed test problem is suitable for further researches on overlap and conflict relation between BBs. And the discussion of the upper-bound of degree of overlap degree guide researchers to know the limitation of test problem for GAs. One of the key topics for future works is to investigate the computation complexity of GAs for solving problems of different degree of overlap and conflict.

References

[1] F. Barahona. On the computational complexity of ising spin glass models. *Journal of Physics A: Mathematical and General*, pages 3241–3253, 1982.

[2] J.-H. Chang, C.-H. Hsueh, H. Lee, T.-L. Yu, and T.-Y. Ho. A test function with full controllability over overlapping and confliction between sub-problems. TEIL Report No. 2011003, National Taiwan University, Taipei, Taiwan, July 2011.

[3] W.-M. Chen, C.-Y. Shao, P.-C. Hsu, and T.-L. Yu. A test problem with adjustable degrees of overlap and conflict among subproblems. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2012)*, 2012.

[4] H. Chernoff. A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations. *Annals of Mathematical Statistics*, 23:493–507, 1952.

[5] K. Deb and D. E. Goldberg. Analyzing deception in trap functions. *Foundations of Genetic Algorithms 2*, pages 93–108, 1993.

[6] K.-C. Fan. Linkage identification by nfe estimation. Technical report, 2011.

[7] G. R. Harik. Finding multimodal solutions using restricted tournament selection. In *ICGA*, pages 24–31, 1995.

[8] G. R. Harik. Linkage learning via probabilistic modeling in the ECGA. IlliGAL Report No. 99010, University of Illinois at Urbana-Champaign, Urbana, IL, February 1999.

[9] H. Lee and T.-L. Yu. Off-line building block identification: Detecting building blocks directly from fitness without genetic algorithms. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2012)*, pages 641–648, 2012.

[10] M. Pelikan and D. E. Goldberg. Escaping hierarchical traps with competent genetic algorithms. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2001)*, pages 511–518, 2001.

[11] M. Pelikan and D. E. Goldberg. Hierarchical BOA solves Ising spin glasses and MAXSAT. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2003)*, pages 1271–1282, 2003.

[12] M. Pelikan, K. Sastry, D. E. Goldberg, M. V. Butz, and M. Hauschild. Performance of evolutionary algorithms on NK landscapes with nearest neighbor interactions and tunable overlap. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2009)*, pages 851–858, 2009.

[13] K. Sastry. Evaluation-relaxation schemes for genetic and evolutionary algorithms. Master thesis, University of Illinois at Urbana-Champaign, Urbana, IL, 2002.

[14] M. Tsuji, M. Munetomo, and K. Akama. A crossover for complex building blocks overlapping. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2006)*, pages 1337–1344, 2006.

[15] T.-L. Yu and D. E. Goldberg. Quality and efficiency of model building for genetic algorithms. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2004)*, pages 367–378, 2004.

[16] T.-L. Yu, D. E. Goldberg, K. Sastry, C. F. Lima, and M. Pelikan. Dependency structure matrix, genetic algorithms ,and effective recombination. *Evolutionary Computation*, vol. 17, no. 4, pp. 595–626, 2009.

[17] T.-L. Yu, D. E. Goldberg, A. Yassine, and Y.-p. Chen. Genetic algorithm design inspired by organizational theory: Pilot study of a dependency structure matrix driven genetic algorithm. *Proceedings of Artificial Neural Networks in Engineering (ANNIE-2003)*, pages 327–332, 2003.

[18] T.-L. Yu, K. Sastry, and D. E. Goldberg. Linkage learning, overlapping building blocks, and systematic strategy for scalable recombination. *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2005)*, pages 1217–1224, 2005.

Algorithm 1: Constructing the test problem

Input: ℓ, k, ω, c
Output: A test problem with the desired overlap and conflict ratio.

