Bootstrap assignment

There will be some functions that start with the word "grader" ex: grader_sampples(), grader_30().. etc, you should not change those function definition.
Every Grader function has to return True.

Importing packages

In [1]:
```
import numpy as np # importing numpy for numerical computation
from sklearn.datasets import load_boston # here we are using sklearn's boston dataset
from sklearn.metrics import mean_squared_error # importing mean_squared_error metric
```

In [2]:
```
boston = load_boston()
x=boston.data #independent variables
y=boston.target #target variable
```

In [3]:
```
x.shape
```

Out[3]:
```
(506, 13)
```

In [4]:
```
x[0]
```

Out[4]:
```
array([6.320e-03, 1.800e+01, 2.310e+00, 0.000e+00, 5.380e-01, 6.575e+00,
       6.520e+01, 4.090e+00, 1.000e+00, 2.960e+02, 1.530e+01, 3.969e+02,
       4.980e+00])
```

In [5]:
```
y[0]
```

Out[5]:
```
24.0
```

Task 1

Step - 1

- **Creating samples**
  Randomly create 30 samples from the whole boston data points
  - **Creating each sample: Consider any random 303(60% of 506) data points from whole data set and then replicate any 203 points from the sampled points**

    For better understanding of this procedure lets check this examples, assume we have 10 data points [1,2,3,4,5,6,7,8,9,10], first we take 6 data points randomly , consider we have selected [4, 5, 7, 8, 9, 3] now we will replicate 4 points from [4, 5, 7, 8, 9, 3], consder they are [5, 8, 3,7] so our final sample will be [4, 5, 7, 8, 9, 3, 5, 8, 3,7]
- **Create 30 samples**
  - **Note that as a part of the Bagging when you are taking the random samples make sure each of the sample will have different set of columns**
    Ex: Assume we have 10 columns[1 ,2 ,3 ,4 ,5 ,6 ,7 ,8 ,9 ,10] for the first sample we will select [3, 4, 5, 9, 1, 2] and for the second sample [7, 9, 1, 4, 5, 6, 2] and so on... Make sure each sample will have atleast 3 feautres/columns/attributes

Step - 2

Building High Variance Models on each of the sample and finding train MSE value

- **Build a regression trees on each of 30 samples.**
- **Computed the predicted values of each data point(506 data points) in your corpus.**
- **Predicted house price of** i^{th} **data point** $y^i_{pred} = \frac{1}{30}\sum_{k=1}^{30}(\text{predicted value of } x^i \text{ with } k^{th} \text{ model})$
- **Now calculate the** $MSE = \frac{1}{506}\sum_{i=1}^{506}(y^i - y^i_{pred})^2$

Step - 3

- **Calculating the OOB score**

- **Predicted house price of** i^{th} **data point** $y^i_{pred} = \frac{1}{k}\sum_{\text{k= model which was buit on samples not included } x^i}(\text{predicted value of } x^i \text{ with } k^{th} \text{ model})$.
- **Now calculate the** $OOBScore = \frac{1}{506}\sum_{i=1}^{506}(y^i - y^i_{pred})^2$.

Task 2

- **Computing CI of OOB Score and Train MSE**
  - **Repeat Task 1 for 35 times, and for each iteration store the Train MSE and OOB score **
  - **After this we will have 35 Train MSE values and 35 OOB scores**
  - **using these 35 values (assume like a sample) find the confidence intravels of MSE and OOB Score**
  - **you need to report CI of MSE and CI of OOB Score**
  - **Note: Refer the Central_Limit_theorem.ipynb to check how to find the confidence intravel **

Task 3

- **Given a single query point predict the price of house.**

Consider xq= [0.18,20.0,5.00,0.0,0.421,5.60,72.2,7.95,7.0,30.0,19.1,372.13,18.60] Predict the house price for this point as mentioned in the step 2 of Task 1.

Task - 1

Step - 1

- **Creating samples**

Algorithm

Pesudo Code for generating Sample

```
def generating_samples(input_data, target_data):

    Selecting_rows <--- Getting 303 random row indices from the input_data

    Replcaing_rows <--- Extracting 206 random row indices from the "Selecting_rows"

   Selecting_columns<--- Getting from 3 to 13 random column indices

   sample_data<--- input_data[Selecting_rows[:,None],Selecting_columns]

   target_of_sample_data <--- target_data[Selecting_rows]

    #Replicating Data

   Replicated_sample_data <--- sample_data [Replaceing_rows]

   target_of_Replicated_sample_data<--- target_data[Replaceing_rows]

    # Concatinating data

    final_sample_data <---  perform vertical stack on  sample_data, Replicated_sample_data

    final_target_data<--- perform vertical stack on target_of_sample_data.reshape(-1,1), target_of_Replicated_sample_data.reshape(-1,1)

    return  final_sample_data,  final_target_data, Selecting_rows, Selecting_columns
```

