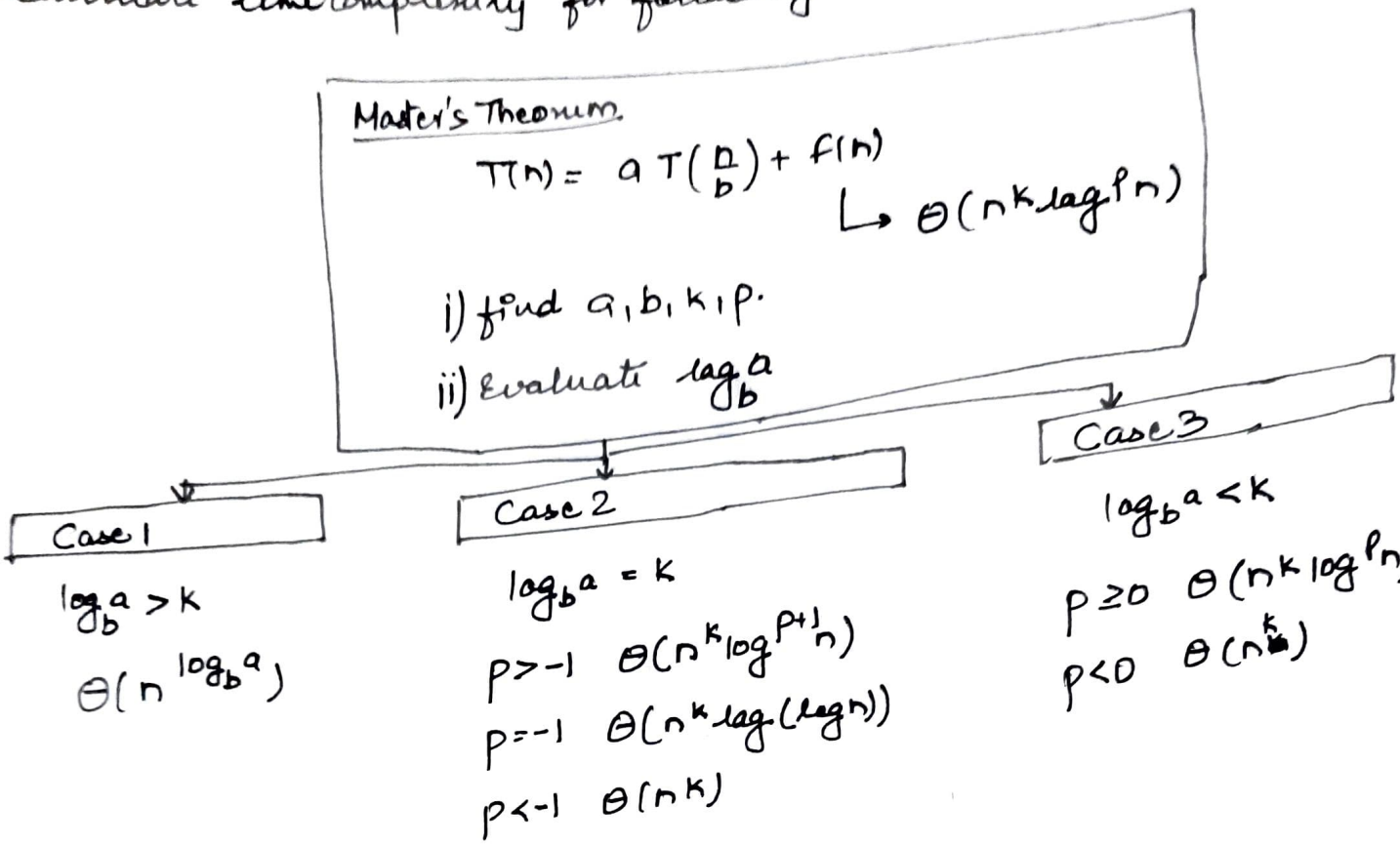
Assignment : 4

Master's Theorem

Calculate time complexity for following recurrence relation :-



Q1. $T(n) = 2T(\frac{n}{2}) + n$

Compair above with $T(n) = a\,T(\frac{n}{b}) + f(n)$ $\hookrightarrow \Theta(n^k \log^p n)$

$a = 2,\ b = 2,\ k = 1,\ p = 0$

Evaluate $\log_b^a$, put values of a, b.

$\Rightarrow \log_2^2 = 1$

Compair $\log_b^a$ with k. $\Rightarrow \log_b^a = k$ case 2 will apply.

check the value of p for case 2. p is 0 which ~~is~~ greater than (-1)

for $p > (-1)$,

Time Complexity $= \Theta(n^k \log^{p+1} n)$

$= \Theta(n^1 \log^{0+1} n)$

$= \Theta(n \log n)$ ANS.

Q2. $2T(\frac{n}{2}) + n\log n = T(n)$

compair above equation with $T(n) = aT(\frac{n}{b}) + f(n)$ ↳ $n^k \log^p n$

$\Rightarrow a = 2, b = 2, k = 1, p = 1$

evaluati value of $\log_b a \Rightarrow \log_2 2 = 1$

compair $\log_b a$ with value of $k \Rightarrow \log_b a = k$ case 2 applied.

check value of $p \rightarrow p > -1$

Time complexity $= \theta(n^k \log^{p+1} n)$

$= \theta(n^1 \log^{1+1} n) \Rightarrow \theta(n \log^2 n)$ ANS.

Q3. $T(n) = 2T(\frac{n}{2}) + n^2$

compair above equation with $T(n) = aT(\frac{n}{b}) + n^k \log^p n$

$\Rightarrow a = 2, b = 2, k = 2, p = 0$

evaluati value of $\log_b a \Rightarrow \log_2 2 = 1$

compair $\log_b a$ with value of $k \Rightarrow \log_b a < k$ case 3 applied

check value of $p \Rightarrow p = 0 \Rightarrow p \geq 0$

$\Rightarrow$ Time complexity $= \theta(n^k \log^p n)$

$= \theta(n^2 \log^0 n)$

$= \theta(n^2)$

Q4 $T(n) = 8T(\frac{n}{2}) + n^2$

compair above equation with $T(n) = aT(\frac{n}{b}) + n^k \log^p n$

$\Rightarrow a = 8, b = 2, k = 2, p = 0$

evaluati value of $\log_b a \Rightarrow \log_2 8 = 3$ which $> k$ case 1 applied

(priority given to $\log_b a$)

Time Complexity $= \theta(n^{\log_b a})$

$= \theta(n^3)$