Time Complexity — Apriori Analysis

find the Order of Magnitude of the following Statements :-

i) $i = n$

    while $i > 2$:

        $i = i^{1/25}$

        print (i)

The stopping criteria will be

$$n^{1/25\,k} = 2$$

$\Rightarrow$ condition stops when $n^{1/25 k}$ will be 2.

$i = n$

$i^{1/25}$

$n^{1/25}$

until k times $n^{1/25}$ repeated.

$\vdots$

$n^{1/25 K}$

In order to find Order of Magnitude i.e. K in above stopping criteria we will take log both side. Specifically we want to make RHS = 1 so we will take log base 2. in order make computation simple.

$$\log_2 n^{1/25 k} = \log_2 2$$

    $[\log_a p^n = n \log_a p] \;\&\; [\log_a a = 1]$

$$\frac{1}{25k} \log_2 n = 1$$

$$\log_2 n = 25^k$$

$$\log_{25}(\log_2 n) = k \log_{25} 25^{1}$$

[remember we want to find k] so again log both side but this time log base 25 in order make RHS = 1 and computation simple.

$$\Rightarrow k = \log_{25}(\log_2 n)$$

$$k_{approx} = \log(\log n)$$

$\Rightarrow$ Time Complexity is $O(\log(\log n))$

(ii) find Order of Magnitude of :

```
i = 29
while i<n:
    i = i^23
```

$$i = 29$$
$$i = i^{23}$$
$$(29)^{23} \quad k \text{ repeat } k \text{ times.}$$

Stopping Criteria is when $(29)^{23^k}$ will be equal to n. (while i<n)

$$(29)^{23^k} = n \qquad [\log_a p^n = n \log_a p] \ \& \ [\log_a a = 1]$$

$$\log_{29}(29)^{23^k} = \log_{29} n$$

$$23^k \ \log_{29} 29^{1} = \log_{29} n$$

$$23^k = \log_{29} n \qquad [\text{remember we are trying to find}$$
$$\text{Order of magnitude i.e. how many}$$
$$k \ \log_{23} 23^{1} = \log_{23}[\log_{29} n] \quad \text{time statement is running i.e. } k]$$
$$\qquad\qquad\qquad\qquad\qquad \text{again take log. both side.}$$
$$k = \log_{23}[\log_{29} n]$$

$$k_{approx} = \log[\log n]$$

Time Complexity $= O[\text{Order of Magnitude}] \Rightarrow O[\log[\log n]]$

iii) find Order of Magnitude of

```
i = 1
while i<n:
    i = 2*i
    i = 3*i
```

$$i = 1$$
$$2*1$$
$$3*1$$
$$\vdots \quad k \text{ times. repeat}$$
$$2^k$$
$$3^k$$

Stopping Criteria :-

$$2^k 3^k = n$$
$$6^k = n \qquad \text{remember we are trying to calculate value of } k.$$
$$k \ \log_6 6^{1} = \log_6 n$$
$$k = \log_6 n$$
$$k_{approx} = \log n$$

Time Complexity $\Rightarrow O[\log n]$