Assignment: 2

Asymptotic Notation

1) $f(n) = n-10$
$g(n) = n+10$

$f(n) = \Theta(g(n))$? Theta?

$n-10 = \Theta(n+10)$?

i) $f(n) <= C_2 g(n)$ $\quad C_2 > 0$

$n-10 <= C_2 (n+10)$

$\downarrow$

1

$n-10 = O(n+10) \Rightarrow$ Big O

ii) $f(n) >= C_1 g(n)$ $\quad C_1 > 0$

$n-10 >= C_1 (n+10)$

$\downarrow$

$1/2$

$n-10 = \Omega(n+10)$

consider large value of n.

True, we can write $n-10 = \Theta(n+10)$. Valid Theta.

2) $f(n) = n$
$g(n) = n$
$f(n) = \Theta(g(n))$? Theta?

i) $n = O(n)$ - ?

$n \leq C_2 n$

$\downarrow$

1

possible

ii) $n = \Omega(n)$ - ?

$n \geq C_1 n$

$\downarrow$

$1/2$

possible

So, we can write

$f(n) = \Theta g(n)$

$n = \Theta(n)$

True.

Valid Theta

3) $f(n) = 64^{\log_2 n} \cdot 32^{\log_2 n}$
$g(n) = n^5$
$f(n) = O(g(n))$ Big O ?

condition for Big O is

$f(n) \leq C\, g(n)$ $\forall n \quad C > 0, n \geq n_0, n_0 \geq 1$

$64^{\log_2 n} \cdot 32^{\log_2 n} \leq C\, n^5$

$[a^{\log_a p} = p^{\log_a a}]$ log property.

$n^{\log_2 64} \cdot n^{\log_2 32} \leq C\, n^5$

$n^6 \cdot n^5 \leq C n^5$

$n^{11} \leq C n^5$

C should be constant.

Not a valid Big O.

4.) $\frac{4^n}{2^n} = O(2^n)$ Check for Valid BigO.

$\frac{4^n}{2^n} \leq c\, 2^n$

$\frac{2^n \cdot 2^n}{2^n} \leq c\, 2^n$

$2^n \leq c\, 2^n$

possible. Valid BigO.

5) $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

$[\log_b a^p = p^{\log_a b}]$

$n^{\log_2 128} \cdot n^2 \leq c\, n^9$

$n^7 \cdot n^2 \leq c\, n^9$

$n^9 \leq c\, n^9$

possible c constant value.

Valid BigO.

Checking for Omega Validy.

$n^9 \geq c\, n^9$

$\downarrow c = 1$

Valid ~~Theta~~ Ω also.

So, Valid Theta.

Tree.