Introduction to the Theory of Computation: Hints

Lwins_Lights

Contents

1 Chapter 1: Regular Languages **2**

2 Chapter 2: Context-Free Languages **3**

1 Chapter 1: Regular Languages

*1.45 Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA such that $L(M) = A$. Consider $\{q \in Q \mid \exists x \in B,\ \delta(q, x) \in F\}$.

*1.56 Let $A = \{2^n \mid n \in \mathbb{N}\}$. You may need to do a bit of algebra or number theory.

*1.57 Let M be a DFA such that $L(M) = A$. Utilizing M, design an NFA which accepts $A_{\frac{1}{2}-}$ by guessing something.

*1.58 Let $A = \mathtt{a}^*\mathtt{\#}\mathtt{b}^*$.

*1.59 Let Q' stands for some subset of Q. Try to design a method that finds a relatively short string w such that $|\delta(Q', w)| < |Q'|$, where $\delta(Q', w) = \{\delta(q, w) \mid q \in Q'\}$.

*1.63 For part a, use the pumping lemma.

*1.65 Consider $B_{n+2} = \Sigma^*\mathtt{10}^n$.

*1.67 For part b, let M be a DFA such that $L(M) = A$. Utilizing M, design an NFA which accepts $RC(A)$ by guessing something.

*1.68 Solve problem 1.67 first.

2 Chapter 2: Context-Free Languages

*2.19 $Y \to (\mathtt{a} \cup \mathtt{b})^*$ and $S \to \mathtt{a}^n(\mathtt{b}Y \cup Y\mathtt{a})\mathtt{b}^n$ where $n \in \mathbb{N}$.

*2.21 Define $\chi : \Sigma^* \to \mathbb{Z}$ by $\chi(x) = n_{\mathtt{a}}(x) - 2n_{\mathtt{b}}(x)$, where $n_{\mathtt{a}}(x)$ counts the number of as in x. Suppose $x = x_1x_2 \dots x_m$ with $x_i \in \Sigma$. What will happen to $x_lx_{l+1} \cdots x_r$ if $\chi(x_1x_2 \cdots x_r) = \chi(x_1x_2 \cdots x_{l-1})$?

*2.22 $C = \{x\#y \mid |x| \neq |y|\} \cup \bigcup_{i \in \mathbb{Z}^+}\{x\#y \mid |x| = |y| \text{ and } x_i \neq y_i\}$, where x_i is denoted as the i-th character of x.

*2.23 Solve problem 2.22 first.

*2.24 $E = \{\mathtt{a}^i\mathtt{b}^j \mid j < i\} \cup \{\mathtt{a}^i\mathtt{b}^j \mid i < j < 2i\} \cup \{\mathtt{a}^i\mathtt{b}^j \mid j > 2i\}$.

*2.27 For part b, try to let every `else` correspond to the nearest `if`, like what C/C++ grammar specifies.

*2.28 Solve problem 2.21 first.

*2.29 Use the pumping lemma.

*2.33 Use the pumping lemma on $\mathtt{a}^{(p+1)^2}\mathtt{b}^{p+1}$.

*2.37 R's appearing twice gives us the pumping lemma for CFL. What if it appears thrice?

*2.40 Use the pumping lemma.