Deep Hedging – Learning to Trade

Chen-Chung Yang[1] De-Jin Huang[2]

[1]Department of Computer Science
Tsing Hua University

[2]Department of Mathematics
Tsing Hua University

URP Midterm Presentation, March 1

Table of Contents

Table of Contents

Motivation

- Derivative Pricing

- Derivative Pricing
- Portfolio Hedging

Challenges

Challenges

- Mathematical finance models often assume "ideal" market conditions and frictionless trading, which do not align with real-world scenarios.

Challenges

- Mathematical finance models often assume "ideal" market conditions and frictionless trading, which do not align with real-world scenarios.
- Practical trading is affected by transaction costs, market impacts, and liquidity issues.

Challenges

- Mathematical finance models often assume "ideal" market conditions and frictionless trading, which do not align with real-world scenarios.
- Practical trading is affected by transaction costs, market impacts, and liquidity issues.
- A significant gap exists in effective, scalable solutions for addressing these real-world trading challenges.

Challenges

- Mathematical finance models often assume "ideal" market conditions and frictionless trading, which do not align with real-world scenarios.
- Practical trading is affected by transaction costs, market impacts, and liquidity issues.
- A significant gap exists in effective, scalable solutions for addressing these real-world trading challenges.

Solution: Deep Hedging!!!

What's This???

Deep hedging

Deep Hedging introduces an innovative method proposed by Bühler-Gonon-Teichmann-Wood [1] by utilizing neural networks to guide trading decisions in hedging strategies.

Deep hedging

Deep Hedging introduces an innovative method proposed by Bühler-Gonon-Teichmann-Wood [1] by utilizing neural networks to guide trading decisions in hedging strategies.

Key Advantages:

Deep hedging

Deep Hedging introduces an innovative method proposed by Bühler-Gonon-Teichmann-Wood [1] by utilizing neural networks to guide trading decisions in hedging strategies.

Key Advantages:

- Efficiency: Facilitates the effective training of hedging strategies, ensuring precise optimization of performance.

Deep hedging

Deep Hedging introduces an innovative method proposed by Bühler-Gonon-Teichmann-Wood [1] by utilizing neural networks to guide trading decisions in hedging strategies.

Key Advantages:

- Efficiency: Facilitates the effective training of hedging strategies, ensuring precise optimization of performance.
- Model-free: Operates free from the constraints of traditional financial models, providing flexibility in facing the unpredictable nature of markets.

Project objective

The primary aim of this project is to:

Project objective

The primary aim of this project is to:

- Delve into the theoretical foundations of hedging and pricing mechanisms.

Project objective

The primary aim of this project is to:

- Delve into the theoretical foundations of hedging and pricing mechanisms.
- Execute the deep hedging algorithm, deriving effective hedging strategies and determining asset pricing.

Table of Contents

Setting

Setting

- a discrete-time financial market with finite time horizon T and trading dates $0 = t_0 < t_1 < ... < t_n = T$.

Setting

- a discrete-time financial market with finite time horizon T and trading dates $0 = t_0 < t_1 < ... < t_n = T$.
- $I_k \in \mathbb{R}^r$ the market information available at time t_k, which represents all known information such as prices, bid, asks, etc.

Setting

- a discrete-time financial market with finite time horizon T and trading dates $0 = t_0 < t_1 < ... < t_n = T$.
- $I_k \in \mathbb{R}^r$ the market information available at time t_k, which represents all known information such as prices, bid, asks, etc.
- d stocks with mid-prices given by an $\mathbb{R}^d$-valued stochastic process $\underline{S} = (S_k)_{k=0,...,n}$.

Setting

- a discrete-time financial market with finite time horizon T and trading dates $0 = t_0 < t_1 < ... < t_n = T$.
- $I_k \in \mathbb{R}^r$ the market information available at time t_k, which represents all known information such as prices, bid, asks, etc.
- d stocks with mid-prices given by an $\mathbb{R}^d$-valued stochastic process $\underline{S} = (S_k)_{k=0,\dots,n}$.
- a random variable Z, which represents a contingent claim with maturity T.

Trading strategies

Trading strategies

- Engage in Trading the assets $\underline{S}$ through an $\mathbb{R}^d$-valued strategy $\underline{\delta} = (\delta_k)_{k=0,\ldots,n-1}$.