- **Write code for generating samples**

In [ ]:

```

```

In [6]:
```
def generating_samples(input_data, target_data):

    '''In this function, we will write code for generating 30 samples '''
    # you can use random.choice to generate random indices without replacement
    # Please have a look at this link https://docs.scipy.org/doc/numpy-1.16.0/reference/generated/numpy.rando
m.choice.html for more details
    # Please follow above pseudo code for generating samples

    Selecting_rows = np.random.choice(np.array(range(len(input_data))), 303, replace=False)
    Replacing_rows = np.random.choice(np.array(range(len(Selecting_rows))), 203, replace=False)
    Selecting_cols = np.random.choice(np.array(range(3,13)), np.random.randint(3,11), replace=False)

    sample_data = input_data[Selecting_rows[:,None], Selecting_cols]

    target_of_sample_data = target_data[Selecting_rows]

    # Replicating Data
    Replicated_sample_data = sample_data[Replacing_rows]
    target_of_Replicated_sample_data = target_data[Replacing_rows]

    # Concatenating data
    final_sample_data = np.vstack((sample_data, Replicated_sample_data))
    final_target_data = np.vstack((target_of_sample_data.reshape(-1,1), target_of_Replicated_sample_data.resh
ape(-1,1)))


    return list(final_sample_data), list(final_target_data), list(Selecting_rows), list(Selecting_cols)
    #note please return as lists
```

Grader function - 1 </fongt>

In [7]:
```
def grader_samples(a,b,c,d):
    length = (len(a)==506  and len(b)==506)
    sampled = (len(a)-len(set([str(i) for i in a]))==203)
    rows_length = (len(c)==303)
    column_length= (len(d)>=3)
    assert(length and sampled and rows_length and column_length)
    return True
a,b,c,d = generating_samples(x, y)
grader_samples(a,b,c,d)
```

Out[7]: True

- **Create 30 samples**

Run this code 30 times, so that you will 30 samples, and store them in a lists as shown below:

```
list_input_data=[]
list_output_data=[]
list_selected_row=[]
list_selected_columns=[]

for i in range(0,30):
    a,b,c,d=generating_sample(input_data,target_data)
   list_input_data.append(a)
   list_output_data.append(b)
   list_selected_row.append(c)
   list_selected_columns.append(d)
```

In [8]:
```
# Use generating_samples function to create 30 samples
# store these created samples in a list
list_input_data =[]
list_output_data =[]
list_selected_row= []
list_selected_columns=[]

for i in range(30):
    a,b,c,d = generating_samples(x,y)
    list_input_data.append(a)
    list_output_data.append(b)
    list_selected_row.append(c)
    list_selected_columns.append(d)
```

