Deep Hedging – Learning to Trade

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Tsing Hua University

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Motivation

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Equation: Optimal hedging

$$\pi(X) := \inf_{\underline{\delta} \in \mathcal{H}} \rho(X + (\underline{\delta} \cdot \underline{S})_T - C_T(\delta))$$

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- $C_T(\delta)$ is transaction cost.

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What we do this semester?

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- Consider different transaction costs impact weight and PnL of deep hedging.
- Use real historical data into my hedging strategy and discuss whether this hedging strategy's effectiveness is significantly improved.

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- stock paths are under BS-model, The initial price is 100, strike price is 100, $\mu = 0$, $\sigma = 0.2$, and generate 5000 paths for our experiment.

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Deep hedging(CVaR) with different α

Our utility functions are

$$CVaR_\alpha(X) := \mathbb{E}[X|X \leq VaR_\alpha(X)]$$

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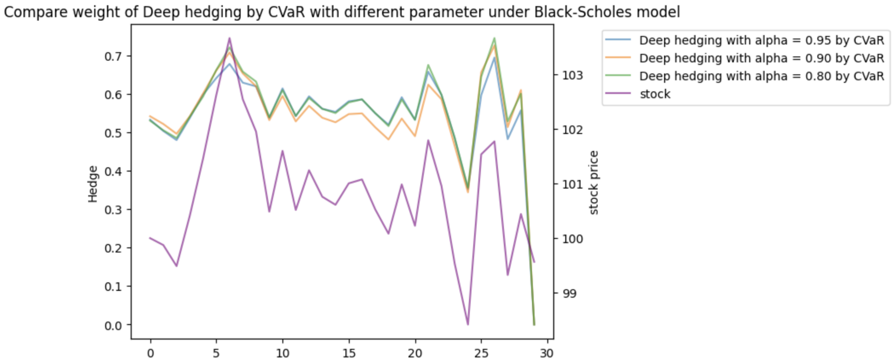
We compare with variable $\alpha = 0.95, 0.90, 0.80$

Deep hedging(CVaR) with different α



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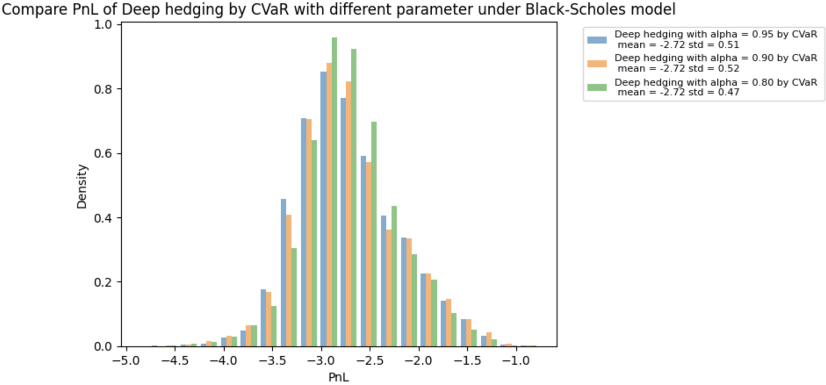

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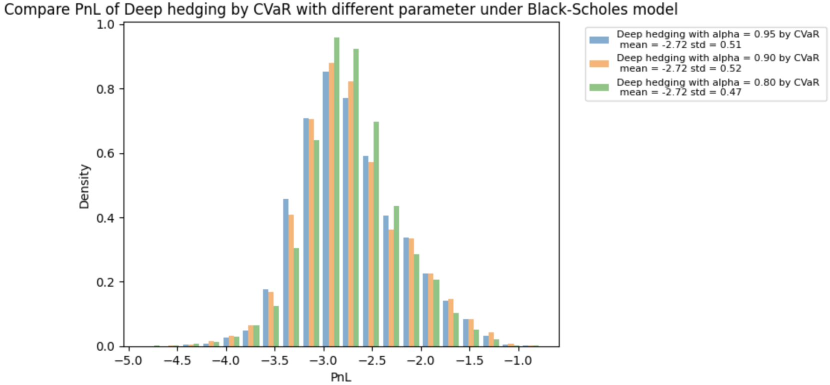

We will use $\alpha = 0.95$ on the following experiment.

Deep hedging with different epoch

We know some function converge very slowly, so we need to insure they have enough training epoch to converge the best solution.

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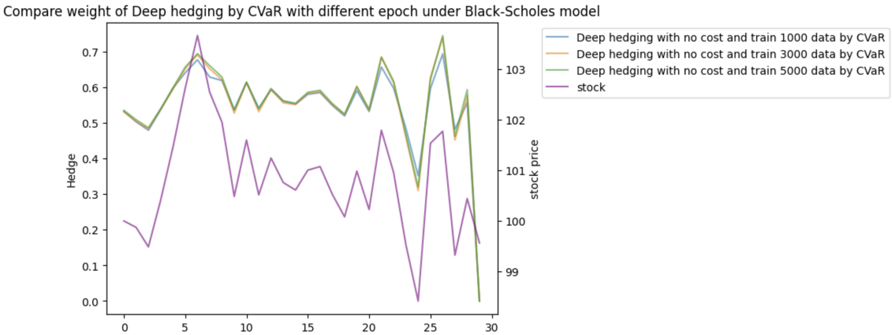
We compare with training epoch $1000, 3000, 5000, 10000$

Deep hedging(CVaR) with different epoch



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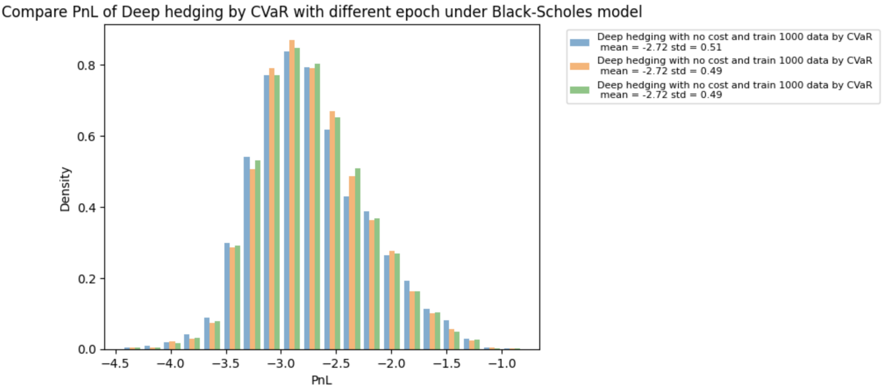

