Monte-Carlo simulation of Heston volatility model using Euler scheme

This code replicates the paper of Kim & Wee (2014). Heston model's asset pricing dynamics are given by

$$dS_t = S_t(rdt + \sqrt{v_t}dW_1)$$

$$dv_t = \kappa\left(\theta - v_t\right)dt + \sigma\sqrt{(v_t)}dW_2$$

$$dW_1 \cdot dW_2 = \rho dt$$

$$dW_2 = \rho \cdot dW_1 + \sqrt{1-\rho^2} \cdot \widetilde{dW_1}$$

where,

S_t: Stock Price at time t
r: Risk free interest rate
v_t: Volatility at time t
σ: Volatility of the volatility
ρ: Correlation between the two Brownian motions
κ: Mean reversion rate
θ: Long run average of v_t
$\widetilde{dW_1}$: Brownian motion independent of dW_1

To apply the Euler discretization method, we first divide the time interval [0,T] into N equally spaced subintervals of length $\delta t = \frac{T}{N}$. We then use the following updated equations:

$$dS_{(t+\delta t)} = S_t + S_t(rdt + \sqrt{v_t^+} \cdot \sqrt{\delta t}Z_1)$$

$$dv_{(t+\delta t)} = v_t + \kappa\left(\theta - v_t\right)dt + \sigma\sqrt{v_t^+} \cdot \sqrt{\delta t} \cdot Z_2$$

$$v_t^+ = \max\left(v_t, 0\right)$$

Where, Z_1, Z_2 are standard normal variables and

$$Z_2 = \rho \cdot Z_1 + \sqrt{1-\rho^2} \cdot \widetilde{Z_2}$$

Where, $\widetilde{Z_2}$ is a random draw from standard normal distribution independent of Z_1

References

Kim, B. & Wee, I.-S. (2014), 'Pricing of geometric Asian options under Heston's stochastic volatility model', Quantitative Finance **14**(10), 1795–1809.
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