Trading strategies

- Engage in Trading the assets $\underline{S}$ through an $\mathbb{R}^d$-valued strategy $\underline{\delta} = (\delta_k)_{k=0,\dots,n-1}$.
- Each entry $\delta_k = (\delta_k^1, \dots, \delta_k^d)$ represents the allocation within the d assets at each time point.

Trading strategies

- Engage in Trading the assets $\underline{S}$ through an $\mathbb{R}^d$-valued strategy $\underline{\delta} = (\delta_k)_{k=0,\dots,n-1}$.
- Each entry $\delta_k = (\delta_k^1, \dots, \delta_k^d)$ represents the allocation within the d assets at each time point.
- δ_k^i quantifies the holdings of the i-th asset at the moment t_k.

Trading strategies

- Engage in Trading the assets $\underline{S}$ through an $\mathbb{R}^d$-valued strategy $\underline{\delta} = (\delta_k)_{k=0,\dots,n-1}$.
- Each entry $\delta_k = (\delta_k^1, \dots, \delta_k^d)$ represents the allocation within the d assets at each time point.
- δ_k^i quantifies the holdings of the i-th asset at the moment t_k.
- For ease of notation, we set initial and final with $\delta_{-1} = \delta_T := 0$.

Trading strategies

- Engage in Trading the assets $\underline{S}$ through an $\mathbb{R}^d$-valued strategy $\underline{\delta} = (\delta_k)_{k=0,\ldots,n-1}$.
- Each entry $\delta_k = (\delta_k^1, \ldots, \delta_k^d)$ represents the allocation within the d assets at each time point.
- δ_k^i quantifies the holdings of the i-th asset at the moment t_k.
- For ease of notation, we set initial and final with $\delta_{-1} = \delta_T := 0$.
- The trading gains or losses under strategy $\underline{\delta}$ is calculated by:

$$(\underline{\delta} \cdot \underline{S})_T := \sum_{k=0}^{T-1} \delta_k \cdot (S_{k+1} - S_k).$$

Trading space

We define the following trading spaces:

Trading space

We define the following trading spaces:

- $\mathcal{H}^u$ is defined as the unconstrained set of trading strategies.

Trading space

We define the following trading spaces:

- $\mathcal{H}^u$ is defined as the unconstrained set of trading strategies.
- For each time step k, $\mathcal{H}_k := H_k(\mathbb{R}^{d(k+1)})$ denotes the set of trading strategies for δ_k that adhere to specific constraints, as image under a mapping $H_k : \mathbb{R}^{d(k+1)} \to \mathbb{R}^d$.

Trading space

We define the following trading spaces:

- $\mathcal{H}^u$ is defined as the unconstrained set of trading strategies.
- For each time step k, $\mathcal{H}_k := H_k(\mathbb{R}^{d(k+1)})$ denotes the set of trading strategies for δ_k that adhere to specific constraints, as image under a mapping $H_k : \mathbb{R}^{d(k+1)} \to \mathbb{R}^d$.
- $\mathcal{H} := (H \circ \mathcal{H}^u) \subset \mathcal{H}^u$ represents the set of restricted trading strategies.

Hedging Strategies

Hedging Strategies

- Let's start with a portfolio whose initial cash position is p_0. Note: A negative p_0 indicates the potential for cash withdrawal from the portfolio.

Hedging Strategies

- Let's start with a portfolio whose initial cash position is p_0. Note: A negative p_0 indicates the potential for cash withdrawal from the portfolio.
- Assume all trading operations are **self-financing**, meaning they rely solely on the portfolio's own assets, eliminating the necessity for external financing.

Hedging Strategies

- Let's start with a portfolio whose initial cash position is p_0. Note: A negative p_0 indicates the potential for cash withdrawal from the portfolio.
- Assume all trading operations are **self-financing**, meaning they rely solely on the portfolio's own assets, eliminating the necessity for external financing.
- Account for **proportional transaction costs** in trading activities. That is, initiating a position $\boldsymbol{n} \in \mathbb{R}^d$ in $\underline{S}$ at time t_k leads to costs

$$c_k(\boldsymbol{n}) := \sum_{i=1}^{d} c_k^i S_k^i |n_i|.$$

Hedging strategies

The terminal value of the portfolio at time T is determined by:

$$\mathrm{PnL}_T(Z, p_0, \underline{\delta}) := -Z + p_0 + (\underline{\delta} \cdot \underline{S})_T - C_T(\underline{\delta}),$$

where:

Hedging strategies

The terminal value of the portfolio at time T is determined by:

$$\mathrm{PnL}_T(Z, p_0, \underline{\delta}) := -Z + p_0 + (\underline{\delta} \cdot \underline{S})_T - C_T(\underline{\delta}),$$

where:

- The sum of the first three terms represents the portfolio's wealth at maturity T, assuming a market without transaction costs.