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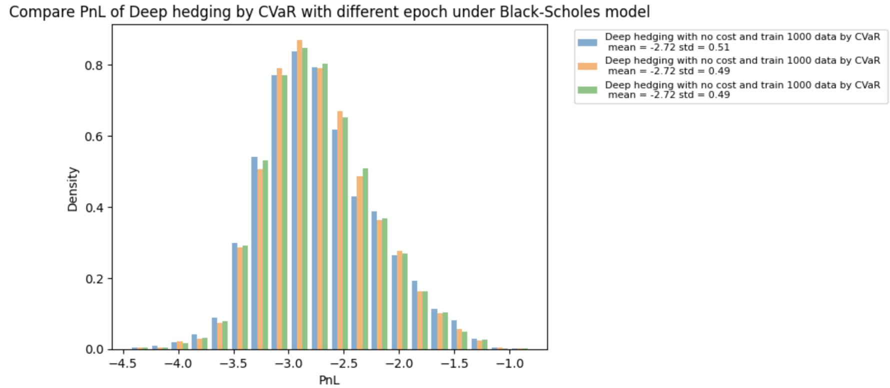
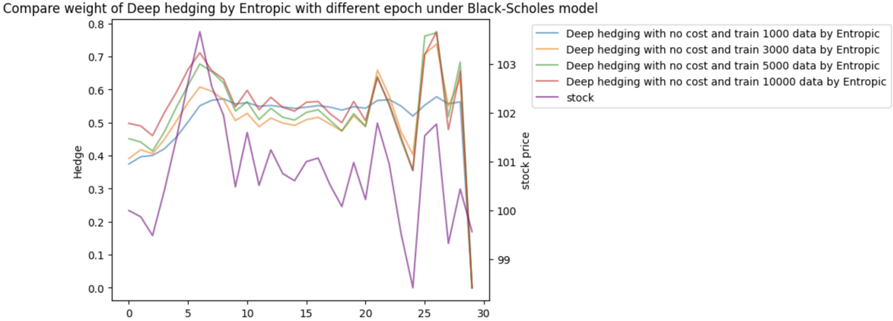


We will use training epoch 1000 on the following experiment for CVaR.

Deep hedging(Entropic) with different epoch



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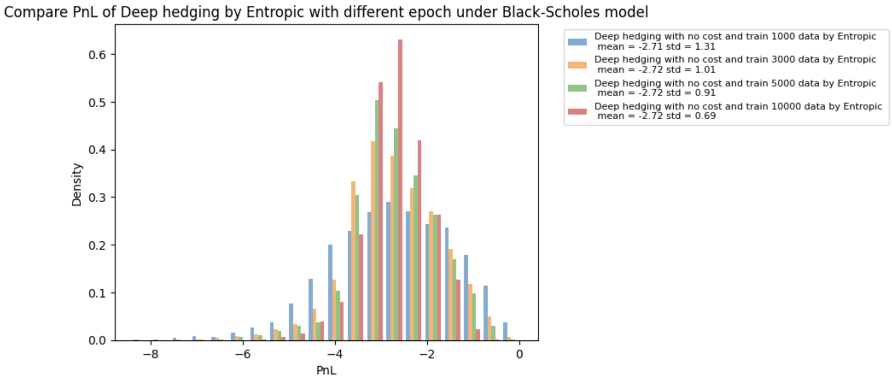

Deep hedging(Entropic) with different epoch

Loss after training

- 1000 epochs: 265.6
- 3000 epochs: 237.1
- 5000 epochs: 204.2
- 10000 epochs: 20.4

We will use training epoch 10000 on the following experiment for Entropic.

Different transaction costs

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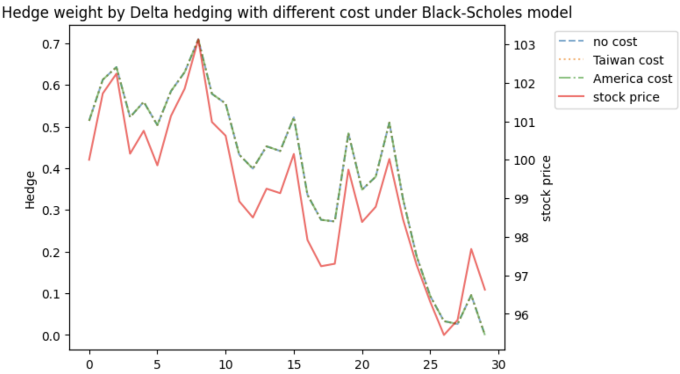

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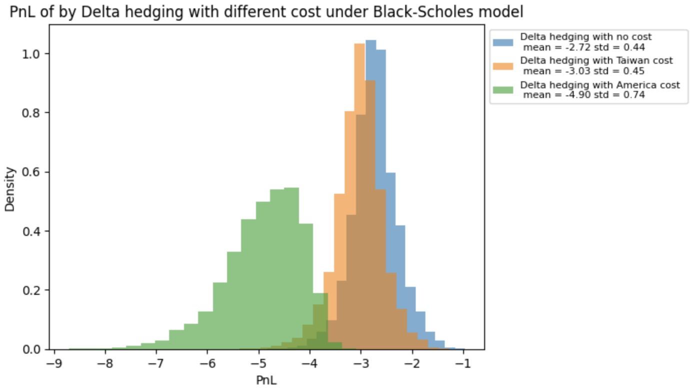

