# Homework 1
# Max Points: 55

**Directions:** Answer each of the questions to the best of your ability. Make sure to show your work in order to receive potential FULL credit. Just stating the answer will result in point deductions (and possibly no credit for the respective problem) and that cannot be disputed through the grade dispute policy. Your answers must typed up and submitted as a PDF. Handwritten solutions will not be accepted.

1. (15 points) Show that the following statement holds true using induction. Please show your steps in order to receive potential full credit. Skipping any steps of how things were derived can lead to not receiving full credit. Your proof must reach the same conclusion based on the given closed form. *Hint: If you get stuck with factoring polynomials that aren't quadratic, consider polynomial division to assist with determining factors.*

$$s_n = \sum_{k=1}^{n} \frac{k+4}{k(k+1)(k+2)} = \frac{n(3n+7)}{2(n+1)(n+2)}$$

2. (5 points) Given sets $A$ and $B$ where $A = \{2, 3, 5, 7\}$ and $B = \{2, 4, 5, 8, 9\}$ and the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
   List each member in set $S$ which is defined as $S = \overline{A} \cap B$.

3. (10 points) Given $\Sigma = \{a, b\}$, construct a DFA that recognizes the following language $L_3$. Note: $n_a(w)$ and $n_b(w)$ means the total number of $a$'s and number of $b$'s in $w$ respectively. Example: If $w = abbaaba$, then $n_a(w) = 4$ and $n_b(w) = 3$.

$$L_3 = \{w : (n_a(w) \bmod 3 \leq n_b(w) \bmod 3\}$$

4. (15 points) In the C programming language, a value can be designated as a character type through a declaration statement where the prefix starts with `char` and ends with the suffix `;`. Between the prefix and suffix is the name of the variable. Let $C$ be the language of valid character declaration statements. A member of $C$ must start with `char` and end with `;`. The member must have at least 2 characters in its variable name and cannot contain the `;` character. For example, the statement `char nums;` is valid since `nums` has 4 characters, but `char x;` is not valid since `x` is only one character. For simplicity, assume that the alphabet for $C$ is $\Sigma = \{a, b, c, d, e, h, i, r, ;\}$. Draw a DFA that recognizes $C$. Note: *This is only to represent a declaration statement. We are not focusing on assigning values. We will also assume for simplicity that there is no maximum size for the variable name.*

5. (5 points) Given the following DFA $M$. Construct a grammar $G$ that generates all strings that the DFA should recognize. Assume that $\Sigma = \{0, 1\}$.
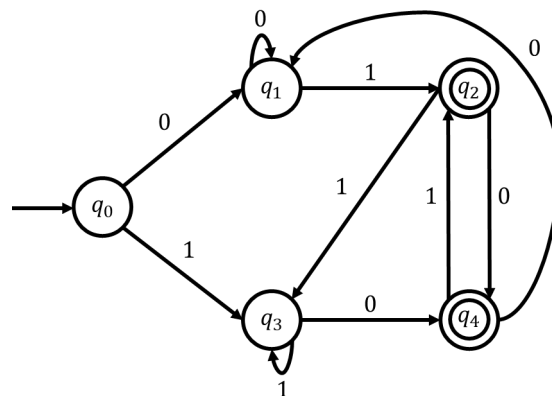


6. (5 points) Describe the language $L$ based on the following grammar $G = (V, \Sigma.R, S)$, where $V = \{S\}$, $\Sigma = \{a\}$, $R$ (which is provided below), starting variable $S$. Based on your answer, please include a description of what led you to the conclusion in order to receive potential full credit on this question. Any key factors missing could result in not receiving full credit.

$S \to aaaaaS \mid aa$