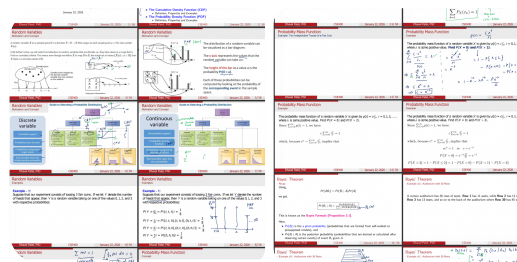


# L6\_Lecture Scribe



L6\_S2\_A (1).pdf  
PDF

Generate a lecture scribe suitable for exam preparation. While writing the scribe, explain the definitions and assumptions explicitly, give step-by-step reasoning for definitions, assumptions, proofs and the example step-by-step solutions from the provided context only. All the reasoning should be based solely on the provided context. Make the logical flow explicit so that a student can follow the reasoning during revision. Do not introduce material that is not present in the context.

Below is a **structured lecture scribe for exam preparation built only from the provided lecture content**.

Every definition, assumption, formula, and example is explained step-by-step with explicit logical flow, exactly as developed in the slides.

## CSE400 – Lecture 6 Scribe

**Discrete Random Variables, PMF, Bayes' Theorem & Standard Discrete Distributions**

### 1. Random Variables – Motivation and Definition

#### Step 1: Start from the sample space

Let:

- $\Omega$  = sample space (set of all possible outcomes of an experiment)
- A **random variable** is a function:

$$X : \Omega \rightarrow \mathbb{R}$$

This means:

👉 Each outcome  $\omega \in \Omega$  is mapped to a real number  $X(\omega)$

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## Step 2: Restriction to discrete random variables

In this lecture:

- We focus on **discrete random variables**

This means:

✓ The set of values taken by  $X$

$$\{X(\omega) : \omega \in \Omega\}$$

is:

- finite OR
- countably infinite

So even though  $X$  maps into real numbers, only **discrete points** occur.

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## Step 3: Distribution visualization (PMF bars)

For a discrete RV:

- x-axis  $\rightarrow$  possible values of the RV
- bar height at value  $a \rightarrow P(X = a)$

Each probability comes from the probability of the corresponding event in the sample space.

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## 2. Discrete vs Continuous (conceptual distinction from slides)

**Discrete RV:**

- Countable support
- Uses **Probability Mass Function (PMF)**
- Probabilities assigned to single values
- Each value has positive probability

## Continuous RV:

- Uncountable support
- Uses **Probability Density Function (PDF)**
- Probabilities assigned to intervals
- Each exact value has probability zero

(Only discrete RVs are used in this lecture's worked material.)

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## 3. Example: Tossing 3 Fair Coins

### Step 1: Define experiment

Toss 3 fair coins.

Each outcome equally likely.

Let:

$Y = \text{number of heads}$

Possible values:

$$Y \in \{0, 1, 2, 3\}$$

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### Step 2: List outcomes by value of Y

$Y = 0$

Only outcome:

$$(t, t, t)$$

So:

$$P(Y = 0) = \frac{1}{8}$$

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$Y = 1$

Outcomes:

$$(t, t, h), (t, h, t), (h, t, t)$$

Total = 3 outcomes

$$P(Y = 1) = \frac{3}{8}$$


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$Y = 2$

Outcomes:

$$(t, h, h), (h, t, h), (h, h, t)$$

Total = 3 outcomes

$$P(Y = 2) = \frac{3}{8}$$


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$Y = 3$

Outcome:

$$(h, h, h)$$

$$P(Y = 3) = \frac{1}{8}$$


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**Step 3: Probability must sum to 1**

$$\sum_{i=0}^3 P(Y = i) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$$

✓ Valid PMF

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## 4. Probability Mass Function (PMF) — Formal Definition

### Step 1: Discrete RV

A random variable is **discrete** if it takes at most countably many values.

Let possible values:

$$R_X = \{x_1, x_2, x_3, \dots\}$$


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## Step 2: Define PMF

$$P_X(x_k) = P(X = x_k)$$

for  $k = 1, 2, 3, \dots$

This function is the **Probability Mass Function (PMF)**.