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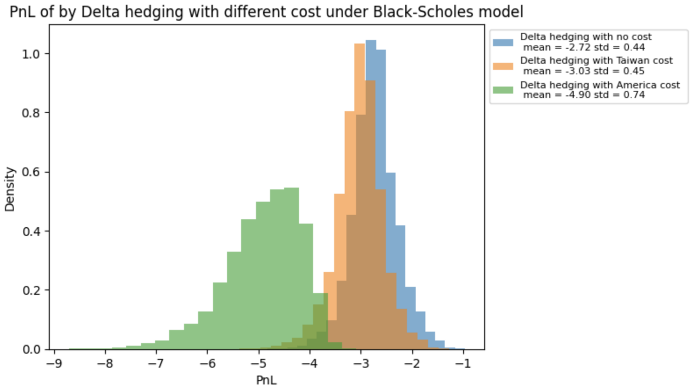
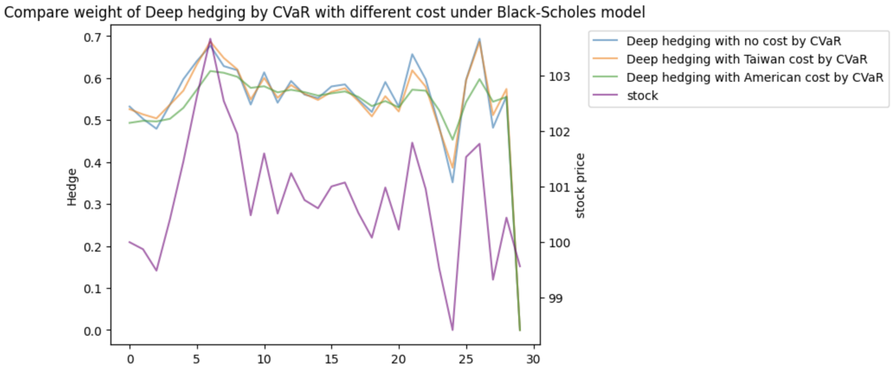


Delta Hedging strategy shows a clear sensitivity to transaction costs.

Deep Hedging (CVaR) with Different Cost



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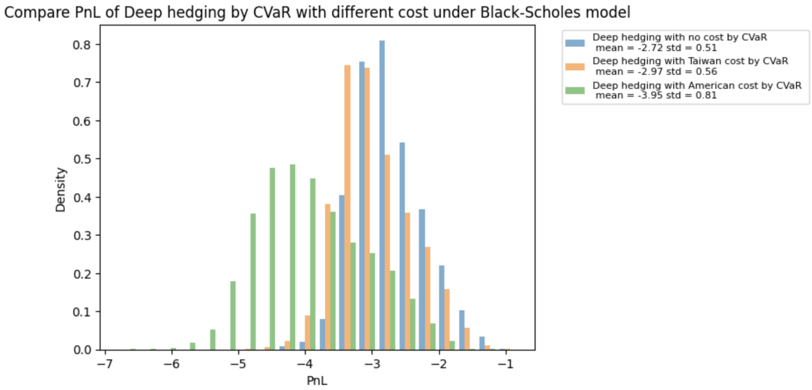

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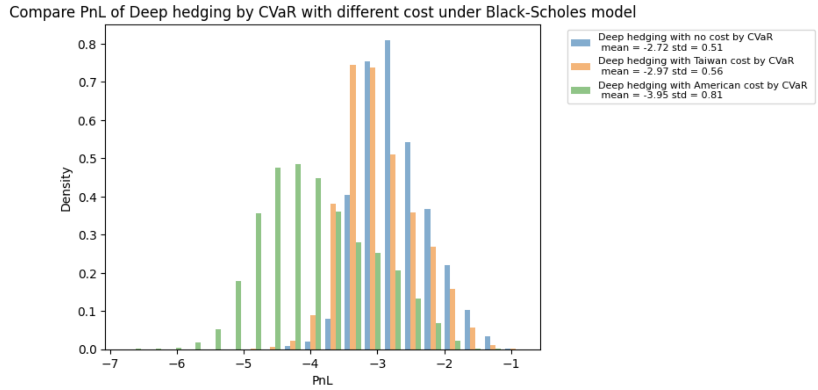
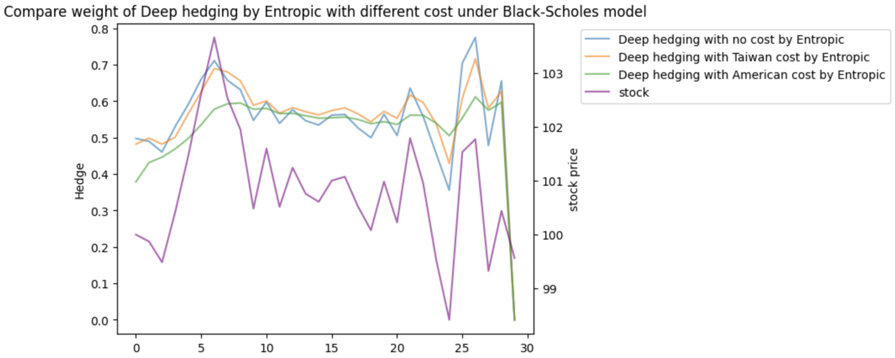


Deep Hedging (CVaR) strategy shows the higher the fee ratio, the lower the PnL, and the fluctuation in weights also becomes smaller.