Hedging strategies

The terminal value of the portfolio at time T is determined by:

$$\mathrm{PnL}_T(Z, p_0, \underline{\delta}) := -Z + p_0 + (\underline{\delta} \cdot \underline{S})_T - C_T(\underline{\delta}),$$

where:

- The sum of the first three terms represents the portfolio's wealth at maturity T, assuming a market without transaction costs.
- The final term,

$$C_T(\underline{\delta}) := \sum_{k=0}^{n} c_k(\delta_k - \delta_{k-1})$$

  accounts for the cumulative transaction costs incurred by implementing strategy $\underline{\delta}$ until maturity.

Table of Contents

Measuring risk

Measuring risk

A hedging strategy aims to reduce the risk associated with a derivative portfolio.

Measuring risk

A hedging strategy aims to reduce the risk associated with a derivative portfolio.

Question: How to quantify risk?

Measuring risk

A hedging strategy aims to reduce the risk associated with a derivative portfolio.

Question: How to quantify risk?

Answer: **Risk Measures**.

Convex risk measure

Convex risk measure

Let Ω be a probability space and $\mathcal{X} : \{\Omega \to \mathbb{R}\}$ be the set of all real-valued random variables over Ω.

Convex risk measure

Let Ω be a probability space and $\mathcal{X} : \{\Omega \to \mathbb{R}\}$ be the set of all real-valued random variables over Ω.

We call $\rho : \mathcal{X} \to \mathbb{R}$ a convex risk measure [2] if it is

Convex risk measure

Let Ω be a probability space and $\mathcal{X} : \{\Omega \to \mathbb{R}\}$ be the set of all real-valued random variables over Ω.

We call $\rho : \mathcal{X} \to \mathbb{R}$ a convex risk measure [2] if it is

1. **Monotone decreasing**: if $X_1 \geq X_2$ then $\rho(X_1) \leq \rho(X_2)$.

Convex risk measure

Let Ω be a probability space and $\mathcal{X} : \{\Omega \to \mathbb{R}\}$ be the set of all real-valued random variables over Ω.

We call $\rho : \mathcal{X} \to \mathbb{R}$ a convex risk measure [2] if it is

1. **Monotone decreasing**: if $X_1 \geq X_2$ then $\rho(X_1) \leq \rho(X_2)$.
2. **Convexity**: $\rho(\alpha X_1 + (1-\alpha)X_2) \leq \alpha\rho(X_1) + (1-\alpha)\rho(X_2)$ for $\alpha \in [0,1]$.

Convex risk measure

Let Ω be a probability space and $\mathcal{X} : \{\Omega \to \mathbb{R}\}$ be the set of all real-valued random variables over Ω.

We call $\rho : \mathcal{X} \to \mathbb{R}$ a convex risk measure [2] if it is

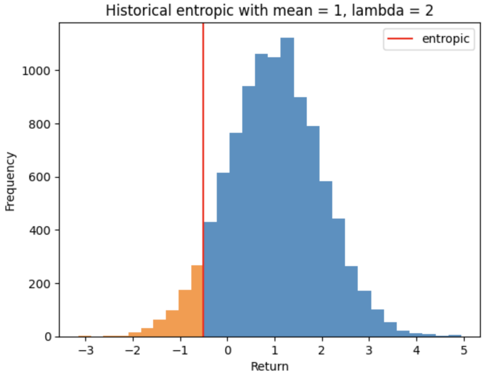
1. **Monotone decreasing**: if $X_1 \geq X_2$ then $\rho(X_1) \leq \rho(X_2)$.
2. **Convexity**: $\rho(\alpha X_1 + (1-\alpha)X_2) \leq \alpha\rho(X_1) + (1-\alpha)\rho(X_2)$ for $\alpha \in [0, 1]$.
3. **Cash-Invariant**: $\rho(X + c) = \rho(X) - c$ for $c \in \mathbb{R}$

Entropic risk measure

Examples (Entropic risk measure)

$$\rho = \frac{1}{\lambda} \log \left(\mathbb{E} \left[\exp(-\lambda X) \right] \right)$$

Here λ is the risk aversion parameter.