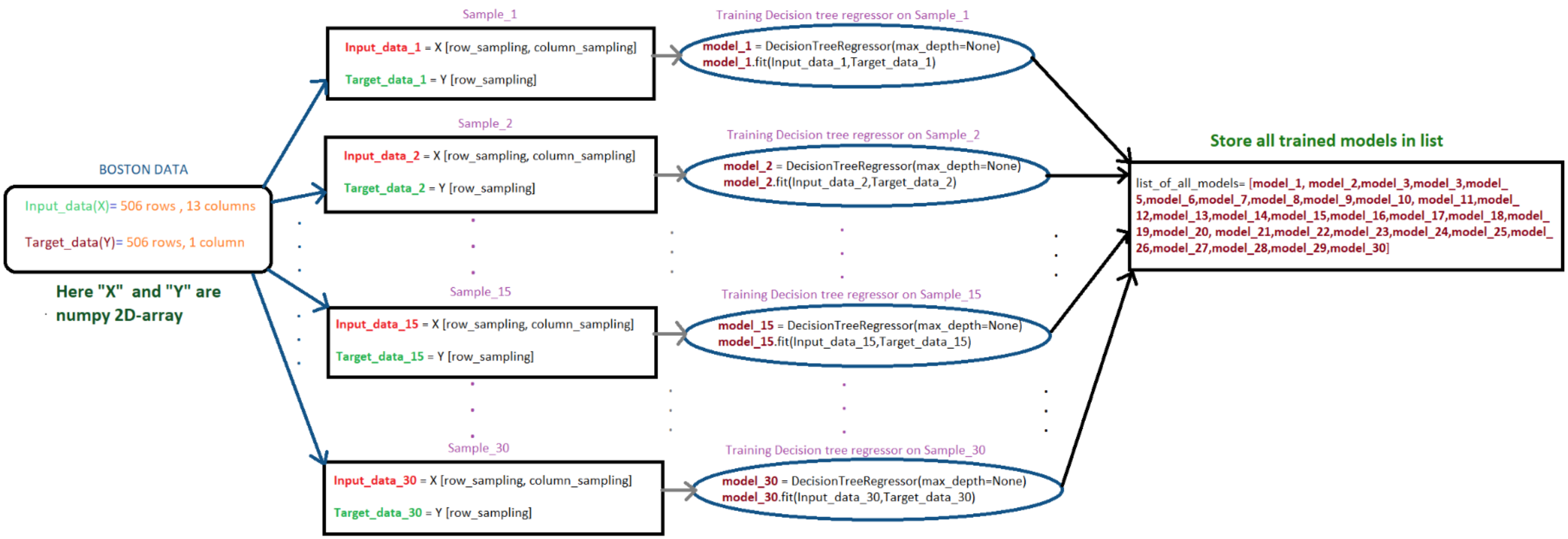
Grader function - 2

In [9]:
```
def grader_30(a):
    assert(len(a)==30 and len(a[0])==506)
    return True
grader_30(list_input_data)
```

Out[9]:
```
True
```

Step - 2

Flowchart for building tree



- **Write code for building regression trees**

In [10]:
```
len(list_input_data[0])
```

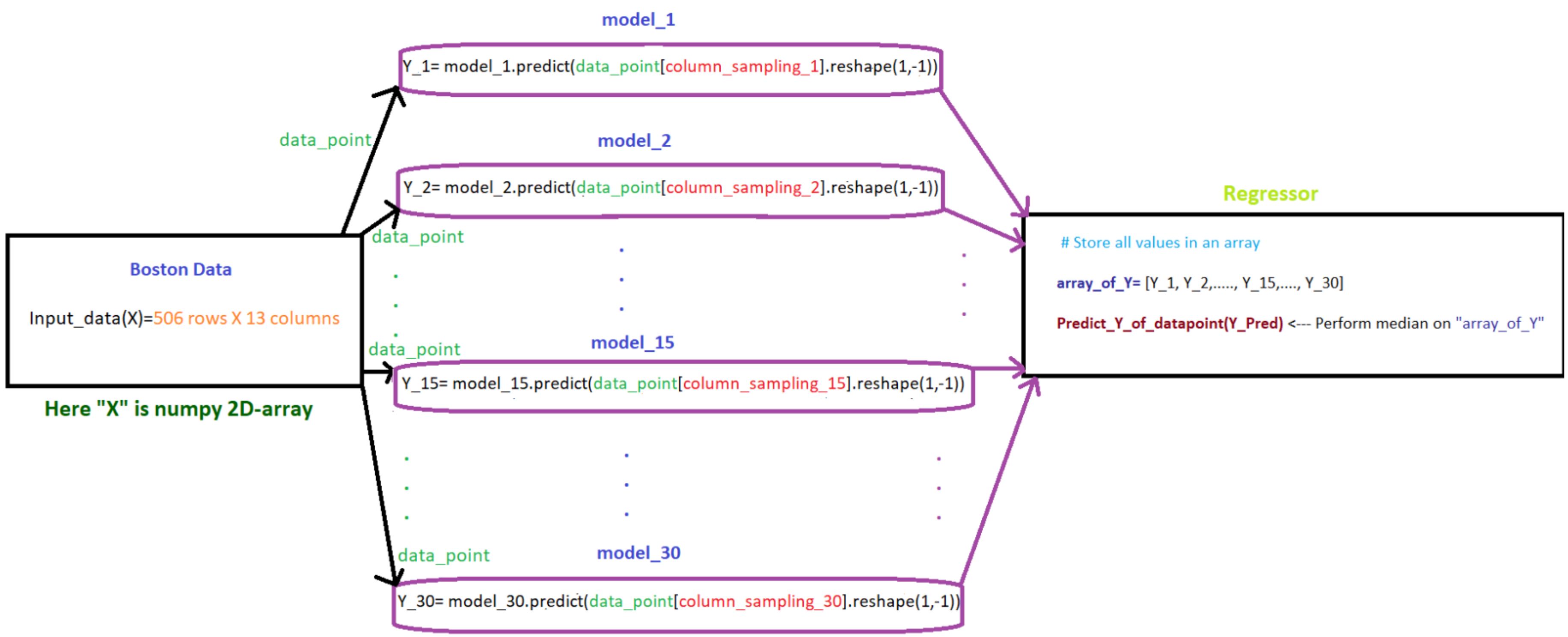
Out[10]:
```
506
```

In [11]:
```
len(list_output_data[0])
```
Out[11]:
```
506
```

In [12]:
```
from sklearn.tree import DecisionTreeRegressor
list_of_all_models = []

for i in range(30):
    regressor = DecisionTreeRegressor(max_depth=None)
    regressor.fit(list_input_data[i], list_output_data[i])
    list_of_all_models.append(regressor)
```