Deep Hedging (Entropic) with Different Cost



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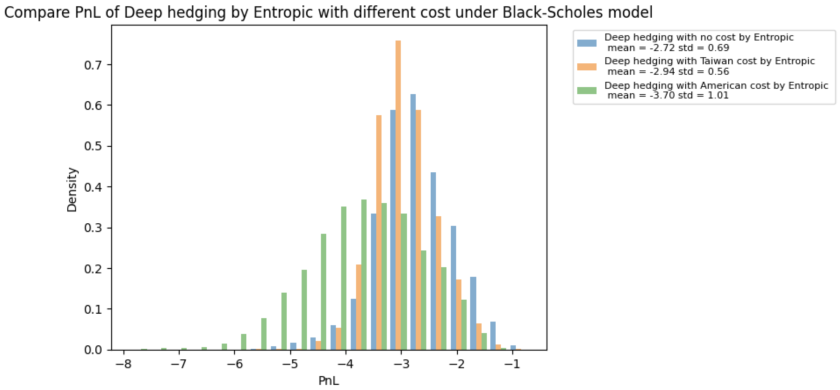

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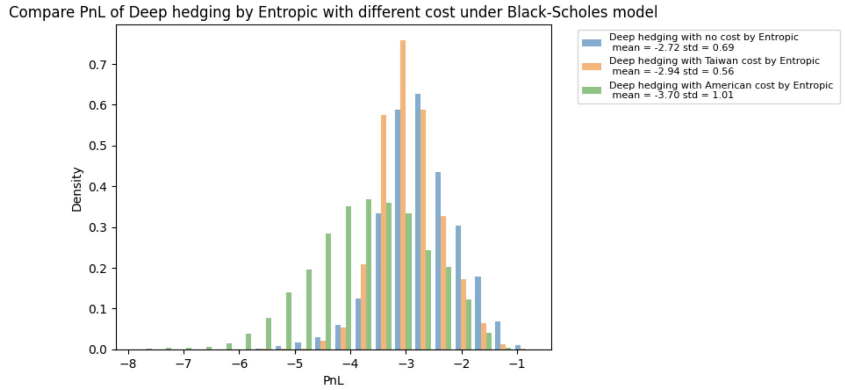

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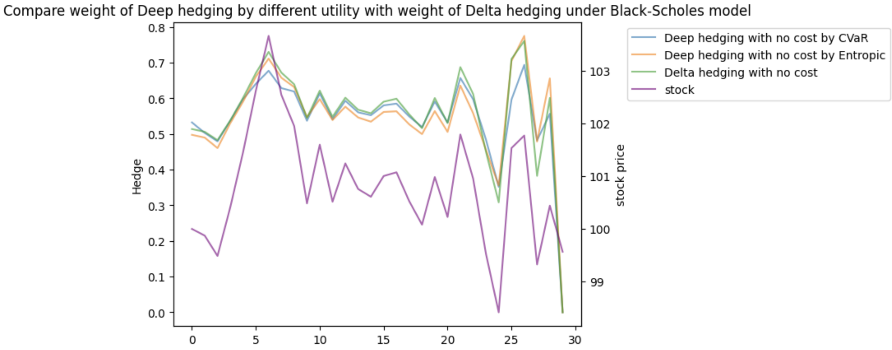
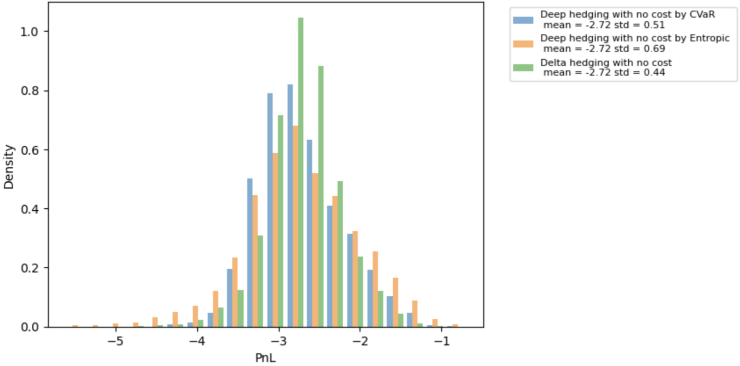


No Cost with Different Strategies

Compare PnL of Deep hedging by different utility with PnL of Delta hedging under Black-Scholes model



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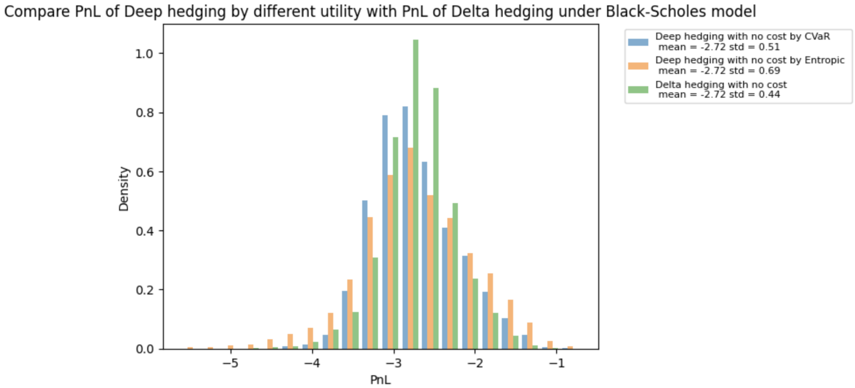
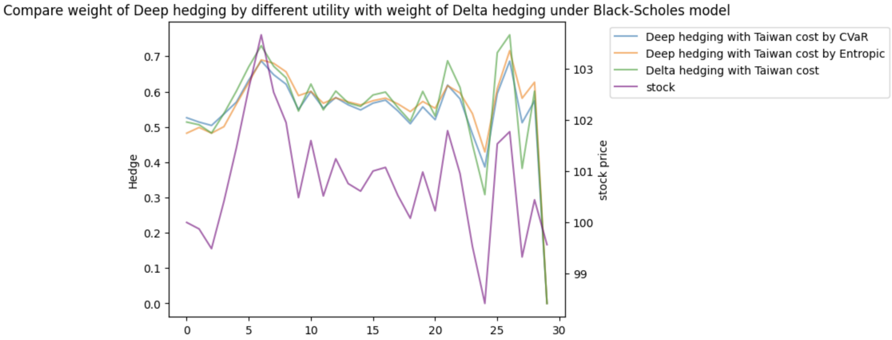


Delta Hedging strategy shows the better outcome.