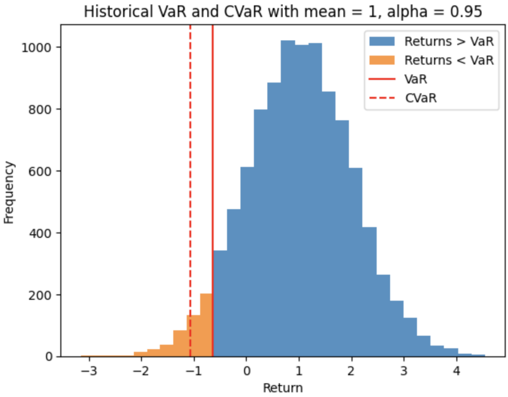


Conditional value at risk

Examples (CVaR)

$$CVaR_\alpha(X) := \mathbb{E}[X|X \leq VaR_\alpha(X)]$$

where $VaR_\alpha(X) := P[X \leq \cdot]^{-1}(1-\alpha)$



Optimal hedging strategy

Optimal hedging strategy

Let ρ be a given convex risk measure.

Optimal hedging strategy

Let ρ be a given convex risk measure.

Consider the optimization problem

$$\pi(X) := \inf_{\underline{\delta} \in \mathcal{H}} \rho(X + (\underline{\delta} \cdot \underline{S})_T - C_T(\delta)). \tag{1}$$

Optimal hedging strategy

Let ρ be a given convex risk measure.

Consider the optimization problem

$$\pi(X) := \inf_{\underline{\delta} \in \mathcal{H}} \rho(X + (\underline{\delta} \cdot \underline{S})_T - C_T(\delta)). \tag{1}$$

We define an **optimal hedging strategy** as δ that minimizes within H as per the definition of $\pi(X)$.

Optimal hedging strategy

Let ρ be a given convex risk measure.

Consider the optimization problem

$$\pi(X) := \inf_{\underline{\delta} \in \mathcal{H}} \rho(X + (\underline{\delta} \cdot \underline{S})_T - C_T(\delta)). \tag{1}$$

We define an **optimal hedging strategy** as δ that minimizes within H as per the definition of $\pi(X)$.

Proposition

π is monotone decreasing and cash-invariant. If moreover C_T and $\mathcal{H}$ are convex, then π is a convex risk measure.

Indifference price

Indifference price

- Indifference pricing is a method of pricing derivatives based on a risk measure (or a utility function).

Indifference price

- Indifference pricing is a method of pricing derivatives based on a risk measure (or a utility function).
- We define the indifference price $p(Z)$ of a contingent claim Z as the amount of cash that one needs to charge in order to be indifferent between the position $-Z$ and not doing so. That is, as the solution p_0 to $\pi(-Z + p_0) = \pi(0)$.

Indifference price

- Indifference pricing is a method of pricing derivatives based on a risk measure (or a utility function).
- We define the indifference price $p(Z)$ of a contingent claim Z as the amount of cash that one needs to charge in order to be indifferent between the position $-Z$ and not doing so. That is, as the solution p_0 to $\pi(-Z + p_0) = \pi(0)$.
- Due to cash-invariance, this condition simplifies to setting $p_0 = p(Z)$, hence,
$$p(Z) = \pi(-Z) - \pi(0).$$

Indifference price of an attainable contingent claim

Lemma

Suppose $C_T \equiv 0$ and $\mathcal{H} = \mathcal{H}^u$. If Z is attainable, i.e. there exists $\underline{\delta}^* \in \mathcal{H}$ and $p_0 \in \mathbb{R}$ such that $Z = p_0 + (\underline{\delta}^* \cdot \underline{S})_T$, then $p(Z) = p_0$.

Indifference price of an attainable contingent claim

Lemma

Suppose $C_T \equiv 0$ and $\mathcal{H} = \mathcal{H}^u$. If Z is attainable, i.e. there exists $\underline{\delta}^* \in \mathcal{H}$ and $p_0 \in \mathbb{R}$ such that $Z = p_0 + (\underline{\delta}^* \cdot \underline{S})_T$, then $p(Z) = p_0$.

Takeaway: A replicable contingent claim can be hedged completely and thus has zero price.