In [13]:
```
len(list_of_all_models)
```
Out[13]:
```
30
```

Flowchart for calculating MSE


model_1
Y_1= model_1.predict(data_point[column_sampling_1].reshape(1,-1))
data_point
model_2
Y_2= model_2.predict(data_point[column_sampling_2].reshape(1,-1))
data_point
Boston Data
Input_data(X)=506 rows X 13 columns
Here "X" is numpy 2D-array
data_point
model_15
Y_15= model_15.predict(data_point[column_sampling_15].reshape(1,-1))
data_point
model_30
Y_30= model_30.predict(data_point[column_sampling_30].reshape(1,-1))
Regressor
# Store all values in an array
array_of_Y= [Y_1, Y_2,....., Y_15,...., Y_30]
Predict_Y_of_datapoint(Y_Pred) <--- Perform median on "array_of_Y"

After getting predicted_y for each data point, we can use sklearns mean_squared_error to calculate the MSE between predicted_y and actual_y.

- **Write code for calculating MSE**

In [14]:
```
import statistics as stats
```

In [15]:
```
y_hat = [] # for all data points
for i in range(len(x)):
    array_of_Y = []
    for j in range(30):
        model = list_of_all_models[j]
        y_predicted = model.predict(x[i][list_selected_columns[j]].reshape(1,-1))
        array_of_Y.append(y_predicted)
    y_hat.append(stats.median(array_of_Y))

y_hat = np.array(y_hat).reshape(-1,1)
```

In [16]:
```
len(y_hat)
```

Out[16]:
```
506
```

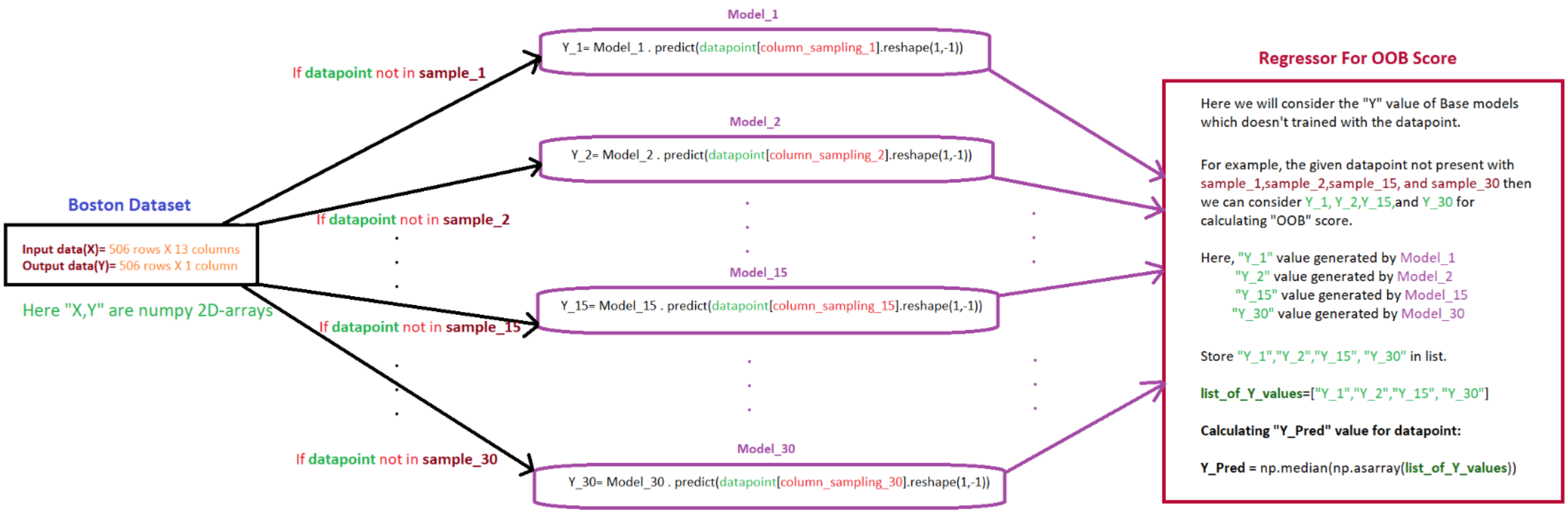
In [17]:
```
MSE = 0
from sklearn.metrics import mean_squared_error
MSE = mean_squared_error(y, y_hat)
MSE
```

Out[17]:
```
13.964854935221782
```

In [ ]:

Step - 3

Flowchart for calculating OOB score



Now calculate the $OOBScore = \frac{1}{506} \sum_{i=1}^{506}(y^i - y^i_{pred})^2$.

