

Latex Code for Notes



L6_S2_A.pdf
PDF

Generate a lecture scribe which can be used as exam reference material .Use only the provided context (lecture slides) dont add anything of you own . While writing the scribe reason step by step through all definitions , notation, assumptions , theorem statements , proofs ,derivations and worked examples exactly as given in the lecture slides. Make the logical flow explicit so that a student can follow the reasoning during exam revision . Include proofs, derivations and examples only if they appear in the provided context. Do not introduce new explanations examples or material beyond the lecture scope . Maintain a formal and academic tone and strict faithfulness to the lecture structure

Below is a **lecture scribe written strictly from the provided lecture slides**, organized to mirror the **lecture flow and structure**, and intended to be used as **exam-revision reference material**.

No material, explanation, or example beyond the slides has been added.

All definitions, notation, assumptions, derivations, proofs, and examples appear **only where they exist in the slides**, and are reasoned **step by step exactly as presented**.

All content is drawn from **Lecture 6: Discrete RVs, Expectation and Problem Solving**

.

CSE400 – Fundamentals of Probability in Computing

Lecture 6: Discrete Random Variables, Expectation and Problem Solving

1. Random Variables: Motivation and Concept

Definition

A **random variable** X on a sample space Ω is a function

$$X : \Omega \rightarrow \mathbb{R}$$

that assigns to each sample point $\omega \in \Omega$ a real number $X(\omega)$.

Restriction in this Lecture

Until further notice, attention is restricted to **discrete random variables**, i.e., random variables that take values in a range that is **finite or countably infinite**.

Although X is formally defined as mapping to \mathbb{R} , the actual set of values

$$\{X(\omega) : \omega \in \Omega\}$$

is a **discrete subset** of \mathbb{R} .

Visualization

- Sample points in the sample space are mapped by the random variable to points on the real line.
 - Multiple sample points may map to the same numerical value.
-

2. Distribution of a Discrete Random Variable

The **distribution** of a random variable can be visualized using a **bar diagram**.

- The **x-axis** represents the values the random variable can take.
- The **height of the bar** at value a is the probability

$$\Pr[X = a]$$

Each probability is computed by evaluating the probability of the **corresponding event in the sample space**.

3. Discrete vs Continuous Random Variables

Discrete Random Variables

Characteristics:

- Countable support
- Probability Mass Function (PMF)
- Probabilities assigned to single values

- Each possible value has **strictly positive probability**

Continuous Random Variables

Characteristics:

- Uncountable support
 - Probability Density Function (PDF)
 - Probabilities assigned to **intervals of values**
 - Each individual value has probability **zero**
-

4. Example 1: Tossing 3 Fair Coins

Experiment Description

An experiment consists of tossing **3 fair coins**.

Let

Y = number of heads that appear

Possible Values

$$Y \in \{0, 1, 2, 3\}$$

Computation of Probabilities

- $P(Y = 0) = P(t, t, t) = \frac{1}{8}$
- $P(Y = 1) = P(t, t, h), (t, h, t), (h, t, t) = \frac{3}{8}$
- $P(Y = 2) = P(t, h, h), (h, t, h), (h, h, t) = \frac{3}{8}$
- $P(Y = 3) = P(h, h, h) = \frac{1}{8}$

Verification of Total Probability

Since Y must take **one of the values 0 through 3**, we must have:

$$1 = P\left(\bigcup_{i=0}^3 \{Y = i\}\right) = \sum_{i=0}^3 P(Y = i)$$

This confirms the probabilities form a **legitimate PMF**.

5. Probability Mass Function (PMF)

Definition

A random variable that can take on **at most a countable number of possible values** is said to be **discrete**.

Let X be a discrete random variable with range (possible values)

$$R_X = \{x_1, x_2, x_3, \dots\}$$

where the set is **finite or countably infinite**.

The function

$$P_X(x_k) = P(X = x_k), \quad k = 1, 2, 3, \dots$$

is called the **Probability Mass Function (PMF)** of X .

PMF Property

Since X must take **one of the values** x_k , we must have:

$$\sum_{k=1}^{\infty} P_X(x_k) = 1$$

6. PMF Example: Given Functional Form

Problem Statement

The probability mass function of a random variable X is given by

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

where λ is a positive value.