Table of Contents

Universal approximation by neural networks

Universal approximation by neural networks

Artificial Neural networks are compositions of multiple simple functions that map real-valued vectors to real-valued vectors.

Universal approximation by neural networks

Artificial Neural networks are compositions of multiple simple functions that map real-valued vectors to real-valued vectors.

Universal Approximation Theorem

Let $\mathfrak{X}$ be a compact subset of $\mathbb{R}^{d_0}$. The set of neural networks mapping from $\mathfrak{X}$ to $\mathbb{R}^{d_1}$ is dense in $C(\mathfrak{X}, \mathbb{R}^{d_1})$.[3]

Optimal hedging using deep neural networks

We now consider neural network hedging strategies.

Optimal hedging using deep neural networks

We now consider neural network hedging strategies.

In order to apply the Universal Approximation Theorem above, we represent the optimization over constrained trading strategies $\underline{\delta} \in \mathcal{H}$ as an optimization over $\underline{\delta} \in \mathcal{H}^u$ with a following modified objective.

Optimal hedging using deep neural networks

We now consider neural network hedging strategies.

In order to apply the Universal Approximation Theorem above, we represent the optimization over constrained trading strategies $\underline{\delta} \in \mathcal{H}$ as an optimization over $\underline{\delta} \in \mathcal{H}^u$ with a following modified objective.

Lemma

We may write the constrained problem (1) as the modified un-constrained problem as

$$\pi(X) = \inf_{\underline{\delta} \in \mathcal{H}^u} \rho\left(X + (H \circ \underline{\delta} \cdot \underline{S})_T - C_T(H \circ \underline{\delta})\right).$$

Optimal hedging using deep neural networks

Optimal hedging using deep neural networks

Define

$$
\begin{aligned}
H_M &= \{(\delta_k)_{k=0,\dots,n-1} \in \mathcal{H}^u : \delta_k = F_k(I_0, \dots, I_k, \delta_{k-1}), F_k \in \mathcal{NN}_{M,r(k+1)+d,d}\} \\
&= \{(\delta_k^\theta)_{k=0,\dots,n-1} \in \mathcal{H}^u : \delta_k^\theta = F_k^\theta(I_0, \dots, I_k, \delta_{k-1}), \theta_k \in \Theta_{M,r(k+1)+d,d}\}
\end{aligned}
$$

Optimal hedging using deep neural networks

Define

$$H_M = \{(\delta_k)_{k=0,\dots,n-1} \in \mathcal{H}^u : \delta_k = F_k(I_0,\dots,I_k,\delta_{k-1}), F_k \in \mathcal{NN}_{M,r(k+1)+d,d}\}$$
$$= \{(\delta_k^\theta)_{k=0,\dots,n-1} \in \mathcal{H}^u : \delta_k^\theta = F_k^\theta(I_0,\dots,I_k,\delta_{k-1}), \theta_k \in \Theta_{M,r(k+1)+d,d}\}$$

We aim to calculate

$$\pi^M(X) = \inf_{\delta \in \mathcal{H}_M} \rho\,(X + (H \circ \underline{\delta} \cdot \underline{S})_T - C_T(H \circ \underline{\delta}))$$
$$= \inf_{\theta \in \Theta_M} \rho\left(X + (H \circ \underline{\delta}^\theta \cdot \underline{S})_T - C_T(H \circ \delta^\theta)\right) \tag{2}$$

where $\Theta_M = \prod_{k=0}^{n-1} \Theta_{M,r(k+1)+d,d}$.

Optimal hedging using deep neural networks

The Universal Approximation Theorem suggests that strategies within $\mathcal{H}$ can be arbitrarily approximated by strategies in $\mathcal{H}_M$. i.e.

The Universal Approximation Theorem suggests that strategies within $\mathcal{H}$ can be arbitrarily approximated by strategies in $\mathcal{H}_M$. i.e.

Proposition

$$\lim_{M\to\infty} \pi^M(X) = \pi(X).$$

Optimal hedging using deep neural networks

The Universal Approximation Theorem suggests that strategies within $\mathcal{H}$ can be arbitrarily approximated by strategies in $\mathcal{H}_M$. i.e.

Proposition

$$\lim_{M\to\infty} \pi^M(X) = \pi(X).$$

Consequently, the neural network price $p^M(Z) := \pi^M(-Z) - \pi^M(0)$ converges to the exact price $p(Z)$.