- **Write code for calculating OOB score**

In [18]:
```
from tqdm import tqdm # to visualize the processing
```

In [19]:
```
Y_pred = [] # to st`ore the predicted value for each datapoint
for i in tqdm(range(len(x))): # for each datapoints in train data


    list_of_Y_values = [] # list to store the predicted value of single datapoint from all different
                          # models which wasn't trained including this perticular datapoint
    count=0
    for j in range(len(list_of_all_models)):# for each model
        sample = list_input_data[j] # ith sampled data
        model  = list_of_all_models[j] # ith model


        # it took me more than an hour...

        if not any((x[i][list_selected_columns[j]]==t).all() for t in sample): # check, if datapoint not in s
ample
            count+=1
            datapoint = x[i][list_selected_columns[j]] # datapoint
            y_pred = model.predict(datapoint.reshape(1,-1)) # predict
            list_of_Y_values.append(y_pred) # store the prediction

#     print(count) # uncomment this to track
    Y_pred.append(np.median(list_of_Y_values))
```

```
100%|██████████████████████████████████████████████████| 506/506 [02:03<00:00,
4.09it/s]
```

In [20]:
```
len(Y_pred)
```

Out[20]:
```
506
```

In [21]:
```
y.shape
```

Out[21]:
```
(506,)
```

In [22]:
```
y_pred = np.array(Y_pred).reshape(-1,1)
```

In [23]:
```
y_pred.shape
```

Out[23]:
```
(506, 1)
```

In [24]:
```
y = y.reshape(-1,1)
```

In [25]:
```
y.shape
```

Out[25]:
```
(506, 1)
```

In [26]:
```
# oobscore = 0
# for i in range(len(x)):
#     oobscore += (y[i] - y_pred[i])**2

# oobscore /= len(x)

oobscore = mean_squared_error(y , y_pred)
```

In [27]:
```
oobscore
```

Out[27]:
```
24.69395202020202
```

Task 2

- Computing CI of OOB Score and Train MSE
  - **Repeat Task 1 for 35 times, and for each iteration store the Train MSE and OOB score **
  - **After this we will have 35 Train MSE values and 35 OOB scores**
  - **using these 35 values (assume like a sample) find the confidence intravels of MSE and OOB Score**
  - **you need to report CI of MSE and CI of OOB Score**
  - **Note: Refer the Central_Limit_theorem.ipynb to check how to find the confidence intravel **

In [28]:
```
train_mse_ci = [] # to store train scores for each iteration
oobscore_ci = [] # to store oobscore scores for each iteration

for k in tqdm(range(35)):

    # Use generating_samples function to create 30 samples
    # store these created samples in a list
    list_input_data =[]
    list_output_data =[]
    list_selected_row= []
    list_selected_columns=[]

    for i in range(30):
        a,b,c,d = generating_samples(x,y)
        list_input_data.append(a)
        list_output_data.append(b)
        list_selected_row.append(c)
        list_selected_columns.append(d)

    ##########################################################
    ###### MODELING ##########################################
    ##########################################################
    from sklearn.tree import DecisionTreeRegressor
    list_of_all_models = []

    for i in range(30):
        regressor = DecisionTreeRegressor(max_depth=None)
        regressor.fit(list_input_data[i], list_output_data[i])
        list_of_all_models.append(regressor)
    ##########################################################

    ##########################################################
    #### CALCULATING MSE AND OOBSCORE #####################################
    y_hat = [] # for all data points
    for i in range(len(x)):
        array_of_Y = []
        for j in range(30):
            model = list_of_all_models[j]
            y_predicted = model.predict(x[i][list_selected_columns[j]].reshape(1,-1))
            array_of_Y.append(y_predicted)
        y_hat.append(stats.median(array_of_Y))
```

```
    y_hat = np.array(y_hat).reshape(-1,1)

    MSE = 0
    MSE = mean_squared_error(y, y_hat)

    ###### Calculating OOBScore ###################################
    Y_pred = [] # to st`ore the predicted value for each datapoint
    for i in range(len(x)): # for each datapoint in train data


        list_of_Y_values = [] # list to store the predicted value of single datapoint from all different
                              # models which wasn't trained including this perticular datapoint
        count=0
        for j in range(len(list_of_all_models)):# for each model
            sample = list_input_data[j] # ith sampled data
            model  = list_of_all_models[j] # ith model


            # it took me more than an hour...

            if not any((x[i][list_selected_columns[j]]==t).all() for t in sample): # check, if datapoint non
in sample
                count+=1
                datapoint = x[i][list_selected_columns[j]] # datapoint
                y_pred = model.predict(datapoint.reshape(1,-1)) # predict
                list_of_Y_values.append(y_pred) # store the prediction

        Y_pred.append(np.median(list_of_Y_values))
        y_pred = np.array(Y_pred).reshape(-1,1)
    y = y.reshape(-1,1)
    oobscore = mean_squared_error(y , y_pred)

    #appending MSE and oobscore
    train_mse_ci.append(MSE)
    oobscore_ci.append(oobscore)
```

```
100%|██████████████████████████████████████████████████| 35/35 [1:09:45<00:00, 1
19.58s/it]
```