Taiwan Cost with Different Strategies



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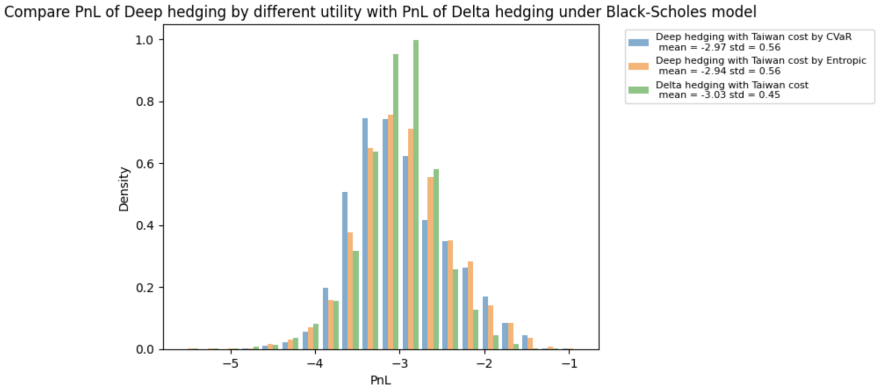

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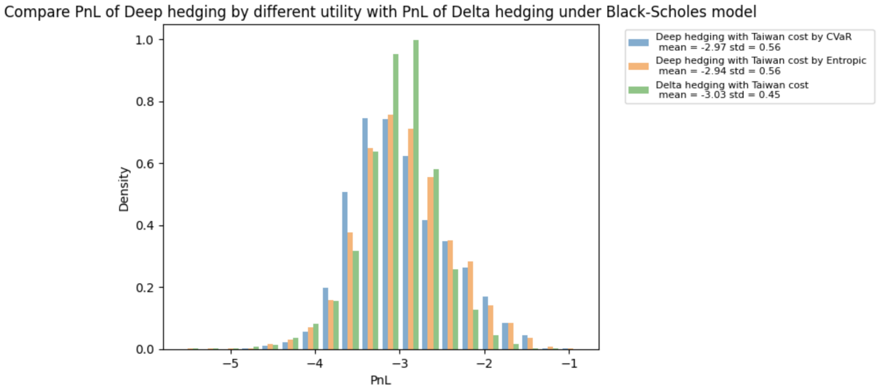

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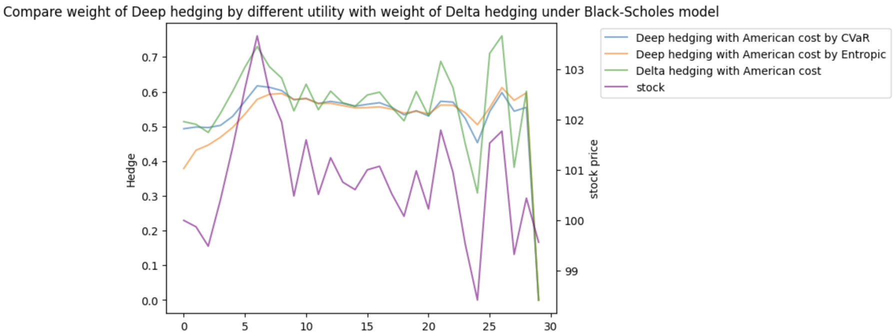

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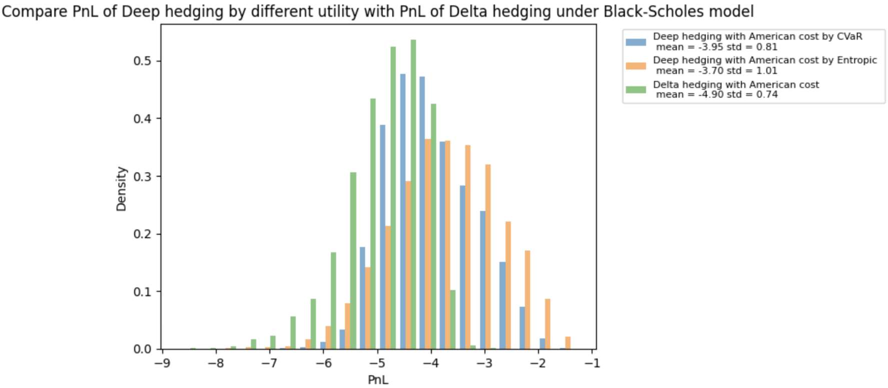

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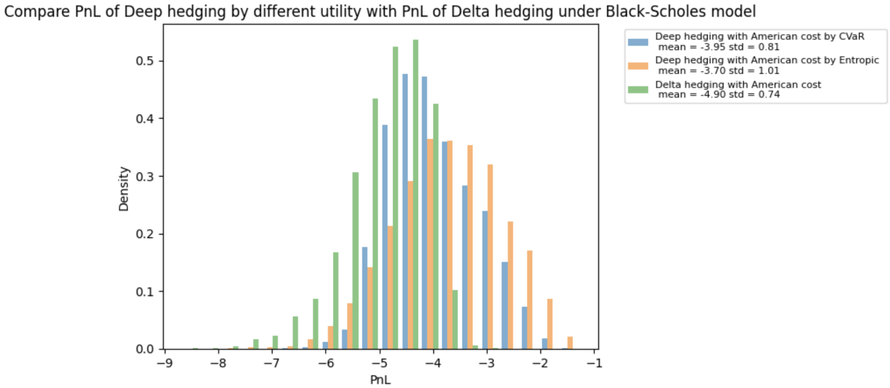

