

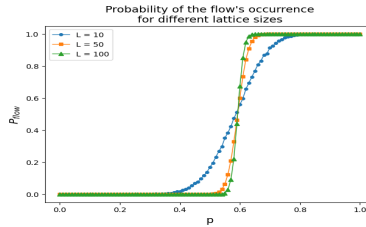
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1 Percolation

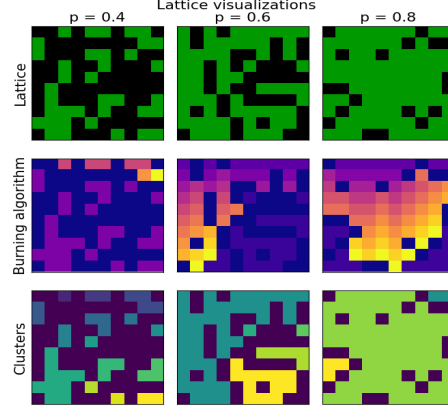
Percolation refers to the movement and filtering of fluids through porous materials. Let us assume that we create a square lattice with side length L (e.g. $L = 10$) such that every cell can either be occupied or empty. The initial configuration is that all fields are empty. We fill each cell with probability p , that is, for each cell random number $z \in [0, 1]$ and compare it to p . If $z \leq p$, the cell will be marked as occupied otherwise the cell remains empty. This can be done for different values of p , leading to different results. It is implemented using python language.

First we are going to implement the burning method, in which we percolate the fire from top to bottom. Every iteration, the fire from a burning tree lights up occupied neighboring cells. The algorithm ends if all (neighboring) blocks are burnt or if the burning method has reached the bottom line. The main part was finding and labelling the clusters present in the lattice. It was achieved with a help of Hoshen–Kopelman algorithm as defined in task. Wrapping probability as a function of the occupation probability for different lattice sizes and a step function for comparison given below

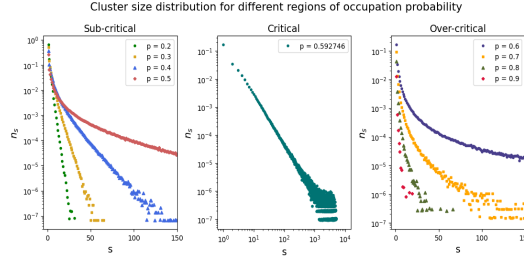


The percolation threshold is a characteristic value for a given type of lattice. One finds different percolation thresholds for different lattices, in this one observes that is nearly $p_c = 0.592746$. The Lattice graphs with different method are

given below:



The algorithm is very efficient since it scales linearly with the number of sites. Once we have run the algorithm for a given lattice we can evaluate the results. We find different behaviors of the relative cluster sizes depending on the occupation probability p . These results are illustrated in Fig below, where the first graph represents the behavior for subcritical occupation probabilities ($p < p_c$), the second graph shows the behavior for the critical occupation probability ($p = p_c$) and the third graph depicts the behavior for overcritical occupation probabilities ($p > p_c$).



One finds that in the subcritical regime ($p < p_c$), $n_p(s)$ obeys a power law multiplied with exponential function, whereas in the overcritical region ($p > p_c$), we observe a distribution that resembles an exponential decay but with an argument that is stretched with a power

$$s^{11/d}$$

Also We can summarize the behavior of the cluster size distribution $n_p(s)$ in the three different regions:

$$n_p(s) : se^{as} \Rightarrow p < p_c$$

$$s^t \Rightarrow p = p_c,$$

$$e^{bs^{11/d}} \Rightarrow p > p_c$$