In [29]:
```
len(train_mse_ci)
```

Out[29]:
```
35
```

In [30]:
```
len(oobscore_ci)
```

Out[30]:
```
35
```

In [31]:
```
train_mse_ci
```

Out[31]:
```
[14.077728626915965,
 14.241258064996899,
 14.602775642902454,
 14.12674493851559,
 14.094529372025491,
 15.651075823847378,
 15.2155556418679,
 14.725736714975847,
 13.43109861532465,
 13.209862929567413,
 14.71104310770751,
 14.424436442050673,
 12.969498594985177,
 13.549703117248583,
 13.41694972826087,
 12.575912670208071,
 15.099948122529645,
 12.327934713987704,
 15.392370927766574,
 13.08452150581906,
 14.763274582784364,
 14.70808437637242,
 14.886042254668792,
 15.041958647898515,
 13.988198184288535,
 13.378420669689335,
 13.317411070582367,
 13.955985157279313,
 14.98591286276361,
 14.038023337999558,
 14.368765362699758,
 13.419134182861221,
 14.222570188734506,
 14.735060952083378,
 13.971089444809955]
```

In [32]:
```
oobscore_ci
```

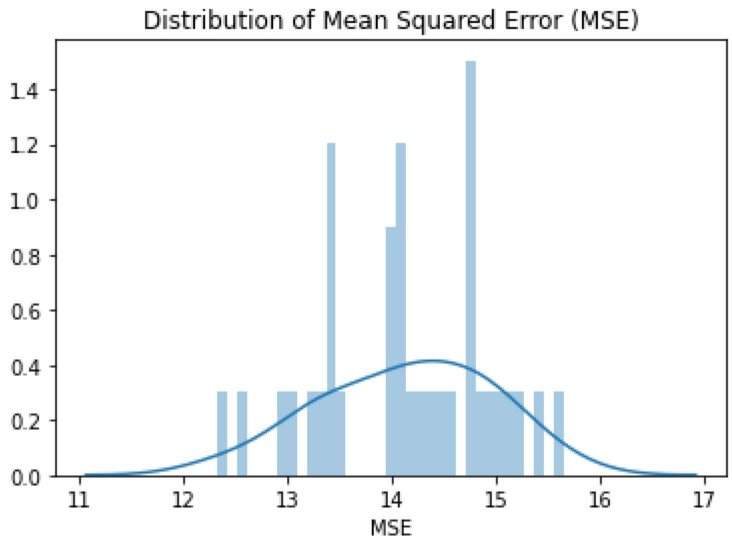
Out[32]:
```
[27.280357927097057,
 24.3679218818621,
 27.73140347084981,
 27.535234683794467,
 26.76888741353755,
 27.174696549568445,
 26.375550203118138,
 24.172465415019765,
 25.800654541337288,
 27.126321640316206,
 27.707547074000882,
 25.01606225296443,
 22.54284584980237,
 27.612256669960473,
 26.49898097826087,
 23.395737104743084,
 27.611316425120773,
 23.657449357707513,
 27.136519817742645,
 26.815434782608694,
 25.438436539306103,
 29.230241477272724,
 24.234595377689942,
 27.87745923913043,
 28.99698795015371,
 25.837791023029762,
 24.29808067083882,
 25.224919713438734,
 27.233121191534913,
 28.643684535573126,
 29.1711351284585,
 25.861175889328067,
 23.978799306283776,
 27.399975093380565,
 27.773785820158103]
```

In [33]:
```
# finding the C.I of MSE and OOBScore
```

In [34]:
```
import seaborn as sns
```

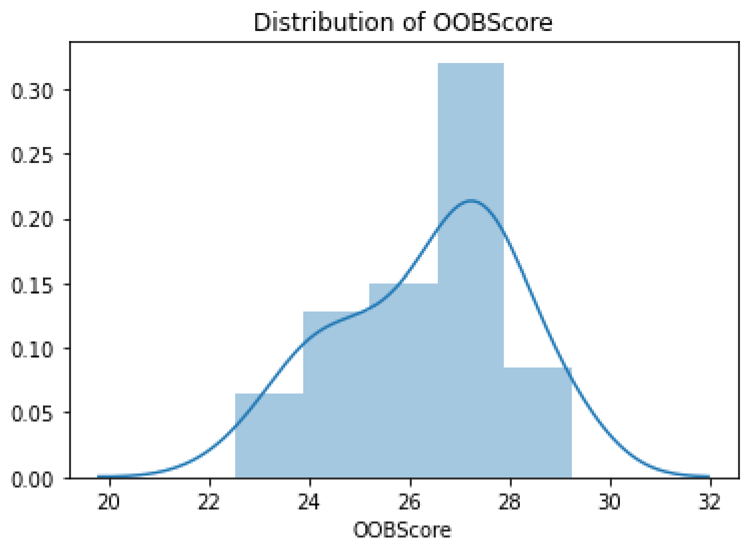
In [35]:
```
import matplotlib.pyplot as plt
```