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AAPL History Data

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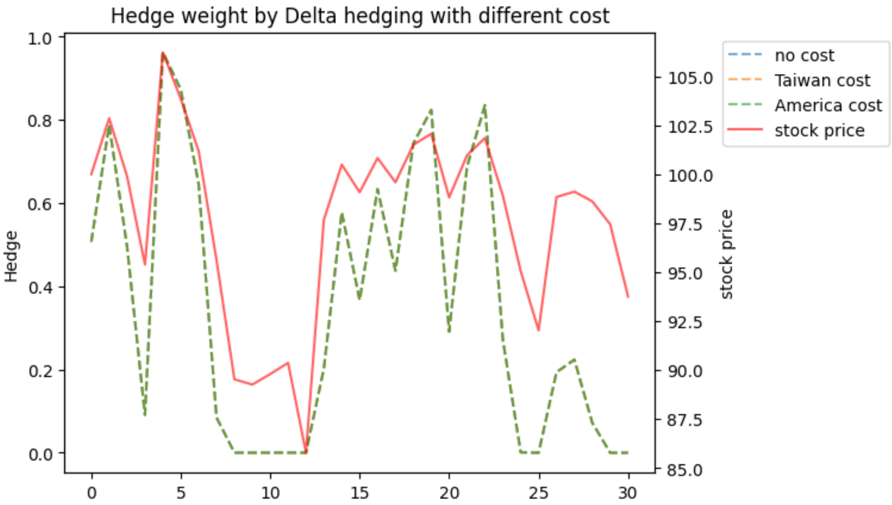

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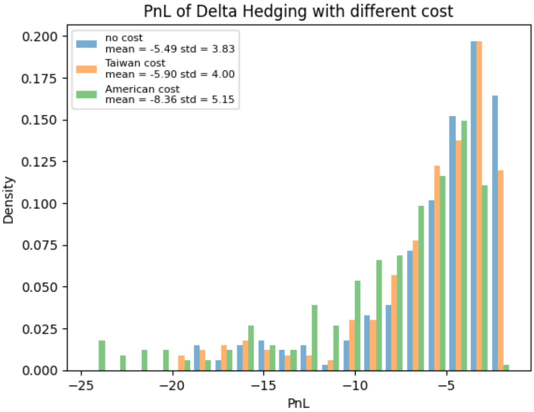

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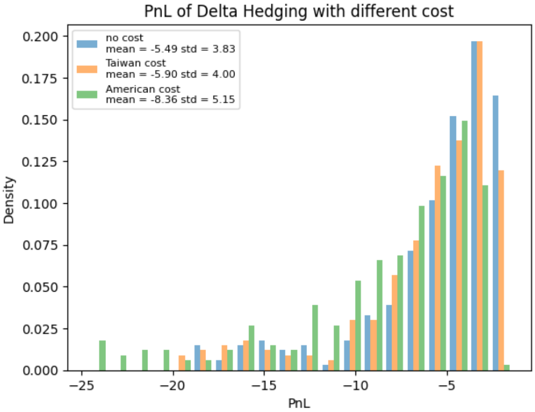

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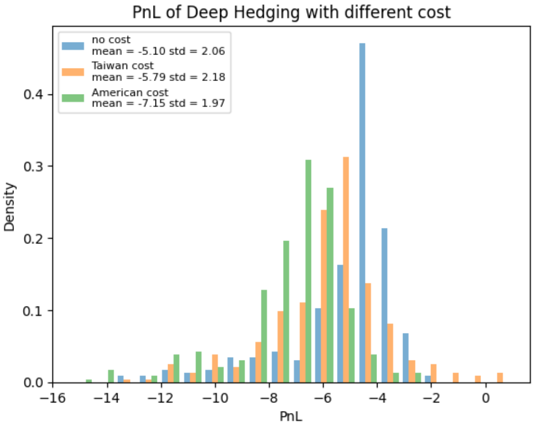

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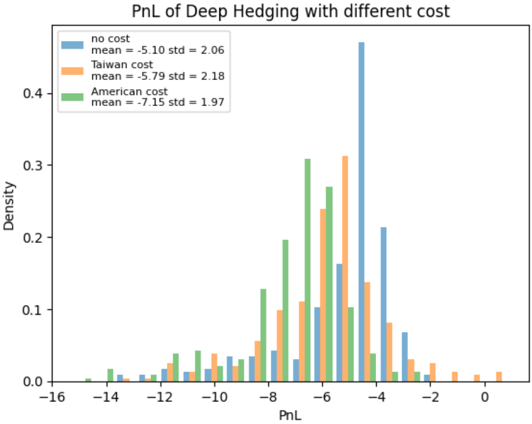

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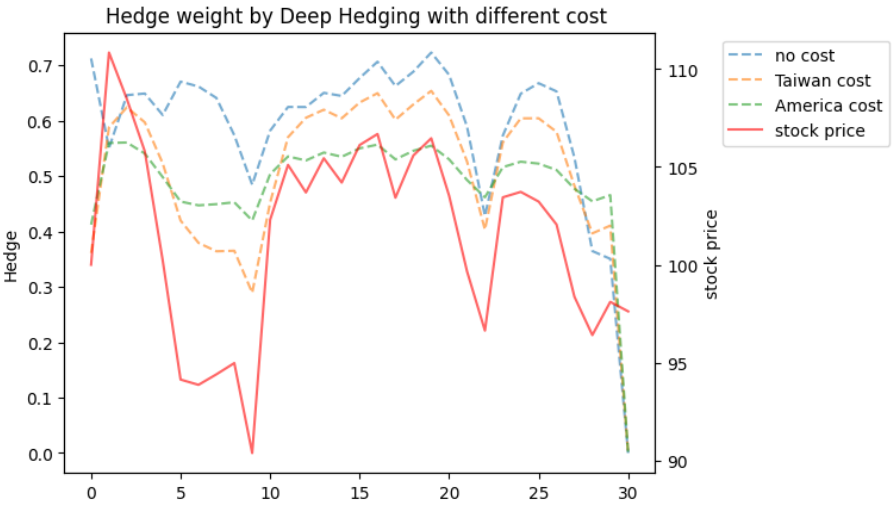

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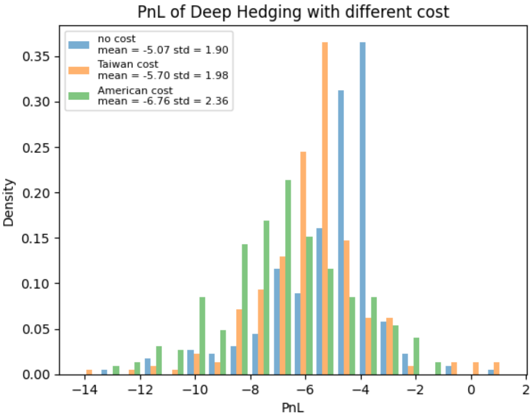

