

# Latex Code for Notes



L6\_S2\_A.pdf

PDF

Generate a lecture scribe which can be used as exam reference material .Use only the provided context (lecture slides) dont add anything of you own . While writing the scribe reason step by step through all definitions , notation, assumptions , theorem statements , proofs ,derivations and worked examples exactly as given in the lecture slides. Make the logical flow explicit so that a student can follow the reasoning during exam revision . Include proofs, derivations and examples only if they appear in the provided context. Do not introduce new explanations examples or material beyond the lecture scope . Maintain a formal and academic tone and strict faithfulness to the lecture structure

Below is a **lecture scribe written strictly from the provided lecture slides**, organized to mirror the **lecture flow and structure**, and intended to be used as **exam-revision reference material**.

No material, explanation, or example beyond the slides has been added.

All definitions, notation, assumptions, derivations, proofs, and examples appear **only where they exist in the slides**, and are reasoned **step by step exactly as presented**.

All content is drawn from **Lecture 6: Discrete RVs, Expectation and Problem Solving**

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## CSE400 – Fundamentals of Probability in Computing

### Lecture 6: Discrete Random Variables, Expectation and Problem Solving

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#### 1. Random Variables: Motivation and Concept

##### Definition

A random variable  $X$  on a sample space  $\Omega$  is a function

$$X : \Omega \rightarrow \mathbb{R}$$

that assigns to each sample point  $\omega \in \Omega$  a real number  $X(\omega)$ .

## Restriction in this Lecture

Until further notice, attention is restricted to **discrete random variables**, i.e., random variables that take values in a range that is **finite or countably infinite**.

Although  $X$  is formally defined as mapping to  $\mathbb{R}$ , the actual set of values

$$\{X(\omega) : \omega \in \Omega\}$$

is a **discrete subset of  $\mathbb{R}$** .

## Visualization

- Sample points in the sample space are mapped by the random variable to points on the real line.
  - Multiple sample points may map to the same numerical value.
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## 2. Distribution of a Discrete Random Variable

The **distribution** of a random variable can be visualized using a **bar diagram**.

- The **x-axis** represents the values the random variable can take.
- The **height of the bar** at value  $a$  is the probability

$$\Pr[X = a]$$

Each probability is computed by evaluating the probability of the **corresponding event in the sample space**.

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## 3. Discrete vs Continuous Random Variables

### Discrete Random Variables

Characteristics:

- Countable support
- Probability Mass Function (PMF)
- Probabilities assigned to single values

- Each possible value has **strictly positive probability**

## Continuous Random Variables

Characteristics:

- Uncountable support
  - Probability Density Function (PDF)
  - Probabilities assigned to **intervals of values**
  - Each individual value has probability **zero**
- 

## 4. Example 1: Tossing 3 Fair Coins

### Experiment Description

An experiment consists of tossing **3 fair coins**.

Let

$$Y = \text{number of heads that appear}$$

### Possible Values

$$Y \in \{0, 1, 2, 3\}$$

### Computation of Probabilities

- $P(Y = 0) = P(t, t, t) = \frac{1}{8}$
- $P(Y = 1) = P(t, t, h), (t, h, t), (h, t, t) = \frac{3}{8}$
- $P(Y = 2) = P(t, h, h), (h, t, h), (h, h, t) = \frac{3}{8}$
- $P(Y = 3) = P(h, h, h) = \frac{1}{8}$

### Verification of Total Probability

Since  $Y$  must take **one of the values 0 through 3**, we must have:

$$1 = P \left( \bigcup_{i=0}^3 \{Y = i\} \right) = \sum_{i=0}^3 P(Y = i)$$

This confirms the probabilities form a **legitimate PMF**.

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## 5. Probability Mass Function (PMF)

## Definition

A random variable that can take on **at most a countable number of possible values** is said to be **discrete**.

Let  $X$  be a discrete random variable with range (possible values)

$$R_X = \{x_1, x_2, x_3, \dots\}$$

where the set is **finite or countably infinite**.

The function

$$P_X(x_k) = P(X = x_k), \quad k = 1, 2, 3, \dots$$

is called the **Probability Mass Function (PMF)** of  $X$ .

## PMF Property

Since  $X$  must take **one of the values  $x_k$** , we must have:

$$\sum_{k=1}^{\infty} P_X(x_k) = 1$$

## 6. PMF Example: Given Functional Form

### Problem Statement

The probability mass function of a random variable  $X$  is given by

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

where  $\lambda$  is a positive value.