In [36]:
```
sns.distplot(train_mse_ci,bins=35)
plt.title("Distribution of Mean Squared Error (MSE)")
plt.xlabel('MSE')
plt.show()
```



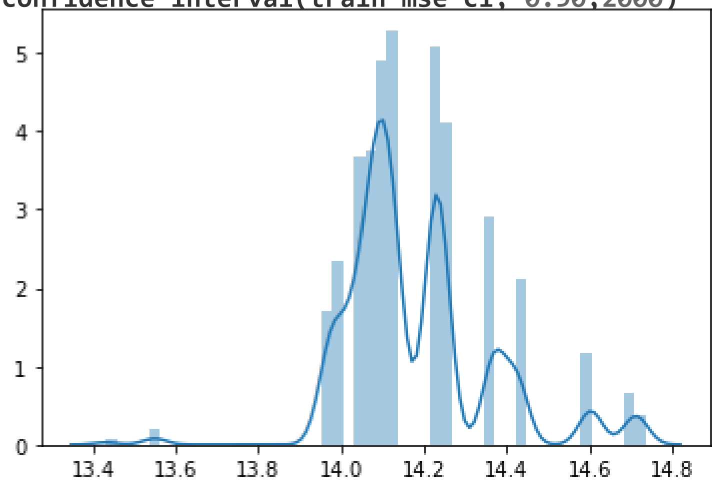
In [37]:
```
sns.distplot(oobscore_ci)
plt.title("Distribution of OOBScore")
plt.xlabel("OOBScore")
```

Out[37]: `Text(0.5, 0, 'OOBScore')`



In [38]:
```
# confindence interval for MSE

def confidence_interval(data, confidence_value=0.95, n_iter=1000):
    '''this function returns Confidence Interval of given
        data and given confidence_value(default=0.95)'''

    import numpy as np
    import matplotlib.pyplot as plt
    import seaborn as sns
    from sklearn.utils import resample

    # fix the randomization
    np.random.seed(42)
    # confidence bootstrap
    n_iterations = n_iter
    n_size=len(data)

    medians = []

    for i in range(n_iterations):
        # creating samples using list MSE scores
        s = resample(data, n_samples=n_size)
        m = np.median(s)

        medians.append(m)

    # plot scores
    sns.distplot(medians)
    plt.show() # enforce to show plot first

    # confidence intervals (95)
    alpha = confidence_value # override it and find any confidence intervals

    p = ((1.0-alpha)/2.0) * 100
    lower =  np.percentile(medians, p)

    p = (alpha+((1.0-alpha)/2.0)) * 100
    upper =  np.percentile(medians, p)

    print('%.1f confidence interval %.1f and %.1f' % (alpha*100, lower, upper))
    return (lower, upper)
```

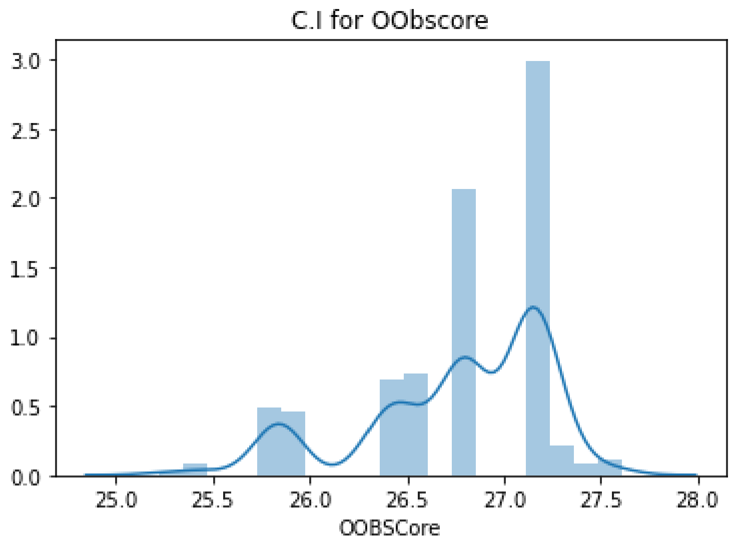
```
confidence_interval(train_mse_ci, 0.90,2000)
```




```
90.0 confidence interval 14.0 and 14.6
```

Out[38]:
```
(13.971089444809955, 14.602775642902454)
```

In [39]:
```
# C.I for OOBscore
plt.title('C.I for OObscore')
plt.xlabel('OOBSCore')
confidence_interval(oobscore_ci,0.95, 1000)
```