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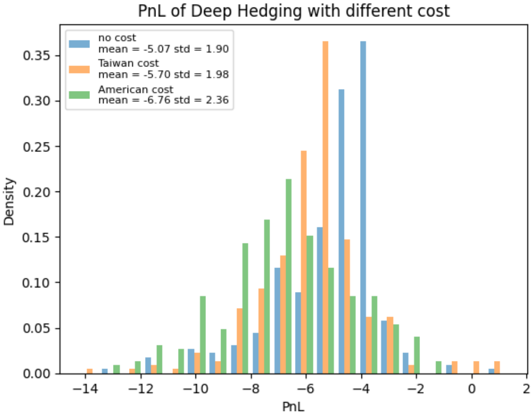

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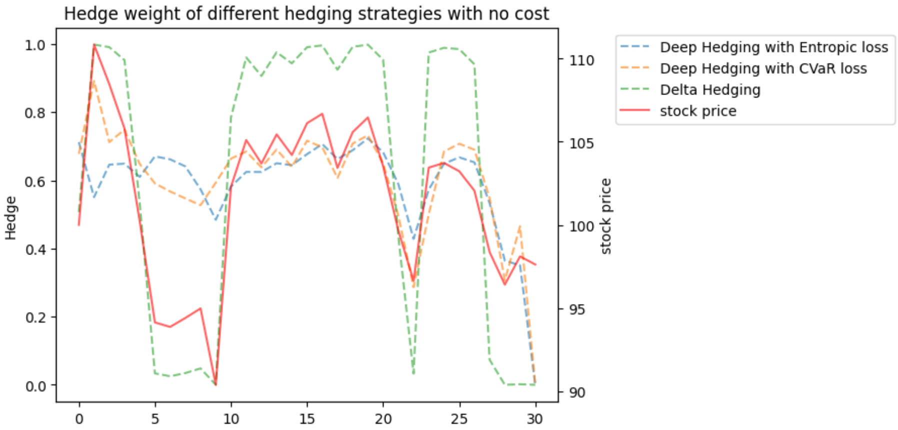

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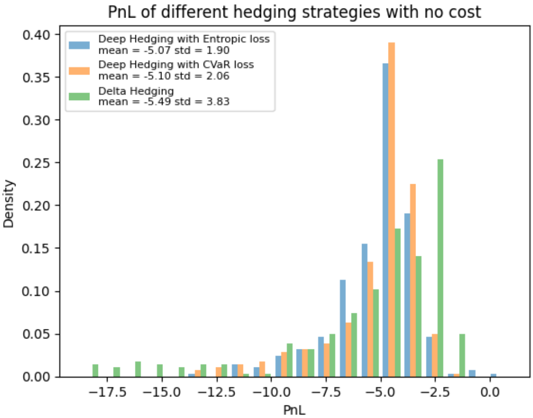

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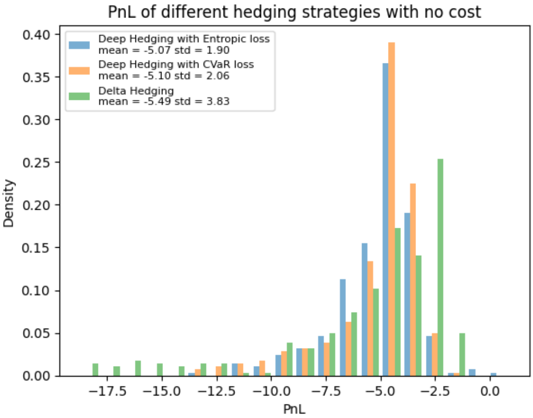

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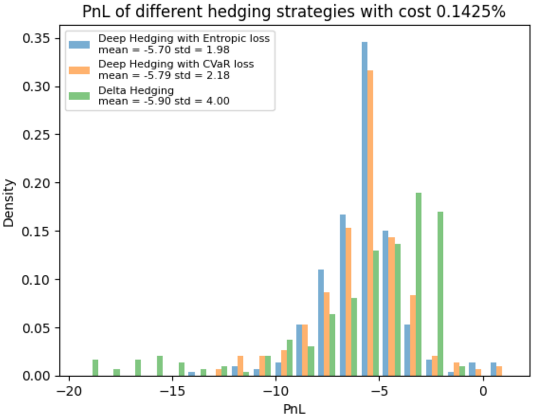

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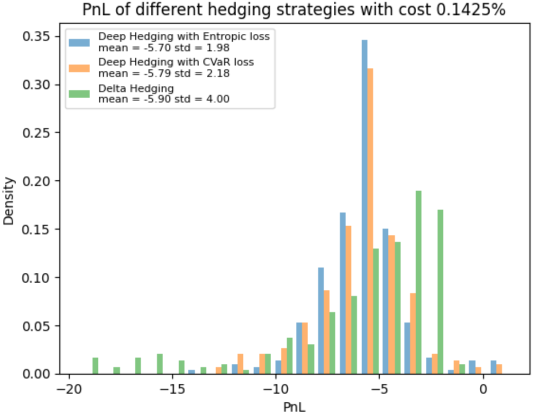

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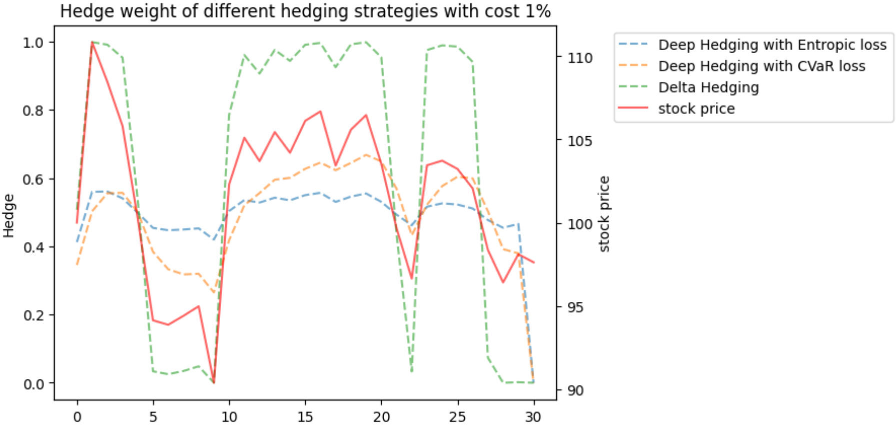