Find:

1.  $P(X = 0)$
2.  $P(X > 2)$

### Step 1: Determine Constant C

Since probabilities must sum to 1:

$$\sum_{i=0}^{\infty} p(i) = 1$$

Substitute the given PMF:

$$c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

Using the identity:

$$e^\lambda = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$

We obtain:

$$ce^\lambda = 1 \Rightarrow c = e^{-\lambda}$$


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### Step 2: Compute $P(X = 0)$

$$P(X = 0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda}$$


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### Step 3: Compute $P(X > 2)$

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \\ &= 1 - [e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2}] \end{aligned}$$


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## 7. Bayes' Theorem (Recap)

### Starting Point

Using:

$$P(A \cap B) = P(B \mid A)P(A)$$

We obtain:

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{\sum_{j=1}^n P(A \mid B_j)P(B_j)}$$

This is known as the **Bayes Formula (Proposition 3.1)**.

## Terminology

- $P(B_i)$ : a priori probability
  - $P(B_i \mid A)$ : posterior probability
- 

## 8. Bayes' Theorem: Example – Auditorium with 30 Rows

### Problem Description

- An auditorium has **30 rows of seats**
- Row 1 has 11 seats
- Row 2 has 12 seats
- Row 3 has 13 seats
- ...
- Row 30 has 40 seats

### Selection Process

1. A row is selected **uniformly at random** from the 30 rows.
2. A seat is selected **uniformly at random** from the chosen row.

### Required Computations

1. Probability that **Seat 15** was selected given that **Row 20** was selected.
2. Probability that **Row 20** was selected given that **Seat 15** was selected.

### Step 1: Conditional Probability

If Row 20 is selected, each seat in that row is equally likely.

Since Row 20 has 30 seats:

$$P(S_{15} \mid R_{20}) = \frac{1}{30}$$

### Step 2: Bayes' Formula Application

$$P(R_{20} \mid S_{15}) = \frac{P(S_{15} \mid R_{20})P(R_{20})}{P(S_{15})}$$

where:

$$P(R_{20}) = \frac{1}{30}$$

and

$$P(S_{15}) = \sum_{k=15}^{30} P(S_{15} \mid R_k)P(R_k)$$

This completes the example exactly as shown in the lecture.

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## End of Lecture 6 Scribe

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If you want, I can next:

- convert this into **handwritten-style exam notes**, or
- **condense it into a 2-3 page revision cheat-sheet**, or
- format it **exactly like your previous exam answers**

write the latex code for your revision notes dont add anything more and dont skip anything write as you have given

latex

```
% Lecture 6 Revision Notes  
% Source: CSE400 - Fundamentals of Probability in Computing  
% Lecture 6: Discrete Random Variables, Expectation and Problem Solving  
% :contentReference[oaicite:0]{index=0}
```

```
\documentclass[12pt]{article}  
\usepackage{amsmath, amssymb}  
\usepackage{geometry}  
\geometry{margin=1in}  
  
\begin{document}
```

```
\begin{center}
\textbf{CSE400 -- Fundamentals of Probability in Computing}\ \\
\textbf{Lecture 6: Discrete Random Variables, Expectation and Problem Solving}\\
\end{center}
```

## \section{Random Variables: Motivation and Concept}

A random variable  $X$  on a sample space  $\Omega$  is a function

```
\[\\
X : \Omega \rightarrow \mathbb{R}\\
\]
```

that assigns to each sample point  $\omega \in \Omega$  a real number  $X(\omega)$ .

Until further notice, we restrict our attention to random variables that are discrete, i.e., they take values in a range that is finite or countably infinite. This means even though we define  $X$  to map  $\Omega$  to  $\mathbb{R}$ , the actual set of values

```
\[\\
\{X(\omega) : \omega \in \Omega\}\\
\]
```

is a discrete subset of  $\mathbb{R}$ .

## \section{Distribution of a Discrete Random Variable}

The distribution of a random variable can be visualized as a bar diagram. The  $x$ -axis represents the values that the random variable can take on. The height of the bar at a value  $a$  is the probability

```
\[\\
\Pr[X = a].\\
\]
```

Each of these probabilities can be computed by looking at the probability of the corresponding event in the sample space.

