Lecture Notes Scribe

L6_S2_A.pdf
PDF

Generate a lecture scribe notes which can be used as exam reference material .Use only the provided context (lecture slides) dont add anything of you own . While writing the scribe reason step by step through all definitions , notation, assumptions , theorem statements , proofs ,derivations and worked examples exactly as given in the lecture slides. Make the logical flow explicit so that a student can follow the reasoning during exam revision . Include proofs, derivations and examples only if they appear in the provided context. Do not introduce new explanations examples or material beyond the lecture scope . Maintain a formal and academic tone and strict faithfulness to the lecture structure

Below is a **formal lecture scribe (exam-ready reference notes)** reconstructed **strictly and only** from the provided Lecture 6 slides.
No material, intuition, notation, examples, or explanations beyond the slides has been added.
All definitions, assumptions, statements, derivations, and worked examples follow the **exact logical order and scope of the lecture**.

CSE400: Fundamentals of Probability in Computing

Lecture 6: Discrete Random Variables, Expectation and Problem Solving

(Based entirely on the provided lecture slides)

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1. Random Variables: Motivation and Concept

Definition

A **random variable** X on a sample space Ω is a function

$$X : \Omega \to \mathbb{R}$$

that assigns to each sample point $\omega \in \Omega$ a real number $X(\omega)$.

Restriction

In this lecture, attention is restricted to **discrete random variables**, i.e., random variables that take values in a range that is **finite or countably infinite**.

Although X maps into $\mathbb{R}$, the actual set of values

$$\{X(\omega) : \omega \in \Omega\}$$

forms a **discrete subset of** $\mathbb{R}$.

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2. Visualization of a Discrete Random Variable

The distribution of a discrete random variable can be visualized using a **bar diagram**:

- The **x-axis** represents the possible values the random variable can take.
- The **height of the bar** at value a represents

$$\Pr[X = a].$$

Each probability is computed by evaluating the probability of the **corresponding event** in the sample space.

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3. Discrete vs Continuous Random Variables

Discrete Random Variable

- Countable support
- Defined using a **Probability Mass Function (PMF)**
- Probabilities are assigned to **single values**
- Each possible value has **strictly positive probability**

Continuous Random Variable

- Uncountable support
- Defined using a **Probability Density Function (PDF)**
- Probabilities are assigned to **intervals**
- Each individual value has **zero probability**

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4. Example 1: Tossing Three Fair Coins

Experiment

An experiment consists of tossing **3 fair coins**.

Let Y denote the **number of heads** that appear.

Possible Values

$$Y \in \{0, 1, 2, 3\}$$

Probabilities

$$\Pr(Y = 0) = \Pr(t, t, t) = \frac{1}{8}$$

$$\Pr(Y = 1) = \Pr(t, t, h), (t, h, t), (h, t, t) = \frac{3}{8}$$

$$\Pr(Y = 2) = \Pr(t, h, h), (h, t, h), (h, h, t) = \frac{3}{8}$$

$$\Pr(Y = 3) = \Pr(h, h, h) = \frac{1}{8}$$

Thus,

$$\Pr(Y = 0) = \frac{1}{8}, \quad \Pr(Y = 1) = \frac{3}{8}, \quad \Pr(Y = 2) = \frac{3}{8}, \quad \Pr(Y = 3) = \frac{1}{8}$$

Since Y must take one of these values,

$$1 = \Pr\left(\bigcup_{i=0}^{3}(Y = i)\right) = \sum_{i=0}^{3} \Pr(Y = i).$$

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5. Probability Mass Function (PMF)

Definition

A random variable that can take on at most a **countable number of possible values** is said to be **discrete**.

Let X be a discrete random variable with range

$$R_X = \{x_1, x_2, x_3, \ldots\}$$

(finite or countably infinite).

The function

$$P_X(x_k) = \Pr(X = x_k), \quad k = 1, 2, 3, \ldots$$

is called the **Probability Mass Function (PMF)** of X.

Normalization Property

Since X must take one of the values x_k,

$$\sum_{k=1}^{\infty} P_X(x_k) = 1.$$

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6. PMF Example

Given

The probability mass function of a random variable X is

$$p(i) = c\frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \ldots$$

where $\lambda > 0$.

Step 1: Determine c

Since

$$\sum_{i=0}^{\infty} p(i) = 1,$$

we have

$$c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1.$$

Using

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!},$$

it follows that

$$ce^{\lambda} = 1 \quad \Rightarrow \quad c = e^{-\lambda}.$$

Step 2: Compute $P(X = 0)$

$$P(X = 0) = e^{-\lambda}\frac{\lambda^0}{0!} = e^{-\lambda}.$$

Step 3: Compute $P(X > 2)$

$$P(X > 2) = 1 - P(X \leqslant 2)$$

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

$$= 1 - \left(e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2}{2}e^{-\lambda}\right).$$

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7. Bayes' Theorem (Recap)

Using

$$\Pr(A \mid B) = \frac{\Pr(B \mid A)\Pr(A)}{\Pr(B)},$$

we obtain

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^{n}\Pr(A \mid B_j)\Pr(B_j)}.$$

This result is known as **Bayes' Formula (Proposition 3.1)**.

Terminology

- $\Pr(B_i)$: **Prior probability**
- $\Pr(B_i \mid A)$: **Posterior probability**

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8. Bayes' Theorem: Example — Auditorium with 30 Rows

Setup

- An auditorium has **30 rows**.
- Row 1 has **11 seats**, Row 2 has **12 seats**, ..., Row 30 has **40 seats**.
- A door prize is awarded by:
  1. Randomly selecting a row (each row equally likely),

2. Randomly selecting a seat within that row (each seat equally likely).

Tasks

1. Compute the probability that **Seat 15** was selected given that **Row 20** was selected.
2. Compute the probability that **Row 20** was selected given that **Seat 15** was selected.

Computations (as shown in slides)

$$\Pr(S_{15} \mid R_{20}) = \frac{1}{30}$$

$$\Pr(R_{20} \mid S_{15}) = \frac{\Pr(S_{15} \mid R_{20})\Pr(R_{20})}{\Pr(S_{15})}$$

with

$$\Pr(S_{15}) = \sum_{k=15}^{30} \Pr(S_{15} \mid R_k)\Pr(R_k).$$

End of Lecture 6 Scribe

This scribe is a **faithful reconstruction** of the lecture slides and is suitable for **exam revision**.

If you want, I can next:

- Convert this into **clean LaTeX exam notes**, or
- Split it into **definition–theorem–example sheets** for quick memorization.