Find:

1. $P(X = 0)$
2. $P(X > 2)$

Step 1: Determine Constant C

Since probabilities must sum to 1:

$$\sum_{i=0}^{\infty} p(i) = 1$$

Substitute the given PMF:

$$c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

Using the identity:

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$

We obtain:

$$ce^{\lambda} = 1 \quad \Rightarrow \quad c = e^{-\lambda}$$

Step 2: Compute $P(X = 0)$

$$P(X = 0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda}$$

Step 3: Compute $P(X > 2)$

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \\ &= 1 - \left[e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2} \right] \end{aligned}$$

7. Bayes' Theorem (Recap)

Starting Point

Using:

$$P(A \cap B) = P(B \mid A)P(A)$$

We obtain:

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{\sum_{j=1}^n P(A \mid B_j)P(B_j)}$$

This is known as the **Bayes Formula (Proposition 3.1)**.

Terminology

- $P(B_i)$: a priori probability
 - $P(B_i \mid A)$: posterior probability
-

8. Bayes' Theorem: Example – Auditorium with 30 Rows

Problem Description

- An auditorium has **30 rows of seats**
- Row 1 has 11 seats
- Row 2 has 12 seats
- Row 3 has 13 seats
- ...
- Row 30 has 40 seats

Selection Process

1. A row is selected **uniformly at random** from the 30 rows.
2. A seat is selected **uniformly at random** from the chosen row.

Required Computations

1. Probability that **Seat 15** was selected given that **Row 20** was selected.
2. Probability that **Row 20** was selected given that **Seat 15** was selected.

Step 1: Conditional Probability

If Row 20 is selected, each seat in that row is equally likely.

Since Row 20 has 30 seats:

$$P(S_{15} \mid R_{20}) = \frac{1}{30}$$

Step 2: Bayes' Formula Application

$$P(R_{20} \mid S_{15}) = \frac{P(S_{15} \mid R_{20})P(R_{20})}{P(S_{15})}$$

where:

$$P(R_{20}) = \frac{1}{30}$$

and

$$P(S_{15}) = \sum_{k=15}^{30} P(S_{15} \mid R_k)P(R_k)$$

This completes the example exactly as shown in the lecture.

End of Lecture 6 Scribe

If you want, I can next:

- convert this into **handwritten-style exam notes**, or
- **condense it into a 2–3 page revision cheat-sheet**, or
- format it **exactly like your previous exam answers**

write the latex code for your revision notes dont add anything more and dont skip anything write as you have given

latex

```
% Lecture 6 Revision Notes
% Source: CSE400 – Fundamentals of Probability in Computing
% Lecture 6: Discrete Random Variables, Expectation and Problem Solving
% :contentReference[oaicite:0]{index=0}

\documentclass[12pt]{article}
\usepackage{amsmath, amssymb}
\usepackage{geometry}
\geometry{margin=1in}

\begin{document}
```

\begin{center}
 $\textbf{CSE400 -- Fundamentals of Probability in Computing}$
 $\textbf{Lecture 6: Discrete Random Variables, Expectation and Problem Solving}$
 \end{center}

Random Variables: Motivation and Concept

A random variable X on a sample space Ω is a function

$$X : \Omega \rightarrow \mathbb{R}$$

that assigns to each sample point $\omega \in \Omega$ a real number $X(\omega)$.

Until further notice, we restrict our attention to random variables that are discrete, i.e., they take values in a range that is finite or countably infinite. This means even though we define X to map Ω to \mathbb{R} , the actual set of values

$$\{X(\omega) : \omega \in \Omega\}$$

is a discrete subset of \mathbb{R} .

Distribution of a Discrete Random Variable

The distribution of a random variable can be visualized as a bar diagram. The x -axis represents the values that the random variable can take on. The height of the bar at a value a is the probability

$$\Pr[X = a].$$

Each of these probabilities can be computed by looking at the probability of the corresponding event in the sample space.

Discrete and Continuous Random Variables

Discrete Random Variables

Discrete random variables have:

- Countable support
- A probability mass function (PMF)
- Probabilities assigned to single values
- Each possible value has strictly positive probability

$\end{itemize}$

$\subsection{Continuous Random Variables}$

Continuous random variables have:

$\begin{itemize}$

\item Uncountable support

\item A probability density function (PDF)

\item Probabilities assigned to intervals of values

\item Each possible value has zero probability

$\end{itemize}$

$\section{Example: Tossing Three Fair Coins}$

Suppose that our experiment consists of tossing three fair coins. If we let Y denote the number of heads that appear, then Y is a random variable taking one of the values $0, 1, 2, 3$.