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## Step 3: Required property

Since  $X$  must take one of these values:

$$\sum_k P_X(x_k) = 1$$

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## 5. PMF Example Using Exponential Series

Given:

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

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### Step 1: Use total probability = 1

$$\sum_{i=0}^{\infty} p(i) = c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

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### Step 2: Recognize series

From slides:

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$

So:

$$ce^{\lambda} = 1 \Rightarrow c = e^{-\lambda}$$

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**Step 3: Find probabilities**

$$P(X = 0) = e^{-\lambda}$$

$$P(X > 2) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$


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**6. Bayes' Theorem (from recap)****Step 1: Start from joint probability**

$$P(A, B_i) = P(B_i|A)P(A)$$


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**Step 2: Solve for conditional probability**

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_j P(A|B_j)P(B_j)}$$


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**Step 3: Interpretation**

- $P(B_i)$  = prior probability
  - $P(B_i|A)$  = posterior probability (after observing A)
- 

**7. Bayes Example — Auditorium****Given:**

- 30 rows
- Row 1 has 11 seats, Row 2 has 12 seats, ... Row 30 has 40 seats

**Procedure:**

1. Randomly select a row (each with probability  $1/30$ )
  2. Randomly select a seat within that row
- 

**(a) Probability Seat 15 given Row 20**

Row 20 has 30 seats.

Each seat equally likely:

$$P(\text{Seat 15}|\text{Row 20}) = \frac{1}{30}$$


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### (b) Probability Row 20 given Seat 15

Use Bayes:

$$P(R_{20}|S_{15}) = \frac{P(S_{15}|R_{20})P(R_{20})}{P(S_{15})}$$

Where:

- $P(R_{20}) = 1/30$
- $P(S_{15}|R_{20}) = 1/30$
- $P(S_{15}) = \sum_{k=15}^{30} \frac{1}{30} \cdot \frac{1}{(k+10)}$

(from rows that actually contain seat 15)

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## 8. Independent Events

### Definition (from slides)

Two events A and B are independent if:

$$P(A|B) = P(A), \quad P(B|A) = P(B)$$

which implies:

$$P(A, B) = P(A)P(B)$$


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### For three events:

A, B, C are mutually independent if:

$$P(A, B) = P(A)P(B)$$

$$P(A, C) = P(A)P(C)$$

$$P(B, C) = P(B)P(C)$$

$$P(A, B, C) = P(A)P(B)P(C)$$


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## 9. Bernoulli Random Variable

### Step 1: Define experiment

Outcome = Success or Failure

Define RV:

$$X = 1 \text{ if success, } X = 0 \text{ if failure}$$

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### Step 2: PMF

$$P(X = 1) = p$$

$$P(X = 0) = 1 - p$$

where  $0 < p < 1$

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### Applications (from slides):

- single coin toss
  - spam/not spam
  - classification outcomes
- 

## 10. Binomial Random Variable

### Step 1: Experiment

- n independent trials
  - each success probability = p
- 

### Step 2: Define RV

$X$  = number of successes in n trials

Notation:

$$X \sim B(n, p)$$

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**Step 3: PMF**

$$P(X = i) = \binom{n}{i} p^i (1 - p)^{n-i}, \quad i = 0, 1, \dots, n$$

Reason:

- $\binom{n}{i}$  ways to choose  $i$  successes
  - $p^i$  probability of successes
  - $(1 - p)^{n-i}$  probability of failures
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**11. Geometric Random Variable****Step 1: Experiment**

Repeated independent trials until first success

Each trial success probability =  $p$

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**Step 2: Define RV**

$X$  = number of trials until first success

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**Step 3: PMF**

To get success on trial  $n$ :

- first  $n-1$  must be failures  $\rightarrow (1 - p)^{n-1}$
- $n$ th must be success  $\rightarrow p$

So:

$$P(X = n) = (1 - p)^{n-1} p$$


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**Example (urn)**

With replacement:

$$p = \frac{M}{M + N}$$

$$P(X = n) = \left( \frac{N}{M + N} \right)^{n-1} \frac{M}{M + N}$$

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## 12. Poisson Random Variable

### Step 1: Values

$$X = 0, 1, 2, \dots$$

Parameter:

$$\lambda > 0$$

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### Step 2: PMF

$$P(X = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

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### Step 3: Validity

$$\sum_{i=0}^{\infty} P(X = i) = e^{-\lambda} \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = e^{-\lambda} e^{\lambda} = 1$$

✓ Proper PMF

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### Note from slides:

Poisson approximates Binomial when:

- n large
  - p small
  - np moderate
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### ✓ Final Logical Flow for Revision

1. Random variable = mapping outcomes → numbers
2. Discrete RV → finite/countable values

3. PMF assigns probability to each value
  4. Probabilities must sum to 1
  5. Examples reinforce PMF construction
  6. Bayes' theorem updates probabilities using evidence
  7. Independence formalized via conditional probabilities
  8. Bernoulli → single trial
  9. Binomial → fixed number of trials
  10. Geometric → trials until first success
  11. Poisson → counts of rare events
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If you'd like, I can:

- ✓ Convert this into short exam-ready formula sheets
- ✓ Add common pitfalls professors test
- ✓ Create practice problems based strictly on these slides

Just tell me 👍