Procedure GNFA to REX

Let's first start with the actual outline of the procedure[1]. We are already provided as an input a finite automata called G. We will assume it is already a GNFA.

Procedure for Converting a GNFA to a Regular Expression

1. Let k be the number of states of G.

2. If $k = 2$, then G must consist of a start sate, an accept state, and a single arrow connecting them and labeled with a regular expression R. Return the expression R.

3. If $k > 2$, we select any state $q_{rip} \in Q$ different from q_{start} and q_{accept} and let G' be a GNFA $(Q', \Sigma', \delta', q_{start}, q_{accept})$, where

$$Q' = Q - \{q_{rip}\},$$

and for any $q_i \in Q' - \{q_{accept}\}$ and any $q_j \in Q' - \{q_{start}\}$, let

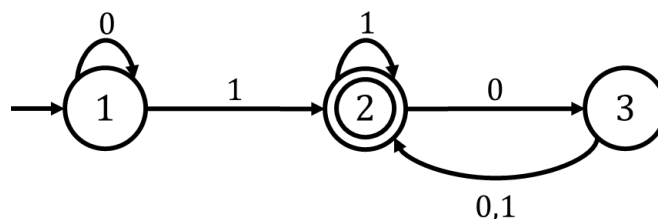$$\delta'(q_i, q_j) = (R_1)(R_2)^*(R_3) \cup (R_4)$$

for $R_1 = \delta(q_i, q_{rip})$, $R_2 = \delta(q_{rip}, q_{rip})$, $R_3 = \delta(q_{rip}, q_j)$, $R_4 = \delta(q_i, q_j)$

4. Repeat step 3 until the number of states in the GNFA is 2.

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[1]Procedure comes from the book *Introduction to the Theory of Computation* by Michael Sipser Third Edition.

Example

Convert the DFA into a Regular Expression.

1. We need to convert the DFA into a GNFA. In the example we have 1 start state and 1 final state. We will name the start state S and the final state A. Below is the resulting GNFA.
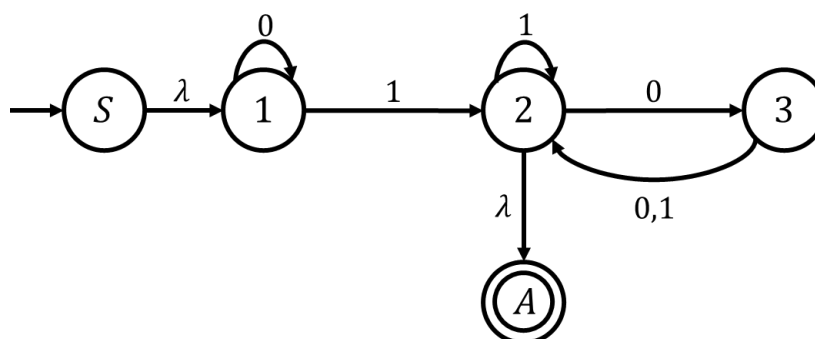
2. Now we need to select a state to rip from the GNFA. We can choose any state as long as it is not S or A. We will rip state 2.

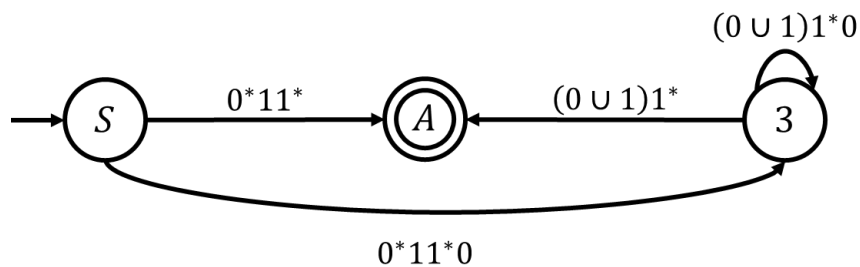| a	r	b	R_{ar}	R_r	R_{rb}	R_{ab}	$R_{ar}R_r^*R_{rb} \cup R_{ab}$
1	2	A	1	1	λ	$\varnothing$	$11^* \cup \varnothing = 11^*$
1	2	3	1	1	0	$\varnothing$	$11^*0 \cup \varnothing = 11^*0$
3	2	A	$0 \cup 1$	1	λ	$\varnothing$	$(0 \cup 1)1^*\lambda \cup \varnothing = (0 \cup 1)1^*$
3	2	3	$0 \cup 1$	1	0	$\varnothing$	$(0 \cup 1)1^*0 \cup \varnothing = (0 \cup 1)1^*0$

Now we will update the GNFA since we were able to repair it successfully. The following figure shows the resulting GNFA.
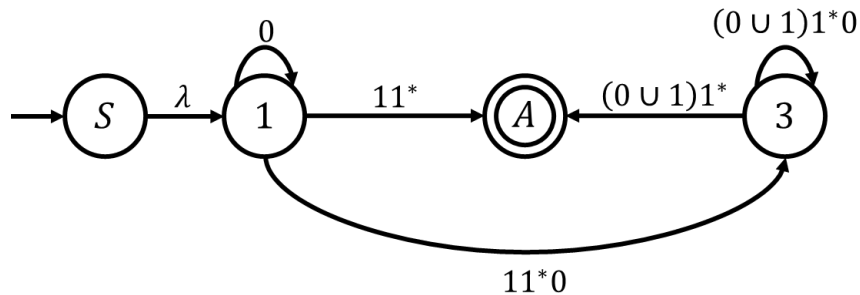
3. Now we can choose another state to rip. We have two options. We can rip state 1 or state 3. We will rip state 1.

| a	r	b	R_{ar}	R_r	R_{rb}	R_{ab}	$R_{ar}R_r^*R_{rb} \cup R_{ab}$
S	1	A	λ	0	11^*	$\varnothing$	$\lambda 0^*11^* \cup \varnothing = 0^*11^*$
S	1	3	λ	0	11^*0	$\varnothing$	$\lambda 0^*11^*0 \cup \varnothing = 0^*11^*0$

Now we will update the GNFA since we were able to repair it successfully. The following figure shows the resulting GNFA.

4. Now we can choose another state to rip. We have only one option. We will rip state 3.

a	r	b	R_{ar}	R_r	R_{rb}	R_{ab}	$R_{ar}R_r^*R_{rb} \cup R_{ab}$
S	3	A	0*11*0	$(0 \cup 1)1^*0$	$(0 \cup 1)1^*$	0*11*	$(0^*11^*0((0 \cup 1)1^*0)^*(0 \cup 1)1^*) \cup 0^*11^*$

Now we will update the GNFA since we were able to repair it successfully. The following figure shows the resulting final GNFA.
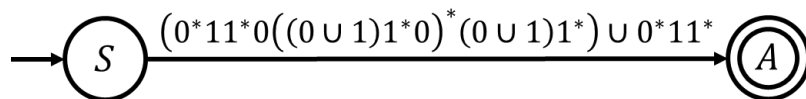
5. The regular expression is

$$r = (0^*11^*0((0 \cup 1)1^*0)^*(0 \cup 1)1^*) \cup 0^*11^*$$