Q-3) Using bilinear transformation design a high pass filter, monotonic in pass band with cut OFF freqⁿ of 1000 Hz and down 10 dB at 350 Hz. The sampling freqⁿ is 5000 Hz.

Ans → Given:-

passband attenuation $\alpha_p = 3$ dB

stop band attenuation $\alpha_s = 10$ dB.

pass band frequency $\omega_p = 2\pi * 1000$

$= 2000\pi$ rad/sec

stop band frequency $\omega_s = 2\pi * 350$

$= 700\pi$ rad/sec.

$$T = \frac{1}{f} = \frac{1}{5000} = 2 * 10^{-4} \text{ sec.}$$

prewarping the digital frequencies, we have

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{2}{2 * 10^{-4}} \tan \frac{(2000\pi * 2 * 10^{-4})}{2}$$

$$= 10^{+4} \tan(0.2\pi)$$

$$= 7265 \text{ rad/second.}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2} = \frac{2}{2 * 10^{-4}} \tan \frac{(700\pi * 2 * 10^{-4})}{2}$$

$$= 10^{4} \tan(0.07\pi)$$

$$= 2235 \text{ rad/second.}$$

The order of the filter

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log \sqrt{\frac{10^{0.1\times10}-1}{10^{0.1\times3}-1}}}{\log \frac{7265}{2235}} = \frac{\log(3)}{\log(3.25)}$$

$$= \frac{0.4771}{0.5118} = 0.932$$

$N = 1$

The first order butterworth filter for $\Omega c = 1$ rad/sec is $H(s) = \frac{1}{(s+1)}$

The high pass filter for $\Omega c = \Omega p = 7265$ rad/sec can be obtained by using the transformation

$$s \rightarrow \frac{\Omega c}{s}$$

$$s \rightarrow \frac{7265}{s}$$

The transfer function of high pass filter

$$H(s) = \left. \frac{1}{s+1} \right|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation -

$$H(z) = H(s) \Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{s}{s+7265}$$

$$s = \frac{2}{2\times10^{-4}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$H(z) = \frac{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+7265}$$

$$\boxed{H(z) = \frac{0.5792\,(1-z^{-1})}{1-0.1584\,z^{-1}}}$$

$$\frac{y(z)}{x(z)} = \frac{0.5792(1-z^{-1})}{1-0.1584\,z^{-1}}$$

$$y(z) = \frac{0.5792(1-z^{-1})}{1-0.1584\,z^{-1}}\,x(z)$$