Assignment : 3

Calculate time complexity :- ~~Recursive~~ Recurrence Relation

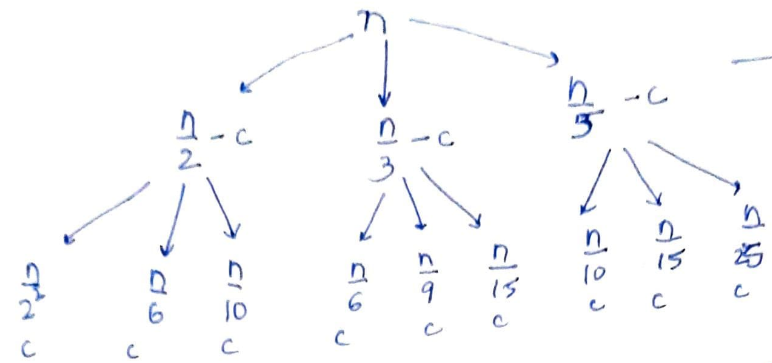
$$T(n) = T(n/2) + T(n/3) + T(n/5) + c$$

Different Cost

$\boxed{c \to 3^0 c}$

$\boxed{\text{Level 1} \quad 3c = 3^1 c}$

$\boxed{\text{Level 2} \quad 9c = 3^2 c}$



we get an Idea that we have a GP series for cost.

$$3^0c + 3^1c + 3^2c + \dots + 3^K c = \text{Overall } \cancel{\text{time Complexity}} \text{ Cost}$$

$K \to$ Order of Magnitude (No. of times recursion)

Lets talk about base condition : stopping critria.

$$\frac{n}{2^{K_L}} = 1 \qquad \frac{n}{3^{K_M}} = 1 \qquad \frac{n}{5^{K_R}} = 1$$

$$n = 2^{K_L} \qquad n = 3^{K_M} \qquad n = 5^{K_R}$$

$$\Rightarrow K_L = \log_2 n \qquad \Rightarrow K_M = \log_3 n \qquad \Rightarrow K_R = \log_5 n$$

Lower the BASE Higher the Value.

Overall time Complexity $= O(\text{cost} \ \cancel{\times K_L})$

$$= O(3^0c + 3^1c + 3^2c + \dots + 3^{K_L} c \ \cancel{\times K_L})$$

GP series: $\text{sum} = \dfrac{a(r^n - 1)}{r-1} \Rightarrow r > 1$

$$= \frac{1(3^{\log_2 n} - 1)}{3-1} = \frac{(n^{\log_2 3} - 1)}{3-1}$$

$$\cong (n^{1.58})$$

Value $\log_2 3 = 1.58$

Time Complexity $= O(n^{1.58})$ Ans.

(2) Find the time complexity of recurrence relation by Substitution Method.

$$T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right)+n & n>1 \end{cases}$$

stopping criteria $T(1) = 1$

$T(n) = 2T\left(\frac{n}{2}\right) + n \longrightarrow$ level 1

$= 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2n \longrightarrow$ level 2

$= 2^2\left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n$

$= 2^3 T\left(\frac{n}{2^3}\right) + 3n \longrightarrow$ level 3

$\downarrow$ k times

$= 2^k T\left(\frac{n}{2^k}\right) + kn \longrightarrow$ kth level. Order of Magnitude.

base condition $\longrightarrow T(1) = 1$

$T\left(\frac{n}{2^k}\right)\Big|_{\frac{n}{2^k}=1} = 1 \Rightarrow \frac{n}{2^k} = 1$

$\boxed{k = \log_2 n}$

$T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + n\log_2 n$

$= n^{\cancel{\log_2 2}^{1}} T\left(\frac{n}{n^{\cancel{\log_2 2}^{1}}}\right) + n\log_2 n$

$= n\,T(1) + n\log_2 n$

$= n + n\log_2 n$

$O(n) < O(n\log_2 n)$

Time Complexity $= O(n\log_2 n)$

(3) Calculate time complexity of recursive relation by substitution method.

$$T(n) = \begin{cases} 1 & n \geq 1 \\ 8T\left(\frac{n}{2}\right) + n^2 & n > 1 \end{cases}$$

stopping criteria $\longrightarrow T(1) = 1$

$T(n) = 8T\left(\frac{n}{2}\right) + n^2 \longrightarrow$ level 1

$= 8\left[8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$

$= 8^2 T\left(\frac{n}{2^2}\right) + 3n^2 \longrightarrow$ level 2

$= 8^2\left[8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 3n^2$ $\qquad 8^2 = (2^3)^2 = 2^6$

$= 8^3 T\left(\frac{n}{2^3}\right) + 8^2 \frac{n^2}{2^4} + 3n^2$ $\qquad \frac{2^6 n^2}{2^4} = 4n^2$

$= 8^3 T\left(\frac{n}{2^3}\right) + 2^2 n^2 + 3n^2$

$= 8^3 T\left(\frac{n}{2^3}\right) + 7n^2 \longrightarrow$ level 3

$\downarrow$ k times.

$1, 3, 7, \ldots k$

$(2^k - 1)_{k=0,1,2\ldots}$

$T(n) = 8^k T\left(\frac{n}{2^k}\right) + (2^k - 1)n^2 \longrightarrow k^{th}$ level.

stopping criteria $T(1) = 1$

base condition $\Rightarrow T\left(\frac{n}{2^k}\right)\Big|_{n/2^k = 1} = 1 \Rightarrow \frac{n}{2^k} = 1 \Rightarrow \boxed{k = \log_2 n}$

substitute value of k in $T(n)$

$T(n) = 8^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + (2^{\log_2 n} - 1)n^2$

$= n^{\log_2 8} T\left(\frac{n}{n^{\log_2 2}}\right) + (n^{\log_2 2} - 1)n^2$

$= n^3 T\left(\frac{n}{n}\right) + (n-1)n^2$

$= n^3 + n^3 + n^2$

$O(n^2) < O(n^3)$ $\qquad$ Time Complexity $= O(n^3)$