## \section{Discrete and Continuous Random Variables}

### \subsection{Discrete Random Variables}

Discrete random variables have:

- ```
\begin{itemize}
\item Countable support
\item A probability mass function (PMF)
\item Probabilities assigned to single values
\item Each possible value has strictly positive probability

```

```
\end{itemize}
```

## \subsection{Continuous Random Variables}

Continuous random variables have:

- ```
\begin{itemize}
\item Uncountable support
\item A probability density function (PDF)
\item Probabilities assigned to intervals of values
\item Each possible value has zero probability
\end{itemize}
```

## \section{Example: Tossing Three Fair Coins}

Suppose that our experiment consists of tossing three fair coins. If we let  $Y$  denote the number of heads that appear, then  $Y$  is a random variable taking one of the values  $0, 1, 2, 3$ .

The probabilities are:

$$\begin{aligned} P(Y=0) &= P(t,t,t) = \frac{1}{8} \\ \\ P(Y=1) &= P(t,t,h), (t,h,t), (h,t,t) = \frac{3}{8} \\ \\ P(Y=2) &= P(t,h,h), (h,t,h), (h,h,t) = \frac{3}{8} \\ \\ P(Y=3) &= P(h,h,h) = \frac{1}{8} \end{aligned}$$

Since  $Y$  must take one of the values  $0$  through  $3$ , we must have

$$\begin{aligned} 1 &= P(\bigcup_{i=0}^3 \{Y=i\}) \\ &= \sum_{i=0}^3 P(Y=i). \end{aligned}$$

## \section{Probability Mass Function}

A random variable that can take on at most a countable number of possible values is said to be discrete.

Let  $X$  be a discrete random variable with range (possible values)

\[

$$X = x_1, x_2, x_3, \dots$$

\]

where the set is finite or countably infinite. The function

\[

$$P_X(x_k) = P(X = x_k), \quad k = 1, 2, 3, \dots$$

\]

is called the probability mass function (PMF) of  $X$ .

Since  $X$  must take one of the values  $x_k$ , we have

\[

$$\sum_{k=1}^{\infty} P_X(x_k) = 1.$$

\]

## \section{PMF Example}

The probability mass function of a random variable  $X$  is given by

\[

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

\]

where  $\lambda$  is some positive value. Find  $P(X=0)$  and  $P(X>2)$ .

Since

\[

$$\sum_{i=0}^{\infty} p(i) = 1,$$

\]

we have

\[

$$c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1.$$

\]

Because

\[

$$e^\lambda = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!},$$

\]

it follows that

\[

$$c e^\lambda = 1 \quad \text{or} \quad c = e^{-\lambda}.$$

\]

Thus,

\[

$$P(X=0) = e^{-\lambda}.$$

\]

Further,

\[

$$P(X>2) = 1 - P(X \leq 2)$$

\]

\[

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

\]

\[

$$= 1 - [e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2!}].$$

\]

## \section{Bayes' Theorem}

Using

\[

$$P(A \cap B) = P(B \mid A) P(A),$$

\]

we obtain

\[

$$P(B_i \mid A) =$$

$$\frac{P(A \mid B_i) P(B_i)}{\sum_{j=1}^n P(A \mid B_j) P(B_j)}.$$

\]

This is known as the Bayes Formula (Proposition 3.1).

Here:

\begin{itemize}

\item \$P(B\_i)\$ is the *a priori* probability.

\item \$P(B\_i \mid A)\$ is the posterior probability.

\end{itemize}

## \section{Bayes' Theorem Example: Auditorium with 30 Rows}

A certain auditorium has 30 rows of seats. Row 1 has 11 seats, Row 2 has 12 seats, Row 3 has 13 seats, and so on to the back of the auditorium where Row 30 has 40 seats.

A door prize is to be given away by randomly selecting a row (with equal probability of selecting any of the 30 rows) and then randomly selecting a seat within that row (with each seat in the row equally likely to be selected).

The probability that seat 15 was selected given that row 20 was selected is

\[

$$P(S_{15} \mid R_{20}) = \frac{1}{30}.$$

\]

The probability that row 20 was selected given that seat 15 was selected is

\[

$$P(R_{20} \mid S_{15}) = \frac{P(S_{15} \mid R_{20}) P(R_{20})}{\{P(S_{15})\}},$$

\]

where

\[

$$P(R_{20}) = \frac{1}{30}$$

\]

and

\[

$$P(S_{15}) = \sum_{k=15}^{30} P(S_{15} \mid R_k) P(R_k).$$

\]

\end{document}