Numerical solution approach

As a result, the focus shifts to solving for π^M effectively.

Numerical solution approach

As a result, the focus shifts to solving for π^M effectively.

Key Question: How do we calculate a (close-to) optimal $\theta \in \Theta_M$ for our neural network-based hedging strategy (2)?

Numerical solution approach

As a result, the focus shifts to solving for π^M effectively.

Key Question: How do we calculate a (close-to) optimal $\theta \in \Theta_M$ for our neural network-based hedging strategy (2)?

Brief Answer: Apply standard deep learning techniques to robust representation of convex risk measures, including:

Numerical solution approach

As a result, the focus shifts to solving for π^M effectively.

Key Question: How do we calculate a (close-to) optimal $\theta \in \Theta_M$ for our neural network-based hedging strategy (2)?

Brief Answer: Apply standard deep learning techniques to robust representation of convex risk measures, including:

1. Minibatch stochastic gradient descent for iterative optimization.

Numerical solution approach

As a result, the focus shifts to solving for π^M effectively.

Key Question: How do we calculate a (close-to) optimal $\theta \in \Theta_M$ for our neural network-based hedging strategy (2)?

Brief Answer: Apply standard deep learning techniques to robust representation of convex risk measures, including:

1. Minibatch stochastic gradient descent for iterative optimization.
2. Backpropagation algorithm for efficient gradient calculation.

Table of Contents

Setup

Setup

Market generator :

- Black-Scholes model
- Heston model

Setup

Market generator :

- Black-Scholes model
- Heston model

Hedging target : European call option

$$\text{payoff}\ (S, K) = max(S - K, 0).$$

Setup

Market generator :

- Black-Scholes model
- Heston model

Hedging target : European call option

$$\text{payoff}\,(S,K) = max(S-K,0).$$

Risk measure : CVaR with $\alpha = 0.95$.

Setup

Parameters definition and settings :

- Simulate 5000 paths
- Trading days $T = 30/252$
- Trading every day, $\Delta t = 1/252$
- S_t is the stock price at time t, $S_0 = 100$
- K is strike price, K $= 100$
- V_t is the variance at time t
- δ_t is asset position at time t
- F^θ is neural network with parameters θ

Black-Scholes model

Asset price paths :

$$\Delta S_t = S_{t-1} \times (\mu \Delta t + \sigma \sqrt{\Delta t} \mathcal{N}), \quad \mathit{for}\ t > 0$$

where :

- μ is a constant means expected return
- σ is a constant means variance
- $\mathcal{N}$ is standard normal distribution

Hedging strategies

Black-Scholes model delta hedging :

$$\text{Hedge Ratio } \delta = \frac{\partial C}{\partial S} = N(d_1)$$

where

- C is the call option price
- $N(x)$ is the CDF of standard normal distribution
- $d_1 = \frac{\log\left(\frac{S_0 e^{-rt}}{K}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$

Black-Scholes model delta hedging :

$$\text{Hedge Ratio } \delta = \frac{\partial C}{\partial S} = N(d_1)$$

where

- C is the call option price
- $N(x)$ is the CDF of standard normal distribution
- $d_1 = \frac{\log\left(\frac{S_0 e^{-rt}}{K}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$

Deep Hedging :

$$\text{Hedge Ratio } \delta_t^{\theta} = F^{\theta}(I_t, \delta_{t-1}^{\theta})$$

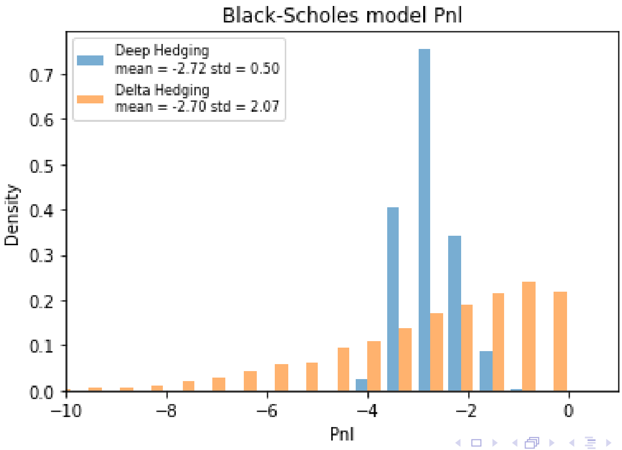
where

- Input $I_t = (\log(S_t/K),\ \sigma,\ T-t)$ at time t

Simulation result

$CVaR_{0.95}$ value

- Deep Hedging: -3.63
- Delta Hedging: -7.96



Heston model

Asset price paths :

$$\Delta S_t = S_{t-1} \times (\mu \Delta t + \sqrt{V_{t-1} \Delta t} \mathcal{N}_S), \quad \textit{for } t > 0$$
$$\Delta V_t = \alpha (b - V_{t-1}) \Delta t + \sigma \sqrt{V_{t-1} \Delta t} \mathcal{N}_V, \quad \textit{for } t > 0$$

where :