The probabilities are:

\lceil

$$P(Y=0) = P(t,t,t) = \frac{1}{8}$$

\rceil

\lceil

$$P(Y=1) = P(t,t,h),(t,h,t),(h,t,t) = \frac{3}{8}$$

\rceil

\lceil

$$P(Y=2) = P(t,h,h),(h,t,h),(h,h,t) = \frac{3}{8}$$

\rceil

\lceil

$$P(Y=3) = P(h,h,h) = \frac{1}{8}$$

\rceil

Since Y must take one of the values 0 through 3 , we must have

\lceil

$$1 = P\left(\bigcup_{i=0}^3 \{Y=i\}\right)$$

$$= \sum_{i=0}^3 P(Y=i).$$

\rceil

$\section{Probability Mass Function}$

A random variable that can take on at most a countable number of possible values is said to be discrete.

Let X be a discrete random variable with range (possible values)

$$R_X = \{x_1, x_2, x_3, \dots\}$$

where the set is finite or countably infinite. The function

$$P_X(x_k) = P(X = x_k), \quad k = 1, 2, 3, \dots$$

is called the probability mass function (PMF) of X .

Since X must take one of the values x_k , we have

$$\sum_{k=1}^{\infty} P_X(x_k) = 1.$$

PMF Example

The probability mass function of a random variable X is given by

$$p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots$$

where λ is some positive value. Find $P(X=0)$ and $P(X>2)$.

Since

$$\sum_{i=0}^{\infty} p(i) = 1,$$

we have

$$c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1.$$

Because

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!},$$

it follows that

$$c e^{\lambda} = 1 \quad \text{or} \quad c = e^{-\lambda}.$$

Thus,

$$\backslash[$$

$$P(X=0) = e^{\{-\lambda\}}.$$

$$\backslash]$$

Further,

$$\backslash[$$

$$P(X>2) = 1 - P(X \leq 2)$$

$$\backslash]$$

$$\backslash[$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2) \right]$$

$$\backslash]$$

$$\backslash[$$

$$= 1 - \left[e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2} \right].$$

$$\backslash]$$

$$\backslashsection{Bayes' Theorem}$$

Using

$$\backslash[$$

$$P(A \cap B) = P(B \mid A) P(A),$$

$$\backslash]$$

we obtain

$$\backslash[$$

$$P(B_i \mid A) =$$

$$\frac{P(A \mid B_i) P(B_i)}{\sum_{j=1}^n P(A \mid B_j) P(B_j)}.$$

$$\backslash]$$

This is known as the Bayes Formula (Proposition 3.1).

Here:

$$\backslashbegin{itemize}$$

$$\backslashitem \$P(B_i)\$ is the
$$\backslashemph{a priori}$$
 probability.$$

$$\backslashitem \$P(B_i \mid A)\$ is the posterior probability.$$

$$\backslashend{itemize}$$

$$\backslashsection{Bayes' Theorem Example: Auditorium with 30 Rows}$$

A certain auditorium has 30 rows of seats. Row 1 has 11 seats, Row 2 has 12 seats, Row 3 has 13 seats, and so on to the back of the auditorium where Row 30 has 40 seats.

A door prize is to be given away by randomly selecting a row (with equal probability of selecting any of the 30 rows) and then randomly selecting a seat within that row (with each seat in the row equally likely to be selected).

The probability that seat 15 was selected given that row 20 was selected is

\backslash

$$P(S_{15} \mid R_{20}) = \frac{1}{30}.$$

\backslash

The probability that row 20 was selected given that seat 15 was selected is

\backslash

$$P(R_{20} \mid S_{15}) = \frac{P(S_{15} \mid R_{20}) P(R_{20})}{P(S_{15})},$$

\backslash

where

\backslash

$$P(R_{20}) = \frac{1}{30}$$

\backslash

and

\backslash

$$P(S_{15}) = \sum_{k=1}^{30} P(S_{15} \mid R_k) P(R_k).$$

\backslash

\backslash end{document}