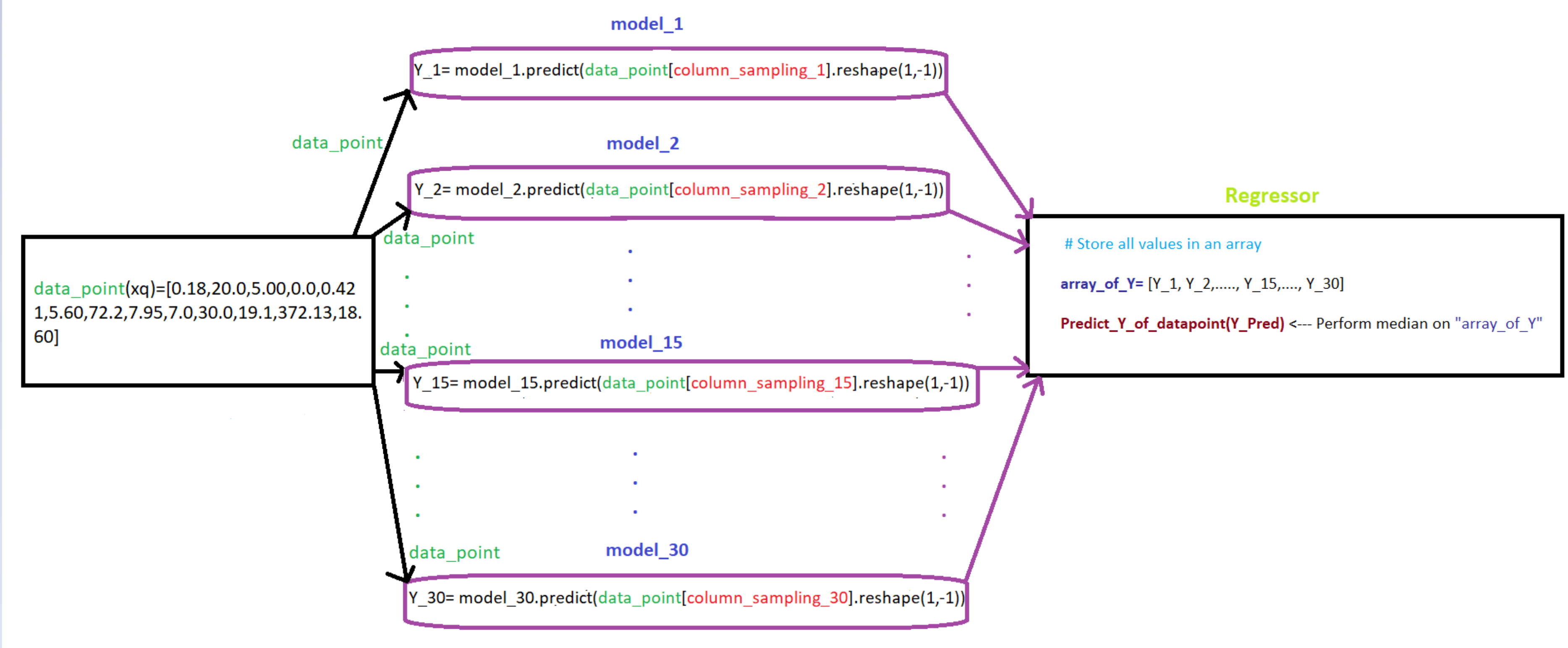

```
95.0 confidence interval 25.8 and 27.3
```

Out[39]:
```
(25.800654541337288, 27.280357927097057)
```

Task 3

Flowchart for Task 3

Hint: We created 30 models by using 30 samples in TASK-1. Here, we need send query point "xq" to 30 models and perform the regression on the output generated by 30 models.



- **Write code for TASK 3**

In [47]:
```
xq = [0.18, 20.0, 5.0, 0.0, 0.421, 5.60, 72.2, 7.95, 7.0, 30.0, 19.1, 372.13, 18.60]
len(xq)
```
Out[47]:
```
13
```

In [ ]:
```
```

In [48]:
```
def find_prediction(datapoint, models):
    from tqdm import tqdm

    list_of_all_models = models
    yq_hat = []

    for i in tqdm(range(len(list_of_all_models))):
        xq = np.array(datapoint).reshape(1,-1)
        xq = xq[:,list_selected_columns[i]]

        model = list_of_all_models[i]

        y_predicted = model.predict(xq.reshape(1,-1))
        yq_hat.append(y_predicted)

    return np.median(yq_hat)
```

In [49]:
```
y_hat = find_prediction(xq, list_of_all_models)
```

```
100%|██████████████████████████████████████████████████| 30/30 [00:00<00:00, 60
03.01it/s]
```

In [50]:
```
y_hat
```

Out[50]:
```
21.775
```

Write observations for task 1, task 2, task 3 indetail

Observations:

In all three Task, I understand the internal details that how everything is working internally of Bagging model(Random Forest).

In [ ]:

```

```