- μ is the expected return
- $\mathcal{N}_S$ and $\mathcal{N}_V$ are standard normal distribution with correlation $\rho \in [-1, 1]$
- $\alpha, b, \sigma, v_0, s_0$ are positive constants.

Hedging strategies

Heston model delta hedging :

$$\text{Hedge Ratio } \delta = \frac{\partial C}{\partial S}$$

where C is the call option price.

Hedging strategies

Heston model delta hedging :

$$\text{Hedge Ratio } \delta = \frac{\partial C}{\partial S}$$

where C is the call option price.

Deep Hedging :

$$\text{Hedge Ratio } \delta_t^{\theta}, V_t^{\theta} = F^{\theta}(I_t, \delta_{t-1}^{\theta}, V_{t-1}^{\theta})$$

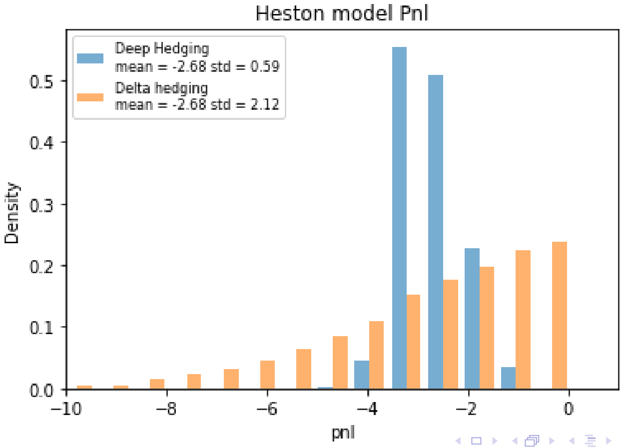
where

- Input $I_t = (\log(S_t/K), T - t)$ at time t

Simulation result

$CVaR_{0.95}$ value

- Deep Hedging: -3.76
- Delta Hedging: -8.13

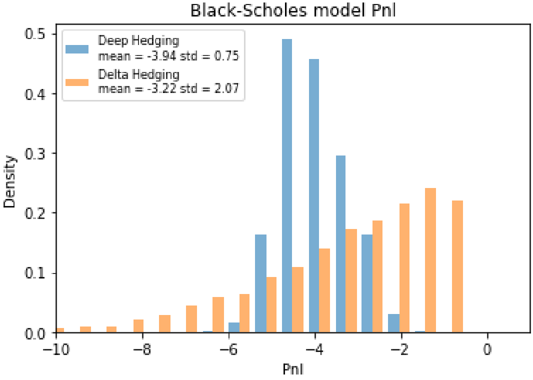


Transaction cost

Transaction cost $c_t(x) = \epsilon |x| S_t,$ with $\epsilon = 0.01$

$CVaR_{0.95}$ value

- Deep Hedging: -5.27
- Delta Hedging: -8.49



Future plan

Future plan

- Replace model-based paths with synthetic paths generated by "GAN"

Future plan

- Replace model-based paths with synthetic paths generated by "GAN"
- Explore signature hedging strategies

Future plan

- Replace model-based paths with synthetic paths generated by "GAN"
- Explore signature hedging strategies
- Delve into deep Bellman hedging approaches i.e. leveraging reinforcement learning algorithms for dynamic decision-making

References

[1] H. Buehler, L. Gonon, J. Teichmann, and B. Wood, "Deep hedging," *Quantitative Finance*, vol. 19, no. 8, pp. 1271–1291, 2019.

[2] F. Hans and A. Schied, *Stochastic Finance: An Introduction in Discrete Time by*. De Gruyter, 2016.

[3] K. Hornik, "Approximation capabilities of multilayer feedforward networks," *Neural networks*, vol. 4, no. 2, pp. 251–257, 1991.