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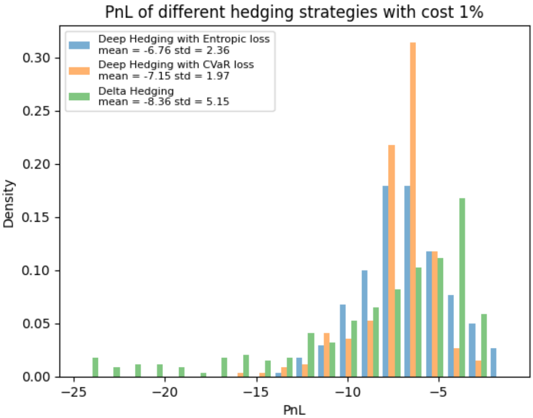

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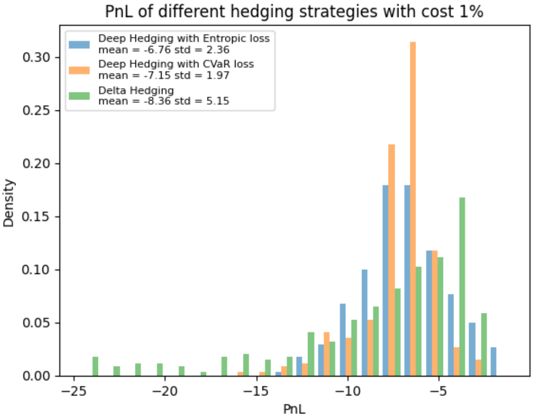

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Deep Hedging strategies offer superior performance and resilience compared to traditional Delta Hedging, particularly in the presence of transaction costs.

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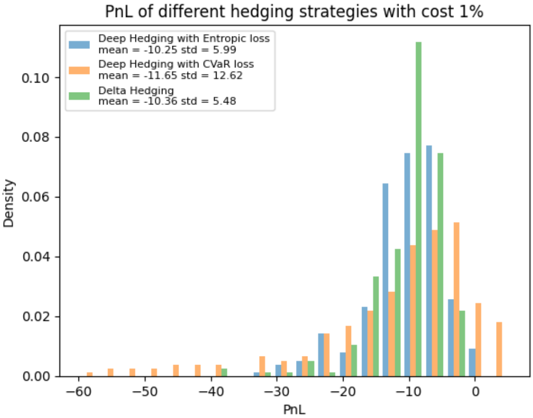
Amazon stock data

Observation: Deep hedging underperforms Black-Scholes delta hedging.

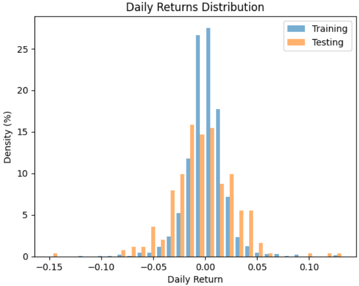


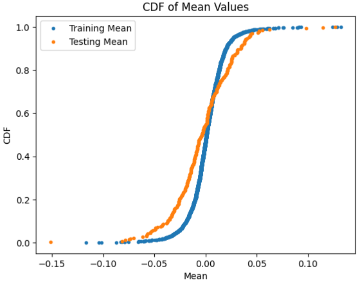
Amazon stock data

Action: Changed the time period of the Amazon data.

Amazon stock data

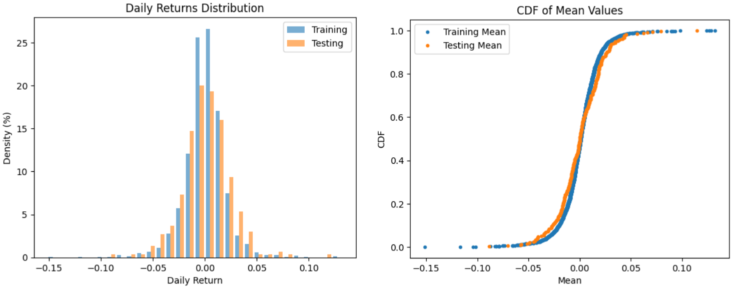
Observation: The distribution of daily returns differs significantly between the training and testing datasets.





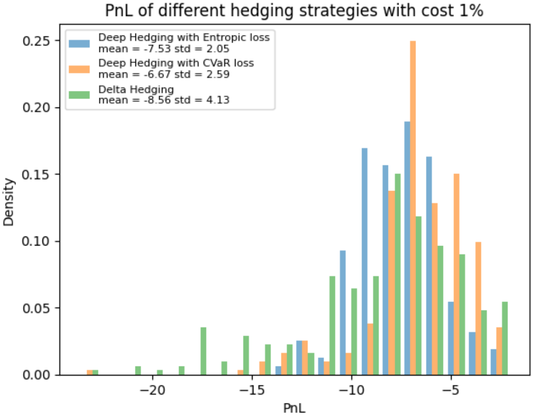
Amazon stock data

Action: Reevaluated the distribution differences between the training and testing datasets. The differences are not substantial.



Amazon stock data

Observation: In the revised period, deep hedging outperforms delta hedging, similar to the previous results with Apple stock data.



Amazon stock data

Conclusion

- Deep Hedging strategies may underperform traditional delta hedging in unforeseen market conditions.

Amazon stock data

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- Deep Hedging strategies perform optimally when the training and testing data distributions are similar.
- Proper preprocessing of training data is essential, with potential use of GANs to generate synthetic data for better performance.