Initialize Problem $P(BB_S, BB_U, E_O, E_C) = (\phi, \phi, \phi, \phi)$
Initialize *Genes* with ℓ genes belonging to 0 BB
Initialize $G_{Reserved} = \phi$
while —GetGenes($Genes, 0, P.BB_S \cup P.BB_U$)—$\neq$*0* **do**
    Compute ω_{cur} and c_{cur} by Equation 5 and 8
    $G_{Chosen} \leftarrow \phi$
    $AddS \leftarrow ((|\omega - \omega_{cur}| \geq |c - c_{cur}|) \vee (|P.BB_S| = |P.BB_U|))$
    if *AddS* ChooseBBsat(P,$Genes$,G_{Chosen})
    else ChooseBBunsat(P,$Genes$,G_{Chosen})
    if $AddS \wedge |G_{Chosen}| = k$ **then**
        Add a new BBsat with the genes G_{Chosen} to $P.BB_S$
    else Move genes in G_{Chosen} from Genes to $G_{Reserved}$
Equally assign genes in $G_{Reserved}$ to BBs in $P.BB_U$
Choose the rest genes for BBs in $P.BB_U$
return P
Function ChooseBBsat(P,$Genes$,G_{Chosen}) **begin**
    $e_r \leftarrow \omega \cdot (|P.BB_S| + 1) \cdot \frac{d_{max}}{2} - |P.E_O|$
    $\bar{e} \leftarrow \lceil e_r/k \rceil$
    $G_{Chosen} \leftarrow \phi$
    while $|G_{Chosen}| < k$ **do**
        $\bar{e}_{old} \leftarrow \bar{e}$
        $G_{Cand} \leftarrow$ GetGenes($Genes, \bar{e}, P.BB_S$)
        while $(|G_{Chosen}| < k) \wedge (\bar{e} = \bar{e}_{old}) \wedge (e_r > 0)$ **do**
            $g \leftarrow$ the genes in G_{Cand} with minimum $f(g)$ which is calculated by Equation 10
            Move g from G_{Cand} to G_{Chosen}
            $e_r \leftarrow e_r - \bar{e}$
            $\bar{e} \leftarrow e_r/(k - |G_{Chosen}|)$
        if $\bar{e} = \bar{e}_{old}$ **then** $\bar{e} \leftarrow \bar{e} - 1$
    if $k - |G_{Chosen}| > 0$ **then**
        $G_{Cand} \leftarrow$ GetGenes($Genes, 0, P.BB_S$)
        ChooseGene (G_{Cand},G_{Chosen},$k - |G_{Chosen}|$)
Function ChooseBBunsat(P,Genes) **begin**
    $e_r \leftarrow c \cdot (|P.BB_S|) \cdot k - |P.E_C|$
    if e_r¡1 **then** $e_r = 1$
    if e_r¿k **then** $e_r = k$
    $G_{Cand} \leftarrow$ GetGenes($Genes, 0, P.BB_S$)
    ChooseGene (G_{Cand},G_{Chosen},$k - e_r|$)
Function ChooseGeneG_{Cand},G_{Chosen}, Num **begin**
    Randomly choose Num genes from G_{Cand} to G_{Chosen}
Function GetGenes(Genes, d, BBset) **begin**
    return $\{g | g \in Genes$ and g belongs to d BBs in BBset$\}$

	c		
ω	0	0.05	0.1
0	363430	514524	CNS
0.05	458964	722956	CNS
0.1	CNS	CNS	CNS

Table 3: NFE require by ECGA, CNS denotes that NFE exceeds 10^7

ω	c					
	0	0.2	0.4	0.6	0.8	1.0
0	181391	196323	244589	299466	616849	CNS
0.2	300411	291251	330206	474103	775187	CNS
0.4	299907	304918	375540	539495	912040	CNS
0.6	327607	301941	408889	637715	CNS	CNS
0.8	322940	347340	413490	782565	CNS	CNS
1.0	327196	347853	405867	814429	CNS	CNS

Table 4: NFE require by DSMGA, CNS denotes that NFE exceeds 10^7

ω	c					
	0	0.2	0.4	0.6	0.8	1
0	134810	126578	135387	129318	155106	209928
0.2	164525	162064	157824	158470	181224	215679
0.4	179282	167442	168467	174481	194376	247625
0.6	186646	177185	177884	183108	188661	243542
0.8	179417	188046	172059	205425	220221	265534
1	185665	210221	183935	215654	214115	253241

Table 5: NFE require by hBOA